Identifying Optimal Contingent Fiscal Policies in a Business Cycle Model

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Abstract

Optimal fiscal policy is indeterminate in a dynamic and stochastic environment. The complete characterization of the fiscal policy requires the use of identification constraints. In the literature either capital taxes or debt have been restricted to be not contingent on the state of nature. We propose a different type of identification constraints to have both policy variables state-contingent. Three alternative identification conditions are considered: (i) restrictions on the dynamic and stochastic behavior of the debt path; (ii) an exogenous debt path, and (iii) an exogenous belief function. The main result indicates that the optimal capital tax is zero and constant over the business cycle for any of the identification conditions used, suggesting that is optimal for the government to use debt return as a shock absorber, keeping capital taxes constant. The result is quite different from the previous literature, which obtains very volatile capital taxes.

Keywords: optimal taxation, indeterminacy.

JEL codes: E62, H21.

1. Introduction

Optimal fiscal policy deals with the combination of taxes and debt return that maximizes welfare and is consistent with the government spending path. The work of Chamley (1986) allowed discussing the problem in a dynamic framework and the development of simulation techniques extended it to the stochastic setting. A survey of the most recent development in this field can be found in Manzano and Ruiz (2004).

The stochastic framework is interesting because it addresses the question of the optimal contingent fiscal policy, that is, how should fiscal policy be set over the business cycle? An important feature is that introducing uncertainty yields the indeterminacy of the optimal fiscal policy. As Zhu (1992) shows, the indeterminacy issue arises from the Euler conditions for capital and bonds implying that expected after-tax returns on capital and bonds must be equal. There are infinite paths of contingent capital income taxes and contingent bonds that can be implemented by the government, decentralizing optimal allocations and satisfying this ex-ante arbitrage condition, that is, different capital income tax rates induce the same expected after-tax return on capital. It is remarkable that the indeterminacy does not affect the optimal allocations, the expected capital income tax rate or the labor income tax rate, but it does affect the state-contingent capital income taxes and bonds.

Given the indeterminacy issue, the complete characterization of the fiscal policy, requires the use of identification constraints. Chari, Christiano and Kehoe (1994) analyze those properties by restricting either capital taxes or debt to be not contingent on the state of nature. When the government issue uncontingent debt, the path of state-contingent capital income tax rates can be obtained. Alternatively, under uncontingent capital income taxation, the state-contingent path of debt supporting optimal allocations can be computed.

The properties of the capital income tax rate depend crucially on the identification constraint used. Chari, Christiano and Kehoe (1994) report the properties of the capital tax rate under the two decentralizations of the optimal allocations. In the first case, when capital taxes are uncontingent, optimal capital tax rates are simply the expected tax rates, which are zero in the models with log utility, and close to zero on average, with low volatility and high persistence when the risk aversion is large. In the second case, when the debt return is assumed to be not state-contingent, optimal capital taxes are very volatile and serially uncorrelated. Therefore optimal capital taxes would have a wide range of variation, from constant to very volatile tax rates, and from i.i.d. to close to random walk stochastic processes, depending on the identification assumption.

In this paper we propose a different type of identification constraints in order to pin down one of the infinite state-contingent policies, each one supporting the same optimal allocations. Under each identification constraint we characterize the cyclical properties of the policy variables. Then, we will be able to study the differences with the Ramsey policies of Chari, Christiano and Kehoe (1994), who use the assumption of uncontingent policies as identification constraints. Under our identification constraints, both capital taxes and debt return are state-contingent.

Optimal allocations, expected capital income taxes and labor income taxes are not affected by indeterminacy; hence our main interest is to compute the cyclical properties of the optimal capital income taxes, showing that these properties are very different from those under state-uncontingent policies of previous papers.

Since optimal allocations are implementable with the Chari, Christiano and Kehoe (1994) policies, why would it be worth to care about fully state-contingent optimal fiscal policy? The interest is twofold. On the one hand, when the government issue public debt, the nominal return of public debt is announced, but the real return is uncertain because it depends on the inflation rate; so the assumption of state-contingent debt return would be more appropriate, either in nominal or real models. Something similar can be argued about capital income taxes; the complexity of tax credits, exemptions and deductions in most countries affects the effective tax rate, in contrast to the expected tax rate, so it would be realistic to consider state-contingent taxes. On the other hand, the identification constraints that we assume are theoretically and empirically relevant. Two kinds of alternative constraints are assumed in this paper: i) restrictions on the stability of the debt path, and ii) restrictions on the expectations mechanism. The first identification assumption prevents debt from exploding by imposing an endogenous stability condition that limits the debt path to not grow faster than output in the long-run. This condition summarizes the common concern of governments about the control of the debt/GDP ratio[1]. Such a rule resembles the one used by Sims (1994), who proposed a theoretical relation between taxes and debt to guarantee the debt path stability. In order to evaluate how restrictive this constraint

[1] The debt/GDP ratio is one of the important indices of an economy's stability in the eyes of foreign investors, and every decline in the ratio is viewed by them as indicating greater economic stability. In addition, one of the central conditions stipulated in the Maastricht criteria for the access of new members to the European Monetary Union is a maximum debt of 60% of their GDP or a distinctly downward trend converging to that figure.

is, in terms of the properties of the optimal policies, we compare the results with those obtained under different exogenous rules for the debt path. In particular, we assume that the level of debt follows a first order autoregressive stochastic process under different degrees of persistence.

The second kind of identification is in line with Zhu (1992), who proposes a way for obtaining alternative feasible state-contingent policies that implement the same competitive allocation: there is a variety of alternative capital taxes, together with an appropriate debt restructuring, consistent with the same competitive allocation. This alternative policy does not change the household's intertemporal consumption choice, although such fiscal policy does generate different expectation errors. Therefore, a way to generate state-contingent policies, in the spirit of Zhu (1992), is to impose expectation errors compatible with the competitive allocations.These errors then enable us to identify the feasible optimal fiscal policy, which is consistent with the competitive equilibrium. Thus, our identification constraint imposes a belief function for the representative agent, being consistent with the hypothesis of rational expectations. In particular, we assume that one of the expectation errors associated with the Euler conditions of the household is exogenous, following a white noise process uncorrelated with the information set. Imposing such a belief function is enough to obtain a unique stochastic path of the optimal fiscal policy. The economic interpretation behind this identification assumption is as follows: if agents endogenously change their beliefs about the future fiscal policy, it is sufficient that the government issue debt such that it absorbs that shock or belief. This can be done by the government because optimal fiscal policy is indeterminate; thus, for every sequence of optimal capital income tax rates there is an associated sequence for issued debt and debt return.

The results obtained in this paper are robust to the alternative identification schemes proposed, and different from Chari, Christiano and Kehoe's (1994) findings. We obtain that contingent tax rates on capital should be set to zero and display no volatility, since it is optimal for the government to use the debt return as a shock absorber instead of capital income taxes. The result would be explained by the greater effectiveness of using the debt return instead of capital taxes to stabilize the stock of debt.

The paper is organized as follows: section 2 describes the model. Section 3 presents the Ramsey problem and the simulation results under the different identification constraints. Finally, section 4 concludes by summarizing the main findings.

2. The model

The economy consists of households, firms, and the government, represented by the neoclassical stochastic growth model. We assume a representative household and a representative firm that produces a single good.

2.1. Households

The household makes decisions by maximizing an expected flow of utility, subject to the budget constraint and taking wages and interest rates as given. Preferences at each period are represented by a utility function that includes consumption ($\tilde{c}_t$) and leisure ($1-n_t$), where the household is endowed with one unit of time. We assume a standard utility function[2]:

$$U(\tilde{c}_t, 1-n_t) = \frac{\left(\tilde{c}_t^{\,1-\theta}(1-n_t)^{\theta}\right)^{1-\sigma}}{1-\sigma} , \tag{1}$$

where $\sigma>0$ is the relative risk aversion, and $\theta\in(0,1)$ is the preference for leisure. Future utility is discounted at a rate $\beta\in(0,1)$.

Household income arises from renting capital and labor to the firm and from the bond returns. Labor and capital income are taxed. After-tax income is spent on consumption, investment and government bonds ($\tilde{b}_t$). The household budget constraint is:

$$\tilde{c}_t + \tilde{k}_t + \tilde{b}_t = (1-\tau_{w_t})\tilde{w}_t n_t + R_t \tilde{b}_{t-1} + \left[1+(r_t-\delta)(1-\tau_{k_t})\right]\tilde{k}_{t-1} , \tag{2}$$

where δ is the depreciation rate of the capital stock, and τ_{w_t} and τ_{k_t} are tax rates on labor and capital income. R_t is the return on government bonds. In equation (2), the term in brackets on the right hand side represents the gross return of capital after taxes and depreciation, where taxation on capital income has a depreciation tax credit.

2.2. Firms

The production function of the firm exhibits constant returns to scale, using labor and capital as inputs. This function incorporates a stochastic productivity shock (z_t):

$$\tilde{y}_t = F(n_t, \tilde{k}_{t-1}; z_t) , \tag{3}$$

[2] $\tilde{x}$ indicates that the variable x grows in the steady state at a constant and exogenous rate.

where $F(\cdot)$ is a Cobb-Douglas production function with labor augmenting technological change:

$$\tilde{y}_t = \left(e^{\rho \cdot t + z_t} n_t\right)^{\alpha} \tilde{k}_{t-1}^{1-\alpha} \ , \tag{4}$$

where ρ represents the exogenous growth rate. The productivity shock follows a stochastic process:

$$z_t = \phi_z z_{t-1} + \varepsilon_{z_t} \ , \quad \varepsilon_{z_t} \underset{iid}{\sim} N(0, \sigma^2_{\varepsilon_z}) \ , \quad |\phi_z| < 1 \ , \quad z_{-1} = 0 \ . \tag{5}$$

The competitive behavior of the firm ensures that input prices equal marginal productivities:

$$\tilde{w}_t = F_n(n_t, \tilde{k}_{t-1}; z_t) \ , \tag{6}$$

$$r_t = F_k(n_t, \tilde{k}_{t-1}; z_t) \ . \tag{7}$$

2.3. Government

The government finances an exogenous flow of government consumption by taxing labor and capital income and by issuing debt. The government budget constraint is:

$$\tilde{G}_t + R_t \tilde{b}_{t-1} = \tau_{w_t} \tilde{w}_t n_t + \tau_{k_t} [r_t - \delta] \tilde{k}_{t-1} + \tilde{b}_t \ . \tag{8}$$

Government consumption is given by:

$$\tilde{G}_t = G e^{\rho t + g_t} \ , \tag{9}$$

where G is a constant and g_t is a shock that affects government consumption and follows a stochastic process:

$$g_t = \phi_g g_{t-1} + \varepsilon_{g_t} \ , \quad \varepsilon_{g_t} \underset{iid}{\sim} N(0, \sigma^2_{\varepsilon_g}) \ , \quad |\phi_g| < 1 \ , \quad g_{-1} = 0 \ . \tag{10}$$

2.4. Competitive equilibrium

In order to analyze the competitive equilibrium of the economy, the optimization problem of the household can be easily converted into stationary by dividing variables by the gross rate of growth: $x_t = (\tilde{x}_t / e^{\rho t})$, $x = \{c, k, w, y, G, b\}$, and modifying the discount rate appropriately[3]:

[3] Under CRRA preferences, as those specified in (1), the discount rate is: $\hat{\beta} = \beta e^{\rho(1-\sigma)(1-\theta)}$

$$\underset{\{c_t,\, n_t,\, k_t,\, b_t\}_{t=0}^{\infty}}{Max} \qquad E_0 \sum_{t=0}^{\infty} \hat{\beta}^t \, U\left(c_t,\, 1-n_t\right) \tag{11}$$

subject to:

$$c_t + e^{\rho} k_t + e^{\rho} b_t = (1-\tau_{w_t})\, w_t n_t + R_t b_{t-1} + \left[1 + (r_t - \delta)(1-\tau_{k_t})\right] k_{t-1} \ , \tag{12}$$

$$k_{-1},\ b_{-1} \ \ given \ ,$$

$$c_t,\ n_t,\ k_t \ \geq \ 0 \ .$$

The competitive equilibrium of this economy is the set of paths $\{c_t, n_t, k_t, b_t\}_{t=0}^{\infty}$, prices $\{w_t, r_t\}_{t=0}^{\infty}$ and government policies $\{\tau_{w_t}, \tau_{k_t}, R_t, G_t\}_{t=0}^{\infty}$ that satisfy:

(i) Allocations $\{c_t, n_t, k_t, b_t\}_{t=0}^{\infty}$, solve the household problem given prices $\{w_t, r_t\}_{t=0}^{\infty}$ and policies $\{\tau_{w_t}, \tau_{k_t}, R_t\}_{t=0}^{\infty}$.

(ii) Allocations $\{n_t, k_t\}_{t=0}^{\infty}$, maximize the firm profits given the input prices $\{w_t, r_t\}_{t=0}^{\infty}$ and the productivity shock $\{z_t\}_{t=0}^{\infty}$.

(iii) The government budget constraint is fulfilled at each period.

(iv) Goods, labor, capital and bond markets clear at each period. So, the aggregate resources constraint is satisfied:

$$c_t + e^{\rho} k_t - (1-\delta) k_{t-1} + G_t = \ F(n_t, k_{t-1}; z_t) \ . \tag{13}$$

This expression indicates that output in the economy is spent on private consumption, investment and public consumption.

3. The Ramsey problem

3.1. Ramsey allocations

The government solves the Ramsey problem in order to select optimally the fiscal policy tools. We adopt the primal approach, characterizing the optimal allocations that can be implemented as a competitive equilibrium with distorting taxation, subject to the feasibility constraint (13) and the so-called implementability constraint:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[U_{c_t} c_t + U_{n_t} n_t\right] = U_{c_0}\left[R_0 b_{-1} + \left(1+(r_0-\delta)(1-\tau_{k_0})\right) k_{-1}\right] . \tag{14}$$

Such a constraint represents the present value of the budget constraint of the household, eliminating prices and policy variables by using the Euler conditions of the competitive equilibrium. Then it is possible to calculate the optimal allocations separately from the fiscal policy variables.

Allocations of consumption, hours and capital, initial tax rate on capital income and initial debt return arise from:

$$\underset{\{c_t, n_t, k_t\}}{Max}\ E_0 \sum_{t=0}^{\infty} \hat{\beta}^t\, W(c_t, 1-n_t, \lambda) - \lambda\, U_{c_0}\left[R_0 b_{-1} + \left(1+(r_0-\delta)(1-\tau_{k_0})\right) k_{-1}\right] , \tag{15}$$

subject to:

$$c_t + e^{\rho} k_t - (1-\delta) k_{t-1} + G_t = F(n_t, k_{t-1}; z_t) , \tag{16}$$

$$c_t,\ n_t,\ k_t \geq 0, \qquad b_{-1}, k_{-1} \ \ given,$$

given the path of public consumption, $r_0 = F_{k_0}$[4] and where $W(c_t, 1-n_t, \lambda) = U(c_t, 1-n_t) + \lambda(U_{c_t} c_t + U_{n_t} n_t)$, $\forall t \geq 0$, with λ representing the Lagrange multiplier that discounts the implementability constraint.

The objective function is an increasing function of τ_{k_0} and decreasing in R_0. The reason is that τ_{k_0} taxes capital returns and R_0 rewards the debt stock, both at $t = -1$, so the individual cannot react to the tax and debt return by varying investment and debt stock decisions. Therefore the government has incentives to set an initial capital income tax rate as high as possible and an initial debt return as low as possible. In order to have an interesting problem, we follow Chari, Christiano and Kehoe (1994) in assuming that the initial tax rate on capital income and debt return, τ_{k_0} and $R_0 b_{-1}$, are fixed.

The optimal allocations $\{c_t, n_t, k_t\}_{t=0}^{\infty}$ that satisfy optimal conditions of the problem given by (15)-(16), depend on the multiplier λ, which discounts the implementability constraint (13). Those paths are the *Ramsey allocations*, for such a λ, so that the paths of consumption, hours and capital stock satisfy the optimal conditions of the Ramsey problem and the implementability constraint.

In order to solve the Ramsey allocations numerically, the solution method proposed by

[4] $F_{k_0} = \partial F(n_0, k_{-1}; z_0)/\partial k_{-1}$

Sims (2002) is implemented, extending it to the case of non-linear rational expectations systems[5].

3.2. Ramsey policies

Since the Ramsey allocations are computed, we obtain the set of policies (*Ramsey policies*) that support optimal allocations from the conditions of competitive equilibrium. Throughout the analysis we assume that the government can commit itself to follow the fiscal policy plan.

Given the Ramsey allocations, the optimal labor income tax rate is pinned down from the competitive equilibrium condition that equals the consumption-leisure marginal substitution rate to the inverse of labor marginal productivity after taxes:

$$\tau_{w_t} = 1 - \frac{U_{n_t}}{w_t U_{c_t}} \quad . \tag{17}$$

Nevertheless, in a stochastic framework an indeterminacy arises that makes it impossible to obtain the tax rate on capital income and the debt return simultaneously, both contingent on the state of nature. Competitive equilibrium first order conditions for capital and debt are:

$$e^{\rho} = E_t\left(\hat{\beta}\frac{U_{c_{t+1}}}{U_{c_t}}\left[1 + (r_{t+1} - \delta)(1 - \tau_{k_{t+1}}) \right] \right) = \hat{\beta}\left(\frac{U_{c_{t+1}}}{U_{c_t}}\left[1 + (r_{t+1} - \delta)(1 - \tau_{k_{t+1}}) \right] \right) - \nu_{1,t+1} \ , \tag{18}$$

$$e^{\rho} = E_t\left(\hat{\beta}\,\frac{U_{c_{t+1}}}{U_{c_t}} R_{t+1} \right) = \hat{\beta}\,\frac{U_{c_{t+1}}}{U_{c_t}} R_{t+1} - \nu_{2,t+1} \ , \tag{19}$$

where $\nu_{1,t}$ and $\nu_{2,t}$ are the expectation errors associated with Euler conditions (18) and (19).

The expectation operator in (18) and (19) imply that the after-tax returns on capital and bonds (weighted by marginal utility) must be equal on average. The government can implement many paths of capital income tax rates and debt return to decentralize the Ramsey allocations satisfying this ex-ante arbitrage condition. Thus, from conditions (18) and (19), the state-contingent paths of capital income tax rate and debt return cannot be computed. From a computational point of view, the indeterminacy of fiscal policy implies that equations (8), (12),

[5] See Novales et al. (1999) for detailed applications of this solution method to standard models of real business cycles. Appendix A describes the application of this methodology to the Ramsey problem and shows how to compute λ.

(18) and (19), are not enough to calculate the optimal paths of $\{\tau_{k_t}, b_t, R_t\}_{t=1}^{T}$ along with the paths of the expectation errors $\{\nu_{1,t}, \nu_{2,t}\}_{t=1}^{T}$.

The indeterminacy is the same showed by Zhu (1992) and Chari, Christiano and Kehoe (1994) in a similar model. A more rigorous way to show this policy indeterminacy is the followed by Zhu (1992). Let e_t be an i.i.d. stochastic process with $E_t(e_{t+1})=0$ and $Cov_t\left(e_{t+1}, U_{c_{t+1}}(r_{t+1}-\delta)\right)=0$. Given a path for the capital tax rate $\{\tau_{k_t}\}_{t=0}^{\infty}$ and the process for e_t, an alternative fiscal policy for the tax rate and the debt stock both contingent to the state of nature can be implemented, compatible with the optimal allocations and hence fulfilling (8), (12) (18) and (19). Let the new policy be:

$$\begin{aligned}
&\hat{\tau}_{k_0}=\tau_{k_0},\\
&\hat{\tau}_{k_t}=\tau_{k_t}+e_t, \quad \forall t\geq 1,\\
&\hat{\tau}_{w_t}=\tau_{w_t}, \quad \forall t\geq 0,\\
&e^{\rho}\hat{b}_t=e^{\rho}b_t-e_t\left(r_t-\delta\right)k_{t-1}, \quad \forall t\geq 0.
\end{aligned}$$

The new policy does not change expression (19) or expression (18):

$$e^{\rho}\beta E_t\left\{\frac{U_{c_{t+1}}}{U_{c_t}}\left[1+(r_{t+1}-\delta)(1-\hat{\tau}_{k_{t+1}})\right]\right\}=\hat{\beta}E_t\left\{\frac{U_{c_{t+1}}}{U_{c_t}}\left[1+(r_{t+1}-\delta)(1-\tau_{k_{t+1}})\right]\right\}-\frac{\hat{\beta}}{U_{c_t}}E_t\left\{U_{c_{t+1}}(r_{t+1}-\delta)e_{t+1}\right\}=$$
$$=\hat{\beta}E_t\left\{\frac{U_{c_{t+1}}}{U_{c_t}}\left[1+(r_{t+1}-\delta)(1-\tau_{k_{t+1}})\right]\right\}.$$

The government budget constraint is fulfilled, given the debt restructuring proposed:

$$G_t+R_tb_{t-1}=\tau_{w_t}w_tn_t+\hat{\tau}_{k_t}(r_t-\delta)k_{t-1}+e^{\rho}\hat{b}_t=\tau_{w_t}w_tn_t+\tau_{k_t}(r_t-\delta)k_{t-1}+e^{\rho}b_t.$$

It can be argued likewise for the household budget constraint.

It is clear that there is a continuum of different sequences of $\{e_{t+1}\}$ satisfying $E_t(e_{t+1})=0$ and $Cov_t\left(e_{t+1}, U_{c_{t+1}}(r_{t+1}-\delta)\right)=0$. For example, a stochastic process such e_t can be selected, such that either the new capital income tax rate $\hat{\tau}_{k_{t+1}}$ is known in period *t*, or the new debt payment $\hat{b}_t$ is known in period *t*. This second possibility is the one used by Chari, Christiano and Kehoe (1994), who identify optimal fiscal policy by restricting the debt return to be not contingent on the state of nature (that is, e_t is selected such that the new debt payment $\hat{b}_t$ is known in period *t*), then equation (19) can be transformed into:

$$R_{t+1} = \frac{e^{\rho}}{E_t \hat{\beta} \frac{U_{c_{t+1}}}{U_{c_t}}} , \tag{20}$$

allowing them to obtain the debt return. Therefore, these authors compute optimal fiscal policy under the assumption of uncontigent debt return. In other words, they assume that private agents know the real return of debt for the next period with certainty.

However, the expected tax rate on capital income can always be computed. Following Chari, Christiano and Kehoe (1994), the ex-ante tax rate is defined as the ratio of the expected value of revenues from capital income taxation to the expected value of the net return of capital, both terms weighted by the marginal substitution rate between consumption today and tomorrow:

$$\tau^e_{k_t} = \frac{E_t\left[\hat{\beta}\frac{U_{c_{t+1}}}{U_{c_t}}\tau_{k_{t+1}}(F_{k_{t+1}} - \delta)\right]}{E_t\left[\hat{\beta}\frac{U_{c_{t+1}}}{U_{c_t}}(F_{k_{t+1}} - \delta)\right]} . \tag{21}$$

So long as the ex-ante tax rate is a ratio of expected values, if we have continual support shocks, having approximations of these expectation terms is not trivial. However the solution method proposed allows us to evaluate this tax rate, as we describe in appendix A. In this appendix, we also show, from the solution method, that the ex-ante tax rate on capital income is zero for all $t \geq 1$ under logarithmic preferences, and fluctuates around zero for a risk aversion coefficient different than one.

In contrast to the work of Chari, Christiano and Kehoe (1994), we consider a set of alternative identification restrictions yielding optimal fiscal policies contingent on the state of nature. Three kinds of restrictions are considered: (i) restrictions on the dynamic and stochastic behavior of debt path; (ii) an exogenous debt path and (iii) an exogenous belief function. The model is simulated under the different identification constraints in order to analyze the stochastic properties of the optimal contingent fiscal policies obtained.

3.2.1. Restrictions on the dynamic and stochastic behavior of debt path.

Since the optimal tax rate on labor income is computed, the linear system consisting of (8), (12), (18) and (19) represents an approximation to the dynamic evolution of the remainder of the policy variables, given the *Ramsey allocations*. Because of the inherent indeterminacy of the fiscal policy, there is a continuum of paths for variables $\{\tau_{k_t}, R_t, b_t, \nu_{1,t}, \nu_{2,t}\}$ that solves such a system. $\{\tau_{k_t}, R_t, \nu_{1,t}, \nu_{2,t}\}$ evolves to a stochastic steady state, while the debt stock can follow an explosive path[6], being optimal and compatible with a stable and stationary equilibrium for the remaining variables.

There are examples in the literature that limit the dynamic evolution of debt, to guarantee a stable time path for debt and a viable equilibrium. In Sims (1994) the government budget constraint is: $B_t - R_t B_{t-1} = T_t$, where T_t are government lump-sum transfer payments (if positive) or taxes (if negative). This constraint is an unstable linear difference equation in the debt. With the tax policy set as: $T_t = \bar{T} - \eta B_{t-1}$, the government budget constraint becomes: $B_t = (R_t - \eta) B_{t-1} + T_t$. This expression is a stable linear difference equation for $R_t - 1 < \eta < R_t$; thus, with η chosen in this range, any initial value for debt generates a stable time path for debt and a viable equilibrium. Leeper (1991) also argues in this sense, considering a class of rules suggested by actual policies, whose objective could be to smooth distorting direct taxes over time. Barro (1979) showed that tax smoothing creates a role for public debt, perhaps as a shock absorber.

We use a condition for the dynamic evolution of debt similar to the ones used by Sims and Leeper. However, our condition is not ad-hoc but arises from the elimination of the unstable subspace associated to the eigenvalue $R>1$ from the system consisting of equations (8), (12), (18) and (19), that determines the Ramsey policies. Thus, we obtain a condition that relates the tax rates, the debt return and the bonds, and guarantees a stable time path for the debt, and that can be interpreted as a policy rule. By avoiding the unstable behavior of the debt, we enforce the debt path to not grow faster than the other variables in the economy in the long-run.[7]

As we mentioned above, it will be sufficient to introduce the condition of eliminating the unstable subspace as an identification constraint for the policy variables. In order to do this, the government budget constraint:

$$e^{\rho} b_t = R_t b_{t-1} + G_t - \tau_{k_t}(r_t - \delta) k_{t-1} - \tau_{w_t} w_t n_t \ , \tag{22}$$

[6] Although fulfilling the non-Ponzi condition.

[7] It is clear that this stability condition is stronger than the standard transversality condition:

$$\lim_{j \to \infty} E_t \frac{b_{t+j} \hat{\beta}^{t+j}}{\prod_{s=0}^{t+j-1} R_s} = 0 \ .$$

is linearized around the deterministic steady state[8], taking into account that optimal allocations are known at any period:

$$b_t=\frac{R_{ss}}{e^{\rho}}(b_{t-1}-b_{ss})+\frac{b_{ss}}{e^{\rho}}(R_t-R_{ss})+\frac{G_t}{e^{\rho}}-\frac{\tau_{k_t}}{e^{\rho}}(r_t-\delta)k_{t-1}-\frac{\tau_{w_t}}{e^{\rho}}w_t n_t \ . \tag{23}$$

The linear approximation given by (23) implies that the debt path is not stable because $(R_{ss}>e^{\rho}>1)$. The debt path is not explosive if we eliminate the unstable path. In order to find the unstable path we linearize the dynamic system consisting of (8), (12), (18) and (19). We then take the eigenvector of the transition matrix associated with the unstable eigenvalue as the direction to eliminate. For the selected parameterization, given by table 1, the stability condition is[9]:

$$b_t-b_{ss}=.15(R_t-R_{ss})+.26(\tau_{w_t}-\tau_{w_{ss}})+.04(\tau_{k_t}-\tau_{k_{ss}}) \ , \tag{24}$$

which guarantees that b_t is not explosive and is compatible with both the budget constraints of the household and the government, and satisfies the transversality condition of private and public assets[10].

It must be noticed that condition (24) enforces not only stability of the debt path, but it also imposes stationarity. Since debt return and tax rates are stationary (do not grow in the steady state), then b_t fluctuates around b_{ss}.

Condition (24) establishes that the debt return will be stable whenever deviations of debt between the value at period *t* and the steady state are due to variations either in debt return or in the tax rates, according to the parameters in (24). Thus, if $R_t>R_{ss}$, the debt stock increases

[8] Note that the deterministic steady state is determined, since:

$$\nu_{1_{ss}}=\nu_{2_{ss}}=0\,,\ \tau_{k_{ss}}=0\,,\ R_{ss}=1/\beta\,,\ b_{ss}=\frac{1}{e^{\rho}-R_{ss}}\left(G_{ss}-\tau_{ss}w_{ss}b_{ss}\right).$$

[9] Calibrated parameters and initial conditions for capital stock, debt return and the capital tax rate are the same as those discussed by Chari, Christiano and Kehoe (1994), when calibrating the model with U.S. economy data.

[10] Given the transversality condition of the competitive equilibrium:

$$\lim_{j\to\infty} E_{t-1+j}\,\beta^{t+j}\,U_{c_{t+j}}(k_{t+j}+b_{t+j})=0 \ ,$$

constraint (24) implies that $\lim_{j\to\infty} E_{t-1+j}\,\hat{\beta}^{t+j}\,U_{c_{t+j}}b_{t+j}=0$, since the path b_{t+j} is stable, that is, the growth rate of b_{t+j} is lower than that of $\hat{\beta}^{t+j}U_{c_{t+j}}$, and this last growth rate is lower than the gross debt return (*R*). The Ramsey allocations solve the optimal path of capital, so the transversality condition for capital in the Ramsey problem holds. Thus, the transversality condition of the competitive equilibrium holds.

because more debt needs to be issued to repay the outstanding debt. Moreover when $b_t > b_{ss}$, higher tax rates are needed in order to repay outstanding debt.

Given the use of the stability condition as an identification constraint, we are able to undertake the stochastic simulation of the model. The model has been simulated 100 times with a length of 1200 periods. Table 1 shows the calibrated parameters, that are those discussed by Chari, Christiano and Kehoe (1994) when calibrating the model with U.S. economy data. The baseline model considers logarithmic preferences (σ=1). Other versions of the model are also simulated: a high risk aversion model (σ=9), a model with i.i.d. shocks, and finally a model with only technology shocks.

To compute the properties of the stochastic simulation, the first 200 periods are dropped. That ensures the stationarity of the statistics of the optimal policies. Table 2 reports the simulation statistics.

By analyzing taxation statistics, we can see that the average ex-post capital income tax rate is zero and constant over the business cycle (standard deviation of simulated tax rate is zero). Moreover, the optimal capital tax rate is uncorrelated with both the productivity shock and government consumption, and it exhibits no persistence. The different stochastic processes implemented for shocks and risk aversion do not change the properties described above. Notice that the standard deviations of the statistics are very low, suggesting high precision when estimating those moments.

The results are quite different from those of Chari, Christiano and Kehoe (1994) which report a non-zero average and a very volatile ex-post capital income tax rate, correlated with both the technology shock and government consumption. From an empirical point of view, there is not a regularity about the volatility of capital taxes relative to labor taxes. For example, from the updated[11] effective tax rate data reported by Mendoza, Razin and Tesar (1994), it follows that countries like Italy, Japan and the U.K. exhibit more volatility on capital taxes, whilst in other countries such as Canada, France, Germany and the U.S. labor taxes are more volatile.

From a theoretical point of view, the difference in results comes from the available state-contingent fiscal tools. Chari, Christiano and Kehoe (1994) restrict the debt return to be uncontingent, therefore it is known with certainty in the previous period. Thus, the optimal capital income tax rate becomes very volatile because the government cannot use debt return as a shock

[11] http:/www.econ.duke.edu/~mendozae/pdfs/taxdata.pdf

absorber, and capital income taxation contingent on the state of nature must be used. However, the identification condition we use allows the government to set both the debt return and the capital income tax rate as state-contingent, that is, the government could use both policy variables as shock absorbers.[12] The government finds it optimal to keep this tax rate constant over the business cycle at the expense of the debt return becoming more volatile. The explanation comes from expression (24): using the debt return is roughly 4 times more effective than using the capital tax rate in order to stabilize the debt stock (.15 versus .04). In the end, the stability condition could be interpreted as a policy rule whose objective is to smooth capital taxes over time. According to Barro (1979), smoothing the capital tax rate creates a shock absorber role for public debt.

With regard to the stochastic properties of labor income taxation, there are not differences with Chari, Christiano and Kehoe (1994) results. The reason for this is that we also use the relationship of the marginal substitution rate of consumption-leisure with after tax wages given in (17) to compute the labor income tax rate, and the differences with our paper do not affect the computation of the *Ramsey allocations*.

3.2.2. Exogenous debt path

An interesting issue to address is whether alternative identification conditions change the properties of contingent capital income taxation. In order to assess to what extent imposing a stationary path for the debt can restrict the properties of the *Ramsey policies*, we use an alternative identification condition: we evaluate exogenous stationarity stochastic processes for the debt path so that the processes do not violate the transversality condition for the debt path. We assume first order autoregressive stochastic processes:

$$b_t = b_{ss}(1-\phi) + \phi b_{t-1} + \varepsilon_t^b \ , \quad \varepsilon_t^b \underset{iid}{\sim} N(0,\sigma_b) \ , \tag{25}$$

for several degrees of persistence (ϕ)[13]. Unconditional mean (b_{ss}) is selected to mimic the steady state value of debt when it is decided endogenously. Ludvigson (1996) settled exogenous first

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[12] Notice that the labor income tax rate cannot play the role of debt stabilizer, because is uniquely determined by the optimal allocations, according to (17).

[13] A particular case of (25) would be b_t=b_{ss} $\forall t$, (ϕ=0, ε_t^b=0, $\forall t$), ensuring that the debt is not explosive because it is constant over time.

order autoregressive processes for the debt paths in order to analyze how the degree of persistence affects the competitive decisions of private agents. An ad-hoc hypothesis such as this one also generates stable time series for the debt. This kind of constraint provides a large set of debt processes to study its implications on the properties of the *Ramsey policies*.

Table 3 summarizes simulation results under several stochastic processes for the debt path. The results point out that, independently of persistence, ex-post capital income tax rate properties do not differ from those reported when the stability condition is used to identify optimal fiscal policy. This result indicates that imposing a stability condition does not constrain the optimal properties shown by contingent capital income taxation.

3.2.3. Exogenous belief function

Alternatively, the last set of constraints we impose is an exogenous belief function for private agents, fulfilling the government budget constraint and computing the debt path residually from such a constraint.

The set of optimal policies consistent with the competitive allocations, proposed by Zhu (1992), are such that:

$$E_t\left\{\frac{U_{c_{t+1}}}{U_{c_t}}\left[1+(r_{t+1}-\delta)(1-\hat{\tau}_{k_{t+1}})\right]\right\}=E_t\left\{\frac{U_{c_{t+1}}}{U_{c_t}}\left[1+(r_{t+1}-\delta)(1-\tau_{k_{t+1}})\right]\right\}, \tag{26}$$

where $\hat{\tau}_{k_t}=\tau_{k_t}+e_t$ and e_t is an i.i.d. stochastic process with $E_t(e_{t+1})=0$ and $Cov_t\left(e_{t+1},U_{c_{t+1}}(r_{t+1}-\delta)\right)=0$. Expression (26) implies that the expected terms in brackets are equal, but not the realized values. So the expectation errors associated to each term are different, although both errors follow a white noise process uncorrelated with the information set. Therefore, one way to have determinate optimal state-contingent policies is to compute one of the expectation error exogenously, obtaining a correspondence between that expectation error and stochastic process e_t used to build the optimal feasible fiscal policy.

The prophecies of agents, which do not depend on the fundamentals of the model, can be useful to determine the Ramsey policies. Along these lines it is interesting to characterize the optimal policies compatible with the optimal allocations under this kind of identification assumption. Thus, we can analyze two issues of interest: (i) how the perceptions of private agents about fiscal policy lead the government to change optimal fiscal policy so that, in effect, the beliefs of agents are self-fulfilling. We can then identify the fiscal instrument (capital income taxes or debt return) acting as a shock absorber; and (ii) whether expectation errors of both Euler conditions are similar or not in the sense that they both have a significant correlation or similar volatilities. We show that private agents interpret that the expectation errors associated with the Euler equations of each asset (with and without risk) have a different nature.

The *Ramsey policies* are simulated again, assuming that expectation error ν_{2t} associated with the Euler condition of bonds (19), follows a white noise stochastic process with different standard deviation sizes. The results reported in table 4 confirm the stochastic properties of optimal capital income taxation obtained under the previous identification conditions, as well as the role of the debt return as a shock absorber.

As we mentioned above, there is a correspondence between ν_{2t} and e_t, implying that the variance of e_t is a function of the variance of ν_{2t}. Since $\hat{\tau}_{k_t}=\tau_{k_t}+e_t$, then, the variance of $\hat{\tau}_{k_t}$ is bounded, explaining the shock absorber role of the debt return.

An interesting simulation result is that the endogenous expectation error is uncorrelated with the exogenous expectation error, and its size, measured by the standard deviation, does not depend on the standard deviation assumed for the exogenous error. Then, in spite of both Euler conditions implying the same expected after-tax returns for capital and bonds, the way in which agents forecast both returns is quite different (expectation errors are orthogonal), denoting the different character of these two assets.

4. Conclusions

In the public finance literature it has been established that optimal fiscal policy is indeterminate in a dynamic and stochastic environment, allowing the existence of a capital market along with the government bond market. As a consequence, there are infinite combinations of capital income tax rates and debt return, both contingent on the state of nature, supporting the optimal allocations.

Given the indeterminacy issue, the complete characterization of the fiscal policy, requires the use of identification constraints. Chari, Christiano and Kehoe (1994) analyze those properties by restricting either capital taxes or debt to be not contingent on the state of nature.

In this paper we propose a different type of identification constraints in order to pin down one of the infinite state-contingent policies, each one supporting the same optimal allocations.

Under each identification constraint we characterize the cyclical properties of the policy variables. Under our identification constraints, both capital taxes and the debt return are state-contingent.

Three alternative kinds of identification conditions are considered: (i) restrictions on the dynamic and stochastic behavior of the debt path; (ii) an exogenous debt path, and (iii) an exogenous belief function.

The main result of this paper indicates that ex-post capital income taxation is zero and constant over the business cycle for any of the identification conditions used. Our analysis suggest that in the optimum, the government uses debt return as a shock absorber, keeping the capital income tax rate constant. The result is quite different from that of Chari, Christiano and Kehoe (1994) who, assuming uncontingent debt return, obtain a very volatile capital income tax rate.

Therefore, for a wide class of identification constraints making both capital income taxes and the debt return state-contingent, the optimal properties of those policy variables are robust, but they radically differ from the properties reported in previous papers with not fully state-contingent policies.

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Appendix . Solution method for Ramsey allocations.

For $t \geq 1$, the equations described below, (A.1)-(A.5), along with expressions (5), (9), (10) and (15) represent the set of conditions that allocations $\{c_t, n_t, k_t\}_{t=0}^{\infty}$ must fulfill, with the exogenous $\{z_t, g_t\}_{t=0}^{\infty}$, given realizations for the innovations of structural shocks (ε_{z_t}, ε_{g_t}):

$$-\frac{W_{c_t}}{W_{n_t}} = \frac{1}{F_{n_t}} \quad , \tag{A.1}$$

$$e^{\rho}\left[1 + \lambda\left(\frac{U_{cc_t}}{U_{c_t}}c_t + 1 + \frac{U_{cn_t}}{U_{c_t}}n_t\right)\right] = X_{1,t} + X_{2,t} + X_{3,t} \quad , \tag{A.2}$$

$$X_{1,t-1} = \hat{\beta}\frac{U_{c_t}}{U_{c_{t-1}}}(F_{k_t} - \delta) - \eta_{1,t} \quad , \tag{A.3}$$

$$X_{2,t-1} = \hat{\beta}\frac{U_{c_t}}{U_{c_{t-1}}} - \eta_{2,t} \quad , \tag{A.4}$$

$$X_{3,t-1} = \hat{\beta}\lambda\left(\frac{U_{cc_t}}{U_{c_{t-1}}}c_t + \frac{U_{c_t}}{U_{c_{t-1}}} + \frac{U_{cn_t}}{U_{c_{t-1}}}n_t\right)(1 - \delta + F_{k_{t-1}}) - \eta_{3,t} \quad , \tag{A.5}$$

where $X_{1,t}, X_{2,t}, X_{3,t}$ in expressions (A.2) to (A.5) represent expectations that arise when the global conditional expectation in the Euler condition of the problem defined by (14)-(15) is partitioned. The expectation term decomposition will be very useful when computing the ex-ante capital income tax rate later. $\eta_{1,t}, \eta_{2,t}, \eta_{3,t}$ represent the forecasting errors associated with the expectations. Summing up, we have twelve variables $\{c_t, n_t, k_t, G_t, g_t, z_t, X_{1,t}, X_{2,t}, X_{3,t}, \eta_{1,t}, \eta_{2,t}, \eta_{3,t}\}$ to be solved and nine equations (A.1)-(A.5) , (5), (9), (10) and (15) in each period. To compute all the system variables at each period, we need three additional conditions which will eliminate the non-converging subspace to the steady state.

An approximation to those conditions that eliminate such subspaces emerges from the first order approximation of the previously mentioned system of nine equations around the steady state:

$$\Gamma_0 y_t = \Gamma_1 y_{t-1} + \Psi\varepsilon_t + \Pi\eta_t \quad , \tag{A.6}$$

where matrices (□□□) Γ_0, Γ_1 contain the partial derivatives of each equation (A.1)-(A.5), (5), (9),

(10) and (15) with respect to each variable $(c_t, n_t, k_t, z_t, G_t, g_t, X_{1,t}, X_{2,t}, X_{3,t})$, evaluated in the deterministic steady state.

Vector y_t contains steady state deviations from the deterministic steady state $(c_t - c_{ss}, n_t - n_{ss}, k_t - k_{ss}, z_t, G_t - G, g_t, X_{1,t} - X_{1,ss}, X_{2,t} - X_{2,ss}, X_{3,t} - X_{3,ss})$. Innovations and expectation errors are contained in vectors $\varepsilon_t = (\varepsilon_{zt}, \varepsilon_{gt})'$, $\eta_t = (\eta_{1,t}, \eta_{2,t}, \eta_{3,t})'$.

Equations that describe the dynamic stochastic path of allocations can be simplified with logarithmic preferences, in particular it can be shown that if risk aversion is 1, then $X_{3,t-1} = 0 \ \forall t$, and therefore $\eta_{3,t} = 0 \ \forall t$.

The system consisting of (A.1)-(A.5), (5), (9), (10) and (15) can be simplified under this kind of preferences, eliminating equation (A.5) and rewriting equation (A.2) as:

$$e^{\rho} = X_{1,t} + X_{2,t} \ . \tag{A.7}$$

In such a case, we have two expectation errors, and to identify all the variables in the dynamic system two stability conditions are required to eliminate the non-converging subspace to the stochastic stationary equilibrium.

For a risk aversion strictly larger than one, the system linearization comes from (A.6). Given the partition of the Euler condition in the Ramsey problem, it is obvious that Γ_0 is non invertible[14].

It can be shown that Γ_0 is of order $(9{\times}9)$, and it has a maximum range of 7 when $\sigma \neq 1$, while with $\sigma = 1$ the matrix is of order $(8{\times}8)$, with a range of 7 as maximum. Since matrix Γ_0 is singular, it is necessary to compute a *QZ* decomposition to obtain generalized eigenvalues and eigenvectors.

For any pair of square matrices (Γ_0, Γ_1) there exist orthonormal matrices Q, Z, $(QQ' = ZZ' = I)$ and upper triangular matrices Λ, Ω such that $\Gamma_0 = Q'\Lambda Z'$ $\Gamma_1 = Q'\Omega Z'$.

Premultiplying the system (A.6) by Q and replacing $Z'y_t$ with u_t, we obtain:

$$\Lambda u_t = \Omega u_{t-1} + Q(\Psi \varepsilon_t + \Pi \eta_t) \ . \tag{A.8}$$

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[14] Notice that if the expectation were not partitioned, there would not be any problem of singularity for Γ_0 to compute the stability conditions. However, as we see later, this partition will be extremely useful to evaluate the ex-ante capital income tax rate.

We can rearrange matrices Λ, Ω, in order to partition (A.8) in such a way that the lower block corresponds to the equations associated with the unstable eigenvalues (those larger than $\hat{\beta}^{-1/2}$):

A zero element in the diagonal of matrix Λ implies some identification lack in the system;

$$\begin{bmatrix} \Lambda_{11} & \Lambda_{12} \\ \mathbf{0} & \Lambda_{22} \end{bmatrix} \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix} = \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \mathbf{0} & \Omega_{22} \end{bmatrix} \begin{bmatrix} u_{1,t-1} \\ u_{2,t-1} \end{bmatrix} + \begin{bmatrix} Q_{1\cdot} \\ Q_{2\cdot} \end{bmatrix} (\Psi \varepsilon_t + \Pi \eta_t) \quad . \tag{A.9}$$

we have two zero elements in the diagonal of Λ, with $\sigma \neq 1$. Since the elements in the diagonal of Ω in the same position are not zero, we have two infinite eigenvalues (so larger than $\hat{\beta}^{-1/2}$) that solve the system identification, that is, two stability conditions emerge, along with the remaining finite eigenvalue larger than $\hat{\beta}^{-1/2}$ (typical of saddle point solutions). This analysis allows us to identify the three expectation errors[15]. Therefore, constraining the space of solutions implies canceling the unstable paths associated with the unstable eigenvalues, that is:

$$u_{2t} = Z_{2\cdot}^{\prime} \; y_t = 0 \quad , \quad \forall t \quad , \tag{A.10}$$

that provides an approximated structure of relationships between the expectation errors and the innovations of the structural shocks:

$$Q_{2\cdot} \, \Psi \varepsilon_t = - Q_{2\cdot} \, \Pi \, \eta_t \quad . \tag{A.11}$$

Since we have two infinite eigenvalues and another finite but unstable eigenvalue, expression (A.10) is a set of three equations that eliminates the unstable subspaces. As was pointed out, the optimal allocations follow different rules at t=0 than from then onwards, so the computational algorithm is different from what we describe above.

Algorithm:

1. Given a value for λ :

i) At period t=0, *given τ_{k_0} and $_{R\,0}b_{-1}$, z_{-1}, g_{-1} and the realization of the innovations of structural shocks (ε_{z_0}, ε_{g_0}), from (5), (9) and (10) particularized at period 0, the three stability*

[15] In the case of σ=1, there are two eigenvalues larger than $\hat{\beta}^{-1/2}$: one is finite and the remaining is infinite; so the two expectation errors are identified.

conditions and the Euler conditions of the Ramsey problem at $t=0$ *and:*

$$\frac{W_{c_0}-\lambda U_{cc_0}[R_0 b_{-1}+\left(1+(F_{k_0}-\delta)(1-\tau_{k_0})\right)k_{-1}]}{W_{n_0}-\lambda U_{cn_0}[R_0 b_{-1}+\left(1+(F_{k_0}-\delta)(1-\tau_{k_0})\right)k_{-1}]-\lambda U_{c_0}F_{nk_0}(1-\tau_{k_0})k_0}=-\frac{1}{F_{n_0}}\ , \tag{A.12}$$

$$\frac{e^{\rho}}{U_{c_0}}\left\{W_0-\lambda U_{cc_0}\left[R_0 b_{-1}+\left(1+(r_0-\delta)(1-\tau_{k_0})\right)k_{-1}\right]\right\}=X_{1,0}+X_{2,0}+X_{3,0}\ , \tag{A.13}$$

we can compute $\{c_0(\lambda),\ n_0(\lambda),\ z_0,\ g_0,\ G_0,\ X_{1,0}(\lambda),\ X_{2,0}(\lambda),\ X_{3,0}(\lambda)\}$. *Equation (A.13) corresponds with (A.2) at period* $t=0$. *Finally, the capital stock at* $t=0$ *is computed from the aggregate resources constraint (15).*

ii) At $t=1$, *from* $\{c_0(\lambda),\ n_0(\lambda),\ k_0(\lambda),\ z_0,\ g_0,\ G_0,\ X_{1,0}(\lambda),\ X_{2,0}(\lambda),\ X_{3,0}(\lambda)\}$, *the realization of structural innovations* $\varepsilon_{z_1},\varepsilon_{g_1}$, *and using (A.1), (A.2), (5), (9), (10) and the three stability conditions,* $\{c_1(\lambda),\ n_1(\lambda),\ z_1,\ g_1,\ G_1,\ X_{1,1}(\lambda),\ X_{2,1}(\lambda),\ X_{3,1}(\lambda)\}$ *are computed. Capital stock at period 1 emerges from (15). The solution is computed recursively for the periods from here on.*

iii) Once the allocation paths are computed, the expectation error paths associated with the expectational terms are obtained from (A.3)-(A.5)[16].

2. From the described solution method, we obtain allocations of $\{c_t, n_t, k_t\}_{t=0}^{T}$ *as a function of λ. We then check whether the implementability constraint is satisfied. If so then we stop there, otherwise we iterate in λ and go back to 1, until finding those allocations that fulfill the implementability constraint, given the following convergence criterion:*

Following Aiyagari, Marcet, Sargent and Seppälä (2002), we iterate in the value of λ, by using the Gauss-Newton algorithm. We evaluate the implementability constraint (I.C.) numerically across 100 simulations as:

$$I.C.=\frac{1}{100}\sum_{i=1}^{100}\left[\sum_{t=0}^{T}\hat{\beta}^t(U_{c_{t,i}}c_{t,i}+U_{n_{t,i}}n_{t,i})-U_{c_{0,i}}\left(R_0 b_{-1}+\left(1+(F_{k_{0,i}}-\delta)(1-\tau_{k_0})\right)k_{-1}\right)\right] \tag{A.14}$$

where the conditional expectation has been approximated at t=0 as the average of the 100

[16] Notice that (A.6) system equations are the only approximation in the solution method.

*simulations for a large enough T (*T=1000*).*

The convergence criterion used to find the multiplier λ is reached when the numerical value of (18) is put into the interval: **(** *- dt(I.C.) , d.t.(I.C.)* **)***, where dt(I.C.) is the standard deviation of (A.14), obtained from the 100 simulations.*

As a by-product of the solution method, we can demonstrate that the ex-ante capital income tax rate is zero for all $t \geq 1$ when risk aversion is one, and it fluctuates around zero when risk aversion is different than one.

Proof: *In the definition of the ex-ante capital income tax rate, given by (20), we can see that the denominator corresponds with expectation* $X_{1,t}$, *defined by (A.3).*

The numerator of (20) can be obtained from the competitive equilibrium conditions (that must be fulfilled by the previously computed Ramsey allocations). In particular, partitioning the expectation term of the Euler condition of capital of the problem defined by (11)-(12) and comparing it with (A.3) and (A.4), we have:

$$\begin{aligned} e^{\rho} &= E_t \hat{\beta} \frac{U_{c_{t+1}}}{U_{c_t}} + E_t \hat{\beta} \frac{U_{c_{t+1}}}{U_{c_t}} (F_{k_{t+1}} - \delta) - E_t \hat{\beta} \frac{U_{c_{t+1}}}{U_{c_t}} (F_{k_{t+1}} - \delta) \tau_{k_{t+1}} \\ &= X_{2,t} + X_{1,t} - E_t \hat{\beta} \frac{U_{c_{t+1}}}{U_{c_t}} (F_{k_{t+1}} - \delta) \tau_{k_{t+1}} \ . \end{aligned} \tag{A.15}$$

Therefore:

$$E_t \hat{\beta} \frac{U_{c_{t+1}}}{U_{c_t}} (F_{k_{t+1}} - \delta) \tau_{k_{t+1}} = X_{1,t} + X_{2,t} - e^{\rho} \ , \tag{A.16}$$

identifying terms with (A.2):

$$E_t \hat{\beta} \frac{U_{c_{t+1}}}{U_{c_t}} (F_{k_{t+1}} - \delta) \tau_{k_{t+1}} = -X_{3,t} + e^{\rho} \lambda \left(\frac{U_{cc_t}}{U_{c_t}} c_t + 1 + \frac{U_{cn_t}}{U_{c_t}} n_t \right) \ . \tag{A.17}$$

With $\sigma=1$ *it can be shown that* $X_{3,t}=0$ $\forall t$*. Moreover, it is clear that under logarithmic preferences, the term inside brackets is zero. Therefore, the ex-ante capital income tax rate is zero for all* $t \geq 1$.

When $\sigma \neq 1$, *the ex-ante capital income tax rate is zero on average because, in the deterministic steady state, we get from the Euler condition of the Ramsey problem:*

$$\frac{e^{\rho}}{\hat{\beta}} = 1 - \delta + F_{k_{ss}} \ , \tag{A.18}$$

and the Euler condition for capital stock in the competitive equilibrium:

$$\frac{e^{\rho}}{\hat{\beta}} = 1 + (F_{k_{ss}} - \delta)(1 - \tau_{k_{ss}}) \ , \tag{A.19}$$

then, $\tau_{k_{ss}} = 0$. ■

Table 1. Baseline parameters.

Preferences:	
Discount rate (β)	.98
Risk aversion (σ)	1
Preference for leisure (θ)	.75
Technology:	
Output elasticity of labor (α)	.66
Growth rate (ρ)	.016
Capital depreciation rate (δ)	.08
Stochastic process of public consumption:	
Steady state public consumption (G)	.07
Autocorrelation of public consumption shock (ϕ_g)	.89
Standard deviation of innovation of public consumption shock (σ_{ε_g})	.07
Stochastic process of productivity shock:	
Autocorrelation of productivity shock (ϕ_z)	.81
Standard deviation of innovation of productivity shock (σ_{ε_z})	.04
Initial conditions:	
Outstanding debt ($R_0 b_{-1}$)	.20
Capital stock (k_{-1})	1.05
Capital income tax rate (τ_{k_0})	27.1%

Table 2. Stochastic simulation under stable behavior of the debt path. Properties of optimal tax rates. Statistics computed are means of 100 simulations of 1200 periods, where the first 200 periods are dropped. Standard deviation of statistics is in parentheses. NA indicates that the corresponding statistic is not well defined. Means and standard deviations are in percentage terms.

			Alternative stochastic processes for shocks	
	Baseline model	High risk aversion	Only technology shock	I.I.D.
			Labor income tax rate	
Mean	25.198 (.019)	22.588 (.012)	25.191 (.004)	25.198 (.002)
Standard deviation	.190 (.010)	.096 (.005)	.128 (.006)	.149 (.004)
Autocorrelation	.800 (.021)	.860 (.015)	.688 (.025)	-.069 (.025)
Correlation with public consumption	.731 (.033)	-.813 (.030)	NA	NA
Correlation with technology shock	.433 (.064)	-.468 (.067)	.541 (.038)	.929 (.007)
			Ex-post capital income tax rate	
Mean	.000 (.001)	.000 (.000)	.000 (.000)	.000 (.000)
Standard deviation	.002 (.022)	.000 (.001)	.000 (.000)	.000 (.001)
Autocorrelation	-.003 (.029)	-.002 (.015)	-.005 (.038)	-.002 (.014)
Correlation with public consumption	.000 (.047)	.008 (.037)	NA	-.005 (.035)
Correlation with technology shock	-.003 (.029)	.001 (.033)	.002 (.042)	-.001 (.021)

Table 3. Stochastic simulation under exogenous processes for the debt path. Properties of optimal capital income tax rate. Statistics computed are means of 100 simulations of 1200 periods, where the first 200 periods are dropped. Standard deviation of statistics is in parentheses. Means and standard deviations are in percentage terms. The exogenous stochastic path of the debt is: $b_t = b_{ss}(1-\phi) + \phi\, b_{t-1} + \varepsilon_t^b \qquad \varepsilon_t^b \sim N(0\ ,\sigma_b\ )$

	σ_b=.05			σ_b=.5		
	φ=1	φ=.95	φ=0	φ=1	φ=.95	φ=0
Mean	.000 (.000)	.000 (.002)	.000 (.000)	.000 (.000)	.000 (.001)	.000 (.002)
Standard deviation	.001 (.005)	.009 (.062)	.002 (.005)	.003 (.009)	.007 (.017)	.021 (.055)
Autocorrelation	-.003 (.050)	.000 (.013)	.002 (.025)	.003 (.055)	-.004 (.059)	.014 (.067)
Correlation with public consumption	-.001 (.029)	.002 (.032)	-.001 (.032)	.001 (.033)	.003 (.032)	.000 (.034)
Correlation with technology shock	.001 (.031)	-.002 (.035)	-.001 (.030)	-.005 (.032)	.006 (.027)	.001 (.029)

Table 4. Exogenous expectation error in the Euler condition of debt. Stochastic simulation under different sizes of standard deviation of expectation error. Properties of optimal capital income tax rate and endogenous expectation error. Statistics computed are means of 100 simulations of 1200 periods, where the first 200 periods are dropped. Standard deviation of statistics is in parentheses. Means and standard deviations are in percentage terms.

	Ex-post capital income tax rate		
	σ error=.5	σ error=.005	σ error=.00005
Mean	.000 (.000)	.000 (.000)	.000 (.000)
Standard deviation	.000 (.000)	.000 (.000)	.000 (.000)
Autocorrelation	-.003 (.013)	.002 (.025)	-.002 (.030)
Correlation with public consumption	.000 (.034)	.001 (.033)	-.002 (.029)
Correlation with technology shock	-.001 (.029)	.001 (.030)	-.003 (.030)
	Endogenous expectation error(optimality condition of capital)		
	σ error=.5	σ error=.005	σ error=.000005
Standard deviation	.014 (.000)	.015 (.000)	.014 (.000)
Correlation with exogenous expectation error	.004 (.034)	.002 (.029)	-.002 (.033)