DEPARTAMENTO DE ANÁLISIS MATEMÁTICO Y MATEMÁTICA APLICADA

Instituto de Matemática Interdisciplinar

SEMINARIO DE MATEMÁTICA APLICADA

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Initial trace of positive solutions to fractional diffusion equation with absorption

This is a joint work[1] *with Huyuan Chen, Jiangxi Normal University, Nanchang, RPC.*

We prove the existence of an initial trace $\mathcal{T}_u$ of a positive solution u to the semilinear fractional diffusion equation (H)

$$\partial_t u + (-\Delta)^\alpha u + f(t,x,u) = 0 \quad \text{in} \quad \mathbb{R}^*_+ \times \mathbb{R}^N,$$

where $N \geq 1$ where the operator $(-\Delta)^\alpha$ with $\alpha \in (0,1)$ is the fractional Laplacian and $f : \mathbb{R}_+ \times \mathbb{R}^N \times \mathbb{R} \mapsto \mathbb{R}$ is a Caratheodory function satisfying $f(t,x,u)u \geq 0$ for all $(t,x,u) \in \mathbb{R}_+ \times \mathbb{R}^N \times \mathbb{R}$. We define the regular set of the trace $\mathcal{T}_u$ as an open subset of $\mathcal{R}_u \subset \mathbb{R}^N$ carrying a nonnegative Radon measure ν_u such that

$$\lim_{t\to 0} \int_{\mathcal{R}_u} u(t,x)\zeta(x)dx = \int_{\mathcal{R}_u} \zeta d\nu \qquad \forall \zeta \in C_0^2(\mathcal{R}_u),$$

and the singular set $\mathcal{S}_u = \mathbb{R}^N \setminus \mathcal{R}_u$ as the set points a such that

$$\limsup_{t\to 0} \int_{B_\rho(a)} u(t,x)dx = \infty \qquad \forall \rho > 0.$$

We give several estimates of solutions and study the reverse problem of constructing a positive solution to (H) with a given initial trace $(\mathcal{S},\nu)$ where $\mathcal{S} \subset \mathbb{R}^N$ is a closed set and ν is a positive Radon measure on $\mathcal{R} = \mathbb{R}^N \setminus \mathcal{S}$. We develop the study of the model case $f(t.x.u) = t^\beta u^p$ where $\beta > -1$ and $p > 1$.

[1] Journal of Functional Analysis, Volume 276, (2019), 1145-1200

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