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DOCTORAL THESIS
SUMMARY

Sample size, skewness and leverage effects in Value at Risk and Expected Shortfall estimation

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Summary

The objective of this thesis is to compare the performance of a variety of models for VaR and ES estimation for a collection of assets of different nature: stock indexes, individual stocks, bonds, exchange rates, and commodities. Throughout the thesis, by a VaR or an ES "model" is meant a given specification for conditional volatility, combined with an assumption on the probability distribution of return innovations.

Specifically, Chapter 1 considers the concept of unbiasedness in VaR estimation. Franconi and Herzog (2012) (FH) [12] showed that there exists an analytical bias correction for VaR when returns are Normally distributed. In this chapter the FH analysis is extended to the Student-t distribution as well as to Mixtures of two Normal distributions, using a bootstrapping algorithm proposed by FH. The use of the probability-unbiased VaR avoids the systematic underestimation of risk implied by the bias of standard VaR measures. The magnitude of the distortion that needs to be exerted on the quantile to move from the standard VaR to the probability-unbiased VaR depends on the sample size and on the distribution assumption on returns. Since financial returns usually have thick tails, the smaller the sample size and the lower the heaviness of the tail of the assumed distribution in estimation, the higher will be the distortion to be applied to achieve unbiasedness. This VaR adjustment allows us to work with small samples knowing that the estimated VaR will generally display a good performance. Furthermore, the results in the thesis show that using a small sample may easily lead to more accurate VaR estimates than longer samples according to the Exceedance Probability and to the Observed Absolute Deviation per year (mean of the absolute differences between the expected number of exceedances and the number of observed exceedances). The good performance of the probability-unbiased VaR follows from the fact that a short sample size allows for capturing better the structural changes that arise over time in financial returns due to trading behaviour.

Chapter 2 analyzes how the efficiency of VaR depends on the volatility specification and the assumption on the probability distribution for return innovations. This question is crucial for risk managers, since there are so many potential choices for volatility model and probability distributions that it would be very convenient to establish some priorities in modelling returns for risk estimation. We consider different conditional VaR models for assets of different nature, using symmetric and asymmetric probability distributions for the innovations and volatility models with and without leverage. We calculate VaR estimates following the parametric approach. The ability to explain sample return moments might be considered a natural condition to obtain a good VaR performance. However, even though significant effort is usually placed in selecting an appropriate combination of probability distribution and volatility specification in VaR estimation, the ability to explain sample return moments is seldom examined. After using simulation methods to calculate implied return moments from estimated models, we compare the implied levels of skewness and kurtosis of returns with the analogue sample moments. We show that the ability to explain sample moments is in fact linked to performance in VaR estimation. Such performance is examined through standard tests: the unconditional coverage test of Kupiec (1995) [17], the independence and conditional coverage tests of Christoffersen (1998) [7], the Dynamic Quantile test of Engle and Manganelli (2004) [10], as well as the loss functions proposed by Lopez (1998, 1999) [18] [19] and Sarma et al. (2003) [22] and that of Giacomini and Komunjer (2005) [13].

Relative to an ever increasing literature, we contribute in different ways:

i) considering a set of probability distributions that have recently been rendered to be appropriate for capturing the skewness and kurtosis of financial data, but whose performance for VaR estimation has not been compared previously on a common dataset: Skewed Student-t [11], Skewed Generalized Error [11], Johnson S_U [15], Skewed Generalized-t [23] and Generalized Hyperbolic Skew Student-t [1] distributions, with the Normal and symmetric Student-t distributions as benchmark,

ii) considering three volatility specifications with leverage, GJR-GARCH, APARCH and FGARCH, as well as the standard symmetric GARCH model as benchmark. FGARCH and APARCH are increasingly being appreciated as being adequate for financial returns because they are specified for a power of the conditional standard deviation of the innovations, which provides more flexibility to the dynamics of volatility,

iii) explicitly evaluating the fit to return data, relating that fitting ability to VaR performance, and

iv) by introducing a dominance criterion to establish a ranking of models on the basis of their behavior under standard VaR validation tests and loss functions.

We obtain the following results:

i) VaR models that assume asymmetric probability distributions for the innovations, like the Skewed Student-t distribution, Skewed Generalized Error distribution, Johnson SU distribution, and Skewed Generalized-t distribution provide a better fit to sample return moments than symmetric distributions and achieve a better VaR performance,

ii) volatility models with leverage, like APARCH and FGARCH, show a better VaR performance than more standard GARCH and GJR-GARCH volatility specifications,

iii) our out-of-sample simulation results suggest that the important assumption for VaR performance is that of the probability distribution of return innovations, with the choice of volatility model playing a secondary role,

iv) dealing with the power of the conditional standard deviation as a free parameter is an important feature of the APARCH/FGARCH volatility specifications because our estimates suggest that for a number of financial assets the squared conditional deviation specification is inappropriate,

v) a good fit to return moments usually leads to a good VaR performance. APARCH or FGARCH models with Skewed Generalized Error, Skewed Generalized-t and Johnson S_U distributions are preferred to other asymmetric distributions, like Skewed Student-t and Generalized Hyperbolic Skew Student-t, and symmetric distributions, like Student-t and Normal distributions, and

vi) alternative VaR models seem to provide a distinct performance for different classes of assets.

In Chapter 3 we estimate the conditional Expected Shortfall based on the Extreme Value Theory (EVT) approach using asymmetric probability distributions for return innovations, and we analyze the accuracy of our estimates before and during the 2008 financial crisis using daily data for 1- and 10-day horizons. We take into account volatility clustering and leverage effects in return volatility by using the APARCH model under different probability distributions assumed for the standardized innovations: Gaussian, Student-t, skewed Student-t, skewed generalized error and Johnson S_U and under EVT methods, following the two-step procedure of McNeil & Frey (2000) [20]. This two-step procedure fits a Generalized Pareto Distribution to the extreme values of the standardized residuals generated by APARCH models. Then, we compare the one-step-ahead out-of-sample ES forecast performance of all these models for different significance levels (α). Previously existing backtesting tests for ES have been shown have serious limitations [as indicated by McNeil & Frey (2000) [20], Berkowitz (2001) [6], Kerkhof and Melenberg (2004) [16] and Wong (2008) [24]]. Such limitations are overcome by some recent ES backtesting proposals that we use for ES evaluation: the Righi & Ceretta (2013) [21] test, two tests by Acerbi & Szekely (2014) [2] that are straightforward but require simulation analysis (like the Rigui & Ceretta test), the test of Graham & Pál (2014) [14], which is an extension of the Lugannani-Rice approach of Wong, the quantile-space unconditional coverage test of Costanzino & Curran (2015) [8] for the family of Spectral Risk Measures, of which ES is a member and, finally, the conditional test of Du & Escanciano (2015) [9].

This chapter contributes to the literature in different ways:

i) considering the APARCH volatility specification in an EVT model using Filtered Historical Simulation (FHS) [3] [4] to take into account volatility clustering and asymmetric returns,

ii) comparing conditional EVT models that incorporate conditional models with asymmetric probability distributions rarely used in the financial literature for ES estimation,

iii) by analyzing the performance of VaR and ES estimates over 10-day horizons for risk liquidity management, as proposed in Basel capital requirements [5],

iv) by focusing on the accuracy of our risk models for VaR and ES estimation during the pre-crisis and crisis periods as well as under different significance levels (α), and

v) by evaluating ES performance using the most recent ES backtesting proposals in the same study.

We obtain the following conclusions:

i) Extreme Value Theory produces a good ES performance regardless of the probability distribution assumed for return innovations in estimation. This is due to the fact that the tail is modelled with a Generalized Pareto Distribution not only with 1-day but also 10-day horizons,

ii) if we consider conditional models without the EVT approach, we observe that the Skewed Generalized Error distribution and the Johnson S_U distribution play an important role in capturing tail risk in 1-day and 10-day horizons. This is because the stylized facts of financial returns such as volatility clusters, heavy tails and asymmetry are suitably captured by these asymmetric distributions,

iii) even during the crisis period, conditional EVT models are more accurate and reliable for predicting asset risk losses than conditional models that do not incorporate the EVT approach, and

iv) sometimes conditional EVT models produce a strong ES overestimation.

References

[1] Aas, K. and Haff, I.H., 2006 *The Generalized Hyperbolic Skew Student's t-Distribution.* Journal of Financial Econometrics, Vol.4, No. 2, pp. 275-309.

[2] Acerbi, C. and Szekely, B., 2014. *Backtesting Expected Shortfall.* Publication of MSCI.

[3] Barone-Adesi, Giannopoulus, K. and Vosper, L., 1999. *VaR without correlations for portfolios of derivative securities.* Journal of Futures Markets, Vol. 19, pp. 583-602.

[4] Barone-Adesi, G., Giannopoulos, K. and Vosper, L., 2002. *Backtesting derivative portfolios with filtered historical simulation (FHS).* European Financial Management, Vol. 8, No. 1, pp. 31-58.

[5] Basel Committee on Banking Supervision, 2016. Standards: Minimum Capital requirements for market risk. Bank for International Settlements.

[6] Berkowitz, J., 2001. *Testing density forecasts, with applications to risk management.* Review of Financial Studies, Vol. 14, pp. 371-405.

[7] Christoffersen, P., 1998. *Evaluating internal Forecasting.* International Economic Review. Vol. 39, pp. 841-862.

[8] Costanzino, N. and Curran, M., 2015. *Backtesting General Spectral Risk Measures with application to Expected Shortfall.* Available at SSRN 2514403

[9] Du, Z. and Escanciano, J.C., 2015. *Backtesting Expected Shortfall: Accounting for Tail Risk.* Available at SSRN 2548544.

[10] Engle R.F. and Manganelli, S., 2004. *CAViaR: conditional autoregressive value at risk by regression quantiles.* Journal of Business & Economic Statistics, Vol. 22, pp. 367-381.

[11] Fernandez, C. and Steel, M., 1998. *On Bayesian Modelling of Fat Tails and Skewness.* Journal of the American Statistical Association, Vol. 93, No. 441, pp. 359-371.

[12] Francioni, I. and Herzog, F., 2010. *Probability-unbiased Value-at-Risk estimators*, Quantitative Finance, Vol. 12, pp. 755-768.

[13] Giacomini, R. and White, H., 2006. *Tests of conditional predictive ability.* Econometrica, Vol. 74, No. 6, pp. 1545-1578.

[14] Graham, A. and Pál, J., 2014. *Backtesting value-at-risk tail losses on a dynamic portfolio.* Journal of Risk Model Validation, Vol. 8, No. 2, pp. 59-96.

[15] Johnson, N.L., 1949. *Systems of frequency curves generated by methods of translations.* Biometrika, Vol. 36, pp. 149-176.

[16] Kerkhof, J. and Melenberg, B., 1995. *Backtesting for risk-based regulatory capital.* Journal of Banking and Finance, Vol. 28, pp. 1845-1865.

[17] Kupiec, P., 1995. *Techniques for Verifying the Accuracy of Risk Measurement models.* Journal of Derivatives, Vol. 2, pp. 174-184.

[18] Lopez, J.A., 1998. *Testing your risk tests.* Financial Survey (May-Jun), pp. 18-20.

[19] Lopez, J.A., 1999. *Methods for evaluating Value-at-Risk estimates.* Federal Reserve Bank of San Francisco Economic Review, Vol. 2, pp. 3-17.

[20] McNeil, A. and Frey, R., 2000. *Estimation of Tail-Related Risk Measures for Heteroscedastic Financial Time Series: an Extreme Value Approach.* Journal of Empirical Finance, Vol. 7, pp. 271-300.

[21] Righi, M.B. and Ceretta, P.S., 2013. *Individual and Flexible Expected Shortfall Backtesting.* Journal of Risk Model Validation, Vol. 7, No. 3, pp. 3-20.

[22] Sarma, M., Thomas, S., and Shah, A., 2003. *Selection of value at risk models.* Journal of Forecasting, Vol. 22, pp. 337-358.

[23] Theodossiou, P., 1998. *Financial data and skewed generalized t distribution.*. Management Science, Vol. 44, pp. 1650-1661.

[24] Wong, W.K., 2008. *Backtesting trading risk of commercial banks using expected shortfall.* Journal of Banking and Finance, Vol. 3, pp. 1404-1415.