Three essays on the cutting edge of credit spread modeling

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Motivation

Question

Why would a financial entity need to model the behavior of credit spreads?

- Three main reasons:
  1. Accurate pricing of derivatives (CVA-DVA adjustments).
  2. Parametric CVA VaR calculation.
  3. Risk factor modeling under IMM approach.

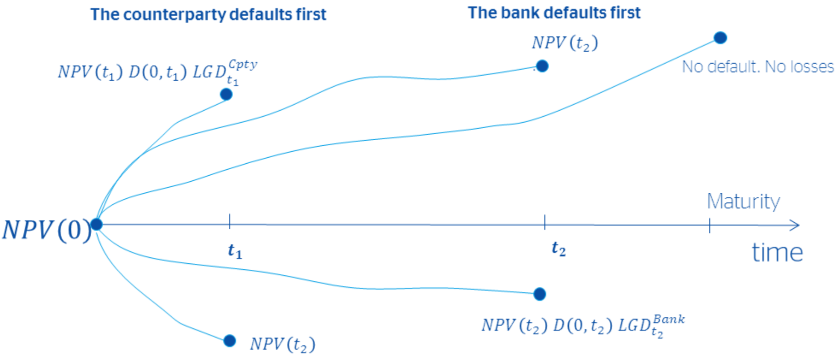
CVA-DVA

- Recent accounting standards demand adjustments on the price of a derivative.
- If in a given transaction the counterparty of a bank defaulted and the PV at default was positive for the bank, it would only be able to get a recovery fraction.
- However, if such PV was negative, the bank would have to pay in full.
- Such asymmetry translated into a **Credit Valuation Adjustment** term (CVA).
- This term needed to be subtracted from the net present value of the equivalent derivative with no counterparty risk.

CVA-DVA

- If two counterparties subtracted the CVA of the other to the price of the default-free portfolio, no deal would be closed (unless one of them was default-free).
- This led to the introduction of a **Debit Valuation Adjustment** (DVA) to take into account the own default probability.
- This term raised paradoxical situations: DVAs get larger when the own credit quality worsens, increasing the value of our portfolio since the chances that we will not fully pay the due amounts are greater.

Example of CVA-DVA



CVA-VaR

- Changes in CVA-DVA can end up generating serious losses.
- Basel III incorporated CVA capital charges based on a Value-at-Risk (VaR) estimate for CVA.
- This CVA-VaR is exclusively focused extreme credit spread oscillations.

Risk factor modeling

- Financial entities are typically forced to use conservative approaches when calculating credit exposures.
- Upon regulatory approval, financial entities can adopt the Internal Model Method (IMM) approach, intended to better capture the risk profile of each particular entity.
- To do so, they must show their local regulators their ability to describe the set of risk factors they are exposed to.
- One of these factors is the behavior of credit quality.
- Thus, realistic and simple credit spread modeling is a mandatory milestone for any bank willing to undertake IMM approval.

Section 1

Credit spread modeling effects on counterparty risk valuation adjustments: a Spanish case study

Introduction

- Financial entities must accurately model default probabilities of all their counterparties with which they have booked a deal in the trading book.
- This can easily mean having to compute default probabilities for thousands of different names.
- Banks need to find an equilibrium between model complexity and model accuracy.
- Moreover, current accountancy standards require the use of as much market information as possible when calculating the fair value of a derivative.
- Therefore, the possibility of calibrating the desired model to market instruments must also be taken into account.

The model

- In order to compute accurate counterparty valuation adjustments, we will follow the arbitrage-free valuation framework presented in Brigo *et al.* (2009).
- We will consider a model with stochastic Gaussian interest rates...

Interest rates

$$\begin{cases} r_t = x_t + z_t + \varphi(t,\alpha) \\ dx_t = -a x_t dt + \sigma dW_{1t}, \qquad x_0 = 0 \\ dz_t = -b z_t dt + \eta dW_{2t}, \qquad z_0 = 0 \end{cases} \tag{1}$$

The model

- ... and CIR++ default intensities:

Default intensity

$$\begin{cases} \lambda_t = y_t + \psi(t, \beta) \\ dy_t = \kappa(\mu - y_t)dt + \nu\sqrt{y_t}dW_{3t} \end{cases} \tag{2}$$

The model

- Each participant in the transaction will have its own default intensity y_t^i, $i \in \{I, C\}$.
- These intensities will be correlated between them.
- Moreover, there will also be correlation between intensities and interest rates.
- Finally, we will also allow for default time correlation.
- If we name τ_i, $i \in \{I, C\}$, the defaulting time, they will be connected through a Gaussian copula function with correlation parameter ρ_{Cop}.

Calibrating interest rates

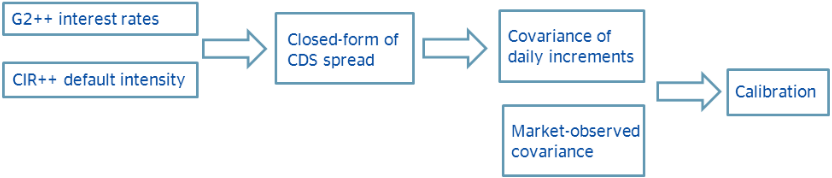
- We will first focus on the calibration of interest rates, that will be performed as in Brigo *et al.* (2009).
- We can calibrate our model to the surface of at-the-money (ATM) swaption volatilities.
- The expression for European swaption prices under G2++ is known. Using this formula, ATM swaption prices can be fitted to the market data surface by minimizing squared errors.

$$\widehat{\alpha} = \operatorname*{argmin}_{\alpha} \sum_{i,j} [Swaption_{ATM}^{Mkt}(0, T_i, \mathcal{T}_j) - Swaption_{ATM}^{G2++}(0, T_i, \mathcal{T}_j; \alpha)]^2$$

Calibrating default intensities

- Default intensities represent a more challenging stage in the calibration process.
- There are no liquid credit instruments to infer implied volatilities.
- We propose a method to satisfactorily calibrate the dynamics of default intensities.
- We achieved closed form solutions of Credit Default Swaps (CDS) spreads in our framework.
- Then, we obtained the variance-covariance matrix of the daily increments of these spreads and fitted it to the one observed in the market.

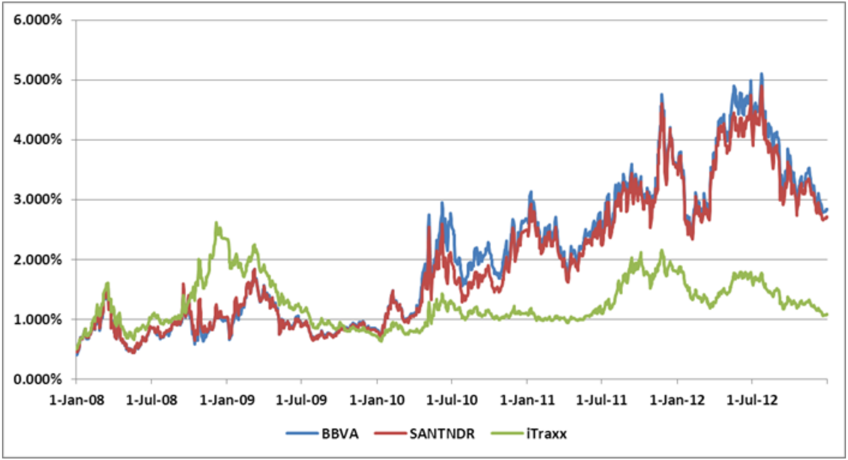
Calibrating default intensities



Calibration window

- We intend to capture counterparty valuation adjustments in two rather different scenarios.
- The first date, December 15, 2008, can be regarded as the last moment when a Spanish financial institution was considered to be significantly safer than an average European company.
- The second one, June 28, 2012, is placed in the most acute phase of the Spanish recession, when Spain had to accept a financial bailout from the Eurogroup.
- We will observe the credit quality of three references to track their behavior in these two dates: BBVA, Santander and iTraxx Europe.

Calibration window



CVA-DVA calculations

- We computed counterparty valuation adjustments for a 10y par IRS and a 5y CDS contract.
- The modeling of the dynamics of credit spreads plays a significant role in the calculation CVA-DVA when compared to simpler setups (no credit volatility).
- Thus, correctly modeling the behavior of credit quality allows financial entities to price their derivatives accurately.

CVA-DVA calculations: IRS

	2008		2012	
iTraxx vs BBVA	No Vol	Vol	No Vol	Vol
CVA	-0.34274	-0.30463	-1.37362	-1.25786
DVA	0.32796	0.43124	0.29015	0.38058
iTraxx vs Santander	No Vol	Vol	No Vol	Vol
CVA	-0.35579	-0.31230	-1.32088	-1.17327
DVA	0.32802	0.43142	0.29021	0.38066
BBVA vs Santander	No Vol	Vol	No Vol	Vol
CVA	-0.35588	-0.31246	-1.32024	-1.17226
DVA	0.18031	0.32321	0.62199	0.74061

Section 2

Model risk in credit valuation adjustments: a Spanish case study

Introduction

- Regulators are increasingly worrying about sources of uncertainty around the fair value of a derivative.
- The European Banking Authority (EBA) intends that banks calculate additional valuation adjustments (AVAs) for determining the prudent value of fair valued positions.
- One of these AVAs pretends to address model risk when calculating Counterparty Valuation Adjustments (CVA) on the fair value of a derivative.
- In this chapter, we will address two sources of model risk under the framework presented in the previous chapter.

Default correlation

- In our modeling framework, we connected defaulting times with a Gaussian copula with correlation parameter ρ_{Cop}.
- We had no way to discern which value does ρ_{Cop} take.
- We could also question the use of a Gaussian copula.
- However, due to the popularity of Gaussian copulas, we will address model risk when correlating defaulting times through the correlation parameter ρ_{Cop}.

Default correlation

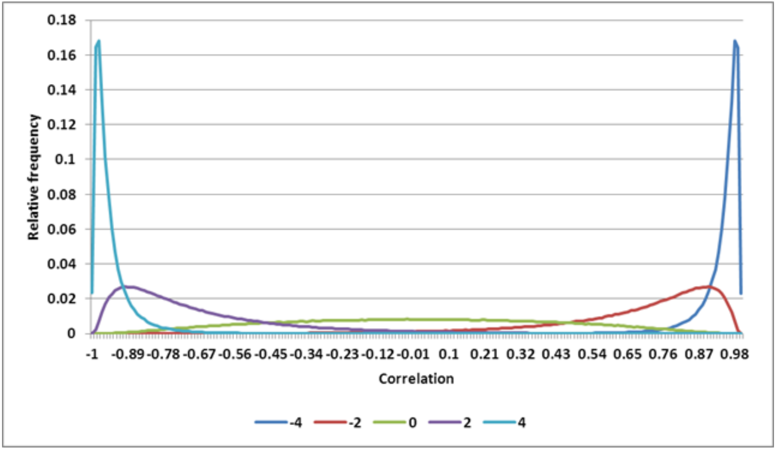
- We model ρ_{Cop} with the following distribution:

$$\rho_{Cop} = \frac{2}{1 + e^{\varsigma - \varphi}} - 1$$

- Due to underlying economic variables, we can assume that correlation in defaulting time will be close to correlation in default probability (already calibrated).

$$E[\rho_{Cop}] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \left[\frac{2}{1 + e^{\varsigma - x}} - 1 \right] e^{-\frac{1}{2}x^2} dx = \rho_{Spread}$$

Default correlation



Default correlation

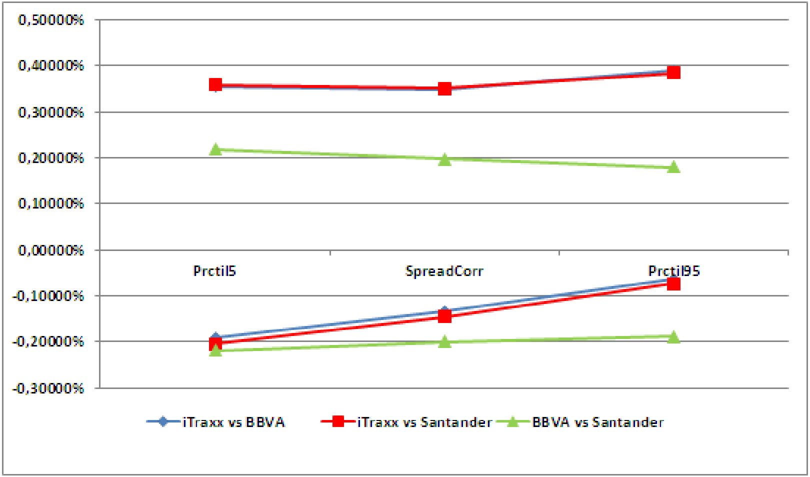
- We found ς such that $E[\rho_{Cop}] = \rho_{Spread}$.
- Once calibrated, we can obtain percentiles for the correlation between defaulting times.
- We can compute counterparty valuation adjustments with these percentiles to obtain a confidence interval for CVA-DVA.
- Counterparty valuation adjustments are monotone in default correlation most of the time.
- However, there is not a common pattern, either increasing or decreasing, since they depend also on the survival probabilities in the corresponding period.

Default correlation

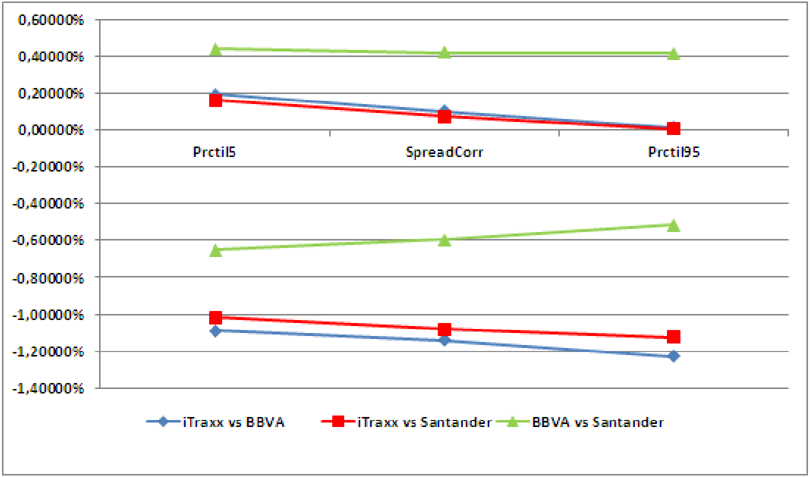
2007-2008	BBVA-Santander	BBVA-iTraxx	Santander-iTraxx
ρ_{Spread}	91.519%	83.009%	80.445%
ς	-3.554	-2.761	-2.592
Percentile 5%	74.1889%	50.6593%	44.0956%
Percentile 95%	98.9017%	97.5886%	97.1497%
2010-2012	BBVA-Santander	BBVA-iTraxx	Santander-iTraxx
ρ_{Spread}	97.389%	81.982%	81.463%
ς	-4.804	-2.691	-2.657
Percentile 5%	91.8550%	48.0020%	46.6733%
Percentile 95%	99.6841%	97.4154%	97.3265%

Table: Defaulting time correlation parameters. ς has been set such that the expected value of default correlation ρ_{Cop} equals the correlation between default intensities ρ_{Spread}.

Default correlation (2008)



Default correlation (2012)



Spread modeling

- Default intensities are typically modeled either under a CIR or a Gaussian framework.
- In the previous chapter, we used the CIR model to prevent excessively negative default probabilities.
- However, we might have used the Gaussian framework due to its higher analytical tractability.
- Accountancy rules define fair value as an exit price, so we should take into account which assumptions may be taking other market players.
- Thus, we will alternatively model default intensities with a H&W model.

Spread modeling

- Interest rates will be modeled in a G2++ setup, as before.
- Default intensities will be described as:

$$\begin{cases} \lambda_t = y_t + \psi(t, \beta) \\ dy_t = -\kappa y_t dt + \nu dW_{3t} \\ y_0 = 0 \end{cases}$$

- Again, in order to estimate the parameters, we will find a formula for instantaneous CDS spread increments and calibrate them to those observed in the market.

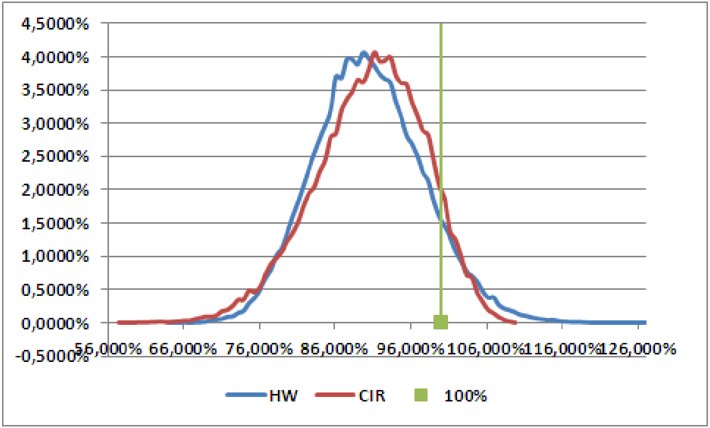
Spread modeling

- When comparing both distributions (CIR and H&W), we observe that they yield similar masses of negative default probabilities.
- However, the H&W model produces more extreme values of negative probabilities.
- These differences end up translating into variations of counterparty valuation adjustments.

Survival probabilities (2012)

	H&W++			CIR++		
$PS(5y,6y)$	BBVA	Santander	iTraxx	BBVA	Santander	iTraxx
Mean	0.9012	0.9072	0.9618	0.9019	0.9076	0.9619
Std	0.0725	0.0708	0.0255	0.0722	0.0709	0.0250
Ex. kurtosis	0.1200	0.1065	0.0380	0.0588	0.1777	0.6220
Skewness	0.2384	0.2354	0.0721	-0.3972	-0.4893	-0.7114
Min	0.6328	0.6490	0.8486	0.5679	0.5197	0.8106
Max	1.2698	1.2497	1.0802	1.0931	1.0754	1.0137
Percentile 1%	0.7458	0.7548	0.9036	0.7166	0.7215	0.8917
Percentile 5%	0.7865	0.7958	0.9206	0.7747	0.7808	0.9160
Percentile 95%	1.0247	1.0277	1.0042	1.0113	1.0132	0.9971
Percentile 99%	1.0833	1.0841	1.0225	1.0440	1.0422	1.0055
$\mathbb{Q}\{PS<1\}$	0.9097	0.9010	0.9305	0.9254	0.9166	0.9684

Survival probabilities (2012)



Section 3

Credit spread modeling from a regulatory perspective

Calculating VaR for CVA

- Basel III presents a metric to address potential CVA-related losses.
- Based upon a given formula, Basel III proposes the calculation of a CVA-VaR.
- Although CVA is dependent on LGD and expected exposures, VaR shall rely **exclusively** on credit spreads.

Basel III recipe for CVA

$$
\begin{aligned}
CVA = \quad & LGD \cdot \sum_{i=1}^{T} \max\left(0; \exp\left(-\frac{s_{i-1}t_{i-1}}{LGD}\right) - \right. \\
& \left. \exp\left(-\frac{s_i t_i}{LGD}\right)\right) \cdot \left(\frac{EE_{i-1}D_{i-1} + EE_i D_i}{2}\right)
\end{aligned}
$$

Calculating VaR for CVA

- We calculated a parametric and a historical CVA-VaR along two years (2012-2013).
- We calibrate the model for interest rates and default intensities on a monthly basis.
- Three references are studied: BBVA, Santander and iTraxx Europe.
- Expected positive exposure profile is that of a par 10-year swap, held constant along the year.
- We will test the VaR model along a one-year horizon.

A parametric VaR

- Get the interest rate curve and CDS spreads for day i.
- Get the corresponding G2++ and CIR++ model parameters.
- Simulate 10,000 times one day of interest rates and default intensities and obtain a distribution of CDS spreads for day $i+1$.
- From these possible CDS spreads, calculate a distribution of possible CVA values.
- Take percentile 99% of these CVA values as the CVA VaR.
- Take the *realized* CDS spreads of day $i+1$ and compute CVA from these spreads. If it is greater than the previously calculated VaR, it will be considered an exception.

Calculating VaR for CVA

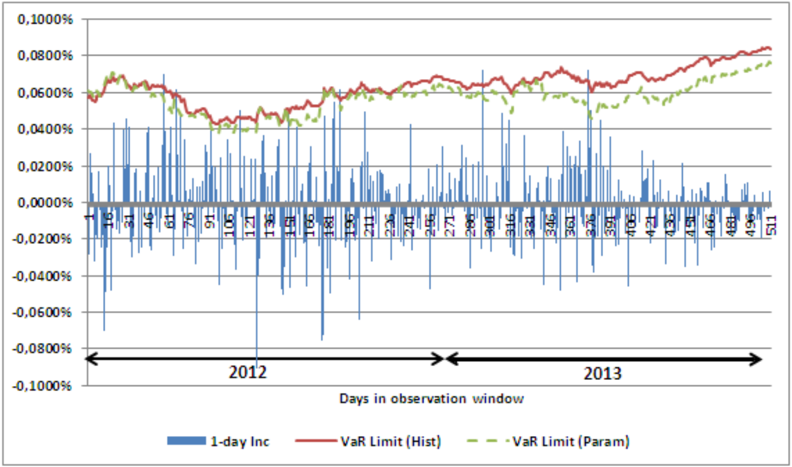
- The historical VaR appears to be conservative when compared to the parametric one.
- The parametric VaR scores yellow for BBVA and Santander in 2012.
- That year was specially troubled for Spanish financial institutions.
- The historical VaR was also close to fall in the yellow zone in 2012 for these references, but narrowly missed it.

Calculating VaR for CVA

2012	Historical		Parametric	
Entity	Nb Exceptions	Zone	Nb Exceptions	Zone
BBVA	4	Green	6	Yellow
Santander	2	Green	5	Yellow
iTraxx	0	Green	0	Green
2013	Historical		Parametric	
Entity	Nb Exceptions	Zone	Nb Exceptions	Zone
BBVA	2	Green	3	Green
Santander	2	Green	3	Green
iTraxx	1	Green	2	Green

Table: Number of exceptions for CVA VaR (2012-2013)

Calculating VaR for CVA (BBVA)



Introduction

- Financial entities have an incentive to migrate to Internal Model Method (IMM) in order to optimize their capital requirements.
- To do so, banks must show their regulators the suitability of their models to describe the set of risk factors they are exposed to.
- Backtesting a risk factor means comparing the realized historical distributions with the ones generated via simulation under the proposed model.
- We need to focus on the **whole** exposure distribution, rather than the tails as in a VaR approach.

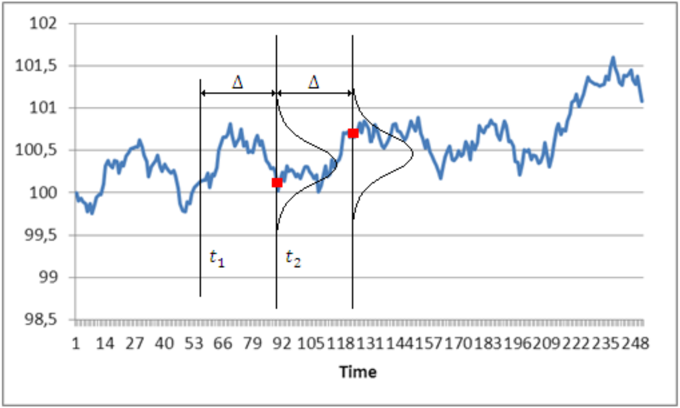
A first approximation to backtesting

- We define:
  - A historical window $T = t_{end} - t_{start}$
  - A recalibration frequency $t = [t_{start}, t_1 = t_0 + \Delta, ..., t_{end}]$
  - A time horizon Δ over which study our model.
- We then proceed as follows:
  - Go to recalibration node t_i. Calculate the model risk factor distribution at point $t_i + \Delta$.
  - Observe the realized value of the risk factor at $t_i + \Delta$ and calculate its probability F_i.
  - Repeat the above until $t_i + \Delta$ reaches t_{end}.

Remark

If reality behaved as our model, the collection $\{F_i\}_{i=1}^{N}$ would be uniformly distributed

A first approximation to backtesting



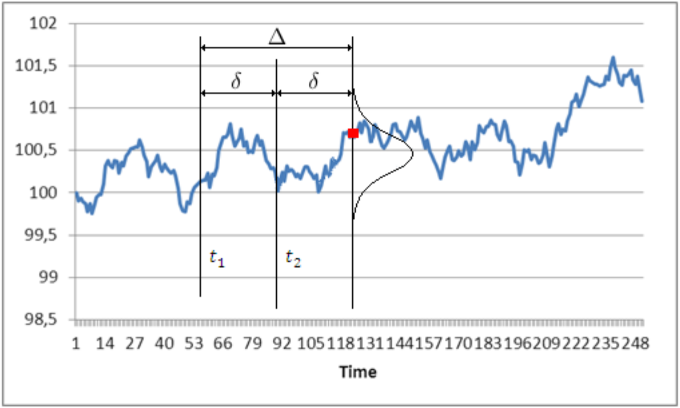
A first approximation to backtesting

- We define a distance function D between distributions to check how far $\{F_i\}_{i=1}^N$ is from a uniform distribution.
- Now, Basel III recommends to backtest horizons of up to one year.
- In a two-year sample, this would leave us with only two observations.
- We need to extend the backtesting methodology following Kenyon (2012) and Ruiz (2012).

Extending the backtesting method

- We define:
  - A historical window $T = t_{end} - t_{start}$
  - A recalibration frequency $t = [t_{start}, t_1 = t_0 + \delta, ..., t_{end}]$
  - A time horizon Δ over which study our model, with $\delta \leq \Delta$.
- As before, we generate a collection of probabilities $\{F_i\}_{i=1}^N$, that will **not** be uniformly distributed due to auto-correlation effects:
  - Eg: $\delta = 1$ month and $\Delta = 1$ year along 2012 and 2013.
  - On January 1, 2012 we estimate for January 1, 2013, observe the realized value on the latter date and get a probability of 40%.
  - On the following recalibration date, February 1, 2012, we estimate for February 1, 2013.
  - The estimated probability will be again close to 40%.

Extending the backtesting method



Extending the backtesting method

- For very large historical windows, this effect would eventually vanish, but not in finite samples (as it is our case).
- We will create M pseudo-historical paths with our model and compute the corresponding distances to a uniform distribution $\{D_k\}_{k=1}^{M}$, compatible with a perfect model.
- This collection will follow a distribution $\Upsilon(D)$.
- This allows us to assign a probability to the historical distance.

Results

- We performed a backtesting exercise along 2012-2013 with monthly calibrations for the CIR model.
- Applying the first methodology to 1m horizon, we did not reject the CIR distribution for the CDS 5y.
- Applying the extended methodology to larger horizons, we did not reject the CIR distribution.

Results

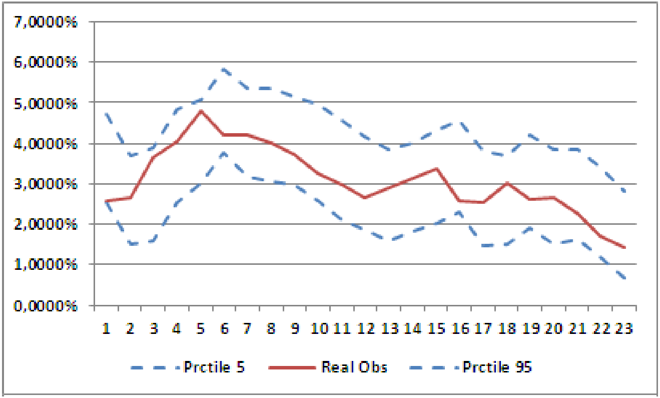


Figure: 5%-95% confidence intervals under the model distribution for CDS5y vs realized value. BBVA. $\Delta = 1$ month

Backtesting results for BBVA (2012-2013)