A Meta-analysis of Systemic Risk Measures for gauging Financial Stability

Massimiliano Caporin[1] **Michele Costola**[2]
Jean-Charles Garibal[3] **Bertrand Maillet**[4]

[1]University of Padova (Italy)

[2]SAFE-Goethe University Frankfurt (Germany)

[3]University of Orléans (France)

[4]EMLyon Business School (France)

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Paper contributions

- Our approach aims at contributing to the definition and detection of financial stress periods, within an explicit, rational and transparent framework based on public market data, which is, in our opinion, of major interest for macro-prudential regulation and the stability of the financial system as a whole
- From the methodological point of view, following Giglio et al. (2016), we propose an aggregated index, called Index of Systemic Risk Measures (ISRM), based on a Sparse-PCA applied to several systemic risk measures
- We endogeneize the sparsity parameter in such a way ISRM Granger-cause extreme movements on the real side of the economy

Literature

- Many contribution in the area, focused on both the proposal of systemic risk indicators or on their comparison
    - Acharya, Pedersen, Philippon and Richardson, 2016, Measuring Systemic Risk, RFS
    - Adrian and Brunnermeier, 2016, CoVaR, AER
    - Benoit, Colliard, Hurlin and Perignon, 2017, Where the Risks Lie: A Survey on Systemic Risk, RF
    - Benoit, Hurlin and Perignon, 2017, Pitfalls in Systemic-risk Scoring, HEC WP
    - Bisias, Flood, Lo and Valavanis, 2012, A Survey of Systemic Risk Analytics, ARFE
    - Brownlees and Engle, 2017, SRISK: A Conditional Capital Shortfall Index for Systemic Risk Measurement, RFS
    - Giglio, Kelly and Pruitt,2016, Systemic Risk and the Macroeconomy: An Empirical Evaluation, JFE
    - Hildebrand, 2016, Systemic Risk and Financial Regulation: Where Do We Stand? in Progress and Confusion: The State of Macroeconomic Policy, MIT Press

Systemic Risk Measures

- Bisias et al., 2012, report 31 systemic risk measures classified in two families...

1. Individual systemic risk measures that quantify the contribution of a single firm to the risk of the entire system (or to a specific systemic event) or that measure the firms' response to a systemic event in the system
2. Global systemic risk measures that examine the system as a whole, not just the response of a firm to the systemic event or the impact of a firm on the system; in other words, they measure the response of the system to a systemic event

Systemic Risk Measures

- We focus on a set of 16 systemic risk measures and indicators proposed and used in the recent literature, namely...

1. Individual measures: the Value-at-Risk (VaR) [VaR], the CoVaR and ΔCoVaR of Adrian and Brunnermeier (2011) [CoVaR], the Marginal Expected Shortfall (MES) of Acharya et al. (2010) [MES], the Component Expected Shortfall (CES) of Banulescu and Dumitrescu (2012) [CES], the Systemic RISK Measure (SRISK) of Acharya et al. (2012) [SRISK], and the Amihud (2002) Illiquidity Measure (AIM) [AIM]
2. Global measures: the Spillover Index (SI) of Diebold et Yilmaz (2009) [SI], the Herfindalh-Hirschman Index (HHI) [HHI], the Absorption Ratio (AR) of Kritzman et al. (2010) [AR], the Turbulence Index (TI) of Kritzman and Li (2010) [TI], Volatility (Vol) [Vol], the Dynamic Causality Index (DCI) of Billio et al. (2012) [DCI], the Term Spread (TS) [TS], the Default Yield Spread (DYS) [DYS], the TED Spread (TED) [TED]

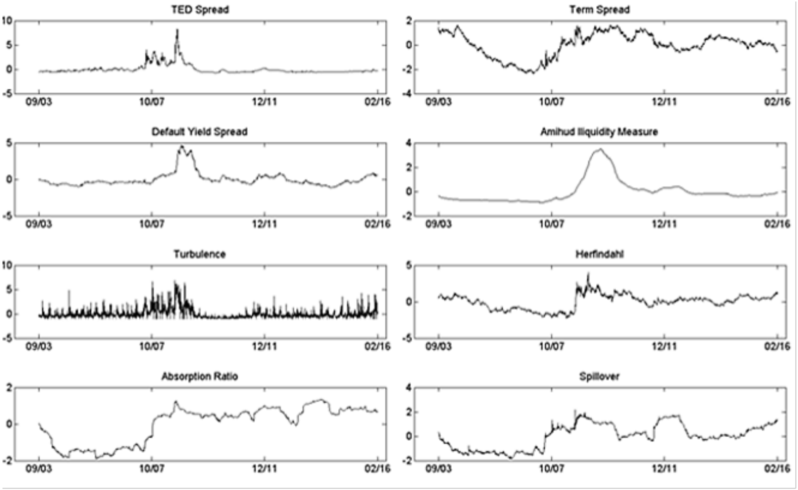
Data and sample

- In a first step, we compute systemic risk measures and indicators on a dataset of historical returns on 60 US financial institutions (Bloomberg: USD, daily quotes) FinInstList
- We consider two *classical* samples, taken from Brownlees and Engle (2017) and Giglio et al. (2016)
  - Period 1: from the 01/2005 to the 12/2012 as in Brownlees and Engle (2017)
  - Period 2: from the 09/2003 to the 12/2011 as in Giglio et al. (2016)
- We combine those data with daily time series of other macro-finance indicators needed to compute global risk measures

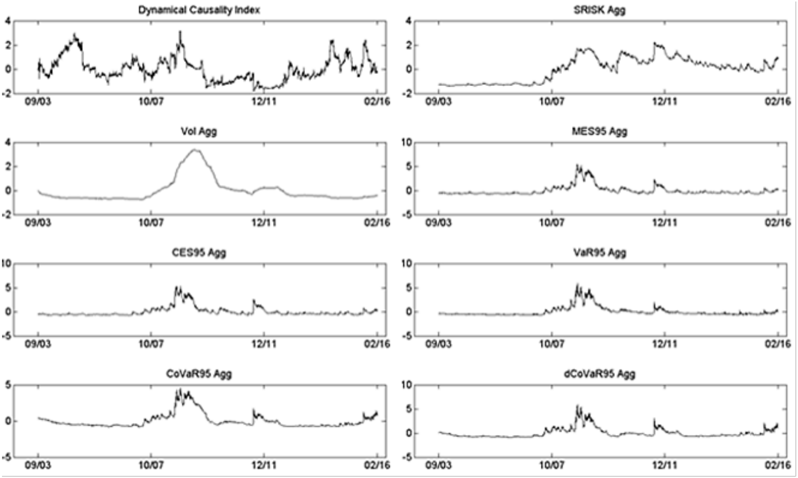
Computing Systemic Risk Indicators

- We compute most risk measures and risk indicators with a rolling window approach and a window size of one year
- Excluded cases are, for example, the spreads which are directly computed
- We aggregate individual risk measures by taking a simple average (an equally weighted combination - *in progress - robustness check: value weighted combination*)
- Giglio et al. (2016) adopt a similar approach but focus on the 20 largest financial institutions
- We transform the indicators into z-scores to equalize mean and scale

Systemic Risk Indicators



Systemic Risk Indicators



Comparing Systemic Risk Indicators

- Several measures and indicators have common patterns and are highly correlated...
- ...but among these measures, is there an *optimal* one? Probably not...
- Systemic risk is a multidimensional phenomenon (losses, capital required during a crisis, liquidity, interconnections, credit, exposures...)
- Each measure insists on one (some) aspect(s) of the systemic risk
- **How can we obtain a composite measure that takes into account the various aspects of the systemic risk?**

Comparing Systemic Risk Indicators

- Recent contributions noted that a definition of a *good* measure of systemic risk is still unresolved
- Two relevant aspects are given by some redundancy in the available systemic risk measure and in their exposure to model risk
- The interdependence between systemic risk rankings and model risk has been addressed by Benoit et al. (2017), Nucera et al. (2016), Kouontchou et al. (2017)
- **How can we mitigate the model risk in evaluating the systemic risk?**

Combining Systemic Risk Indicators

- A possible solution: by aggregation of different systemic risk metrics, Holló et al. (2012), Louzis and Vouldis (2012), Giglio et al (2016)
- Giglio et al. (2016) adopt a classical PCA approach, a dimension reduction method that allows obtaining a composite and more informative measure of the systemic risk
- We start from Giglio et al. (2016) and use Sparse PCA to compute an aggregate index and then assess its link to the real economy (as proxied by IPI growth)
- The Sparse PCA is a variant of classical PCA, which we use to obtain sparse loadings (few indicators are relevant) and a more stable (latent - composite) factor dynamics
- This allows us to identify the systemic risk measures that are most relevant and at the same time to mitigate model risk

Combining Systemic Risk Indicators

- In addition, Sparse PCA is based upon an exogenous parameter that can be fine-tuned to obtained a systemic risk index with the best possible predictive content on severe macroeconomic downturns; this is also of help given the known non-gaussianity and non-linearity of financial returns (in particular during crises)
- The distinctive element of our approach is the endogeneization of the Sparce-PCA parameter which is estimated in such a way the estimated composite index Granger-cause extreme variation on the real economy

Sparse PCA and Regularized SVD

- Notation: M is the $T \times P$ matrix containing the P systemic aggregated risk indicators over a sample of size T; F is the first principal component extracted from M using classical PCA; $||.||_p$ is a $Lp-$norm
- Sparse-PCA comes from the solution of the penalized minimization problem

$$\min_{\beta \in \mathbb{R}^p} ||F - M\beta||_2 + \lambda ||\beta||_1$$

  where λ is the tuning parameter controlling sparsity
- Note that $F - M\beta$ might be read as a regression residual and thus the sparsity constraint corresponds to a LASSO-type shrinking of regression parameters
- Shen and Huang (2008) suggest to obtain the sparse loadings by solving an equivalent problem based on a regularized Singular Value Decomposition of M

Sparse PCA and Regularized SVD

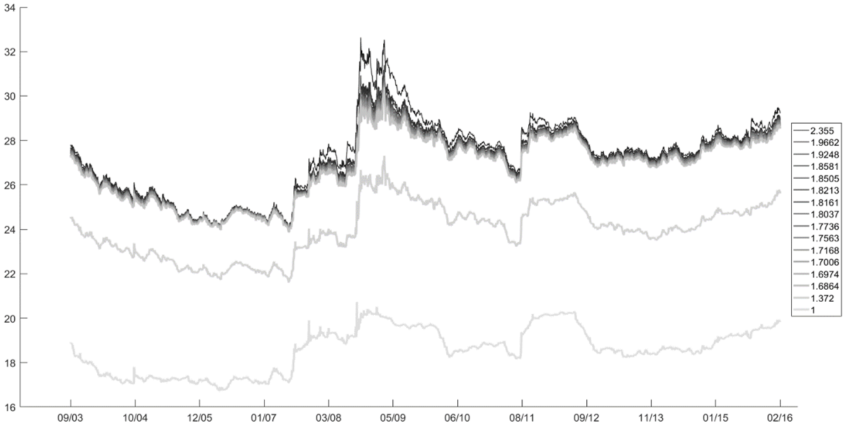
- SVD of M satisfies $M = UDV'$, with U and V left and right singular vectors, respectively, and D the diagonal matrix of singular values
- Let $\tilde{v}_l = d_l v_l$, i.e. the product of right singular vector and corresponding singular value
- The penalized minimization problem used (under a squared Frobenius norm) is

$$\min_{\tilde{v}_1, u_1 \in \mathbb{R}^p} ||M - u_1 \tilde{v}_1||_F^2 + \lambda ||\tilde{v}_1||_1$$

- Shen and Huang (2008) propose an iterative algorithm to solve the minimization problem and recover the loadings for a sparse PCA based on regularized SVD (sPCA-rSVD)
- Advantages of sPCA-rSVD over standard sPCA: computational (not relevant for us), more precise in identifying null loadings i.e. to identify the really relevant variables

Sparse PCA and Regularized SVD

- By varying the penalization parameter λ we recover alternative versions of the first PC (PC1 from sPCA-rSVD) with different *active* systemic risk indicators (choose λ to have 1, 2,. . . active indicators)



Sparse PCA and Regularized SVD

- Common patterns (they are all PC1) but varying composition
- Focus on a subset of the PC1, the table report the PC1 loadings (orthonormal) to systemic risk indicators

k	1	2	3	4	5	6	7
SI	1.00	.86	.71	.71	.71	.71	.71
HHI	.00	.51	.58	.58	.58	.58	.58
AR	.00	.00	.39	.39	.39	.40	.40
DYS	.00	.00	.00	.01	.01	.02	.03
DCI	.00	.00	.00	.00	.01	.01	.02
CoVaR	.00	.00	.00	.00	.00	.01	.01
TS	.00	.00	.00	.00	.00	.00	.01

where SI=Spillover Index, HHI=Herfindahl-Hirschman Index, AR=Absorption Ratio, DYS=Default Yield Spread, DCI=Dynamical Causality Index, TS=Term Spread

Sparse PCA and Regularized SVD

- Three most relevant systemic risk indicators, all of them are *global* indicators and they monitor the system overall spillover (SI) focusing on the diffusion of shocks/contagion, the system concentration (HHI), a proxy of system fragility, and the market co-movement (AR)
- Two open questions: 1) How to choose the optimal value of the penalization parameter? 2) How is the systemic risk indicator selection changing across different forms of the penalization function?
- For the second question, we can easily replace LASSO penalization with Ridge (L_2-norm) or Elastic-net (combination of L_1-norm and L_2-norm) and perform comparative analyses
- For the first question, we might resort to the linear regression representation of the problem and adopt information criteria or (better) use the association between the ISRM and its link to the real side of the economy

Contrasting alternative ISRM

- For the latter, we first need to identify a proxy of the economic activity; we select the IPI and we thus need to aggregate ISRM to a monthly frequency
- We choose to aggregate by taking the ISRM monthly average (*in progress - robustness checks: median, end of month value*)
- Giglio et al. (2016) considered the impact of an aggregate systemic risk index on the quantiles of IPI but we take a different perspective and focus on causality
- To evaluate the link between the ISRM and the economic activity we thus first use a classical Granger causality approach

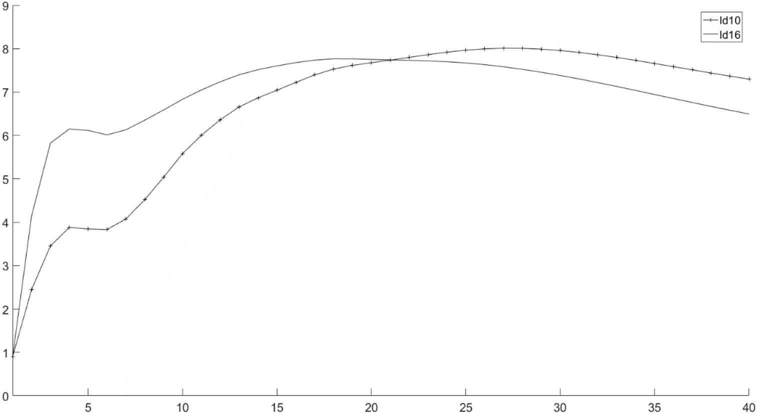
Contrasting alternative ISRM

- However, given that the systemic risk is a highly non-linear phenomenon and thus an approach consistent with those non-linearity should be adopted
- Therefore, we also consider the non-linear Granger causality test proposed by Diks and Panchenko (2006) Diks
- Given the focus of Giglio et al. (2016) on quantiles, we take into account a test for comovements in the tails of the distributions
- In fact, we are interest in the causation from large positive movements in the ISRM (an increase in risk) and large negative movements of IPI
- To that purpose we adopt the approach of Hong et al. (2009) for causality in distributions' tails Hong, the quantile causality test of Jeong et al. (2012) Jeong and the relation between ISRM and IPI within a non-parametric quantile regression approach NPQ

Contrasting alternative ISRM

- Results for the Hong et al. (2009) test *(other approaches in progress)*
- The test depends on a bandwidth that drives results
- Larger bandwidths give more weights to close lags, i.e. observations close in time contribute in a larger way to the construction of the test statistics (a reasonable hypothesis)
- We compare the ISRM with 10 components (10 systemic risk indicators) with the ISRM based on 16 componets (all indicators); the latter corresponds to classical PCA
- In all cases we reject the null of no causation but the maximum values of the test statistics change across bandwidth values and is maximum for the ISDM(10)

Contrasting alternative ISRM



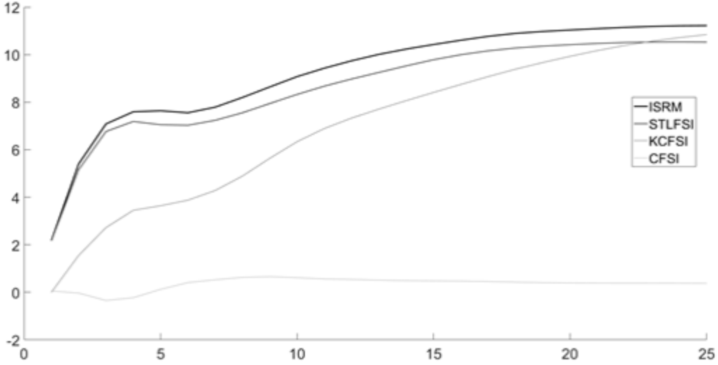
Robustness checks

- ISRM(10) composition across competing Sparse PCA approaches - orthonormal weights

	rSVD	LASSO	Ridge	EL.Net
SI	0.703	0.704	0.704	0.704
HHI	0.583	0.551	0.551	0.551
AR	0.403	0.384	0.384	0.384
DYS	0.036	0.034	0.034	0.034
DCI	0.027	0.000	0.000	0.000
CoVaR	0.025	0.142	0.142	0.142
VaR	0.000	0.157	0.156	0.156
Hong(10)	5.58	4.43	4.43	4.43
Hong(25)	7.97	4.92	4.92	4.92

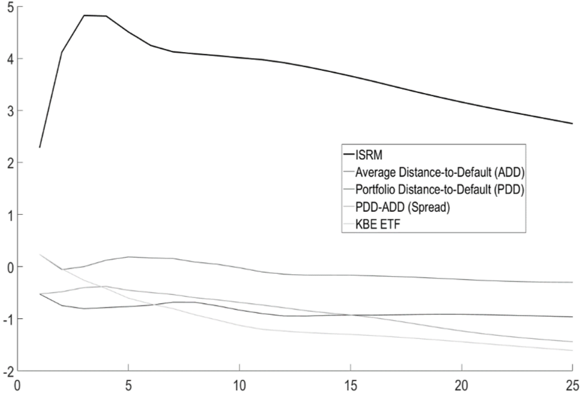
Robustness checks

- Hong et al. (2009) test statistic for causality for ISRM and other financial stress indexes (from the FED of St. Louis, Kansas City, Cleveland)



Robustness checks

- Hong et al. (2009) test statistic for causality for ISRM and other risk indicators (from Cleveland FED and ETF for Banking sector)



Robustness checks

- **...in progress...**
- Different aggregation approaches for individual risk measures...
- Different aggregation approaches for ISRM from daily to monthly...
- Further causality testing tools from ISRM (quantiles) to real activity (quantiles)
- Sub-sample analyses...

Concluding remarks

- Through the analysis of various Systemic Risk Measures computed for different firms, we have been able to build an Index of Systemic Risk Measures (ISRM) which, per construction, encompasses the main common information in the various Systemic Risk Measures
- The ISRM is, among the various possibilities associated with Sparse PCA techniques, the best index according to a number of criteria, namely the causation of economic activity in the tails and (preliminary finding) the predicting power for economic downturn
- Several analyses still in progress to complete the work

Concluding remarks

Thanks!

Appendix

Spillover Index

- SI proposed by Diebold and Yilmaz (2009) uses a forecast error variance decomposition to monitor system contagion
- Starting from a VAR of order p for N variables to forecast H periods ahead as follow SI equals:

$$SI_t = \frac{\sum_{h=0}^{H-1} \sum_{\substack{i,j=1 \\ i \neq j}}^{N} a_{h,ij}^2}{\sum_{h=0}^{H-1} trace(A_h A_h')} \times 100,$$

  with A_h (with elements $a_{h,ij}$) is the FEVD at lag h
- The numerator represents the total spillover in the system whilethe denominator corresponds to the total variance of the forecast error
- In our study, we use a $VAR(2)$ for a horizon of $H = 10$. [RiskList]

Herfindahl-Hirschman Index

- *HHI* is an index quantifying the concentration in the system
- It captures the potential fragility of the system from its concentration and the threat of the defaults of the largest companies
- It is defined as the sum of the squared market values out of the squared sum of these same market values such as:

$$HHI_t = N \frac{\sum_{i=1}^{N} (ME_{i,t})^2}{(\sum_{i=1}^{N} ME_{i,t})^2},$$

with $ME_{i,t}$ the market value of the institution i at time t and N the number of financial institutions [RiskList]

Value-at-Risk

- *VaR* is the Value-at-Risk of the system or for a market index
- It is the maximal potential loss for a given probability on a time horizon,

$$P(r_{i,t} \leq VaR_{i,t}(\alpha)) = \alpha,$$

with $r_{i,t}$ returns of the institution i at time t for a given risk level α (fixed at 5%) [RiskList]

Absorption Ratio

- AR was proposed by Kritzman et al. (2011) and it measures the tendency of the markets to co-move in the same way:

$$AR_t = \frac{\sum_{j=1}^{J} \sigma^2_{E_{j,t}}}{\sum_{i=1}^{N} \sigma^2_{a_i,t}},$$

  with J the number of eigen vectors, $\sigma^2_{E_{j,t}}$ the variance of the eigen vector j and $\sigma^2_{a_i,t}$ the variance of the asset i at time t
- The eigen values and vectors are obtained from the sample covariance matrix of the N asset returns at time t estimated on rolling one-year periods
- Only the J largest eigen values (we use 20% of the number of assets returns series) are summed up to get the numerator while the denominator is the *trace* of the sample covariance matrix. [RiskList]

CoVaR and ΔCoVaR

- *CoVaR* corresponds to the *VaR* of the system conditional on institutions being under distress (Adrian and Brunnermeier, 2016):

$$P(r_{m,t} \leq CoVaR_{i,t}(\alpha) | r_{i,t}(\alpha)) = \alpha,$$

- $\Delta CoVaR$ proposed also by Adrian and Brunnermeier (2016) is the difference between the *CoVaR* of the institution i at a given risk level $\alpha = 5\%$ and the *CoVaR* of the same institution but at $\alpha = 50\%$ (median state).

$$\Delta CoVaR_{i,t}(\alpha) = CoVaR_{i,t}(\alpha) - CoVaR_{i,t}(0.5),$$

[RiskList]

Marginal Expected Shortfall

- *MES* was proposed by Acharya et al. (2012) and is defined as the conditional mean returns of the institution i when the market, as a whole, is in distress,

$$MES_{i,t} = (\alpha = E(r_{i,t}|r_{m,t} \leq VaR_{m,t}(\alpha))) \, ,$$

  with $r_{i,t}$, $r_{m,t}$ are the returns of the institution i and the returns of the market and $VaR_{m,t}(\alpha)$ is the VaR of the market portfolio for a given risk level α, at time t
- The *MES* is equal to the partial derivative of the Expected Shortfall (*ES*) of the market portfolio with respect to the weights of the institution i, and then measures its marginal systemic risk contribution
- We compute it according to Brownlees and Engle (2017) based on a DCC-MVGARCH(1,1,1,1) model. [RiskList]

Term Spread

- It measures the slope of the yield curve and which corresponds to the yield spread between 10-year and 3-month Treasury bills
- This variable is a leading indicator of economic activity (Estrella and Trubin, 2006); [RiskList]

Default Yield Spread

- It represents the difference between the yield of corporate bonds rated BAA and the ones rated AAA by Moody's
- Chen et al. (2009) show that this variable is an aggregated measure of the robust credit risk to frictions (tax and liquidity) on the bond market [RiskList]

Component Expected Shortfall

- *CES* was proposed by Banulescu and Dumitrescu (2015) and quantifies the contribution of an institution to the risk of the system by multiplying the $MES_{i,t}$ of this institution at time t by its weight in the system such as:

$$CES_{i,t}(\alpha) = -w_{i,t} MES_{i,t} \left(\alpha = E(r_{i,t} | r_{m,t} \leq VaR_{m,t}(\alpha)\right)),$$

  with $r_{i,t}$, $r_{m,t}$ are the returns of the institution i and the returns of the market and $VaR_{m,t}(\alpha)$ is the *VaR* of the market portfolio for a given risk level α, at time t
- The weight of the institution i denoted $w_{i,t}$ is simply its market value divided by the total market value of the system.
- We compute it according to Brownlees and Engle (2017) based on a DCC-MVGARCH model [RiskList]

Volatility

- *Vol* is the aggregated volatility of all the financial institutions in the system or simply the volatility of a market index. It is defined as the standard deviation of a one year period of opening days. [RiskList]

Dynamic Causality Index

- It measures the degree of interconnection in the system as the number of significant Granger causalities divided by the total number of Granger causalities (Billio et al., 2012):

$$DCI_t = \frac{\#GC_t^*}{\#GC_t},$$

with $\#GC_t^*$ the number of significant Granger causalities and $\#GC_t$ the total number of Granger causalities at time t between the asset returns (only considering them with no factor augmentation), estimated. The maximum lag considered corresponds to a one-year rolling window. [RiskList]

SRISK

- It corresponds to the amount of capital needed by a firm in distress when the market is also in distress (Acharya et al., 2012 and Brownlees and Engle, 2017),

$$SRISK_{i,t}(1-\alpha) = \max\{0, \gamma D_{i,t} - (1-\gamma) W_{i,t}[1 - LRMES_{i,t}(1-\alpha)]\},$$

  with γ the prudential capital requirement required by the regulator, $D_{i,t}$ the amount of debt and $W_{i,t}$ the amount of liabilities of the institution i at time t

- $LRMES_{i,t}(1-\alpha)$ is the long run approximation (six months) of the $MES_{i,t}(1-\alpha)$ of the institution i at time t and is defined such as:

$$LRMES_{i,t}(1-\alpha) \approx 1 - \exp[18 \times MES_{i,t}(1-\alpha)].$$

[RiskList]

Turbulence Index

- TI reflects the excess volatility and compares the squared realized returns to their historical volatility (Kritzman and Li, 2010),

$$TI_t = (r_t - \mu)'\Sigma^{-1}(r_t - \mu),$$

with r_t the vector of the returns, $\mu = E(r)$ the historical mean returns and $\Sigma = E[(r-\mu)^2]$ the sample covariance matrix estimated over a one-year rolling window of the returns [RiskList]

TED Spread

- It represents the difference between the LIBOR three-month rate and sovereign interest rates to three months
- An increase of this variable is the sign that lenders expect an increase in credit risk in the interbank lending market. [RiskList]

Illiquidity Measure

- *AIM* was proposed by Amihud (2002) and captures the illiquidity level of the trades on a given asset,

$$AIM_{i,t} = \frac{1}{K} \sum_{\tau=t-K}^{t} \frac{|r_{i,\tau}|}{VOLD_{i,\tau}},$$

  where $|r_{i,\tau}|$ is the absolute return of the institution i and $VOLD_{i,\tau}$ the daily volume in dollars of the same asset i at time τ, on a given period from $t - K$ to t
- $VOLD_{i,\tau}$ represents all trade prices multiplied by the number of shares relating to each price; it corresponds to the daily volume in dollar at time τ [RiskList]

Financial Institutions

- Banks: Bank of America, BB&T, Bank of New York Mellon, Citigroup, Comerica Inc, Huntington Bancshares, Hong Kong & Shanghai Banking Corporation, JP Morgan Chase, Keycorp, M & T Bank Corp, Northern Trust, New York Community Bancorp, Peoples United Financial, PNC Financial Services, Regions Financial, Synovus Financial, Suntrust Banks, State Street, US Bancorp, Wells Fargo & Co, Zion
- Insurers: Aflac, American International Group, Allstate Corp, Aon Corp, Berkshire Hathaway, Chubb Corp, CIGNA Corp, Cincinnati Financial Corp, CNA Financial corp, Hartford Financial Group, Health Net, Humana, Lincoln National, MBIA, Marsh & McLennan, Progressive, Torchmark, Travelers, Unitedhealth Group, Unum Group
- Brokers-Dealers: E-Trade Financial, Goldman Sachs, Morgan Stanley, Schwab Charles, T. Rowe Price
- Others: American Capital, TD Ameritrade, American Express, Franklin Resources, Blackrock, Capital One Financial, Eaton Vance, Fifth Third Bancorp, Fannie Mae, Freddie Mac, H&R Block, Legg Mason, SEI Investments Company, SLM Corp

FinInst

Causality test of Jeong et al. (2012)

Let us define $\{y_t\}_{t\in T}$ and $\{y_t\}_{t\in T}$ the two series returns, and denote $Y_{t-1} \equiv (y_{t-1}, \dots, y_{t-p}), X_{t-1} \equiv (x_{t-1}, \dots, x_{t-p})$ and $Z_{t-1} \equiv (z_{t-1}, \dots, z_{t-p})$, with lags p and q being greater than one. The distributions of y_t conditional on Z_{t-1} and X_{t-1} are defined as $F_{y_t|Z_{t-1}}(y_t|Z_{t-1})$ and $F_{y_t|X_{t-1}}(y_t|X_{t-1})$, respectively. For $\tau \in (0,1)$, the τ-th quantile of y_t conditional on Z_{t-1} and on Y_{t-1} is $Q_\tau(Z_{t-1}) \equiv Q_\tau(y_t|Z)_{t-1}))$ and $Q_\tau(Y_(t-1)) \equiv Q_\tau((y_t|Y_{t-1}),$ respectively. Following Jeong et al. (2012), we can say that x_t does not cause y_t in its τ-th quantile if $Q_\tau(Z_{t-1}) \neq Q_\tau(Y_{t-1})$. Therefore, the system of hypotheses to be tested is

$$\begin{cases} H_0 : P[F_{y_t|Z_{t-1}}(Q_\tau(Y_{t-1})|Z_{t-1}) = \tau] = 1, \\ H_0 : P[F_{y_t|Z_{t-1}}(Q_\tau(Y_{t-1})|Z_{t-1}) = \tau] < 1. \end{cases}$$

Causality test of Jeong et al. (2012)/2

The test statistic proposed by Jeong et al. (2012) is equal to

$$\hat{J}_T = \frac{1}{T(T-1)h^m} \sum_{t=1}^{T} \sum K\left(\frac{Z_{t-1} - Z_{t-s}}{h}\right) \widetilde{\varepsilon}_t \widetilde{\varepsilon}_s,$$

where $m = p + q$ and $K(\cdot)$ is the kernel function with bandwidth h and $\widetilde{\varepsilon}_t = \mathbf{1}_{\{y_t \leq \widetilde{Q}_\tau(Y_{y-1})\}}^{-\tau}$. Back

Hong et al. (2009)

Let us note $y_{i,t}$ be a return series or changes in variable, and $Q_{i,t}(\alpha;\theta_i)$ the quantile at the order α of the distribution of $y_{i,t}$, with θ_i a vector of parameters associated with the specification of the dynamic of $y_{i,t}$ for $i = 1, 2$. $Hit_{i,t}(\alpha;\theta_i)$ the dummy variable defined as:

$$Hit_{i,t}(\alpha;\theta_i) = \begin{cases} 1 \text{ if } y_{i,t} \leq Q_{i,t}(\alpha,\theta_i), \\ 0 \text{ otherwise.} \end{cases}$$

The variable equals 1 when the return/change is extreme and negative.

Hong et al. (2009)/2

The null hypothesis testing in Hong et al. (2009) is:

$$E[Hit_{1,t}(\alpha;\theta_1)|\Omega_{t-1}] = E[Hit_{1,t}(\alpha;\theta_1)|\Omega_{1,t-1}]$$

wherein the information sets on the date $t-1$ are defined respectively by:

$$\begin{cases} \Omega_{t-1} = \{(y_{1,s}, y_{2,s}), s \leq t-1\}, \\ \Omega_{1,t-1} = \{y_{1,s} \leq t-1\}. \end{cases}$$

Hong et al. (2009)/3

The test statistic proposed by the authors depends on a weighted sum of the estimated correlations between $Hit_{1,t}(\alpha, \hat{\theta}_1)$ and $Hit_{2,t}(\alpha, \hat{\theta}_2)$ where $\hat{\theta}_1$ and $\hat{\theta}_2$ are consistent estimators of θ_1 and θ_2. This weighted sum is defined by:

$$Z = T \sum_{j=1}^{T-1} \kappa^2(j/d)\hat{\rho}(j),$$

with the function $\kappa(\cdot)$ being a decreasing kernel, d the truncation parameter and $\hat{\rho}(j)$ the cross-correlation of order j between $Hit_{1,t}(\alpha, \hat{\theta}_1)$ and $Hit_{2,t}(\alpha, \hat{\theta}_2)$, that equals to:

$$\hat{\rho}(j) = \frac{\hat{\gamma}(j)}{\hat{s}_1 \hat{s}_2},$$

where $\hat{s}_1$ and $\hat{s}_2$ refer to the standard deviation of $Hit_{1,t}(\alpha, \hat{\theta}_1)$ and $Hit_{2,t}(\alpha, \hat{\theta}_2)$ respectively, and $\hat{\gamma}(j)$ the cross-covariance of order j.

Hong et al. (2009)/4

Under the null hypothesis of no causality in extreme movements, Hong et al. (2009) demonstrate that:

$$U = \frac{Z - C_T(d)}{[D_T(d)]^{1/2}},$$

follows a standard normal distribution, with zero mean and unit variance, where:

$$C_T(d) = \sum_{j=1}^{T-1} (1 - j/T)\kappa^2(j/d),$$

and:

$$D_T(d) = 2\sum_{j=1}^{T-1} (1 - j/T)(1 - (j+1)/T)\kappa^4(j/d).$$

Back

Diks and Panchenko (2006)

Suppose we want to infer about the causality between two variables X and Y using q and p lags of those variables, respectively. Consider the vectors $X_t^q = (X_{t-q+1}, \ldots, X_t)$ and $Y_t^p = (Y_{t-p+1}, \ldots, Y_t)$, with $q, p \geq 1$. The null hypothesis that X_t^q does not contain any additional information about Y_{t+1} is corresponds to:

$$H_0 = Y_{t+1} | (X_t^q ; Y_t^q) \sim Y_{t+1} | Y_t^p .$$

The null hypothesis is equivalent to a statement on the invariance of the distribution of the vector of random variables $W_t(X_t^q; Y_t^p, Z_t)$ where $Z_t = Y_{t+1}$. If we drop the time indexes, the joint probability density function $f_{X,Y,Z}(x, y, z)$ and its marginals must satisfy the following relationship:

$$f_{X,Y,Z}(x, y, z) f_Y(y)^{-1} = f_{X,Y}(X, Y) f_Y(y)^{-1} f_{Y,Z}(y, z) f_Y(y)^{-1},$$

for each vector (x,y,z) in the support of (X,Y,Z).

Diks and Panchenko (2006)/2

Diks and Panchenko (2006) show that, for a proper choice of weight function, $g(x, y, z) = f_Y^2(y)$, the previous relationship is equivalent to:

$$q = E[f_{X,Y,Z}(X, Y, Z) f_Y(Y) - f_{X,Y}(X, Y) f_{Y,Z}(Y, Z)].$$

And they propose the following estimator $\hat{q}$ such as:

$$\hat{q} = (n-1)[n(n-2)]^{-1} \sum_i [\hat{f}_{X,Y,Z}(X_i, Y_i, Z_i) \hat{f}_Y(Y_i) - \hat{f}_{X,Y}(X_i, Y_i) \hat{f}_{Y,Z}(Y_i, Z_i)],$$

where n is the sample size. This estimator is computed using a local density estimator defined such as:

$$\hat{f}_W(W_i) = \frac{(2\varepsilon)^{-d_W}}{n-1} \sum_{j, j \neq i} 1_{i,j}^W.$$

where $\hat{f}_W(\cdot)$ is a local density estimator of a d_W-variate random vector W at W_i based on indicator functions $1_{i,j}^W = (|W_i - W_j| < \varepsilon)$.

Diks and Panchenko (2006)/3

In the case of bivariate causality, the test is consistent if the bandwidth ε is given by $\varepsilon_n = Cn^{-\beta}$, for any positive constant C and $\beta \in (1/4, 1/3)$. The test statistic is asymptotically normally distributed in the absence of dependence of the vectors W_i. For the choice of the bandwidth, Diks and Panchenko (2006) suggest $\varepsilon_n = \max(C_n^{-2/7}, 1.5)$, were C can be calculated based on the ARCH coefficient of the series. Back