Environmental fiscal policies might be ineffective to control pollution

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Abstract

In a one sector growth model with pollution in the utility function, the competitive equilibrium can be indeterminate for plausible values of the intertemporal substitution elasticity of consumption and under constant returns to scale. The tax rate on pollution does not enter the condition characterizing indeterminacy. This means that the government is not able to control emissions in the economy by using environmental policies. Non-separability between private consumption and pollution in the utility function is crucial for this result.

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1 Introduction

We characterize conditions on preferences under which competitive equilibrium can be indeterminate in a single sector growth model with pollution in the utility function, constant returns to scale in production and environmental taxes.

General equilibrium models that display indeterminacy have been focus of attention in recent years. Indeterminacy implies that there will be multiple paths converging to a given steady state. Hence, indeterminacy guarantees existence of a continuum of sunspot stationary equilibria, *i.e.*, stochastic rational expectations equilibria determined by perturbations unrelated to the uncertainty in economic fundamentals. The interest of sunspot equilibria is that they provide a theoretical justification for 'animal spirits' underlying economic instability. In our model, the indeterminacy is able to explain why two economies that share the same preferences, technology and initial capital stock might display different pollution paths regardless of the implemented policies.

Characteristics that produce indeterminacy of equilibria in one- or multi-sector real business cycle models or in endogenous growth models have been widely studied. Initially, it was shown that when these models are extended to include either productivity externalities or some market imperfection, indeterminacy can arise if social returns to scale in production are sufficiently high so that the labor demand curve has a slope which is not only positive, but also greater than that of the labor supply curve (see Benhabib and Farmer (1994), and Farmer and Guo (1994), for one-sector models). These models have been widely criticized because to produce multiple equilibria they require larger returns to scale than observed in actual data [see Aiyagari (1995)]. However, Fernández, Novales and Ruiz (2003) obtain multiple equilibria under constant returns to scale and endogenous government expenditures included in the utility function.

More recent work has described two situations in which increasing returns needed to produce indeterminacy are lower than initially thought, sometimes allowing for negatively sloped labor demand curves: i) two sector economies with externalities in one sector (Benhabib and Farmer (1996), Perli (1998), Benhabib and Nishimura (1998), Weder (1998), who introduces a durable consumption sector in addition to the investment and non-durable consumption sectors, or Barinci and Chéron (2001) in an environment with financial constraints); ii) one sector models with non-standard characteristics, such as non-separable preferences (Bennett and Farmer (2000)), or capacity utilization affecting production (Wen (1998)). These models are able to produce indeterminacy of equilibria with relatively low increasing returns to scale, but the elasticity of intertemporal substitution needed is often too large, relative to values considered in the real business cycle literature (Weder (1998) and Bennett and Farmer (2000), among others). This will generally lead to rather volatile consumption paths, against the observation in most actual economies. The same criticism applies to existing endogenous growth models producing indeterminacy (see Benhabib and Perli (1994), and Xie (1994)).

The contribution of our work is to show that indeterminacy could be present in a simple neoclassical growth model with a single source of externality: pollution entering in the utility function in a non-separable fashion with private consumption. We assume a government whose public expenditures are financed through distorting taxes (a proportional tax on income and an environmental tax), and its budget balances every period. Our economy can display indeterminacy for plausible values of the intertemporal elasticity of substitution of consumption and constant returns to scale. Furthermore, the region of the parameter space producing indeterminacy is independent of the income and pollution tax rates. The implication of this result is crucial for government environmental policies: the government is not able to control pollution level by using an appropiate environmental policy (for example, by implementing the pigouvian pollution tax rate).

The model is described in section 2. In section 3 we characterize the transitional dynamics of the model and the conditions for indeterminacy and we show why environmental fiscal policies might be ineffective to control pollution level. In section 4, we extend the model to an economy with non-separable preferences in pollution and effort and we show that such non-separability is crucial for the competitive equilibrium to be indetermined. The paper closes with some remarks in section 5.

2 The Economy

2.1 Firms

There is a continuum of identical firms operating in a competitive environment. For simplicity, we normalize their number to one. Aggregate production exhibits constant returns to scale:

$$y_t = Ak_t^{\alpha}n_t^{1-\alpha}, \quad \alpha \in (0,1) \quad , \tag{1}$$

A being the level of technology, y aggregate output and k, n the demand for the two production factors: capital and labor. Environmental pollution (P) is regarded as a side product of the capital stock use:

$$P_t = k_t^{\chi}, \qquad \text{with } \chi > 0. \tag{2}$$

To simplify we assume that P_t is a flow quantity. Firms rent capital from households at the interest rate r_t, pay wage ω_t on labor and pay a constant pollution tax τ_P on the level of capital stock. The single period profit function is $\pi_t = y_t - \omega_t n_t - r_t k_t - \tau_P k_t$.

Under perfectly competitive markets for the two factors, profit maximizing conditions are:

$$r_t = \alpha\frac{y_t}{k_t} - \tau_P \quad , \tag{3}$$

$$\omega_t = (1-\alpha)\frac{y_t}{n_t} \quad . \tag{4}$$

2.2 The Consumer's Problem

There is a continuum of identical consumers, all of them with the same preferences defined on private consumption, pollution and leisure, according to the instantaneous utility function:

$$U(c_t, P_t, n_t) = \frac{(c_t P_t^{-\eta})^{1-\sigma}}{1-\sigma} - \gamma n_t \quad , \text{ with } \gamma, \sigma > 0,\ \sigma \neq 1 \ , \tag{5}$$

where c_t is private consumption, P_t aggregate pollution, and n_t labor supply[1]. σ is the inverse of intertemporal substitution elasticity of consumption, η the weight of pollution in utility and γ the marginal disutility of work.

Each consumer receives income on labor and capital, that can be used to consume, save, and pay taxes at a constant rate $\tau \in (0,1)$ on the two sources of income:

$$c_t + \dot{k}_t + \delta k_t = (1-\tau)\left(\omega_t n_t + r_t k_t\right). \tag{6}$$

Hence, the representative consumer faces the optimization problem:

$$\underset{\{c_t, n_t\}}{Max} \quad \int_0^\infty e^{-\rho t} U(c_t, P_t, n_t)\ dt \tag{7}$$

subject to (6) and given k_0 where ρ is the rate of pure time preference. The household ignores the environmental utility externality.

First order conditions for this problem are given by:

$$c_t^{-\sigma} P_t^{-\eta(1-\sigma)} = \lambda_t \quad , \tag{8}$$

$$\gamma = \lambda_t (1-\tau)\omega_t \quad , \tag{9}$$

$$-\frac{\dot{\lambda}_t}{\lambda_t} = (1-\tau) r_t - (\delta + \rho) \quad , \tag{10}$$

$$\lim_{t\to\infty} \ e^{-\rho t}\lambda_t = 0 \quad , \tag{11}$$

λ_t being the multiplier associated to the consumer's budget constraint.

Combining (8) and (9) and using (2):

[1]This function arises from including the assumption of employment indivisibility (Hansen(1985)) into the standard utility function used in the literature studying the relationship between growth and pollution.

$$\gamma = c_t^{-\sigma} k_t^{-\chi\eta(1-\sigma)} (1-\tau)\omega_t. \tag{12}$$

Combining (8) and (10) and using (2):

$$\sigma \frac{\dot{c}_t}{c_t} + \chi\eta\,(1-\sigma)\,\frac{\dot{k}_t}{k_t} = (1-\tau)r_t - (\delta+\rho). \tag{13}$$

2.3 Government

The government chooses a contant income tax rate τ and an environmental tax τ_P and balances its budget every period. Hence, the instantaneous government budget constraint is:

$$g_t = \tau\,(\omega_t n_t + r_t k_t)\; + \tau_P\; k_t, \tag{14}$$

where g_t represents government spending on goods and services which are not an argument of the utility or the production functions.

2.4 Equilibrium

Given τ, τ_P and k_0, a *competitive equilibrium* is a set of trajectories $\{c_t, n_t, k_t, P_t, g_t, \omega_t, r_t\}$ such that: i) given the paths for $\{\omega_t, r_t, P_t\}$, the paths for $\{c_t, n_t, k_t\}$ solve consumer's problem [(6), (8), (11), (12), (13)]; ii) given $\{\omega_t, r_t\}$, the paths for $\{n_t, k_t\}$ solve the firm's problem [(3) and (4)]; iii) government budget constraint (14) holds every period, and iv) markets clear, in particular, from (6), (1) and (14) we obtain the aggregate resources constraint

$$Ak_t^{\alpha} n_t^{1-\alpha} = c_t + \left(\dot{k}_t + \delta\; k_t\right) + g_t\;, \tag{15}$$

where (substituing (3) and (4) in (14)) g_t is

$$g_t = \tau A k_t^{\alpha} n_t^{1-\alpha} + \tau_P\,(1-\tau)\,k_t. \tag{16}$$

3 Local Dynamics

Let us now discuss the local properties of the equilibrium dynamics of the system.

Definition: A *steady state* is a vector $(c_{ss}, k_{ss}, n_{ss}, \lambda_{ss}, P_{ss})$ satisfying the equations for competitive equilibrium such that if it is ever reached, the system will stay at that point forever ($\dot{c}_{ss} = \dot{k}_{ss} = \dot{\lambda}_{ss} = 0$):

$$\frac{k_{ss}}{n_{ss}} = \left[\frac{(1-\tau)\, A\alpha}{\delta + \rho + \tau_P\,(1-\tau)}\right]^{\frac{1}{1-\alpha}}, \tag{17}$$

$$\frac{c_{ss}}{k_{ss}} = \frac{[\delta + \tau_P\,(1-\tau)]\,(1-\alpha) + \rho}{\alpha}, \tag{18}$$

$$n_{ss} = \left[\frac{(1-\tau)\, A\,(1-\alpha)}{\gamma}\right]^{\frac{1}{\sigma+\chi\eta(1-\sigma)}} \left(\frac{k_{ss}}{n_{ss}}\right)^{\frac{\alpha}{\sigma+\chi\eta(1-\sigma)}-1} \left(\frac{c_{ss}}{k_{ss}}\right)^{-\frac{\sigma}{\sigma+\chi\eta(1-\sigma)}}, \tag{19}$$

$$P_{ss} = k_{ss}^{\chi}, \tag{20}$$

$$\lambda_{ss} = c_{ss}^{-\sigma} k_{ss}^{-\chi\eta(1-\sigma)}, \tag{21}$$

where (17) comes from (10), (1) and (3); (18) refers to (15)-(17); (19) comes from (12) together with (1), (4), (17) and (18); (20) comes from (2), and (21) is obtained from (8) and (2). All these equations are evaluated at the steady-state.

To characterize the local dynamics of the system it is enough to analyze the transition of the state and co-state variables (k_t, λ_t). That requires to write the control variables (c_t, n_t) as a function of (k_t, λ_t).

$$n_t = \left[\frac{(1-\alpha)\,(1-\tau)\, A}{\gamma}\lambda_t\right]^{\frac{1}{\alpha}} k_t, \tag{22}$$

$$c_t = \lambda_t^{-\frac{1}{\sigma}} - k_t^{-\frac{\chi\eta(1-\sigma)}{\sigma}} \tag{23}$$

where (22) comes from (9), (1) and (4); and (23) is obtained from (8) together with (2).

The first order approximation to the dynamic system made up by equations (10) and (15) is:

$$\begin{bmatrix} \dot{k}_t \\ \dot{\lambda}_t \end{bmatrix} = \Gamma \begin{bmatrix} k_t - k_{ss} \\ \lambda_t - \lambda_{ss} \end{bmatrix},$$

with transition matrix Γ (see Appendix):

$$\Gamma = \begin{bmatrix} \phi_1 & \left(\frac{k_{ss}}{\lambda_{ss}}\right)\left[\frac{(1-\alpha)(\delta+\rho+\tau_P(1-\tau))}{\alpha^2} + \frac{(1-\alpha)(\delta+\tau_P(1-\tau))+\rho}{\sigma\alpha}\right] \\ 0 & \phi_2 \end{bmatrix}, \tag{24}$$

where $\phi_1 = [\sigma + \chi\eta(1-\sigma)] \frac{(1-\alpha)[\delta+\tau_P(1-\tau)]+\rho}{\sigma\alpha}$ and $\phi_2 = -\frac{(1-\alpha)[\delta+\rho+\tau_P(1-\tau)]}{\alpha} < 0$.

Since one of the variables in the system is predetermined and the other is free, the system will have a unique, locally determined equilibrium when the steady state is a saddle point, which requires the two characteristic roots of matrix Γ to be of opposite sign. Indeterminacy of equilibria arises only when the two roots of Γ have negative real parts.

Proposition 1 *Indeterminacy results if and only if* $\sigma + \chi\eta(1-\sigma) < 0$.

Proof. Since matrix Γ is triangular, ϕ_1 and ϕ_2 are the two eigenvalues. ϕ_2 is always negative and ϕ_1 is negative when $\sigma + \chi\eta(1-\sigma) < 0$. Therefore, indeterminacy arises when this condition is satisfied. ■

From proposition 1, note that $\sigma > 1$ is a necessary condition for indeterminacy. Furthermore, indeterminacy only arises in economies with $\chi\eta > \frac{\sigma}{\sigma-1} > 1$. That is, economies where the weight of pollution in the utility function is high (η) and/or economies where the elasticity of pollution with respect to capital is high (χ). Consider two economies that share the same environmental preference (η) but one of them with a dirtier technology than the other (larger χ). The economy with a larger χ (this is more often the case for developing countries) will experience indeterminacy of equilibria more likely than the other.

Corollary 2 *Indeterminacy can arise for any value of the income and pollution tax rates and hence, for any level of government expenditure.*

Proof. Note that in this economy fiscal policy parameters do not affect the condition guaranteeing indeterminacy of equilibria. ■

Hence, if the structural (preferences and technology) parameters place the economy inside the indeterminacy region, public policies will not be able to move the economy towards the determinacy region and achieve its stabilization. That is, the model could display stationary sunspot equilibria in which there exist fluctuations in employment, consumption, pollution, and so on, despite the absence of fluctuations in the fundamental features of the economy.

3.1 Indeterminacy and pollution dynamics

Now we show that in an economy where indeterminacy of equilibria is present, the government can not control the pollution regardless the policy implemented by the government.

Proposition 3 *When* $\sigma + \chi\eta(1-\sigma) < 0$*, starting from an initial capital stock* k_0*, the economy has a continuun of equilibria, indexed by* n_0*.*

Proof: Note that transition matrix Γ is triangular (24). Furthermore, under $\sigma + \chi\eta(1-\sigma) < 0$, both eigenvalues ϕ_1 and ϕ_2 are negative. Therefore, the local solution is:

$$\lambda_t = \lambda_{ss} + e^{\phi_2 t}(\lambda_0 - \lambda_{ss}), \tag{25}$$

$$k_t = k_{ss} + (k_0 - k_{ss})\, e^{\phi_1 t} + \frac{\phi_1}{\phi_2 - \phi_1}\left(e^{\phi_2 t} - e^{\phi_1 t}\right)(\lambda_0 - \lambda_{ss}), \tag{26}$$

and from (22):

$$\lambda_0 = \frac{\gamma}{(1-\alpha)(1-\tau) A} n_0^{\sigma} \frac{1}{k_0}. \tag{27}$$

Therefore, an initial value of labor determinates the path of capital stock and the path of the marginal utility of consumption. From (2), (22) and (23), an initial value of labor also determinates the path of pollution, labor and consumption. □

Corollary 4: *If* $\sigma + \chi\eta(1-\sigma) < 0$, $k_0 = k_{ss}$ *and* $n_0 < n_{ss}$, *then:*
1) $k_t > k_{ss}$, *for all* $t > 0$, *and*
2) $P_t > P_{ss}$, *for all* $t > 0$,
while if $n_0 > n_{ss}$, *opposite results are found.*

Proof: Imposing $n_0 < n_{ss}$ in (27), we obtain that $\lambda_0 < \lambda_{ss}$. Since $k_0 = k_{ss}$, (26) can be written as:

$$k_t = k_{ss} + \frac{\phi_1}{\phi_2 - \phi_1}\left(e^{\phi_2 t} - e^{\phi_1 t}\right)(\lambda_0 - \lambda_{ss}), \quad \forall t > 0. \tag{28}$$

Assume that $\phi_2 > \phi_1$ and $\phi_1 < 0$, then $\frac{\phi_1}{\phi_2 - \phi_1} < 0$ and $e^{\phi_2 t} > e^{\phi_1 t}$. These facts together with $\lambda_0 < \lambda_{ss}$ imply that $k_t > k_{ss}$, $\forall t > 0$. From (2), since $k_0 = k_{ss}$, then $P_0 = P_{ss}$ and $k_t > k_{ss}$ imply that $P_t > P_{ss}$, $\forall t > 0$. When $\phi_2 < \phi_1$ the same results are obtained.

When $n_0 > n_{ss}$ an analogous proof is applicable.□

Assume two economies which share the same fiscal policy, technology and preferences and both of which are in the steady-state. Assume also these economies are placed on the indeterminacy region of the parameter space and, in any moment of time, the agents have different expectations about future, yielding different equilibrium labor at the present in both economies. In particular, assume that in one case labor is above its steady-state value while in the other economy, labor is below its steady-state value. Corollary 8 has shown that economies 1 and 2 will display different paths of capital and pollution while they are converging back to the steady-state (see figure 1). Since the choice of initial labor does not depend on the implemented policy by the government at the present, the convergence path of pollution will not depend on the implemented income or environmental tax rates. Therefore, the government can not do anything to control the pollution in this economy since the pollution path depends, not only on the tax rates, but also on the initial level of labor supply (n_0). Proposition 5 bellow shows that this fact is not present when there is not indeterminacy of equilibria.

[Insert Figure 1]

Proposition 5: *When* $\sigma + \chi\eta(1-\sigma) > 0$, *government can control the pollution in this economy through a environmental tax since the pollution path does not depend on* n_0.

Proof: Transition matrix Γ is triangular (24). Furthermore, under $\sigma+\chi\eta\,(1-\sigma)>0$, the eigenvalue ϕ_1 is positive while ϕ_2 is negative (saddle-path equilibrium). Therefore, the local solution is (25), together with:

$$k_t = k_{ss} + (k_0 - k_{ss})\, e^{\phi_2 t},$$

and λ_0 can take only the value:

$$\lambda_0 = \lambda_{ss} + \frac{\phi_2 - \phi_1}{\phi_1}\,(k_0 - k_{ss})\,.$$

Since $\phi_2 = -\frac{(1-\alpha)[\delta+\rho+\tau_P(1-\tau)]}{\alpha}$, the path of the capital stock depends on both the income tax rate and the pollution tax. Pollution is a function increasing in capital stock (2); so, the government can control the path of pollution through taxes. □

3.2 Indeterminacy and the labor market

Most work discussing indeterminacy of equilibria in one sector models shows that it is associated with upward sloping labor demand curves, downward sloping labor supply curves, or both (see the survey in Benhabib and Farmer (1999)). In our model, the labor demand schedule is downward sloping as a consequence of the constant returns to scale.

Two definitions of the labor supply curve have been used in previous literature: the standard and the Frisch curve[2]. Several papers have proved that under externalities in the utility function and/or non-separable preferences in private consumption and leisure, the appropiate labor supply curve to understand why indeterminacy arises, is the Frisch one (Fernández, Novales and Ruiz (2003) or Benett and Farmer (2000)). Since our model includes an environmental externality in utility non-separable with consumption, we relate our condicion for indetermination to the Frisch labor supply curve.

In our model, the Frisch labor supply curve is infinitely elastic under indeterminacy and non-indeterminacy of equilibria. From (9), households works any time at the wage given by:

$$\omega_t = \frac{\gamma}{\lambda_t(1-\tau)}.$$

Hence, the appropiate condition for indeterminacy of equilibria to arise in an economy is not related to the slope of the labor supply curve.

From (8) together with (2) it is obtained that, under indeterminacy (that is, when $\sigma+\chi\eta\,(1-\sigma)<0$), the marginal utility of consumption (λ) increases when both consumption and capital stock increases. Therefore, the Frisch labor supply curve shifts downwards. On the contrary, when equilibrium is determinate the marginal utility of consumption

[2]The standard labor supply curve is the quantity of labor supplied as a function of the real wage, holding constant consumption. The Frisch labor supply curve is the quantity of labor supplied as a function of the real wages, holding constant the marginal utility of consumption. Browing (1982) and Browing, Deaton and Irish (1985) introduce the definition of a Frisch demand.

decreases as a consequence of an increase in both consumption and capital stock and the Frisch labor supply curve shifts upwards. Therefore, the condition for indeterminacy of the competitive equilibrium to arise in the economy that we consider is that the Frisch labor supply curve shifts downwards when both consumption and capital stock increases.

The intuition behind the existence of stationary sunspot equilibria is as follows. If agents expect future income tax rates will be above average, then future expected wage net of taxes will be lower and hence, future hours worked will decrease (see (12)). As a consequence, the expected rate of return on capital decreases, since the marginal product of capital is increasing in the labor/capital ratio. The decrease in the expected interest rate produces a decline in current private accumulation of capital stock. Then, future hours worked and capital stock will be lower and hence, future output and consumption level will also be lower.

Only when indeterminacy is present (i.e. $\sigma + \chi\eta\,(1-\sigma) < 0$), the decrease in future consumption and capital stock will yield a new upward shift in the Frisch labor supply curve, and hours worked in equilibrium will positively be lower. Since future capital stock and hours worked are lower, future aggregate income will also be below-average and, given an income tax rate, future government revenues would decrease. As a consequence, given a public spending, the only feasible behaviour for government is to rise the income tax rate in order to satisfy its budget constraint. Thus, private expectations of above-average future income tax rates lead to higher future income tax rates, regardless the government wishes.

4 Extending the model: non-separable preferences in pollution and effort

The model studied is a particular case of an economy with non-separable preferences in pollution and effort. Proposition 6 bellow shows that indeterminacy can also arise in this more general model. Let $W(c_t, P_t, n_t) = \frac{(c_t P_t^{-\eta})^{1-\sigma}}{1-\sigma} - \frac{\gamma}{1+\psi}\left(P_t^{\phi} n_t\right)^{1+\psi}$, $\sigma > 0$, $\sigma \neq 1$, $\eta > 0$, $\psi \geq 0$. The sign of ϕ would be negative in an economy in which pollution reduces the marginal utility of time employed out of work (for example, because of polluted leisure areas), and hence, lowers the marginal disutility of effort; on the contrary, it would be positive for an economy in which pollution worsens the conditions at work and hence increases the marginal disutility of effort (for example, by increasing health costs).

In this model, the labor supply is (in log form):

$$\ln w_t = \ln\left(\frac{\gamma}{1-\tau}\right) + [\phi\,(1+\psi) + \eta\,(1-\sigma)]\ln P_t + \sigma \ln c_t + \psi \ln n_t,$$

where ψ is the labor supply curve slope.

Assuming $\sigma > 1$, note that, if $\phi < 0$ the labor supply curve shifts to the right when the pollution level increases. That is, the labor supply is larger for a given wage. On the contrary, if $\phi > 0$ the labor supply curve could shift to the left (if $[\phi\,(1+\psi) + \eta\,(1-\sigma)] < 0$) after a rise in the pollution level. Hence, the labor supply is lower for a given wage.

Proposition 6: *If preferences of the representative agent in the economy are represented by the utility function* $W(c_t, P_t, n_t)$, *the competitive equilibrium is indeterminate if and only if*

$$\rho\left(\frac{\sigma+\chi\eta\left(1-\sigma\right)}{\sigma}\right)+\frac{\Omega\,\chi}{\alpha\sigma\left(\psi+\alpha\right)}\left[\eta\left(1-\sigma\right)\left(\psi+\alpha\right)-\sigma\phi\left(1+\psi\right)\right]<0,$$

and

$$-\Xi\,\frac{\Omega}{\psi+\alpha}\,\left[\frac{\rho}{\sigma}+\frac{\left(1-\alpha\right)\left(\delta+\tau_P(1-\tau)+\rho\right)}{\alpha\sigma}\right]>0,$$

where

$$\begin{aligned}\Omega &= \left(1-\alpha\right)\left(\delta+\tau_P(1-\tau)+\rho\right),\\ \Xi &= \left(\sigma+\chi\eta(1-\sigma)+\psi+\chi\phi(1+\psi)\right).\end{aligned}$$

Proof: See appendix. □

Corollary 7: *If preferences of the representative agent are represented by the utility function* $W(c_t, P_t, n_t)$ *and* $\phi > 0$, *the competitive equilibrium is indeterminate if and only if* $\sigma+\chi\eta(1-\sigma)+\psi+\chi\phi(1+\psi)<0$.

Proof: See appendix. □

From corollary 7, note that $\sigma > 1$ is also a necessary condition for indeterminacy when $\phi > 0$. Furthermore, the higher ϕ and ψ, the narrower the parameter space for which equilibrium is indeterminate, because higher values for χ and η are required.

Corollary 8: *If* $\phi > 0$, *non-separability of private consumption and pollution in the utility function* $(\eta > 0)$ *is crucial for the competitive equilibrium to be indeterminate.*

Proof: Assume that $\eta = 0$ in utility function $W(c, P, n)$. There is no point in the parameter space for which equilibrium is indeterminate since $\sigma+\chi\eta(1-\sigma)+\psi+\chi\phi(1+\psi)<0$ is never satisfied if $\phi > 0$.□

5 Conclusions

For a class of utility functions characterized by non-separability between pollution and consumption, we have shown that the competitive equilibrium can be indeterminate for plausible values of the intertemporal elasticity of substitution, under government expenditures financed through a fixed income and pollution tax rates, and constant returns to scale in production. The region in the parameter space leading to indeterminacy is characterized by:

i) The tax rates on income and pollution do not enter the condition characterizing indeterminacy. This means that the government cannot control emissions in the economy only by using environmental policies.

ii) Indeterminacy only can arise in economies where the weight of pollution in the utility function is high (η) and/or economies where the elasticity of pollution respect to capital is high (χ).

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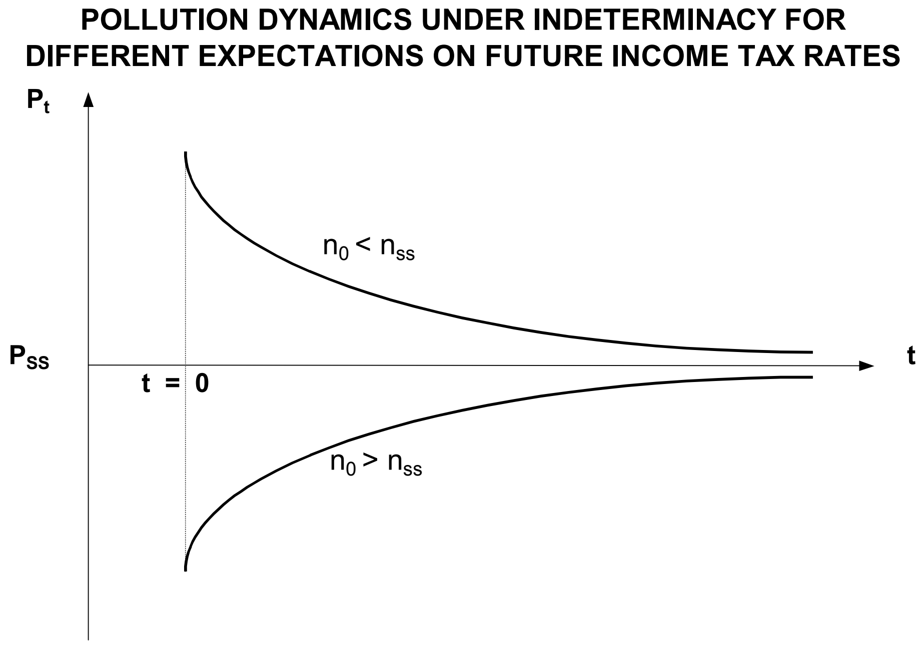

POLLUTION DYNAMICS UNDER INDETERMINACY FOR DIFFERENT EXPECTATIONS ON FUTURE INCOME TAX RATES
P_t
$n_0 < n_{ss}$
P_{SS}
t
t = 0
$n_0 > n_{ss}$

Figure 1.

Appendix

Deriving the elements of Γ **:**

From (15) and (16) we get:

$$\dot{k} = (1-\tau)\, Ak^{\alpha} n(k,\lambda)^{1-\alpha} - (\delta + \tau_P\,(1-\tau))\, k - c(k,\lambda). \tag{29}$$

where $c\,(k,\lambda)$ is given by (23) and $n\,(k,\lambda)$ is given by (22), while from (10), (1) and (3) we get:

$$\dot{\lambda} = -\lambda \left[(1-\tau)\, \alpha A k^{\alpha-1} n\,(k,\lambda)^{1-\alpha} - (\delta + \rho + \tau_P\,(1-\tau)) \right]. \tag{30}$$

The dynamics of system (29)-(30) in (k,λ) around steady state can be characterized through the linear approximation:

$$\begin{bmatrix} \dot{k} \\ \dot{\lambda} \end{bmatrix} = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{bmatrix} \begin{bmatrix} k - k_{ss} \\ \lambda - \lambda_{ss} \end{bmatrix}, \tag{31}$$

with:

$$\Gamma_{11} = (1-\tau)\, A\alpha \left(\frac{k_{ss}}{n_{ss}}\right)^{\alpha-1} - (\delta + \tau_P\,(1-\tau)) + (1-\tau)(1-\alpha)A \left(\frac{k_{ss}}{n_{ss}}\right)^{\alpha} \frac{\partial n}{\partial k}|_{ss} - \frac{\partial c}{\partial k}|_{ss} \tag{32}$$

$$\Gamma_{12} = (1-\tau)(1-\alpha)A \left(\frac{k_{ss}}{n_{ss}}\right)^{\alpha} \frac{\partial n}{\partial \lambda}|_{ss} - \frac{\partial c}{\partial \lambda}|_{ss} \quad , \tag{33}$$

$$\Gamma_{21} = -\lambda_{ss}(1-\tau)\alpha A \left[(\alpha-1)\, k_{ss}^{\alpha-2} n_{ss}^{1-\alpha} + (1-\alpha)\, k_{ss}^{\alpha-1} n_{ss}^{-\alpha} \frac{\partial n}{\partial k}|_{ss} \right] \quad , \tag{34}$$

$$\Gamma_{22} = -\lambda_{ss}(1-\tau)\alpha A\,(1-\alpha)\, k_{ss}^{\alpha-1} n_{ss}^{-\alpha} \frac{\partial n}{\partial \lambda}|_{ss} \quad , \tag{35}$$

with $\frac{\partial x}{\partial z}|_{ss}$, $x = n, c$ and $z = k, \lambda$, denoting partial derivatives, evaluated at steady state.

From (22) and (23), and using (16), (17) and (21), the partial derivatives in (32)-(35) are:

$$\begin{aligned} \frac{\partial n}{\partial \lambda}|_{ss} &= \frac{1}{\alpha} \frac{n_{ss}}{\lambda_{ss}} \ , \\ \frac{\partial c}{\partial \lambda}|_{ss} &= -\frac{1}{\sigma} \frac{c_{ss}}{\lambda_{ss}}, \\ \frac{\partial n}{\partial k}|_{ss} &= \frac{n_{ss}}{k_{ss}} \ , \\ \frac{\partial c}{\partial k}|_{ss} &= -\frac{\chi\eta\,(1-\sigma)}{\sigma} \frac{c_{ss}}{k_{ss}}. \end{aligned}$$

Using (17) to eliminate $\left(\frac{k_{ss}}{n_{ss}}\right)^{\alpha-1} = \frac{\delta+\rho+\tau_P(1-\tau)}{(1-\tau)A\alpha}$, together with these expressions and (18) yields:

$$\begin{aligned}\Gamma_{11} &= \rho + (1-\alpha)\frac{\delta+\rho+\tau_P(1-\tau)}{\alpha} + \frac{\chi\eta(1-\sigma)}{\sigma}\frac{[\delta+\tau_P(1-\tau)](1-\alpha)+\rho}{\alpha} \\ &= [\sigma+\chi\eta(1-\sigma)]\frac{(1-\alpha)(\delta+\tau_P(1-\tau))+\rho}{\sigma\alpha}\end{aligned} \tag{36}$$

$$\Gamma_{12} = \frac{k_{ss}}{\lambda_{ss}}\left[(1-\tau)(1-\alpha)A\left(\frac{k_{ss}}{n_{ss}}\right)^{\alpha-1}\frac{1}{\alpha} + \frac{1}{\sigma}\frac{[\delta+\tau_P(1-\tau)](1-\alpha)+\rho}{\alpha}\right] =$$

$$= \frac{k_{ss}}{\lambda_{ss}}\left[(1-\alpha)\frac{\delta+\rho+\tau_P(1-\tau)}{\alpha^2} + \frac{[\delta+\tau_P(1-\tau)](1-\alpha)+\rho}{\sigma\alpha}\right] \tag{37}$$

$$\Gamma_{21} = -\lambda_{ss}(1-\tau)\alpha A\left[(\alpha-1)k_{ss}^{\alpha-2}n_{ss}^{1-\alpha} + (1-\alpha)k_{ss}^{\alpha-2}n_{ss}^{1-\alpha}\right] = 0, \tag{38}$$

$$\Gamma_{22} = -(1-\tau)\alpha A(1-\alpha)k_{ss}^{\alpha-1}n_{ss}^{1-\alpha}\frac{1}{\alpha} = -\frac{(1-\alpha)[\delta+\rho+\tau_P(1-\tau)]}{\alpha}. \tag{39}$$

Proof of proposition 6:

For utility function $W(c,g,n)$, the control variables (c,n), as functions of (k,λ) are given by (23) and:

$$n = \left[\frac{(1-\alpha)(1-\tau)A}{\gamma}\lambda k^{\alpha-\chi\phi(1+\psi)}\right]^{\frac{1}{\psi+\alpha}},$$

which is analogue of (22), with partial derivatives:

$$\begin{aligned}\frac{\partial n}{\partial k}|_{ss} &= \frac{\alpha-\chi\phi(1+\psi)}{\psi+\alpha}\frac{n_{ss}}{k_{ss}}, \\ \frac{\partial n}{\partial \lambda}|_{ss} &= \frac{1}{\psi+\alpha}\frac{n_{ss}}{\lambda_{ss}},\end{aligned}$$

and, again, $\left(\frac{k_{ss}}{n_{ss}}\right)^{\alpha-1} = \frac{\delta+\rho+\tau_P(1-\tau)}{(1-\tau)A\alpha}$, so that (36)-(39) become:

$$\begin{aligned}
\Gamma_{11} &= \rho+\frac{\chi\eta\,(1-\sigma)}{\alpha\sigma}\left[(1-\alpha)\,(\delta+\tau_P(1-\tau))+\rho\right]+ \\
&\quad \frac{(1-\alpha)\,(\delta+\tau_P(1-\tau)+\rho)\,(\alpha-\chi\phi\,(1+\psi))}{\alpha\,(\psi+\alpha)}, \\
\Gamma_{12} &= \left[\frac{(1-\alpha)\,(\delta+\tau_P(1-\tau)+\rho)}{\alpha\,(\psi+\alpha)}+\frac{(1-\alpha)\,(\delta+\tau_P(1-\tau))+\rho}{\alpha\sigma}\right]\frac{k_{ss}}{\lambda_{ss}} \\
\Gamma_{21} &= \left[\frac{(1-\alpha)\,(\delta+\tau_P(1-\tau)+\rho)\,(\psi+\chi\phi\,(1+\psi))}{\psi+\alpha}\right]\frac{\lambda_{ss}}{k_{ss}}, \\
\Gamma_{22} &= -\frac{(1-\alpha)\,(\delta+\tau_P(1-\tau)+\rho)}{\psi+\alpha}\;.
\end{aligned}$$

Indeterminacy of equilibria arises only when the two roots of Γ are negative. This means that trace of Γ be negative, and its determinant be positive. That is

$Tr(\Gamma)=\Gamma_{11}+\Gamma_{22}=\rho\left(\frac{\sigma+\chi\eta(1-\sigma)}{\sigma}\right)+\frac{\Omega\,\chi}{\alpha\sigma(\psi+\alpha)}\left[\eta\,(1-\sigma)\,(\psi+\alpha)-\sigma\phi\,(1+\psi)\right]<0,$
and
$Det(\Gamma)=-\Xi\,\frac{\Omega}{\psi+\alpha}\,\left[\frac{\rho}{\sigma}+\frac{(1-\alpha)(\delta+\tau_P(1-\tau)+\rho)}{\alpha\sigma}\right]>0,$
where

$$\begin{aligned}
\Omega &= (1-\alpha)\,(\delta+\tau_P(1-\tau)+\rho)\,, \\
\Xi &= (\sigma+\chi\eta(1-\sigma)+\psi+\chi\phi(1+\psi))\,.
\end{aligned}$$

□

Proof of corollary 7: If $\phi>0$ then $Tr(\Gamma)<0$ only if $\sigma>1$ and $\sigma+\chi\eta\,(1-\sigma)<0$, and $Det(\Gamma)>0$ if and only if $\sigma+\chi\eta(1-\sigma)+\psi+\chi\phi(1+\psi)<0$. When $\sigma+\chi\eta(1-\sigma)+\psi+\chi\phi(1+\psi)<0$, then the condition for the trace ($\sigma+\chi\eta\,(1-\sigma)<0$) is also satisfied.□