White dwarfs, black holes and the philosophical incommensurability thesis†

Andrés Rivadulla*

Abstract

Incommensurability has been for about forty years one of the most discussed topics on the contemporary philosophy of science. In order to tackle this issue I assume Howard Sankey's (1997: 425) characterization of incommensurability as "the thesis that the content of some alternative scientific theories is incomparable due to translation failure between the vocabulary the theories employ". This kind of incomparability should prevent for instance the derivation of Newtonian mechanics from relativity theory, as Thomas Kuhn (1970*a* and 1970*b*) maintains. Since I have myself been concerned with the comparison of Newtonian and Einsteinian *mass* concepts in Rivadulla (2004), I focus in this short paper on the comparability of theories of contemporary theoretical physics.

Thus, instead of dealing with the question of whether the theories of contemporary physics are definitely incommensurable with each other, the main aim of this paper is to provide an answer to the question of whether it makes any sense to think about the incommensurability between contemporary physical theories, due to their obvious comparability.

KEY WORDS: Incommensurability, White Dwarfs, Black Holes, Planck Units, Theoretical Physics

Resumen

La inconmensurabilidad ha sido durante unos cuarenta años una de las cuestiones más discutidas de la filosofía contemporánea de la ciencia. Para abordarla asumo la caracterización de Howard Sankey (1997: 425) de la misma como "la tesis de que el contenido de algunas teorías científicas alternativas es incomparable debido a fallos de traducción en el vocabulario que emplean las teorías". Este tipo de incomparabilidad debería impedir, por ejemplo, la derivación de la mecánica newtoniana a partir de la teoría de la relatividad, según mantiene Thomas Kuhn (1970*a* y 1970*b*). Como yo mismo me he ocupado de la comparación de los conceptos newtoniano y einsteiniano de *masa* en

* Universidad Complutense de Madrid, Dpto. de Lógica y Filosofía de la Ciencia, arivadulla@filos.ucm.es

Rivadulla (2004), en este artículo breve me centro en la comparabilidad de teorías de la física teórica contemporánea.

Así, en lugar de tratar la cuestión de si las teorías de la física contemporánea son decididamente inconmensurables entre sí, el objeto principal de este artículo es proporcionar una respuesta al problema de si tiene sentido pensar sobre la inconmensurabilidad entre teorías físicas contemporáneas, dada su obvia comparabilidad.

PALABRAS CLAVE: inconmensurabilidad, enanas blancas, agujeros negros, unidades de Planck, física teórica.

1. Introduction

In his recent, striking book on the history and philosophy of contemporary theoretical physics, *The Trouble with Physics*, Lee Smolin (2007: 255) claims that "What the new spirit of physics cannot tolerate is a presumption that one idea has to succeed, whatever the evidence". Obviously Smolin means here that the idea in question belongs to theoretical physics. What about philosophical ideas? Might a philosophical idea succeed whatever the theoretical and empirical evidence in physics?

In *The Myth of the Framework*, the philosopher of science Karl Popper (1994: 12) affirms the "important thesis that science is capable of solving philosophical problems". Although Popper in general believes that modern science "has something important to say to the philosopher about some of the classical problems of philosophy". I intend to show in my contribution that modern physics also has something important to say to the philosopher of science about the so-called incommensurability problem. The incommensurability thesis might be incompatible with the development of theoretical physics. If this were true, no matter how important and disturbing the idea of incommensurability may be, this thesis could not succeed.

Theoretical astrophysics is a branch of modern physics whose development is inconceivable without the in-depth collaboration of many disciplines and theories of mathematical physics. Indeed, in order to learn about the internal constitution of strange stars such as white dwarfs and black holes, we need to take into account the intense interplay between Newtonian mechanics, relativity theory and quantum mechanics. Moreover, quantum gravity and contemporary theoretical cosmology use the Planck units system – which results from the combination of the fundamental constants G, c, and $\hbar$, belonging to Newtonian mechanics, relativity theory and quantum mechanics respectively – making it difficult to assume that these theories cannot be cross-compared because they are incommensurable.

Thus, according to Smolin, either these theories are not incommensurable, or the incommensurability thesis is wrong. Endorsing Popper, theoretical physics would have contributed to solve a philosophical problem, the incommensurability problem.

2. Entropy and temperature of black holes and Chandrasekhar's mass limit of white dwarfs

Black holes are remnants of supernovae cores, which collapse and concentrate in a sphere of radius

$$r_g = \frac{2G_N M}{c^2},$$ **(1)**

known as the *Schwarzschild radius*. In this formula G_N is Newton's gravitational constant, M is the star's masss and c is the speed of light.

In *The Nature of Space and Time* Stephen Hawking (1966: Chapter 3) claims that in 1973 he had discovered that black holes do have entropy [1974 is the year of the discovery of the entropy of a black hole, according to Hawking's *The Universe in a Nutshell.*] In Planck units – where $G_N = \hbar = c = 1$– the entropy of a black hole is

$$S = \frac{1}{4} A,$$ **(2)**

where A is the surface of the black hole.

Applied to the Sun, a black hole with the Sun's mass would have an entropy $S = 10^{78}$. Since entropy is a measure of disorder or the number of microstates of a physical system that are compatible with the system's macrostate, then there would be 10^{78} different ways the Sun might have been constituted.

According to formula (2) the entropy of a black hole is directly proportional to the black hole's surface: the bigger the black hole, the greater its entropy.
Black holes do also have temperature. In Planck units

$$T = \frac{1}{8fkM},$$ **(3)**

where k is Boltzmann's constant and M the black hole's mass.

As it is inversely related to the mass, the more massive a black hole is, the colder it is. Thus, were the Sun a black hole, then 2×10^{30} kg of its mass would be concentrated in a sphere with a radius of about 4 km, and its temperature would be about 10^{-7} Kelvin, slightly above 0 Kelvin, which is terribly cold.

But if we assume that in the center of our Galaxy a super-massive black hole exists with four million solar masses, its temperature amounts to 10^{-14} Kelvin.

Formulae (2) and (3) are indeed fascinating. Nonetheless, they are not particularly relevant from the viewpoint of the incommensurability problem.

The following formulae, expressing the full content of the entropy and temperature of a black hole are much more interesting:

$$S = \frac{kA}{4}\frac{c^3}{\hbar G_N} \qquad (2')$$

$$T = \frac{1}{8fkM}\frac{\hbar c^3}{G_N} \qquad (3')$$

Forty years before Hawking's discoveries, the Indian astrophysicist Subrahmanyan Chandrasekhar calculated that the mass limit of white dwarfs is given by the formula (Cf. Ostlie and Carroll 1996: 590)

$$M_{Ch} \approx \frac{3\sqrt{2f}}{8}\left(\frac{Z}{A}\frac{1}{m_H}\right)^2\left(\frac{\hbar c}{G_N}\right)^{3/2}, \qquad (4)$$

where Z, A and m_H respectively design the number of protons, the number of nucleons and the hydrogen mass. If $Z/A = 0,5$, then $M_{Ch} = 1,44$ solar masses.

White dwarfs are small stars with a mass of both approximately the Sun's mass and the Earth's size. They are the remnants of small stars in their last life period. The fate of our Sun is to degenerate into a white dwarf.

3. White dwarfs, black holes and Kuhn's incommensurability thesis

Dale A. Ostlie and Bradley W. Carroll (1996: 590) claim that formula (4) "is truly remarkable. It contains three fundamental constants – $\hbar$, c and G – representing the combined effects of quantum mechanics, relativity and Newtonian gravitation on the structure of a white dwarf."

Since formulae (2') and (3') also contain the same fundamental constants – $\hbar$, c and G – representing the combined effects of quantum mechanics, relativity and Newtonian gravitation on the structure of a black hole, it becomes obvious that the combination of quantum mechanics, relativity and Newtonian gravitation is of particular relevance when seeking to find out about the internal structure of both black holes and white dwarfs.

But in agreeing with Kuhn, one should assume that quantum mechanics, relativity theory and Newtonian gravitation are incommensurable with each other. And this

implies that they cannot be compared. Can theories that cannot be combined with each other? That is the question.

Thus either Kuhn is right, and quantum mechanics, relativity theory and Newtonian gravitation are incommensurable with each other, and so they cannot be combined in order to learn about the internal structure of both black holes and white dwarfs, or astrophysicists are right and the combined effects of quantum mechanics, relativity theory and Newtonian gravitation on the structure of white dwarfs and black holes reveal that these theories cannot be incommensurable.

4. Planck units system and the incommensurability problem

Quantum gravity and contemporary theoretical cosmology use the Planck units system, identified by:

$$\text{Planck length: } l_P = \left(\frac{G_N \hbar}{c^3} \right)^{\frac{1}{2}} = 10^{-35} m$$

$$\text{Planck mass: } m_P = \left(\frac{\hbar c}{G_N} \right)^{\frac{1}{2}} = 10^{-8} kg$$

$$\text{Planck time: } t_P = \left(\frac{G_N \hbar}{c^5} \right)^{\frac{1}{2}} = 10^{-43} s$$

These fundamental units result from the combination of the same constants as before. It is difficult to assume that the corresponding theories cannot be compared with each other, because they are incommensurable.

5. Conclusion

In taking theoretical physics seriously, we see that Newtonian gravitation, relativity theory and quantum mechanics are frequently combined for relevant purposes in theoretical physics. Were these theories incommensurable with each other it would be impossible for instance to use them when seeking to find out about the internal structure of white dwarfs and black holes. Since moreover the arithmetical combination of the fundamental constants of these theories produces Planck units system, then we are impelled to conclude that either the fundamental theories of physics are not incommensurable or the incommensurability thesis is wrong. Consequently, theoretical physics would have contributed to solve the philosophical incommensurability problem.

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