IS SPURIOUS REGRESSION AN ISSUE FOR TWO INDEPENDENT STATIONARY AR(1) PROCESSES?

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Abstract

When time series data is used in econometrics serially correlated errors are most likely to appear. Autocorrelation will also be detected in regression analysis as an indication of a false specification between two variables. This study examines the problem of serially correlated errors in the context of spurious regression showing evidence of removing the presence of this phenomenon both theoretically as well as empirically by applying the Cochrane-Orcutt procedure.

Keywords: Spurious Regression, serially correlated errors, Cochrane-Orcutt procedure, stationary AR(1) processes

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1. Introduction

The pioneer work of Granger and Newbold (1974) was simply the introduction of a new concept in regression analysis, known as spurious regression, although this phenomenon was initially presented by Yule (1926). Using a Monte Carlo analysis, Granger and Newbold (1974) showed that the regression of two independent random walk processes without drift will produce strong evidence of a linear relationship. This unique result was left to Phillips (1986) to prove it mathematically and it was extended to two independent stationary processes by Granger *et al.* (2001).

In addition, Granger and Newbold (1974) pointed out that along with the large *t*-values strong evidence of serially correlated errors will appear in regression analysis, stating that, when a low value of the Durbin-Watson statistic is combined with a high value of the coefficient of determination, the relationship is not true. This result is examined by Marmol (1995) who generalized the work of Phillips (1986) and showed that the Durbin-Watson statistic will converge in probability to zero. Moreover, the presence of serially correlated errors in the context of spurious regression was also investigated by Newbold and Davies (1978), for non-stationary moving average processes, by Agiakloglou (2009), for two independent stationary AR(1) processes, and by Agiakloglou *et al*. (2015), for two independent stationary spatial autoregressive SAR(1) processes. Actually, one can get the same misleading statistical results regarding the existence of a linear relationship between two independent variables by simply applying the test associated to the correlation coefficient of these two variables, as Agiakloglou and Tsimpanos (2012) have showed for two independent stationary AR(1) processes. However, in this case, the analyst will have no evidence of false specification, such as the case of serially correlated errors in regression analysis.

The prevailing low values of the Durbin-Watson statistic, in the context of spurious regression, gave the incentive to several analysts to examine even further this phenomenon searching for solutions. Granger *et al.* (2001) proposed a new method, the so called BART method, which improved the statistical behavior of this phenomenon reaching asymptotically the right nominal levels, but for moderate and even for large sample sizes the spuriocity was not removed. Contrary, Agiakloglou (2013) was able to obtain better results for small and moderate sample sizes for two independent stationary AR(1) processes and for two non-stationary I(1) processes by estimating the original simple regression model either with a lagged dependent variable or in first differences respectively.

Therefore, it is still very interesting to further understand this phenomenon and to handle the presence of serially correlated errors in the context of spurious regression. For this purpose, the problem of autocorrelated errors is examined first theoretically, where the use of the correct variance does improve the statistical results and second, empirically, where the Cochrane-Orcutt procedure improves even better the performance of the test, despite the fact that Granger *et al.* (2001) have stated that Cochrane-Orcutt procedure will be inefficient to cure this problem compared to using a wider specification, a statement that was also found in Granger and Newbold (1974) for the case of two non-stationary I(1) processes. The findings of this research are astonishing showing that the Cochrane-Orcutt procedure does resolve the spurious phenomenon for two independent stationary AR(1) processes.

2. Simulation results

Consider the following regression model:

$$Y_t = \alpha + \beta X_t + \varepsilon_t \tag{1}$$

where the error term ε_t is assumed to be *iid* normally distributed with mean zero and constant variance σ_ε^2, i.e., $\varepsilon_t \sim iid\mathrm{N}(0, \sigma_\varepsilon^2)$ and the variables Y_t and X_t are generated by the following DGP:

$$Y_t = \varphi Y_{t-1} + \varepsilon_{yt} \tag{2}$$

$$X_t = \varphi X_{t-1} + \varepsilon_{xt} \tag{3}$$

where the errors ε_{yt} and ε_{xt} are white noise processes independent of each other and the autoregressive parameter is allowed to take values of 0.0, 0.2, 0.5, 0.8, 0.9 and 1.0. Clearly, for all values of the autoregressive parameter less than one both processes are stationary first-order autoregressive processes, i.e., AR(1), whereas for $\varphi = 1$ both processes are non-stationary random walk processes without drift. For $\varphi = 0$ both processes are white noise processes.

The existence of a linear relationship between these two variables is investigated by testing for the significance of the coefficient of the independent variable and the test is executed using the following t statistic:

$$t = \frac{\hat{\beta}}{se(\hat{\beta})} \tag{4}$$

where $\hat{\beta}$ and $se(\hat{\beta})$ are the estimated coefficient of β and its standard error respectively, obtained through OLS estimation of model (1) for given sample size T. The t statistic follows a t distribution with (T - 2) degrees of freedom and the null hypothesis that $\beta = 0$ will be rejected if its absolute value is greater than the critical value, indicating statistical evidence of a linear relationship between the two variables.

Unfortunately, as it is known, false statistical conclusions will be drawn from time series data using independent non-stationary or stationary processes. For the case of two independent non-stationary processes, i.e., for random walk processes without drift, the null hypothesis will be rejected not only too often but also the number of rejections is strongly affected by the sample size, i.e., it increases as sample size increases, showing even stronger evidence of false specifications. For example, the null hypothesis is rejected 76% and 93% at the nominal 5% level for sample sizes of 100 and 1,000 observations respectively, based on 10,000 replications, reaching the level of 100% rejections for larger sample sizes, a result that it highly related to the distribution of the t statistic which does not convert to a standard normal distribution. In fact, the value of its standard deviation diverges from one and it increases as the ample size increases. For example, the standard deviation of the t statistic is equal to 7.294 for sample size of 100 observations and 23.83 for sample size of 1,000 observations, based on 10,000 replications, although its mean value remains close to zero. Phillips (1986) pointed out that the problem of this abnormal behavior of the t statistic arises from the variance of the estimated coefficient.

Equivalently, the null hypothesis will also be rejected very often for two independent stationary AR(1) processes, but in this case the number of rejections is only affected by the magnitude of the autoregressive parameter and not by the sample size, as Table 1 reports (see also Granger *et al.*, 2001, Agiakloglou & Tsimpanos, 2012 and Agiakloglou & Agiropoulos, 2016) and one will get more rejections as the value of the autoregressive parameter increases. For example, for values of φ = 0.5 and 0.9 the null hypothesis will be rejected 13% and 52% respectively at the 5% nominal level, regardless of sample size, a result that it is also related to the distribution of the t statistic which does not convert to a standard normal distribution, as Table 1 reports. In fact, as can be seen from this table, the value of the standard deviation of the t statistic is not equal to one for all values of the autoregressive parameter and it increases as the value of φ increases. For example, the standard deviation of the t statistic is equal to 1.288 and 3.049 for φ = 0.5 and 0.9 respectively, regardless of sample size. Apparently, as in the case of two independent random walk processes without drift (non-stationary processes), the behavior of the t statistic is strongly affected by the variance of the estimated coefficient, which is smaller than it should be, producing larger *t* values, so that the null hypothesis is rejected too often, as Granger *et al.* (2001) have indicated.

Table 1

Percentage of rejections of the null hypothesis that $\beta = 0$ at the nominal 5% level ($|t| > 1.96$) along with the standard deviation of the t statistic for two independent stationary AR(1) processes for all sample sizes based on 10,000 replications

	φ				
	0.0	**0.2**	**0.5**	**0.8**	**0.9**
% of rej.	5	6	13	35	52
St. dev.	1.003	1.046	1.288	2.138	3.049

Table 2

Mean values of the Durbin-Watson statistic based on 10,000 replications

	φ					
Sample Size	**0.0**	**0.2**	**0.5**	**0.8**	**0.9**	**1.0**
50	2.0014	1.6479	1.1237	0.6203	0.4693	0.3332
100	1.9981	1.6222	1.0613	0.5081	0.3216	0.1735
500	2.0003	1.6049	1.0123	0.4265	0.2240	0.0360
1000	1.9995	1.6021	1.0061	0.4102	0.2119	0.0183
10000	2.0009	1.6010	1.0009	0.4010	0.2010	0.0018

Moreover, Agiakloglou (2009) showed that along with the large number of rejections evidence of serially correlated errors will also appear in regression analysis in the context of spurious regression even for two independent stationary AR(1) processes. For this purpose, Table 2 reports the mean values of the Durbin-Watson statistic obtained through the estimation process of model (1), based on 10,000 replications, showing that their magnitude is not only affected by the magnitude of the autoregressive parameter but also by the sample size. In particular, the mean values of the Durbin-Watson statistic decrease as the value of the autoregressive parameter increases and/or as the sample size increases. It seems though that for given value of the autoregressive parameter the mean value of the Durbin-Watson statistic converges to some predetermined value, as Marmol (1995) has indicated that for two non-stationary processes the Durbin-Watson statistic will converge to zero. Indeed, the Durbin-Watson statistic (d) converges to 2, 1.6, 1.0, 0.4, 0.2 and 0 for φ equal to 0, 0.2, 0.5, 0.8, 0.9 and 1 respectively, values that can easily be obtained from $d = 2(1 - \rho)$, i.e., for $\varphi = 1$, $d = 0$, and for $\varphi = 0.9$, $d = 0.2$.

3. Dealing with autocorrelated errors for two independent stationary AR(1) processes

Let us consider now that the errors of model (1) are generated by the following autoregressive AR(1) process:

$$\varepsilon_t = \rho\varepsilon_{t-1} + u_t \tag{5}$$

where the absolute value of the autoregressive coefficient ρ is less than one, i.e., $|\rho| < 1$, and the error term u_t is considered to be *iid* normally distributed with mean zero and constant variance σ_u^2, i.e., $u_t \sim iid\mathrm{N}(0, \sigma_u^2)$.

Estimation of model (1) with OLS, under the assumption of serially correlated errors, given by model (5), will produce the same estimate of the β coefficient, which will still be unbiased, but with different variance. Specifically, the variance of the new estimator $\tilde{\beta}$ will be obtained as:

$$Var(\tilde{\beta}) = \frac{\sigma_\varepsilon^2}{\sum(X_t - \bar{X})^2}\left[1 + 2\rho\frac{\sum(X_t - \bar{X})(X_{t-1} - \bar{X})}{\sum(X_t - \bar{X})^2} + 2\rho^2\frac{\sum(X_t - \bar{X})(X_{t-2} - \bar{X})}{\sum(X_t - \bar{X})^2} + \cdots\right]$$

or equivalently as:

$$Var(\tilde{\beta}) = Var(\hat{\beta})[1 + 2\rho r + 2\rho^2 r^2 + \cdots] \quad (6)$$

where Var($\hat{\beta}$) is the OLS variance of $\hat{\beta}$ without the presence of serially correlated errors, i.e.,

$$Var(\hat{\beta}) = \frac{\sigma_\varepsilon^2}{\sum(X_t - \bar{X})^2}$$

r is the sample autocorrelation of the independent variable, knowing that X_t follows an AR(1) process, and ρ is the correlation coefficient of the error term. Hence, equation (6) can be written as:

$$Var(\tilde{\beta}) = Var(\hat{\beta})A \quad (7)$$

where *A* is a positive number defined as:

$$A = [1 + 2\rho r + 2\rho^2 r^2 + \cdots] \quad (8)$$

and its value can be obtained either as an approximation of the first two terms, i.e., as:

$$A_1 = 1 + 2\rho r$$

or as an infinite sum of a geometric process, i.e., as:

$$A_2 = 1 + 2\rho r(1 + \rho r + \rho^2 r^2 + \cdots) = 1 + 2\rho r \frac{1}{1 - \rho r} = \frac{1 + \rho r}{1 - \rho r}$$

Clearly, the value of A defines the ratio of the two variances, as can be seen from equation (7). For small values of ρ and r both approximations of A will take the same value, i.e., for $\rho = r = 0.2$, $A_1 = A_2 = 1.08$, whereas for large values of ρ and r the two approximations of A, A_1 and A_2, will take different values, i.e., for $\rho = r = 0.8$, $A_1 = 2.28$ and $A_2 = 4.56$. Hence, if the presence of serially correlated errors is ignored and the incorrect variance, Var($\hat{\beta}$), is used to calculate the relevant t statistic, instead of the correct variance, Var($\tilde{\beta}$), the values of the t statistic will be larger, resulting to more rejections of the null hypothesis, since the incorrect variance is smaller than the correct variance. In this case, the variance of the estimator is underestimated, the magnitude of which depends on the approximation of A that is used. For small values of ρ and r the variance will be underestimated equally at a small level, while for large values of ρ and r the underestimation will differ significantly. For example, for $\rho = r = 0.2$ the variance is underestimated 7.4%, whereas for $\rho = r = 0.8$ the variance is underestimated 56% using the A_1 approximation and 78% using the A_2 approximation. Thus, the use of the incorrect variance alters the distribution of the t statistic, which does not convert to a standard normal distribution, and therefore producing misleading statistical results.

As stated, spurious regression is related to serially correlated errors and therefore it makes sense to tackle this phenomenon as an autocorrelated errors problem. The objective in this case is to terminate the false behavior of the t statistic by importing the correct variance so that the test will have the right performance, knowing that the variance that was used was smaller than the correct variance. Thus, keeping the same simulation process alive, the relevant t statistic, for testing the null hypothesis that $\beta = 0$, is now calculated by replacing the OLS incorrect variance with the correct variance, obtained under autocorrelated errors and given by equation (7), using both approximations of A.[1] In other words, the following two t statistics are calculated:

$$t_1 = \frac{\hat{\beta}}{se(\hat{\beta})\sqrt{A_1}} \tag{9}$$

$$t_2 = \frac{\hat{\beta}}{se(\hat{\beta})\sqrt{A_2}} \tag{10}$$

and their standard deviations along with the percentage of rejections of the null hypothesis that $\beta = 0$, based on 10,000 replications, are presented on Table 3.

Perhaps, the most remarkable result of this table is that the performance of the test has significantly improved simply by correcting the variance of the estimator, regardless of the formula that is used. In particular, using the A_1 approximation of A, the relevant t_1 statistic has the same behavior as that of the classical t statistic, meaning that the number of rejections of the null hypothesis is only affected by the magnitude of the autoregressive parameter and not by the sample size, but the null hypothesis is rejected less frequently than using the classical t statistic for every value of the autoregressive parameter and sample size, as can be seen from Tables 3 and 1. For example, using the

[1] The values of A_1 and A_2 are calculated individually in every trial using the sample correlation coefficient r of X_t and the estimated value of ρ obtained by estimating model (5) using the residuals obtained from the OLS estimation of model (1). The whole simulation process is conducted in *R*.

classical t statistic the null hypothesis is rejected 52% and 35% for values of φ equal 0.9 and 0.8 respectively at the 5% nominal level, whereas now the null hypothesis is rejected 30% and 17.5% respectively, using the t_1 statistic. Thus, the problem of getting spurious results has decreased, but not removed, simply because the values that the A_1 approximation of A was taking were not large enough to significantly increase the variance of the estimator and, therefore, the distribution of the t_1 statistic did not convert to a standard normal distribution, as can be seen from the reported on Table 3 standard deviations.

Table 3

Percentage of rejections of the null hypothesis that $\beta = 0$ at the nominal 5% level ($|t| > 1.96$) using the t_1 and t_2 statistics along with their standard deviations for two independent stationary AR(1) processes for sample sizes of 50, 100, 500 and 1000 observations, based on 10,000 replications

			φ				
			0.0	**0.2**	**0.5**	**0.8**	**0.9**
% of rej.	**50**	t_1	5.8	5.9	7.9	18.1	28.4
		t_2	5.8	5.8	7.0	10.4	13.8
	100	t_1	5.6	6.0	7.5	17.7	30.0
		t_2	5.6	5.9	6.4	8.2	10.8
	500	t_1	5.1	5.4	6.9	17.5	31.4
		t_2	5.1	5.4	5.7	6.2	6.8
	1000	t_1	5.0	4.9	6.3	17.3	30.4
		t_2	5.0	4.8	5.0	5.2	5.9
st. dev.	**50**	t_1	1.027	1.038	1.115	1.504	1.907
		t_2	1.027	1.036	1.075	1.234	1.397
	100	t_1	1.025	1.031	1.093	1.465	1.940
		t_2	1.025	1.029	1.047	1.127	1.243
	500	t_1	1.007	1.013	1.073	1.441	1.944
		t_2	1.007	1.011	1.020	1.039	1.068
	1000	t_1	0.998	0.999	1.051	1.421	1.920
		t_2	0.998	0.998	0.998	1.015	1.032

On the other hand, using the A_2 approximation for A, the relevant t_2 statistic produced much better results, than the t_1 statistic, showing even evidence of convergence to the right nominal levels. Hence, the issue of getting spurious results for two independent stationary AR(1) processes can totally be removed asymptotically, as can be seen from Table 3, since for large sample sizes the empirical levels are very close to the nominal levels regardless of the value of the autoregressive parameter and the standard deviations of the t_2 statistic converge to 1. Actually, the performance of the t_2 statistic depends not only on the value of the autoregressive parameter but also on the sample size. For small and moderate sample sizes the percentage of rejections of the null hypothesis increases as the value of the autoregressive parameter increases. For example, for sample size of 100 observations the null hypothesis at the 5% nominal level is rejected 5.9%, 6.4%, 8.2% and 10.8% for values of $\varphi = 0.2, 0.5, 0.8$ and 0.9, respectively. For large sample sizes though the percentage of rejections of the null hypothesis decreases for all values of the autoregressive parameter, reaching happily to the nominal level. For example, for $\varphi = 0.9$ the null hypothesis is rejected 13.8%. 10.8%, 6.8% and 5.9% at the nominal 5% level for sample sizes of 50, 100, 500 and 1000 observations, respectively. It seems, therefore, that in this case the value of A_2 was large enough to alter the magnitude of the variance and produce smaller number of rejections.

Clearly, the use of the correct variance, when serially correlated errors are detected in regression analysis, improves the test results, regardless of the approximation that is used, but it does not remove the spurious regression phenomenon for small sample sizes. The test behaves better using the A_2 approximation of A than the A_1, especially for large sample sizes and large values of the autoregressive parameter. For small values of the autoregressive parameter both approximations work equally well. However, as pointed out for small and moderate sample sizes and for large values of the autoregressive parameter the empirical levels are not close to the nominal ones using the t_2 statistic.

A more thoroughly examination of the simulation process suggests that the source of this problem, along with the asymptotic behavior of the t_2 statistic, comes from the values of ρ and r used to calculate the value of A_2, since these values are not close enough to their theoretical ones for small and moderate sample sizes. In fact, their mean values are smaller than the theoretical ones (see also Agiakloglou and Agiropoulos, 2016), but as sample size increases, these values become identical to the generated ones. This finding suggests that the value of A_2, unlike the value of A_1, will strongly be affected by the values of ρ and r obtained through the simulation process. For example, for known values of $\rho = r = 0.9$, $A_2 = 9.526$ and $A_1 = 2.62$, whereas for sample sizes of $T = 50$, 100, 500 and 1000 observations, based on 10,000 replications, when these values of ρ and r are replaced by their mean values, which are respectively: a) 0.763 & 0.852, b) 0.836 & 0.876, c) 0.888 & 0.895 and d) 0.893 & 0.897, one will get respectively the following values for A_2 and A_1, i.e., a) $A_2 = 4.716$ and $A_1 = 2.300$, b) $A_2 = 6.472$ and $A_1 = 2.465$, c) $A_2 = 8.745$ and $A_1 = 2.590$ and d) $A_2 = 9.097$ and $A_1 = 2.604$.

One way of dealing with this issue is to run the same simulation process using the generated values to calculate A_1 and A_2. In this case, the correct variance is calculated using constant values for A_1 and A_2 throughout the simulation process and the percentage of rejections of the null hypothesis along with the standard deviations of the relevant t_1' and t_2' statistics are reported on Table 4. As can be seen from this table, the performance of test using the t_1' statistic, based on the A_1 approximation of A, has not been affected at all, simply because, as previously discussed, the value of A_1 is small enough to significantly change the magnitude of the variance, whereas using the A_2 approximation of A the convergence was obtained.

Table 4

Percentage of rejections of the null hypothesis that $\beta = 0$ at the nominal 5% level (|t| > 1.96) using the t_1' and t_2' statistics along with their standard deviations for two independent stationary AR(1) processes for sample sizes of 50, 100, 500 and 1000 observations, based on 10,000 replications

			φ				
			0.0	**0.2**	**0.5**	**0.8**	**0.9**
% of rej.	**50**	t_1'	5.7	5.6	6.8	15.1	25.2
		t_2'	5.7	5.6	5.6	4.8	3.3
	100	t_1'	5.6	5.8	6.7	16.3	28.3
		t_2'	5.6	5.8	5.4	5.1	4.4
	500	t_1'	5.1	5.3	6.8	17.2	31.1
		t_2'	5.1	5.3	5.4	5.4	5.2
	1000	t_1'	5.0	5.0	6.4	17.2	30.3
		t_2'	5.0	4.9	4.8	4.9	5.0
st. dev.	**50**	t_1'	1.026	1.026	1.070	1.398	1.751
		t_2'	1.026	1.024	1.015	0.989	0.919
	100	t_1'	1.024	1.024	1.070	1.414	1.864
		t_2'	1.024	1.023	1.015	1.000	0.977
	500	t_1'	1.007	1.011	1.068	1.431	1.929
		t_2'	1.007	1.010	1.013	1.012	1.012
	1000	t_1'	0.998	0.999	1.049	1.416	1.912
		t_2'	0.998	0.997	0.995	1.002	1.003

Indeed, as can be seen from Table 4, the performance of the test using the t_2' statistic, based on the A_2 approximation of A, has produced very astonishing results, since for all values of the autoregressive parameter and for all sample sizes the empirical levels of this test are very close to the nominal levels, indicating that the value of A_2 was large enough to significantly change the magnitude of the variance of the estimator and the relevant statistic did convert to a standard normal distribution.[2] In practice, though, it is very unlikely to visualize a scenario like this, in which the analyst will *a priori* know the true values of ρ and r. Therefore, it is very interesting to investigate further this phenomenon, as an effort to increase even more the magnitude of the correct variance, through the estimation procedure, hoping that the performance of the test will become better, especially, for small sample sizes.

One possible way of improving the size of the test can be obtained from the use of the unbiased estimate of the variance of the error term. In fact, if $\rho = 0$, OLS uses:

$$s_O^2 = \frac{1}{T-2}\sum \hat{\varepsilon}_t^2 \tag{11}$$

since

$$E\left(\sum \hat{\varepsilon}_t^2\right) = (T-2)\sigma_\varepsilon^2 \tag{12}$$

If, on the other hand, $\rho \neq 0$, then

$$E\left(\sum \tilde{\varepsilon}_t^2\right) = [(T-1)-A]\sigma_\varepsilon^2 \tag{13}$$

[2] For small sample size and for large value of the auto regressive parameter, i.e., for $\varphi = 0.9$ and n = 50, the test over performs, i.e., 3.3% rejection of the null hypothesis at the 5% nominal level, a result that is probably related to the mixed use of estimated and true values.

and therefore the unbiased estimate of the variance will be obtained as:

$$s_A^2 = \frac{1}{[(T-1)-A]} \sum \hat{\varepsilon}_t^2 \tag{14}$$

where A is given by equation (8).[3] Notice that if $\rho = 0$, $A = 1$ and both expressions (11) and (14) are identical. Therefore, equation (14) can be written as:

$$s_A^2 = K\, s_O^2 \tag{15}$$

where

$$K = \frac{T-2}{[(T-1)-A]}$$

and K is a positive number greater than one defining the ratio of the two estimated variances of the error term. Clearly, if autocorrelation is not in presence, i.e., if $\rho = 0$, then $K = 1$ and, therefore, both variances are identical, where K will also take the value of one asymptotically, as sample size increases. Thus, if there is any significant contribution to the magnitude of the variance using the unbiased estimate of the variance of the error term that will be expected to happen only for small sample sizes and for large values of the autoregressive parameter. For example, for $\rho = r = 0.8$ and for sample size of 50 observations, $K = 1.08$, whereas for $T = 500$, $K = 1.0072$, using the A_2 approximation of A, declaring an underestimation of the variance at the level of 7.4% and

[3] The proof of equation (13) is similar to the proof of equation (12), where the residuals for both case are identical, since the presence of autocorrelated errors does not affect the estimates of the model.

0.71% respectively, which will be even smaller using the estimated values of ρ and r and/or using the A_1 approximation of A.

Thus,

$$Var(\tilde{\beta}_A) = Var(\hat{\beta})KA \tag{16}$$

and the test is implemented, using the A_1 and A_2 approximations of A, as:

$$t_1'' = \frac{\hat{\beta}}{se(\hat{\beta})\sqrt{K_1 A_1}} \tag{17}$$

$$t_2'' = \frac{\hat{\beta}}{se(\hat{\beta})\sqrt{K_2 A_2}} \tag{18}$$

where their standard deviations along with the percentage of rejections of the null hypothesis that $\beta = 0$, based on 10,000 replications, are presented in Table 5.

Apparently, as can be seen from this table, in lieu with the results reported on Table 3, the performance of the test has improved only for small sample sizes and for large value of the autoregressive parameter, but only by a very small amount. For example, for $T = 50$ and for $\varphi = 0.9$ the percentage of rejections of the null hypothesis at the 5% nominal level is now 12.7% instead of 13.8% that it was before, using the A_2 approximations of A. This result indicates that the use of the unbiased estimator of the variance of the error term did not significantly change the spurious behavior for small sample sizes and for large values of the autoregressive parameter.

Table 5

Percentage of rejections of the null hypothesis that $\beta = 0$ at the nominal 5% level (|t| > 1.96) using the t_1'' and t_2'' statistics along with their standard deviations for two independent stationary AR(1) processes for sample sizes of 50, 100, 500 and 1000 observations, based on 10,000 replications

			φ				
			0.0	**0.2**	**0.5**	**0.8**	**0.9**
% of rej.	**50**	t_1''	5.8	5.9	7.8	17.6	27.8
		t_2''	5.7	5.8	6.9	9.9	12.7
	100	t_1''	5.6	6.0	7.5	17.5	29.7
		t_2''	5.6	5.9	6.3	7.8	10.0
	500	t_1''	5.1	5.4	6.8	17.4	31.3
		t_2''	5.1	5.4	5.7	6.1	6.6
	1000	t_1''	5.0	4.9	6.3	17.3	30.4
		t_2''	5.0	4.8	5.0	5.2	5.8
st. dev.	**50**	t_1''	1.027	1.038	1.110	1.488	1.883
		t_2''	1.027	1.036	1.070	1.209	1.351
	100	t_1''	1.025	1.030	1.090	1.456	1.926
		t_2''	1.025	1.029	1.044	1.112	1.212
	500	t_1''	1.007	1.013	1.072	1.439	1.941
		t_2''	1.007	1.011	1.019	1.035	1.060
	1000	t_1''	0.998	0.999	1.051	1.420	1.918
		t_2''	0.998	0.998	0.998	1.013	1.028

So far this is study has shown that the spurious regression phenomenon, which is related to serially correlated errors, can be asymptotically removed for all values of the autoregressive parameter for two independent stationary AR(1) process, if the correct variance is used to calculate the relevant t statistic. For small and moderate sample sizes and for large values of the autoregressive parameter the performance of the test is still better than the one obtained using the incorrect OLS variance, but the empirical levels are not as close as they should have been to the nominal levels. The answer to this behavior comes from the values of ρ and r used to calculate the correct variance, since, as shown, their values are typically smaller than the theoretical ones for small sample sizes. Hence the next step is to investigate possible alternatives of getting better estimated values for ρ and therefore better sizes for the test.

3. The Cochrane-Orcutt procedure

When autocorrelated errors, generated by an AR(1) process, are observed in time series econometrics the classical way to deal with this issue, in regression analysis, is to apply the Cochrane-Orcutt (CO) procedure by transforming the original model (1) into the following model:.

$$Y_t - \rho Y_{t-1} = \alpha(1-\rho) + \beta(X_t - \rho X_{t-1}) + \varepsilon_t - \rho\varepsilon_{t-1} \qquad (19)$$

known as the generalized difference equation model, where the initial values for both variables are obtained as $Y_0 = \sqrt{1-\rho^2}Y_1$ and $X_0 = \sqrt{1-\rho^2}X_1$.

The Cochrane-Orcutt (1949) procedure is an iterative procedure aiming to obtain better values for ρ, something that has already been discussed as the main problem of the unfavorable behavior of the test. The variance of the estimated coefficient β will equal asymptotically to:

$$Var(\hat{\beta}_{CO}) = \frac{(1-\rho^2)\sigma_\varepsilon^2}{T\sigma_X^2[1+\rho^2-2\rho r]}$$

and the relevant t statistic for testing the null hypothesis that $\beta = 0$ will be calculated as:

$$t_{CO} = \frac{\hat{\beta}_{CO}}{se(\hat{\beta}_{CO})}$$

where $\hat{\beta}_{CO}$ and $se(\hat{\beta}_{CO})$ are the estimated coefficient of β and its standard error respectively obtained through OLS estimation of model (19) for given sample size *T*. The results of this simulation procedure are reported on Table 6.

Table 6

Percentage of rejections of the null hypothesis that β = 0 at the nominal 5% level (|t| > 1.96) using the t_{CO} statistic along with its standard deviation for two independent stationary AR(1) processes for sample sizes of 50, 100, 500 and 1000 observations, based on 10,000 replications

		φ				
		0.0	**0.2**	**0.5**	**0.8**	**0.9**
% of rej.	**50**	6.8	6.8	7.0	7.6	7.9
	100	6.2	6.2	6.2	6.4	6.6
	500	5.3	5.3	5.3	5.3	5.3
	1000	5.0	5.0	5.0	5.0	5.0
st. dev.	**50**	1.069	1.070	1.076	1.100	1.133
	100	1.049	1.044	1.046	1.051	1.058
	500	1.012	1.012	1.012	1.011	1.011
	1000	0.999	0.999	0.999	1.000	1.000

Indeed, as Table 6 reports, the Cochrane-Orcutt procedure has significantly removed or erased the concept of spurious regression. The empirical percentage of rejections of the null hypothesis is very close to the nominal 5% level for all values of the autoregressive parameter and for all sample sizes, while at the same time the distribution of the t statistic has finally reached the standard normal distribution. One possible explanation to this astonishing result is that this iterative procedure produced better estimates of ρ than OLS. In fact, all estimates of ρ were larger than those obtained through OLS and closer to their theoretical values, even for small sample sizes. For example, for sample size of 50 observations the mean value of ρ under OLS was 0.763, where under the Cochrane-Orcutt procedure is 0.807, for value of 0.9 based on 10,000 replications. This difference between these two mean values of ρ goes to zero as sample size increases, i.e. for sample size of 1,000 observations the estimated mean value of ρ under OLS was 0.893, where under the Cochrane-Orcutt procedure is 0.896, for value of 0.9, based on 10,000 replications.

Clearly, the Cochrane-Orcutt procedure is not the only method used to cure serially correlated errors. Alternatively, one may want to use GLS. Simulation, results show that GLS method works very similarly if not identically to Cochrane-Orcutt. Only for small sample size the percentage of rejections is slightly smaller. For example for sample size of 50 observations and for φ = 0.9 the null hypothesis is rejected 7.4% instead of the 7.9% under Cochrane-Orcutt procedure at the 5% nominal level.

Finally, the Cochrane-Orcutt procedure is also applied to two independent random walk processes without drift producing empirical levels 9.5%, 7.5%, 5.4% and 5.1% for sample sizes of 50, 100, 500 and 1,000 observations respectively, based on 10,000 replications, showing that even in this case the analyst will get most likely the right answer. Recall, that Agiakloglou (2013) for this case obtained better test performance for small sample sizes, i.e., 5.3% and 5.2% for T = 50 and 100, by regressing model (1) in first differences.

4. Conclusion

Dealing with spurious regressions is by far one of the most challenging issues in time series regression analysis. This study shows that using the correct variance under the presence of serially correlated errors or the Cochrane_Orcutt procedure the spurious regression phenomenon can totally be removed.

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