Optimal Public Policy *à la Ramsey* in an Endogenous Growth Model

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Abstract

We use an overlapping generations model with physical and human capital to ascertain the consequences for optimality of a social planner adopting a welfare criterion that gives the same weight to all generations and is respectful of individual preferences. In particular, we consider a social planner who maximizes a non-discounted sum of individual utilities à la Ramsey, with consumption levels expressed in terms of output per unit of efficient labour. It is shown that the optimal growth path does not depend on the precise cardinalization of preferences (i.e., the degree of homogeneity of the utility function) and that it converges to the "Golden Rule" defined in this endogenous growth framework. Decentralizing the optimum trajectory requires that subsidies to investment in education be negative (i.e., taxes), and that pensions to the elderly be positive along the entire optimal growth path. Furthermore, this is the case regardless of the initial conditions.

Keywords: endogenous growth, human capital, education policy, intergenerational transfers, pensions.

JEL Classification: D90, H21, H52, H55

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1 Introduction

The choice of an objective function with which to characterize social optimum has always been the subject of controversy. Ramsey (1928) assumed a social planner who maximizes an infinite, non-discounted sum of current and future utilities. The discounting of later enjoyments in comparison with earlier ones was for him an unacceptable practice. In his own words (Ramsey, 1928, p. 543), discounting of future utility is "ethically indefensible and arises merely from the weakness of the imagination." Later, Cass (1965, 1966), Koopmans (1965, 1967), Malinvaud (1965) and Samuelson (1965) generalized Ramsey's approach to allow for the discounting of future utilities. In an explicit overlapping generations setting where individuals are pure life-cyclers, Diamond (1965) adhered to the maximization of the utility level of a representative individual under the constraint that any other achieves the same welfare level, giving rise to what the subsequent literature coined as the Two-Part Golden Rule, i.e., the combination of the Biological Interest Rate and the Golden Rule of (physical) capital accumulation [Samuelson (1968, 1975a, 1975b)]. Adding to this framework the dimension of intergenerational altruism à la Barro (1974), so that individuals behave as if they maximized dynastic utility, the question arises of whether considering the welfare level enjoyed by a representative child only [Carmichael (1982)] or that of all children [Burbidge (1983)].

Clearly, each of these views of social welfare leads to a different optimal resource allocation. But all of them share the feature that they assume economies without productivity growth, in which a steady state is a situation where consumption levels per unit of (natural) labour are kept constant. In contrast, in the presence of productivity growth that translates into consumption growth, these consumption levels will grow without limit. Under these circumstances, a social planner will be unable to choose the stream of consumption levels that maximizes a sum of utilities. The reason is simple: the utility index itself will grow without limit, and will be infinite along the balanced growth path. The standard way to sidestep this is, of course, to assume that the planner maximizes a discounted sum of utilities. This is the approach adopted by Caballé (1995), Kemnitz and Wigger (2000) and Docquier et al. (2007) to characterize the optimal growth path in an endogenous growth model with overlapping generations and individuals who are either dynastic savers or life-cyclers.

There are, however, two uncomfortable features of this approach. First, as mentioned before, the choice of a particular social discount rate can be seen as inherently arbitrary if not, in Ramsey's view, plainly immoral. Second, the precise cardinalization of the utility function (more precisely, its degree of homogeneity) fundamentally affects both the characterization of the social optimum and the policies that support it (Del Rey and Lopez-Garcia, 2012). This means that even if we accept that future utilities should be discounted at an entirely arbitrary rate, we are left with the fact that the optimal policy varies when we use different *mathematical* specifications representing the same *economic* problem. One could reasonably claim that this is also an unpleasant feature of the whole approach.[1]

In light of these facts, it is tempting to ask whether it is possible for a social planner to continue to adhere to the Utilitarian criterion of maximizing a sum of individual utilities but, (i) without having to postulate something as elusive and inherently arbitrary as a social discount rate; and (ii) without having to choose a specific functional form to cardinalize individual preferences. In this paper we argue that the answer to this question is affirmative, accounting for both (i) and (ii), when the social planner maximizes a *non-discounted* sum of individual current and future utilities defined over consumption levels per unit of *efficient* rather than natural labour units. In other words, we show that it is possible for a Utilitarian social planner to give the same weight to all generations and be respectful of individual preferences. On the one hand, in a growth environment, treating all generations alike involves the choice of some utility index that eventually converges. In order to ensure convergence of the utility index we can divide consumption levels by another variable that grows at the same rate along a balanced growth path. Two

[1] This issue has been the subject of some attention in a "static" optimal taxation framework. As pointed out by Stiglitz (1987, p. 1017) "there are many alternative ways of representing the same family of indifference maps [...each yielding...] a different optimal income tax. The literature has developed no persuasive way for choosing among these alternative representations". The relevance of this point seems to be even greater in "dynamic" settings as the current one, where the welfare levels of individuals born at different time periods are involved.

natural candidates to do this are the levels of physical and human capital. However, as consumption and physical capital are denominated in the same units (i.e., output per unit of efficient labour), if we divided consumption levels by physical capital, we would be left with a pure number, with no dimension and with no intuitive interpretation. Dividing by human capital, on the contrary, just "re-scales" consumption, measuring it in terms of units of output per unit of efficient labour. On the other hand, it is important to stress that this "new" utility function is obtained by means of a monotonic transformation of the one guiding individual's behaviour, thus being fully respectul of individual *ordinal* preferences. Notice also that, assuming convergence, the limit of the time sequence of consumption levels expressed in terms of output per unit of efficient labour will be measured in such a dimension, as will be any variable which is constant along a balanced growth path.

This approach to the characterization of the optimal growth path can be related to the discussion in Del Rey and Lopez-Garcia (2013), who focus on the *balanced* growth path that maximizes the lifetime welfare of a representative individual subject to the constraint that everyone else's welfare is fixed at the same level. As the analytics below will show, this "Golden Rule" balanced growth path is the one towards which an economy converges when commanded by a social planner who, given some initial conditions, maximizes an non-discounted sum of individual utilities defined over consumption per unit of efficient labour. They also show that along the optimal, "Golden Rule" balanced growth path, a negative education subsidy and positive pensions to the elderly are called for. However, these results do not seem especially instructive when, as in the present case , the focus turns from balanced growth paths to the whole time trajectory leading to them. In other words, results that pertain to long-run equilibria may be a poor way to approach optimal policy out of such a state. An example is the well known Chamley (1986)-Judd (1985) result of an asymptotical zero capital income tax in the presence of infinitely-lived individuals, which however provides no indication concerning the time sequence of optimal capital income taxes.

In this paper, we explicitly adopt a strict Ramsey approach, i.e., entailing non-discounting, in an endogenous growth model. Our purpose is to characterize those optimal policies that, once superimposed on private behaviour and interaction in the market-place, allow the social planner to decentralize the *entire* optimal growth path. We identify the optimal education subsidies and lump-sum taxes on the working population and the retirees, and enquire about their sing, along the entire growth path. We find that, in the current setting, education expenditures should not only be taxed along the optimal balanced-growth path but also along the *entire* optimal growth path (Proposition 1). Concerning pensions, it is shown that they will be positive also along the transition path leading to the optimal balanced growth path (Proposition 3). Only the sign of the optimal lump-sum tax paid by middle-aged is ambiguous. And importantly, these results hold regardless of the initial conditions for physical and human capital in the situation taken as a starting point.

A negative education subsidy is certainly an odd result from the point of view of the received literature [Boldrin and Montes (2005), Docquier et. al. (2007)]. In our setting, it results from the assumption of the existence of perfect capital markets and from the particular interaction between human and physical capital that takes place when we adopt the aforementioned normative criterion. A clear-cut sign of the pensions received by retirees is also in sharp contrast with the message emerging from overlapping-generations models with exogenous growth à la Diamond (1965). Indeed, in this latter case, whether optimal intergenerational transfers are to or from retirees is in general indeterminate, and depends on whether the laissez-faire capital-labour ratio is greater or less than its Golden-Rule counterpart [Samuelson (1975b)]. However, as we mentioned at the beginning of this introduction for the case of economies without productivity growth, different views of welfare naturally lead to different optimal allocations. And not less importantly, the cardinalization of individual preferences is a matter of indifference when the social planner maximizes a non-discounted sum of utilities defined over consumption levels expressed per unit of labour efficiency.

The rest of the paper is organized as follows. Section 2 presents the model and analyzes the market equilibrium in the presence of government. Section 3 discusses the optimal growth path chosen by a social planner who maximizes an non-discounted sum of individual utilities defined over consumption per unit of efficient labour. Section 4 characterizes the optimal tax policies that allow to decentralize the optimal growth path.

Section 5 concludes.

2 The model and the decentralized equilibrium in the presence of government

The framework of analysis is the overlapping generations model with both human and physical capital and life-cycle saving behaviour used in Boldrin and Montes (2005), Docquier et al. (2007) and Del Rey and Lopez-Garcia (2012,2013). At period t, L_{t+1} individuals are born, and coexist with L_t middle-aged and L_{t-1} old-aged. Population grows at the exogenous rate $n > -1$, so that $L_t = (1+n)L_{t-1}$. Agents are born with the level of human capital of their parents, h_{t-1}, measured in units of efficient labour per unit of natural labour. Human capital in period t results from the interaction of the amount of output that young individuals invest in education, d_{t-1}, and the inherited human capital h_{t-1} according to the production function $h_t = E(d_{t-1}, h_{t-1})$. Assuming constant returns to scale, the production of human capital can be written in intensive terms as $h_t/h_{t-1} = e(\tilde{d}_{t-1})$, where $e(.)$ satisfies the Inada conditions and $\tilde{d}_{t-1} = d_{t-1}/h_{t-1}$ is the amount of output devoted to education per unit of (inherited) human capital. Therefore, the growth rate of productivity from period $t-1$ to period t, g_t, satisfies $h_t/h_{t-1} = e(\tilde{d}_{t-1}) = (1 + g_t)$.

There is a single good, Y_t , that is produced by means of physical capital, K_t, and human capital, H_t, according to a constant returns to scale production function $Y_t = F(K_t, H_t)$. Only the middle-aged work, supplying inelastically one unit of natural labour, so that $H_t = h_t L_t$. Physical capital is assumed to fully depreciate each period. Letting $k_t = K_t/L_t$ be the physical capital per unit of natural labour ratio and $\tilde{k}_t = K_t/H_t = k_t/h_t$ the physical capital per unit of efficient labour ratio, one can write $Y_t/H_t = f(\tilde{k}_t)$, where $f(.)$ also satisfies the Inada conditions.

Perfect competition prevails, so that, if $(1 + r_t)$ and w_t are, respectively, the interest factor and the wage rate per unit of efficient labour,

$$(1 + r_t) = f'(\tilde{k}_t) \tag{1}$$

$$w_t = f(\tilde{k}_t) - \tilde{k}_t f'(\tilde{k}_t) \tag{2}$$

Two policy instruments are assumed to be available to the government: lump-sum taxes, both on the middle-aged and the elderly, and education subsidies. To emphasize the role of credit markets in financing human capital investments and their interaction with public policy, education subsidies are related to the repayment, in the second period of life, of the loans taken in the first one to pay for education and the ensuing interests. Let $z_t^m > 0$ [resp. < 0] be the lump-sum tax [transfer] the middle-aged pay [receive], $z_t^o > 0$ [< 0] the lump-sum tax the old pay [the pension they receive], and let θ_t be the subsidy rate, all of them in period t. The government budget constraint can be written:

$$z_t^m L_t + z_t^o L_{t-1} = \theta_t(1 + r_t)d_{t-1}L_t \tag{3}$$

Individuals are assumed to behave as pure life-cyclers and only consume in their second and third period. The lifetime utility function of an individual born at period $t-1$ is $U_t = U(c_t^m, c_{t+1}^o)$, where c_t^m and c_{t+1}^o denote her consumption levels as middle-aged and old-aged respectively. This function is strictly increasing in both arguments, strictly concave and homogeneous of degree $j < 1$. In their first period, individuals born at $t-1$ borrow in perfect credit markets the amount of output required to pay for the education level d_{t-1} that maximizes the present value of their lifetime resources. In their second period they work, pay taxes z_t^m, pay back the loan net of education subsidies $(1+r_t)d_{t-1}(1-\theta_t)$, consume and save to finance consumption in their third period. In this third period, individuals consume and pay taxes z_{t+1}^o. Letting s_t stand for savings of a middle-aged:

$$c_t^m = w_t h_t - (1 + r_t)d_{t-1}(1 - \theta_t) - z_t^m - s_t \tag{4}$$

$$c_{t+1}^o = (1 + r_{t+1})s_t - z_{t+1}^o \tag{5}$$

As a consequence, the lifetime budget constraint of an individual born at period $t-1$ is:

$$c_t^m + \frac{c_{t+1}^o}{(1 + r_{t+1})} = w_t h_t - (1 + r_t)d_{t-1}(1 - \theta_t) - z_t^m - \frac{z_{t+1}^o}{(1 + r_{t+1})} \tag{6}$$

The first-order conditions associated with the individual decision variables, d_{t-1}, c_t^m and c_{t+1}^o, are:

$$w_t e'(d_{t-1}/h_{t-1}) = (1 + r_t)(1 - \theta_t) \tag{7}$$

$$\frac{\partial U(c_t^m, c_{t+1}^o)/\partial c_t^m}{\partial U(c_t^m, c_{t+1}^o)/\partial c_{t+1}^o} = (1 + r_{t+1}) \tag{8}$$

where use has been made of the homogeneity of degree one of the E function, i.e., $h_t = e(d_{t-1}/h_{t-1})h_{t-1}$. Equation (7) shows that the individual will invest in education up to the point where the marginal benefit in terms of second period income equals the marginal cost of investing in human capital allowing for subsidies. Rewriting (7) as $e'(\tilde{d}_{t-1}) = (1-\theta_t)\ (1+r(\tilde{k}_t))/w(\tilde{k}_t)$, this expression implicitly characterizes the optimal ratio $\tilde{d}_{t-1}$ as a function of $\tilde{k}_t$ and θ_t, i.e., $\tilde{d}_{t-1} = \phi(\tilde{k}_t, \theta_t)$.

Using the government budget constraint (3), (6) becomes

$$c_t^m + \frac{c_{t+1}^o}{(1+r_{t+1})} = \omega_t \tag{9}$$

where ω_t is the present value of the net lifetime income of an individual born at $t-1$:

$$\omega_t = w_t h_t - (1+r_t)d_{t-1}(1-\theta_t) - z_t^m - \frac{(1+n)}{(1+r_{t+1})}[\theta_{t+1}(1+r_{t+1})d_t - z_{t+1}^m] \tag{10}$$

The homogeneity assumption on preferences implies that the c_{t+1}^o/c_t^m ratio is a function of r_{t+1} only. This allows to write consumption in the second period as $c_t^m = \pi(r_{t+1})\omega_t$, where the function $\pi(.)$ depends on the interest rate only. Equilibrium in the market for physical capital will be achieved when the physical capital stock available in $t+1$, K_{t+1}, equals gross savings made by the middle-aged in t, $s_t L_t$, minus the amount of output devoted to human capital investment by the young in $t, (1+n)d_t L_t$. That is, when $K_{t+1} = s_t L_t - (1+n)d_t L_t$, or, equivalently, $\tilde{k}_{t+1} = \tilde{s}_t/e(\tilde{d}_t)(1+n) - \tilde{d}_t/e(\tilde{d}_t)$, where $\tilde{s}_t = s_t/h$. This equilibrium condition can be rewritten as

$$\tilde{k}_{t+1} = \frac{(1-\pi(r_{t+1}))\,\tilde{\omega}_t}{e(\phi(\tilde{k}_{t+1}, \theta_{t+1}))(1+n)} - \frac{\tilde{z}_{t+1}^m}{(1+r_{t+1})} - \frac{(1-\theta_{t+1})\,\phi(\tilde{k}_{t+1}, \theta_{t+1})}{e(\phi(\tilde{k}_{t+1}, \theta_{t+1}))} \tag{11}$$

where $\tilde{z}_{t+1}^m = z_{t+1}^m/h_{t+1}$ and $\tilde{\omega}_t = \omega_t/h_t$ is the present value of lifetime resources expressed in terms of output per unit of efficient labour. Taking into account (1) and (2), this expression implicitly provides $\tilde{k}_{t+1}$ as a function of $\tilde{k}_t$, $\tilde{z}_t^m$, $\tilde{z}_{t+1}^m$, θ_t, and θ_{t+1}, i.e., $\Psi(\tilde{k}_t; \tilde{z}_t^m, \tilde{z}_{t+1}^m, \theta_t, \theta_{t+1})$.

Using factor prices in (1) and (2), the government budget constraint (3), the individual budget constraints in middle and old-age, (4) and (5), and the equilibrium condition (11), one can find the aggregate feasibility constraint expressed in terms of output per unit of efficient labour:

$$\tilde{c}_t^m + \frac{\tilde{c}_t^o}{e(\tilde{d}_{t-1})(1+n)} = f(\tilde{k}_t) - e(\tilde{d}_t)(1+n)\tilde{k}_{t+1} - (1+n)\tilde{d}_t \tag{12}$$

where $\tilde{c}_t^m = c_t^m/h_t$ and $\tilde{c}_t^o = c_t^o/h_{t-1}$ are consumption of a middle-aged and of an old-aged in period t per unit of their respective levels of labour efficiency.[2]

The fact that the utility function is homogeneous implies that the marginal rates of substitution in the $\left(c_t^m, c_{t+1}^o\right)$ and $\left(\tilde{c}_t^m, \tilde{c}_{t+1}^o\right)$ spaces will be the same. Thus, individual behavior in the presence of arbitrary sequences of θ_t and $\tilde{z}_t^m$ will be described along the growth trajectory by:

$$\frac{\partial U(\tilde{c}_t^m, \tilde{c}_{t+1}^o)/\partial \tilde{c}_t^m}{\partial U(\tilde{c}_t^m, \tilde{c}_{t+1}^o)/\partial \tilde{c}_{t+1}^o} = (1 + r_{t+1}) \tag{13}$$

$$w_t e'(\tilde{d}_{t-1}) = (1 + r_t)(1 - \theta_t) \tag{14}$$

$$\tilde{c}_t^m + \frac{\tilde{c}_{t+1}^o}{(1 + r_{t+1})} = \tilde{\omega}_t \tag{15}$$

where

$$\tilde{\omega}_t = w_t - \frac{(1 + r_t)\tilde{d}_{t-1}(1 - \theta_t)}{e(\tilde{d}_{t-1})} - \tilde{z}_t^m - \frac{(1 + n)}{(1 + r_{t+1})}[\theta_{t+1}(1 + r_{t+1})\tilde{d}_t - e(\tilde{d}_{t+1})\tilde{z}_{t+1}^m] \tag{16}$$

is the present value of the individual's lifetime resources at time t expressed in output per unit of efficiency labour.

A balanced growth path is a situation where all variables expressed in terms of output per unit of natural labour grow at a constant rate, and, as a consequence, all variables per unit of efficient labour will remain constant over time. One can then delete the time subscripts in (11) and write $\tilde{k} = \Psi(\tilde{k}; \tilde{z}^m, \theta)$. An equilibrium ratio of physical capital to labour in efficiency units along a balanced growth path will then be a fixed point of the Ψ function, i.e., $\tilde{k} = \Psi(\tilde{k}; \tilde{z}^m, \theta)$. Such an equilibrium will be locally stable provided that $0 < \partial\Psi(\tilde{k}; \tilde{z}^m, \theta)/\partial\tilde{k} < 1$. The amount of output devoted to education per unit of inherited human capital along a balanced growth path, $\tilde{d}$, will be governed by the relationship arising from the education decision (7). We will throughout assume uniqueness and stability of the long-run equilibrium for given values of $\tilde{z}^m$ and θ.[3] Consequently, individual behavior

[2]Note that $c_t^m L_t$ and $c_t^o L_{t-1}$ are measured in units of output. Since middle-aged individuals supply one unit of natural labour, c_t^m and c_t^o are expressed in units of output per unit of *natural* labour. The interpretation of $\tilde{c}_t^m$ and $\tilde{c}_t^o$ in terms of units of output per unit of *efficient* labour follows naturally.

[3]It is well known that in overlapping generations models with only physical capital à la Diamond (1965), providing conditions for existence, uniqueness and stability of equilibria is not straightforward. See

along a balanced growth path can be summarized by expressions (13)-(16) without the time subscripts.

3 The optimum Ramsey growth path

In this section we argue that a social welfare function can be postulated that maximizes a non-discounted sum of utilities without being obliged to adopt a specific cardinalization of individual ordinal preferences. To this end, notice that since the utility function is homogeneous of degree j, we can take the monotonic transformation of $U_t = U(c_t^m, c_{t+1}^o)$ resulting from dividing by h_t and obtain a "new" utility function, $\tilde{U}_t = U(\tilde{c}_t^m, \tilde{c}_{t+1}^o)$, whose arguments are consumptions per unit of labour efficiency. Notice that both of them have the same functional form, thus ensuring that ordinal preferences are respected.[4]

The objective of a social planner à la Ramsey (1928) can then be written as the maximization of a non-discounted sum of the present and future divergences between individual utilities derived from consumption expressed per unit of efficient labour and some "bliss utility" $\tilde{U}_*$:

$$\tilde{W} = \sum_{t=0}^{\infty} \left[U(\tilde{c}_t^m, \tilde{c}_{t+1}^o) - \tilde{U}_* \right], \tag{17}$$

where the bliss level is "the maximum obtainable rate of enjoyment or utility" (Ramsey, 1928, p. 545).[5] This welfare criterion raises two fundamental questions. Firstly, it should be justified that the bliss utility level $\tilde{U}_*$ is a finite value, as otherwise the objective function

Galor and Ryder (1989), De la Croix and Michel (2002) and Li and Lin (2012). The same considerations apply in the current endogenous growth setting.

[4]Formally, we can write $\tilde{U}_t = U(\tilde{c}_t^m, \tilde{c}_{t+1}^o) = U(c_t^m/h_t, c_{t+1}^o/h_t) = (1/h_t^j)U(c_t^m, c_{t+1}^o) = (1/h_t^j)U_t$. It is important to stress that both $U(\tilde{c}_t^m, \tilde{c}_{t+1}^o)$ and $U(c_t^m, c_{t+1}^o)$ have the same functional form and are homogeneous of degree j. As stated above, the curvature and higher derivatives of indifference curves in $(\tilde{c}_t^m, \tilde{c}_{t+1}^o)$ will be the same as their counterparts in the (c_t^m, c_{t+1}^o) space.

[5]Strictly speaking, and to be fully coherent with the approach followed by Ramsey, (17) should be written as $(-\tilde{W}) = \sum_{t=0}^{\infty} \left[\tilde{U}_* - U(\tilde{c}_t^m, \tilde{c}_{t+1}^o) \right]$, so that the purpose of the social planner is to minimize the non-discounted sum of the divergence of the amount by which utility falls short of the bliss level. Obviously both are equivalent, as maximizing $\tilde{W}$ is tantamount to minimizing $(-\tilde{W})$.

(17) is not well defined. And secondly, we have to be sure that the infinite addition in (17) is not infinity when, as it is of course the case, the addends are not discounted. As far as the first question is concerned, the bliss utility level is the one associated with those values $\tilde{c}_*^m$ and $\tilde{c}_*^o$ that maximize $\tilde{U} = U(\tilde{c}^m, \tilde{c}^o)$ under the constraint that the balanced-growth-path version of the feasibility constraint (12) be satisfied. Therefore, $\tilde{U}_*$ is the welfare level $U(\tilde{c}_*^m, \tilde{c}_*^o)$ resulting from the "Golden Rule" discussed in Del Rey and Lopez-Garcia (2013), i.e., the balanced growth path where the utility (as a function of consumptions per unit of efficient labour) of a representative generation is maximized under the constraint that everyone else attains the same level. Concerning the convergence of the infinite sum in (17), since the arguments therein are defined in relation to output per unit of efficient labour (and are therefore constant along a balanced growth path), it can be guaranteed in the same terms as in overlapping generation models without productivity growth [see Samuelson (1968) and De la Croix and Michel (2002), pp. 91-92].

The planner's problem is then to maximize (17) subject to the sequence of aggregate feasibility constraints (12) for given values of $\tilde{k}_0, \tilde{c}_0^o$ and $\tilde{d}_{-1}$ as initial conditions. The socially optimum growth path can be characterized by means of the first-order conditions and the transversality conditions.[6] The Lagrangean function becomes:

$$\begin{aligned}\mathcal{L} \;=\; & \sum_{t=0}^{\infty}\left[U(\tilde{c}_t^m, \tilde{c}_{t+1}^o) - U(\tilde{c}_*^m, \tilde{c}_*^o)\right] \; - \\ & -\sum_{t=0}^{\infty}\mu_t\left(\tilde{c}_t^m + \frac{\tilde{c}_t^o}{e(\tilde{d}_{t-1})(1+n)} - f(\tilde{k}_t) + (1+n)e(\tilde{d}_t)\tilde{k}_{t+1} + (1+n)\tilde{d}_t\right)\end{aligned} \tag{18}$$

where μ_t is the Lagrange multiplier associated with the resource constraint (12) at time t. From the first-order conditions corresponding to $\tilde{c}_t^m$, $\tilde{c}_{t+1}^o$, $\tilde{k}_{t+1}$, $\tilde{d}_t$, and μ_t, and adding the subscript $*$ to denote optimality, we obtain:

$$\frac{\partial U(\tilde{c}_{*t}^m, \tilde{c}_{*t+1}^o)/\partial \tilde{c}_t^m}{\partial U(\tilde{c}_{*t}^m, \tilde{c}_{*t+1}^o)/\partial \tilde{c}_{t+1}^o} = f'(\tilde{k}_{*t+1}) \tag{19}$$

$$\frac{\partial U(\tilde{c}_{*t}^m, \tilde{c}_{*t+1}^o)/\partial \tilde{c}_t^m}{\partial U(\tilde{c}_{*t-1}^m, \tilde{c}_{*t}^o)/\partial \tilde{c}_t^o} = e(\tilde{d}_{*t-1})(1+n) \tag{20}$$

[6]On the precise form of the transversality condition in Ramsey-like optimization problems, see Michel (1990) and De la Croix and Michel (2002).

$$e(\tilde{d}_{*t})\left(\frac{\tilde{c}^{o}_{*t+1}}{f'(\tilde{k}_{*t+1})e(\tilde{d}_{*t})(1+n)}-\tilde{k}_{*t+1}\right)=1 \tag{21}$$

as well as (12). The interpretation of (19) and (20) is straightforward. The former reflects the equality of the intertemporal rates of substitution in consumption (i.e., between second and third period consumptions) and of transformation in production (i.e., the marginal product of physical capital) between periods t and $t+1$. The latter captures the static conditions of optimal distribution of consumption available in period t between middle-aged and old-aged individuals, allowing for growth of both productivity and population.

Although it may seem odd at first glance, expression (21) also has a natural interpretation. Indeed, it is an arbitrage condition between the returns from investing in physical capital and in education. The intuition can be grasped by making use of the fact that μ_t [resp. μ_{t+1}] is the shadow value, in terms of social welfare $\tilde{W}$, of a unit of output per efficient labour in period t [resp. $t+1$]. Suppose that in period t the social planner slightly increases the amount of capital, $\tilde{k}_{t+1}$, it sets aside for the next period. It is clear from (12) that this will affect the aggregate feasibility constraints at periods t and $t+1$: a higher $\tilde{k}_{t+1}$ implies a reduction in the resources left for consumption in period t, given by $e(\tilde{d}_t)(1+n)$, and an increase of the resources available for consumption in period $t+1$, captured by $f'(\tilde{k}_{t+1})$. Thus, the marginal cost of investing in physical capital is $\mu_t(1+n)e(\tilde{d}_t)$ and the marginal benefit is $\mu_{t+1}f'(\tilde{k}_{t+1})$. The first order condition for $\tilde{k}_{t+1}$ imposes that this marginal cost and this marginal benefit should be equal along the optimum growth path. If, instead, the planner in period t increases the amount of output devoted to education, $\tilde{d}_t$, the feasibility constraints at periods t and $t+1$ will also be modified. The cost, incurred in period t, now has two components. On the one hand, there is a direct cost, $(1+n)$, that reduces consumption possibilities. There is, however, also an indirect cost, given by $e'(\tilde{d}_t)(1+n)\tilde{k}_{t+1}$: as a consequence of the effect of $\tilde{d}_t$ on the growth rate, the amount of output devoted to investment in physical capital must be increased if we are to achieve the optimal value of $\tilde{k}_{t+1}$. Using the shadow value μ_t, the marginal cost of an additional unit invested in education is thus $\mu_t[(1+n)+e'(\tilde{d}_t)(1+n)\tilde{k}_{t+1}]$. The benefits, however, do not take place until period $t+1$. Indeed, evaluating (12) at $t+1$, the increased growth rate lowers the marginal rate of transformation between third and second period consumption on the right hand side. This amounts to an expansion of consumption possibilities, so that the marginal benefit is $\mu_{t+1}e'(\tilde{d}_t)(1+n)\tilde{c}^o_{t+1}/[e(\tilde{d}_t)(1+n)]^2$. As before, the first order condition for $\tilde{d}_t$ imposes that these marginal costs and benefits should be equal along the optimum growth path. Both first order conditions involve the same ratio of shadow values, μ_t/μ_{t+1}, so that an arbitrage condition between the returns from investing in $\tilde{k}_{t+1}$ and $\tilde{d}_t$, measured in units of resources at $t+1$ per unit of resources at t, can be derived. This is precisely the way expression (21) is obtained.

We can advance the following definition:

Definition 1 *Given the initial conditions ($\tilde{k}_0$,$\tilde{c}^o_0$,$\tilde{d}_{-1}$), the optimal path $\{\tilde{c}^m_{*t}, \tilde{c}^o_{*t+1}, \tilde{k}_{*t+1}, \tilde{d}_{*t}\}$ that provides the sequence $\{\tilde{U}_{*t}\}$ and maximizes the non-discounted sum (17) defined over consumption per unit of efficient labour from period t=0 to infinity, satisfies conditions (19), (20), (21) and (12).*

Assuming convergence, the optimal balanced growth path can be characterized deleting the time subscripts in (19), (20) and (21), i.e.:

$$\frac{\partial U(\tilde{c}^m_*, \tilde{c}^o_*)/\partial \tilde{c}^m}{\partial U(\tilde{c}^m_*, \tilde{c}^o_*)/\partial \tilde{c}^o} = e(\tilde{d}_*)(1+n) \tag{22}$$

$$f'(\tilde{k}_*) = e(\tilde{d}_*)(1+n) \tag{23}$$

$$e'(\tilde{d}_*)\left(\frac{\tilde{c}^o_*}{\Big[e(\tilde{d}_*)(1+n)\Big]^2} - \tilde{k}_*\right) = 1 \tag{24}$$

in addition to the balanced growth path version of (12). Together, these four equations provide the optimal values $\tilde{c}^m_*$, $\tilde{c}^o_*$, $\tilde{k}_*$ and $\tilde{d}_*$, and thus $\tilde{U}_* = U(\tilde{c}^m_*, \tilde{c}^o_*)$, termed by Del Rey and Lopez-Garcia (2013) as the "Golden Rule" in the current endogenous growth setting. Condition (22) is the equality of the marginal rate of substitution between second and third period consumptions and the economy's growth rate $e(\tilde{d}_*)(1+n)$. In turn, (23) is the equality between the marginal product of physical capital and the growth rate. Finally, (24) is the balanced-growth-path version of the arbitrage condition between investing in physical and human capital. Taken together, (22)-(24) are the counterpart in the current model of the so-called Two-Part Golden Rule [Samuelson (1968, 1975a, 1975b)], i.e., the (now endogenous) Biological Interest Rate [Samuelson (1958)] and the Golden Rule of (physical) capital accumulation [Phelps (1961)]. It is important to emphasize that (23), and along with it the entire system of equations, is *independent* of the specific cardinalization of individual preferences that could have be chosen to describe individual behaviour. In other words, and in contrast to Docquier et al. (2007), the degree of homogeneity j of the utility function is now irrelevant.[7,8]

4 Optimal policy

We are now in a position to discuss the values, and not less important, the signs, of the optimal tax instruments $\left\{\theta_{*t}, \tilde{z}^{m}_{*t}, \tilde{z}^{o}_{*t+1}\right\}$ that allow the social planner to decentralize as a market equilibrium the optimal time path $\left\{\tilde{c}^{m}_{*t}, \tilde{c}^{o}_{*t+1}, \tilde{k}_{*t+1}, \tilde{d}_{*t}\right\}$ discussed in the preceding section. Of course, to do so, the set of tax parameters has to induce individuals to choose the optimal sequence of physical and human capital-labour ratios.

Let us start with the characterization of the optimal education subsidies. Using the feasibility constraint (12) evaluated at t, we can backward the arbitrage condition (21) one period and rewrite it in a way that can be directly compared to the first-order condition

[7] Indeed, using the double subscript $*$ to denote the optimal balanced growth path in Docquier et al. (2007), where the social planner maximizes (with a social discount factor γ) a discounted sum of utilities defined over consumption per unit of natural labour, the optimal balanced growth path is characterized by the Modified Golden Rule, i.e., $\gamma f'(\tilde{k}_{**}) = [e(\tilde{d}_{**})]^{1-j}(1+n)$, and the marginal product of physical capital will be *greater* than the economy's growth rate. The presence of the degree of homogeneity j in this expression makes it apparent that different cardinalizations of the same ordinal preferences will entail different optimal balanced growth paths (and, consequently, different optimal configurations of the tax parameters designed to decentralize them). Notice that the same kind of objection arises in the framework suggested by Caballé (1995) with altruistic individuals.

[8] The degree of homogeneity of the utility function is also irrelevant in the model used in Bishnu (2013), that relates human capital accumulation to consumption externalities. Balanced growth paths therein, however, display no productivity growth so that demography is the only source of long-run growth. As a consequence, the optimal balanced growth path is characterized by the same Modified Golden Rule as in standard overlapping generation models à la Diamond (1965).

of the individual when she chooses the amount of resources invested in education:

$$\left[f(\tilde{k}_{*t}) - \tilde{k}_{*t}f'(\tilde{k}_{*t})\right] e'(\tilde{d}_{*t-1}) = f'(\tilde{k}_{*t}) \left(1 + \frac{e'(\tilde{d}_{*t-1})\Lambda_{*t}}{f'(\tilde{k}_{*t})}\right) \tag{25}$$

where the term $\Lambda_{*t} = (1+n)e(\tilde{d}_{*t})\tilde{k}_{*t+1} + (1+n)\tilde{d}_{*t} + \tilde{c}^m_{*t}$ is strictly positive. Mere comparison of (25) and (7) entails that the optimal education investment tax parameter at period t, θ_{*t}, for all $t \geq 0$ will be:

$$\theta_{*t} = -\frac{e'(\tilde{d}_{*t-1})\Lambda_{*t}}{f'(\tilde{k}_{*t})} < 0. \tag{26}$$

This result can be stated as the following

Proposition 1 *When the social planner maximizes the non-discounted sum of utilities (17) and decentralizes the allocation of resources through the market mechanism, investment in education should be taxed along the entire growth path according to the tax rate given in (26).*

Although the result of a negative education subsidy in (26) may seem hard to explain, the intuition underlying it may easily be grasped with the aid of Figure 1. The downward sloping curve therein depicts the marginal product $e'(.)$ as a function of the amount of resources devoted to education, $\tilde{d}_{t-1}$. When $\theta = 0$, i.e., in the absence of any education tax or subsidy, the rule governing the individual's investment in education, (7), reduces to $e'(\tilde{d}_{t-1}) = f'(\tilde{k}_t)/\left[f(\tilde{k}_t) - \tilde{k}_t f'(\tilde{k}_t)\right]$. It is now clear that, if wages and interest factors were set at their optimal levles, $w(\tilde{k}_{*t}) = f(\tilde{k}_{*t}) - \tilde{k}_{*t}f'(\tilde{k}_{*t})$ and $f'(\tilde{k}_{*t})$, the individual would choose point A in Figure 1. She would then fail to take into account the terms collected by Λ_{*t} in (25) and would choose $\tilde{d}_{LF,t-1}$, thus overinvesting in education. In order to attain $\tilde{d}_{*t-1}$ at B, where $e'(\tilde{d}_{*t-1}) = \left[f'(\tilde{k}_{*t}) + e'(\tilde{d}_{*t-1})\Lambda_{*t}\right]/w(\tilde{k}_{*t})$, the optimal tax on education θ_{*t} in (26) is required.

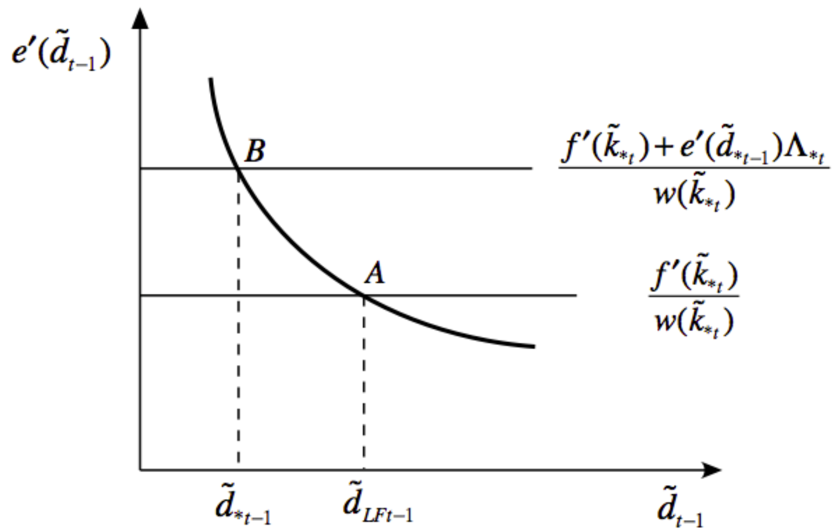

Figure 1: An illustration of Proposition 1

Turning now to the characterization of the optimal lump-sum taxes $\tilde{z}^m_{*t}$ and $\tilde{z}^o_{*t}$, let $\tilde{s}_{*t}$ be the amount of saving per unit of efficient labour made by each middle-aged to support $\tilde{k}_{*t+1}$, according to the physical capital market equilibrium condition, i.e., $\tilde{k}_{t+1} = \tilde{s}_t/e(\tilde{d}_t)(1+n) + \tilde{d}_t/e(\tilde{d}_t)$. One can then use (4) and (5), with the latter backwarded one period, to obtain the required lump-sum taxes and transfers for any period $t \geq 1$.[9]

$$\tilde{z}^m_{*t} = w_{*t} - (1 + r_{*t})\tilde{d}_{*t-1}(1 - \theta_{*t})/e(\tilde{d}_{*t-1}) - \tilde{s}_{*t} - \tilde{c}^m_{*t} \tag{27}$$

$$\tilde{z}^o_{*t} = (1 + r_{*t})\tilde{s}_{*t-1} - \tilde{c}^o_{*t} \tag{28}$$

We can thus state the following

Proposition 2 *When the social planner maximizes the non-discounted sum of utilities (17) and decentralizes the allocation of resources through the market mechanism, the se-*

[9]A full characterization of the optimal policy along this track also requires that the optimal lump-sum taxes and education tax in period 0 be identified. Thus, given $\tilde{k}_0$ (and thus $\tilde{s}_{-1}$, w_0 and r_0), $\tilde{c}_0$ and $\tilde{d}_{-1}$, (26), (27) and (28) become:

$$\theta_{*0} = -\frac{e'(\tilde{d}_{-1})\Lambda_0}{f'(\tilde{k}_0)}$$

$$\tilde{z}^m_{*0} = w_0 - (1 + r_0)\tilde{d}_{-1}(1 - \theta_{*0})/e(\tilde{d}_{-1}) - \tilde{s}_{*0} - \tilde{c}^m_{*0}$$

$$\tilde{z}^o_{*0} = (1 + r_0)\tilde{s}_{-1} - \tilde{c}^o_0$$

Together, they provide three equations to be solved in θ_{*0}, $\tilde{z}^m_{*0}$ and $\tilde{z}^o_{*0}$.

quences of optimal lump-sum taxes or transfers on middle-aged and old-aged along the optimal growth path are given by (27) and (28).

Since $\tilde{z}^m_{*t}$ and $\tilde{z}^o_{*t}$ in (27) and (28) can, at least at first sight, have any sign, Proposition 2 does not seem very instructive, especially to ascertain whether lump-sum taxes in old-age are positive or negative. We can prove, however, that the older generation will always receive a pension along the optimal path.

Proposition 3 *When the social planner maximizes the non-discounted sum of utilities (17) and decentralizes the allocation of resources through the market mechanism, the lump-sum tax paid by the older generation in (28) is always negative, so that pensions received by the elderly are positive along the entire optimal growth path.*

Proof.

The proof starts rewriting the physical capital market equilibrium condition (11) along the optimal growth path as:

$$e(\tilde{d}_{*t})(1+n)\tilde{k}_{*t+1} = f(\tilde{k}_{*t}) - \tilde{k}_{*t}f'(\tilde{k}_{*t}) - \frac{f'(\tilde{k}_{*t})\tilde{d}_{*t-1}(1-\theta_{*t})}{e(\tilde{d}_{*t-1})} - \tilde{z}^m_{*t} - (1+n)\tilde{d}_{*t} - \tilde{c}^m_{*t}, \quad (29)$$

Combining (29) and the individual budget constraint (15) also evaluated along the optimal path, the optimality condition (24) can be rewritten as:

$$e'(\tilde{d}_{*t})\left(-\frac{\theta_{*t+1}\tilde{d}_{*t}}{e(\tilde{d}_{*t})} + \frac{\tilde{z}^m_{*t+1}}{f'(\tilde{k}_{*t+1})}\right) = \left(1 - \frac{e'(\tilde{d}_{*t})}{e(\tilde{d}_{*t})/\tilde{d}_{*t}}\right) > 0 \quad (30)$$

where the inequality follows from the Inada conditions. This ensures that the expression in brackets on the left hand side is positive. Substituting now into the government budget constraint (3) expressed in terms of output per unit of efficient labour, i.e., $\tilde{z}^o_{*t+1} = (1+n)\left[\theta_{*t+1}f'(\tilde{k}_{*t+1})\tilde{d}_{*t} - e(\tilde{d}_{*t})\tilde{z}^m_{*t+1}\right]$, one obtains:

$$\frac{\tilde{z}^o_{*t+1}}{e(\tilde{d}_{*t})(1+n)f'(\tilde{k}_{*t+1})} = -\left(-\frac{\theta_{*t+1}\tilde{d}_{*t}}{e(\tilde{d}_{*t})} + \frac{\tilde{z}^m_{*t+1}}{f'(\tilde{k}_{*t+1})}\right) < 0, \quad (31)$$

which implies that $\tilde{z}^o_{*t+1} < 0$, i.e., that pensions are positive. This proves Proposition 3. ■

With respect to the sign of the sequence of optimal lump-sum taxes paid by the middle-aged, $\tilde{z}^m_{*t}$, nothing can be said in general. Indeed, from (30) one gets:

$$\tilde{z}^m_{*t+1}\frac{e'(\tilde{d}_{*t})}{f'(\tilde{k}_{*t+1})} = \left(1 - \frac{e'(\tilde{d}_{*t})}{e(\tilde{d}_{*t})/\tilde{d}_{*t}}\right) + \frac{\theta_{*t+1}e'(\tilde{d}_{*t})}{e(\tilde{d}_{*t})/\tilde{d}_{*t}} \gtrless 0, \tag{32}$$

The first term on the right hand side is positive by virtue of the Inada conditions, but the second one is negative from Proposition 1. This leaves the sign of $\tilde{z}^m_{*t+1}$ as indeterminate.

To end this section, it is important to emphasize that in proving Propositions 1 and 3 no consideration has been made about the initial conditions for physical and human capital. As a consequence, they hold regardless of the initial values of $\tilde{k}_0, \tilde{c}^o_0$ and $\tilde{d}_{-1}$ associated with the decentralized economy in the presence of government.

5 Concluding remarks

In this paper we have used an overlapping generations model with physical and human capital where individuals save for strict life-cycle reasons to ascertain the consequences for optimality of a social planner adopting a welfare criterion that gives the same weight to all generations and is respectful of individual preferences. In particular, we have considered a social planner who maximizes an non-discounted sum of individual utilities à la Ramsey defined over consumption per unit of efficient labour. We have shown that, given some arbitrary initial conditions, an economy commanded by such a social planner converges to the "Golden Rule" defined in this endogenous growth model. We have also shown that decentralizing the optimal trajectory requires taxing education investment along the entire optimal growth path. With respect to lump-sum taxes on the middle-aged and old-aged individuals, no general result can be derived for the former, but the latter are unambiguously negative (thus entailing positive pensions) along the entire optimal growth path.

Admittedly, there is nothing sacrosanct in both the social welfare function we have posited (a non-discounted sum of utilities derived from consumption per unit of efficient labour) or the sign of the resulting optimal tax instruments (taxing investment in education and paying positive pensions). But the standard welfare objective (an arbitrarily discounted sum of utilities derived from consumption per unit of natural labour) or the policy prescriptions arising from it (education subsidies and pensions of an indeterminate sign) are not irremovable either. Be it as it may, it is worth emphasizing that, unlike what happens when we adopt alternative objectives, our optimal policy is independent of the cardinalization (in particular, the degree of homogeneity) of the utility function. It does not require either a social planner who adheres to an arbitrarily chosen discount rate. The importance of the latter point cannot be exaggerated, as one could claim that discounting is nothing else but a contrivance to solve the mathematical inconveniences posited by an infinite sum not converging. Our approach, however, closely following Ramsey's footprints, is not liable to these formal problems. To conclude, some assumptions of the model are quite unrealistic, in particular the assumption that individuals have access to perfect credit markets. It may well be the case that the insights emerging from the analysis differ when individuals face constraints when trying to borrow to finance their educational investments. We leave these issues for further research.

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