Selecting the Cointegration Rank with the Dickey-Fuller Test

by Antonio Aznar
University of Zaragoza

Contents

1.Introduction

- In this paper, we propose a procedure to test the cointegration rank of a multivariate dynamic system. In their seminal paper, Engel and Granger (1987) used the Dickey-Fuller test to determine the existence of cointegration. But they restrict their approach to only one cointegration relation.

1.Introduction

- The most well-known procedure for the general setting has been that proposed by Johansen (1988, 1991 and 1995) based on maximum-likelihood inference on vector autoregressive (VAR) error correction models. The identification of the model is purely statistic without any reference to restrictions with an economic sense. These identification restrictions are $\beta' S_{11}\beta = I$

2.The Recursive model

- If we have n variables, we can define $\binom{n}{2}+\binom{n}{3}+....\binom{n}{n-1}+1$ different relations between these n variables. For example, with 3 variables we have $\binom{3}{2}+1=4$ different relations, , where (y_{2t}, y_{1t}) is the regression of y_{2t} on y_{1t} ,$(y_{3t}, y_{1t}y_{2t})$ is the regression of y_{3t} on $(y_{1t}y_{2t})$ and so on;

2.The Recursive model

- Assuming four I(1) variables with no deterministic terms the recursive model is

$$\Delta y_{1t} = v_{1t}$$
$$y_{2t} = \beta_{21} y_{1t} + v_{2t}$$
$$y_{3t} = \beta_{31} y_{1t} + \beta_{32} y_{2t} + v_{3t}$$
$$y_{4t} = \beta_{41} y_{1t} + \beta_{42} y_{2t} + \beta_{43} y_{2t} + v_{3t}$$

- Where the $v's$ are stationary

2.The Recursive model

Consider the case with 4 variables

(2,1)

(3,1) (3,12)

(3,2)

(4,1) (4,12) (4,123)

(4,2) (4,13)

(4,3) (4,23)

3. Cointegration Test

- Point of Departure: (n-1) DF values of the recursive model.
- S1) The null r=0 is rejected if there exists a DF value such that

$$DFj<=CVj(S1)$$

The CV(S1) calculated assuming no cointegration

- S2)The null r=1 is rejected if there exists a DF value, different from that used in S1, that satisfies

$$DFj<=CV(S2)$$

3. Cointegration Test

The CV(S2)’s are calculated assuming that, under the null hypothesis, the cointegration relation is the relation with the smaller DF.

- S3) The null r=2 is rejected when there exists a DF value, different from the two first, satisfies

  DFj<=CVj(S3)

The CV(S3)’s are calculated assuming that the two cointegrated relations are those corresponding to the two previous DF values.

3. Cointegration Test

- In the last step the null r=n-2 is rejected when the remaining DF value satisfies

  DFj<=CVj(S(n-1))

The CV's are calculated assuming that the n-2 cointegration relations are those corresponding to the n-2 previous DF values.

The ilustration in the Appendix will make more clear the process. We obtain a simulation of a system with four variables no deterministic terms and different cointegration ranks.

4. Critical Points

- We have simulated a system with four variables:
- Three Sample Sizes, T=100,200,500.
- Deterministic Terms, Two cases: No deterministic terms and non-cointegrated time trends.

4.Critical Points

Case 1. No determinis. $\alpha = 5\% \quad T = 100$

α	Cointegra.	CV2(S1)	CV3(S1)	CV4(S1)
5%		-3.11	-3.57	-3.92
		CV2(S2)	CV3(S2)	CV4(S2)
	(2,1)	-4.68	-2.99	-3.30
5%	(3,12)	-3.11	-4.78	-3.40
	(4,123)	-3.11	-3.57	-5.10
		CV2(S3)	CV3(S3)	CV4(S3)
	(2,1)(3,12)	-4.68	-4.59	-3
5%	(2,1)(4,123)	-4.68	-2.99	-4.99
	(3,12)(4,123)	-3.11	-4.78	-5

4. Critical Points
Case 1: Non-cointegrated time trends.

$$\Delta y_{1t} = \delta_{11} + v_{1t}$$
$$y_{2t} = \delta_2 + \beta_{21} y_{1t} + v_{2t}$$
$$y_{3t} = \delta_3 + \beta_{31} y_{1t} + \beta_{32} y_{2t} + v_{3t}$$
$$y_{4t} = \delta_4 + \beta_{41} y_{1t} + \beta_{42} y_{2t} + \beta_{43} y_{2t} + v_{3t}$$
$$If\ \ no\ co\mathrm{int}egration \quad \Delta y_{it} = \delta_{ii} + v_{it},\ i = 2,3,4$$
$$with \quad \delta_{11} = \delta_{22} = \delta_{33} = \delta_{44} = 1 \quad and \quad \delta_2 = \delta_3 = \delta_4 = .5$$

4. Critical Points
Case 2: Non-cointegrated time trends. T=100

α	Cointegra.	CV2(S1)	CV3(S1)	CV4(S1)
5%		-3.61	-3.94	-4.25
		CV2(S2)	CV3(S2)	CV4(S2)
	(2,1)	-4.9	-3.37	-3.77
5%	(3,12)	-3.61	-5.07	-3.74
	(4,123)	-3.61	-3.94	-5.44
		CV2(S3)	CV3(S3)	CV4(S3)
	(2,1)(3,12)	-4.90	-3.37	-3.77
5%	(2,1)(4,123)	-4.90	-3.37	-5.35
	(3,12)(4,123)	-3.62	-5.10	-5.28

5.Empirical size and Power

- To evaluate the power of the process described in Sections 3 and 4 we adopt the same Data Generating Process commented in the previous section for the case with no deterministic terms. The results are presented in Tables 5.1a, 5.1b and 5.1c. The structure of these tables is as follows.

Table 5.1a. Empirical size and power. n=4, T=100(500).

Cointegration	P1	P2	P3
r=0	.049 (.048)		
(2,1)	.36 (.99)	.048 (.048)	
(2,1)(3,12)		.66 (.99)	.05 (.05)
(2,1)(3,12)(4,123)			.70 (.99)

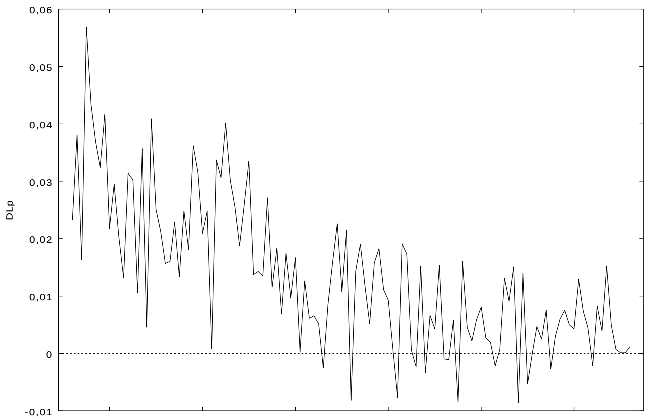
7. Empirical Illustration

- The data set is that used in Juselius (2006) except the short term interest rate. It is Danish quarterly data from 1973:1 to 2003:1. The data vector is $y_t = \left(Lyr_t, Lm3r_t, Dlp_t, Rb_t\right)'$, where Lyr_t is the log of the real Gross National Product, $Lm3r_t$ is the log of real M3, Dlp_t is the rate of inflation and Rb_t is the long-term government bond rate.

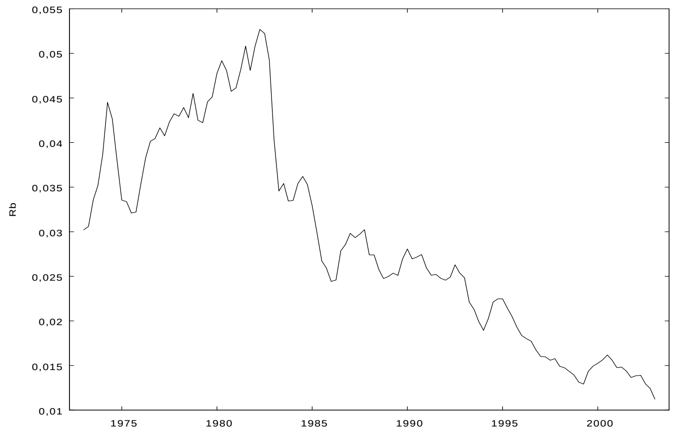
7. Empirical Illustration

Lyr

Lm3r



DLp



Rb

7. Empirical Application

Variable	DF
Lyr	-2.72
Lm3r	-2.53
DLp	-3.72
Rb	-2.42

$$Rb_t \rightarrow Lyr_t \rightarrow Lm3r_t \rightarrow DLp_t$$

7. Empirical Application

- The three DF values are

$$DF2=-2.44,\ DF3=-2.79,\ DF4=-4.74$$

The null hypothesis of no cointegration is rejected when there exists a DF value smaller than the corresponding critical value that can be seen in Table 4.2a as CVj(S1). Since DF4< - 4.25 the null of no cointegration is rejected. Since the value of DF4 is the smallest we assume that under r=1, (4,123) is the cointegrated relation so that in the second step we have to use the critical values corresponding to this relation. It is seen that neither of the two remaining DF values, DF2 and DF3, are smaller than the corresponding critical points, -3.61 and -3.94 . So, the null hypothesis that r=1 is not rejected and we conclude saying that the cointegration rank is one.

7.Empirical Application

- Johansen’s Approach

Rank	Eigenvalues	Test Statistic(p value)
0	0.369	84.18(0.0003)
1	0.118	29.30(0.54)
2	0.077	14.25(0.64)
3	0.038	4.64(0.65)

Appendix. Illustration

- Consider the following simulated results. T=200. No deterministic terms.

DF2	DF3	DF4
	No cointegration	
-1.95(-3.2)	-2.06(-3.57)	-2.42(-3.91)
	r=1 (2,1)	
-5.71(-3.2)	-2.04(-2.99)	-1.73(-3.32)
	r=2 (2,1),(3,12)	
-5.71(-3.2)	-4.23(-2.99)	-1.61(-3.1)
	r=3 (2,1)(3,12)(4,123)	
-5.71(-3.2)	-4.23(-2.99)	-5.23(-3.32)

Each case should be analyzed independently from the others.