Double Dividend in an Endogenous Growth Model with Pollution and Abatement*, †

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Abstract

This paper discusses whether by implementing an environmental tax reform, a government may achieve a double dividend. We consider the simplest endogenous growth model (the AK model) and include a negative environmental externality in the utility function. Pollution flow can be reduced by means of private abatement activities. There is a predetermined non-optimal level of public spending financed by environmental taxes and pre-existing taxes on income and consumption. The major contribution of the paper is to show that, under this simple framework, a double dividend may arise if tax reform consists of substituing environmental tax for income tax, in such a way that the government budget constraint holds in a present value sense, that is allowing public debt issuing.

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1 Introduction

A revenue neutral green tax reform involves introducing or increasing an environmentally oriented tax and using the revenues to finance reductions in pre-existing distorsionary taxes. Thus, environmental taxes seem to be an attractive instrument to potentially enhance environmental quality without damaging economic growth. An environmental tax reform will yield a double dividend if it is able to achieve a cleaner environment and a less distortionary tax system, leading to an increase in welfare from environmental amenities (so-called *green dividend*) and from private commodities (so-called *blue dividend* or private welfare). This is a *strong version* of the double dividend provided by Goulder (1995).

We analyze whether a double-dividend is possible in an AK economy (Rebelo (1990)). We consider revenue-neutral environmental tax reforms: pollution taxes are increased, devoting the proceeds to cut distortionary taxes on income, in such a way that the alternative tax systems can finance the same path of public expenditures, perhaps through short-term deficit financing while still allowing the government to balance its budget in the long run.

The literature on environmental tax reform has often explored the double dividend issue in a static framework (see e.g. Ulph (1992), Bovenberg and de Mooij (1994), Bovenberg and van der Ploeg (1994), Parry (1995), Goulder (1995), Parry and Bento (2000) or Proost and van Regemorter (1995)). Bovenberg and de Mooij (1997) (BM from now on) have analyzed whether environmental fiscal policies can achieve a double dividend within a dynamic model of endogenous growth (a modified Barro (1990) model). They model the environment as a public consumption good as well as a public production factor by incorporating the positive effects of lower aggregate pollution on the productivity of capital. They show that a double dividend may emerge only if the environmental externality in production is powerful enough.

The major contribution of our paper is to show that, without a pollution externality on production (that is, in a simpler model than BM's), a strong version of the double dividend may emerge. This result is achieved by implementing the revenue-neutral tax reform by substituing environmental taxes for income taxes in such a way that the present value of revenues finances the present value of the *predetermined* path for government expenditures (that is, public debt issuing is allowed). This strategy is not usual in the previous literature: BM or Hettich (2000) analyze a revenue-neutral tax reform where the pollution tax partly replaces non-environmental taxes, assuming that the government budget is balanced every period and that government revenues and expenditures must grow at the same rate than the capital stock, so that the expenditure path changes as a result of the tax reform.

We start from an AK economy where a predetermined path for government expenditures is financed by distorting income taxes together with a pollution tax. We model the environment assuming that the level of pollution created by the production process enters negatively in consumers' utility (Smulders and Gradus (1993), Ligthart and van der Ploeg (1994), or BM[1], are some examples). This externality creates an opportunity for environmental corrective taxation. In addition, we assume that the pollution flow can be reduced through private abatement activities, which in turn consume some output. In this paper we show that pollution externality on productivity is not essential for the double dividend to arise: the pollution externality on preferences is enough to obtain double dividend if the period-by-period government budget balance assumption is relaxed (that is, deficit-financed green tax reforms are allowed).

We explore how a tax reform affects the economy, starting from an initial equilibrium with a non-optimal predetermined path of public expenditure (that is, if the pollution tax rate was the only tax in the economy, revenues from the Pigouvian tax rate would be too low to finance the predetermined path of expenditure). Therefore, our analysis is explored in a second-best world.

Initially, a predetermined path for government expenditures (redistributed to households in a lump-sum fashion) is financed by income taxes, consumption taxes and the pigouvian environmental tax[2]. Tax reform consists of increasing the environmental tax and decreasing the income tax so that the present value of revenues matches the present value of expenditures. To that end, the government is allowed to finance a certain deficit in any period t by issuing one-period pure discount bonds.

An increase in pollution tax together with a reduction in the income tax rate could have a positive and permanent impact on the growth rate of the economy, expanding the tax base in the following periods. If these increased revenues allowed for the government budget constraint to hold in a present value sense, then the tax reform would be feasible.[3]

We show that a double dividend arises when the elasticity of pollution

[1]They include environmental quality as a positive externality in the utility function. However, environmental quality is inversely related to pollution, leading to the same welfare function employed in our model.

[2]Initially the tax system consists of the pigouvian environmental tax, among others, because we want to keep the welfare gains due to increased allocative efficiency apart from those corresponding to increased tax efficiency.

[3]A dynamic Laffer effect of this kind has been analyzed in models without pollution: Ireland (1994), Pecorino (1995) and Novales and Ruiz (2002).

with respect to the capital stock is not higher than the elasticity of pollution with respect to private abatement activities. Then, pollution falls and the growth rate of the economy increases following a green tax reform. Otherwise, the double dividend also arises althought it lasts only a few periods (the green dividend extingishes after some periods).

In this framework debt issuing is key to achieve a double dividend: under the mentioned conditions, increasing the pollution tax and reducing income and consumption tax rates to achieve the same revenues path than under the initial tax structure, yields only a green dividend but not a blue one. Debt issuing allows for larger reductions in distortionary taxes for any rise in the pollution tax. Consequently, a larger increase in economic growth is obtained, jointly with significant environmental improvement. Therefore, a blue dividend and a green dividend could be jointly obtained.

Two additional results are found: first, the optimal environmental tax will be higher than the Pigouvian level, given by the marginal social damage from pollution. This result holds for the two types of tax reform analyzed (with and without debt issuing). Since the environmental tax is less distortionary than the income tax[4], such additional welfare gain reduces the net cost of environmental taxation, rising the optimal environmental tax above marginal social damage.

Second, the level of public debt needed to finance a green tax reform yielding a double dividend could be very low. In particular, this is true when green tax reforms lead to a very large increase in the pollution tax.

The remainder of the paper is organized as follows. We describe the model economy in section 2. In section 3, in a second best world, we explore two types of revenue-neutral green tax reforms in which: i) the government is not allowed to issue debt (we show that this green tax reform only yields a green dividend but not a blue dividend); ii) the government is allowed to issue debt (we show that such tax reform yields a double dividend in a strong version). The paper ends with some conclusions.

2 The Model

2.1 Households

We consider an economy populated with an infinitely-lived, representative household who obtains income from capital renting. His/her utility depends positively on consumption and negatively on aggregate pollution :

[4]Pollution taxes are less distortionary than alternative distortionary taxes because a redistribution of government revenues towards pollution taxes increases welfare.

$$U(C_t, P_t) = \begin{cases} \frac{\left(C_t P_t^{-\eta}\right)^{1-\sigma} - 1}{1-\sigma} & \text{for} \quad \sigma > 0, \sigma \neq 1 \\ \ln C_t - \eta \ln P_t, & \text{for} \quad \sigma = 1 \end{cases}$$

where C_t, P_t are the levels of consumption and pollution, σ is the inverse of the intertemporal elasticity of substitution and η is the weight of pollution in utility. Utility is increasing in consumption at a decreasing rate, $U_C > 0$, $U_{CC} < 0$, and decreasing in aggregate pollution, $U_P < 0$. If σ is higher (lower) than $1 + 1/\eta$, an increase in pollution leads to a reduction (an increase) in marginal pollution desutility. This utility function is standard in models that study the relationship between growth and pollution or the effects of green reforms given distortionary taxes.[5]

The household chooses the levels for C_t, K_{t+1} (physical capital) and B_{t+1} (bonds) that solve the following problem:

$$\underset{\{C_t, K_{t+1}, B_{t+1}\}}{Max} \sum_{t=0}^{\infty} \beta^t \; U(C_t, P_t) \tag{1}$$

such that

$$K_{t+1} = (1 - \tau_K) \; r_t K_t + T_t - (1 + \tau_C) C_t + (1 - \delta) K_t - \frac{B_{t+1}}{R_t} + B_t \tag{2}$$

$$K_0, \; B_0 \quad \text{given},$$

where τ_K and τ_C are the tax rates on income and consumption, r_t is the return on capital. T_t are lump-sum transfers received from government, B_t is the stock of public debt owned by the household in period t, R_t is the interest rate on debt and δ is the rate of physical capital depreciation. The household ignores the environmental utility externality.

2.2 Firms

Firms produce the only good in the economy using only physical capital. We assume that the technology is linear: $Y_t = AK_t$. Environmental pollution (P_t) is regarded as a side product of the production process. Abatement (Z_t) enables firms to increase output without causing more pollution. To

[5]An alternative specification would be to consider that environmental quality, instead of pollution, is an argument of the utility function (i.e. Boverberg and de Mooij (1997)).

simplify, we assume that both P_t and Z_t are flow quantities. The pollution function is:

$$P_t = \frac{K_t^{\chi_1}}{Z_t^{\chi_2}}, \qquad \text{with} \;\; \chi_1, \chi_2 > 0. \tag{3}$$

Firms rent capital from households at the interest rate r_t and pay a time-varying pollution tax $\tau_{P,t}$ on the level of pollution.

The firm chooses the path for K_t and Z_t to maximize its profits every period ($\pi_t = Y_t - r_t \; K_t - \tau_{P,t} \; P_t - Z_t$), taking as given the market price of inputs. Without a pollution tax ($\tau_{P,t} = 0$), firms would ignore the negative side-effect of capital in the production process and abatement activities would be zero.

2.3 Government

The government raises taxes on consumption, income and pollution and issues public debt. Income and consumption tax rates are constant, but the tax rate on pollution grows at a rate that guarantees a constant share of pollution tax revenues on output, $(\tau_{P,t} \; P_t)/Y_t$. Public revenues are redistributed to households through lump-sum transfers.

The government budget constraint is:

$$\frac{B_{t+1}}{R_t} + \tau_K \; r_t K_t + \tau_{P,t} \; P_t + \tau_C \; C_t = T_t + B_t \; . \tag{4}$$

With a terminal constraint on the government's ability to issue debt,

$$\lim_{T \to \infty} \left[\frac{B_{T+1}/R_T}{\prod_{s=0}^{T-1} R_s} \right] = 0, \tag{5}$$

which guarantees that the period by period constraints (4) can be combined into a single infinite horizon, present value budget constraint:

$$\sum_{t=0}^{\infty} \frac{\tau_K \; r_t K_t + \tau_{P,t} \; P_t + \tau_C \; C_t}{\prod_{s=0}^{t} R_s} = \sum_{t=0}^{\infty} \frac{T_t}{\prod_{s=0}^{t} R_s}. \tag{6}$$

2.4 The Competitive Equilibrium and the Planner Solution

2.4.1 The Competitive Equilibrium

Definition 1 *A competitive equilibrium for this economy is a set of allocations* $\{C_t, K_{t+1}, B_{t+1}, Z_t\}_{t=0}^{\infty}$ *and a price system* $\{r_t, R_t\}_{t=0}^{\infty}$ *such that given a price system and a fiscal policy* $\left\{\tau_K, \tau_C, \{\tau_{P,t}, T_t\}_{t=0}^{\infty}\right\}$*: i)* $\{C_t, B_{t+1}, K_{t+1}\}_{t=0}^{\infty}$ *maximizes households' utility (1), subject to (2), and taking the pollution level* $\{P_t\}_{t=0}^{\infty}$ *and* $\{K_0, B_0\}$ *as given; ii)* $\{K_t, Z_t\}_{t=0}^{\infty}$ *satisfies the firms' profit maximization conditions, and iii)* $\{C_t, K_{t+1}, Z_t\}_{t=0}^{\infty}$ *satisfies the aggregate resources constraint:*

$$K_{t+1} = AK_t - C_t + (1-\delta)\, K_t - Z_t. \tag{7}$$

It is well known that this model lacks transitional dynamics, that is, the competitive equilibrium takes the form of a balanced growth path.

Competitive equilibrium is characterized in appendix A, where the following properties of the balanced growth path are shown:

1. The abatement to capital ratio is constant,

$$\left(\frac{Z_t}{K_t}\right)_M = \left(\chi_2 \bar{\tau}_P\right)^{1/(1+\chi_2)}, \tag{8}$$

   where the M index refers to market equilibrium values and

$$\bar{\tau}_P = \frac{\tau_{P,t}}{K_t^{1-(\chi_1-\chi_2)}} \tag{9}$$

   is the detrended pollution tax, which is constant along a balanced growth path.

2. Return on capital renting:

$$r_t = A - \xi\, \bar{\tau}_P^{1/(1+\chi_2)}, \tag{10}$$

   where $\xi = \chi_1 \cdot \chi_2^{-\frac{\chi_2}{1+\chi_2}}$.

3. Market return on public debt:

$$R_t = (1-\tau_K)\, r_{t+1} + 1 - \delta. \tag{11}$$

4. Growth rate:

$$g_M = \{\beta\,[\Phi(\tau_K, \bar{\tau}_P) + 1 - \delta]\}^{1/(\sigma + (\chi_1 - \chi_2)\eta(1-\sigma))}\,, \tag{12}$$

where

$$\Phi(\tau_K, \bar{\tau}_P) = (1 - \tau_K)\left(A - \xi\, \bar{\tau}_P^{1/(1+\chi_2)}\right),$$

and $\sigma + (\chi_1 - \chi_2)\,\eta(1-\sigma) > 0$ so that the growth rate depends positively on the productivity parameter, A.

5. Consumption to capital ratio:

$$\left(\frac{C_t}{K_t}\right)_M = A + 1 - \delta - g_M\left(\bar{\tau}_P, \tau_K\right) - \left(\chi_2 \bar{\tau}_P\right)^{1/(1+\chi_2)}. \tag{13}$$

6. Pollution:

$$P_t = P_0\, g_M^{(\chi_1 - \chi_2)t} = K_0^{\chi_1 - \chi_2} \left(\frac{Z_t}{K_t}\right)_M^{-\chi_2} g_M^{(\chi_1 - \chi_2)t}, \tag{14}$$

showing that if $\chi_1 = \chi_2$, pollution remains constant along the balanced growth path, while if $\chi_1 > (<)\chi_2$ pollution will increase (decrease) along the balanced growth path.

7. Government tax revenues[6]

$$\begin{aligned} \Psi_t &= \tau_K\, r_t K_t + \tau_{P,t}\, P_t + \tau_C\, C_t \\ &= K_0 \cdot \Psi\left(\tau_K,\ \bar{\tau}_P,\ \tau_C\right) \cdot \left[g_M\left(\tau_K,\ \bar{\tau}_P\right)\right]^t, \end{aligned} \tag{15}$$

where,

$$\Psi\left(\tau_K,\ \bar{\tau}_P,\ \tau_C\right) = \tau_K \left(A - \frac{\chi_1}{\chi_2}\left(\frac{Z_t}{K_t}\right)_M\right) + \bar{\tau}_P \left[\left(\frac{Z_t}{K_t}\right)_M^{-\chi_2}\right] + \tau_C \left(\frac{C_t}{K_t}\right)_M \tag{16}$$

are the detrended tax revenues.

These equations allow us to preview the effects on the economy of changes in the different tax rates.

[6]The final equation has been obtained by substitution of the interest rate r_t (10) and the level of pollution P_t (3).

First, a higher detrended pollution tax rate reduces the after-tax marginal product of capital (see equation 10) and hence the incentive to invest, while enhancing the abatement activities by the firms. As a consequence, the abatement/capital ratio increases (see 8) and the growth rate decreases (12), the response of the consumption/capital ratio being indeterminate (13).

Second, a higher income tax rate has a similar effect on investment. Higher income tax stimulates consumption relative to investment, while not affecting the abatement to capital ratio. As a consequence, the growth rate decreases and the consumption/capital ratio increases.

Finally, the consumption tax does not affect investment since the after-tax marginal product of capital is unaffected. It does not affect the abatement activities of the firms either. Finally, it does not alter the consumption/capital ratio as long as consumption tax revenues are distributed as a lump-sum to households. Consequently, the consumption tax acts as a lump-sum tax that does not affect the intratemporal or the intertemporal allocation of resources.

2.4.2 The Planner Solution

Definition 2 *The central planner equilibrium is a set of allocations* $\{C_t, K_{t+1}, Z_t\}_{t=0}^{\infty}$ *that maximizes the lifetime utility of households, subject to the aggregate constraint of resources (7) and the pollution equation (3).*

The rate of growth for the planner solution along the balanced growth path is (see Hettich (2000) for the case $\chi_1 = \chi_2$):

$$g_P = \left[\beta\left(A + 1 - \delta - \frac{\chi_1}{\chi_2}\left(\frac{Z_{t+1}}{K_{t+1}}\right)_P\right)\right]^{\frac{1}{\sigma+(\chi_1-\chi_2)\eta(1-\sigma)}}, \tag{17}$$

where P index denotes planner equilibrium, and the abatement to capital ratio is: [7]

$$\left(\frac{Z_t}{K_t}\right)_P = (A + 1 - \delta - g_P)\frac{\eta\chi_2}{1+\eta\chi_2}. \tag{18}$$

[7]Taking (17) and (18), the planner growth rate is computed from:

$$g_P = \left\{\beta\left[A + 1 - \delta - \frac{\eta\chi_1}{1+\eta\chi_2}(A + 1 - \delta - g_P)\right]\right\}^{\frac{1}{\sigma+(\chi_1-\chi_2)\eta(1-\sigma)}},$$

which depends on parameters $A, \delta, \chi_1, \chi_2, \eta$ and σ.

Finally, the efficient value for consumption to capital ratio is:

$$\left(\frac{C_t}{K_t}\right)_P = (A + 1 - \delta - g_P)\,\frac{1}{1+\eta\chi_2}. \tag{19}$$

2.4.3 First-best fiscal policy

The first-best fiscal policy allows for the efficient path to be attained as a competitive equilibrium. Therefore, in order to characterize the first-best tax policy, we compare the growth rate, consumption to capital ratio and abatement to capital ratio for the market solution with those for the central planner solution.

Proposition 3 *The first-best fiscal policy is defined by an income tax rate equal to zero ($\tau_K^* = 0$), any value of the consumption tax rate and a pigouvian pollution tax rate equal to:*

$$\tau_{P,t}^* = \bar{\tau}_P^* \ (g_P)^{[1-(\chi_1-\chi_2)]\ t} \ K_0, \tag{20}$$

where

$$\bar{\tau}_P^* = \frac{1}{\chi_2}\ \left(\frac{Z_t}{K_t}\right)_P^{1+\chi_2}. \tag{21}$$

Proof. See Appendix B. ■

3 Revenue-neutral green tax reform

In this section, we explore the consequences of several tax reforms, all of them financing an exogenously predetermined path of public expenditures. Such spending requirements are supposed to be higher than the revenues raised by the pollution tax rate fixed at the Pigouvian level. That way, additional taxes are needed for expenditure financing.

In endogenous growth models, two different types of reforms allow for financing a predetermined public expenditure path:

- An increase in the pollution tax, devoting the increased revenues to finance reductions in income and consumption tax rates so that the new policy raises exactly the same revenues than the initial tax system. This is the case, not only at the time of the reform but also in the followings periods. In this case, the goverment budget is balanced every period and consequently $B_t = 0$ for all t, even after the tax reform.

- Increasing the environmental tax and decreasing the capital income tax so that the present value of revenues allows for financing the present value of predetermined public expenditures. In this case, the government is allowed to finance a certain deficit in any period t by issuing one-period pure discount bonds.

Both of them are 'green reforms' because the abatement activities are enhanced by a higher pollution tax. They are revenue-neutral because the new tax mix finances the same path of public expenditures. The analysis that we carry out departs from Bovenberg and de Mooij (1997)'s who only consider green tax reforms under which government revenues and expenditures grow at the same rate than the capital stock. That is, their public expenditure path changes as the economy growth rate is altered by the tax reform.

Consider, as a benchmark, a situation in which the predetermined path of public expenditures is financed through the pollution tax rate fixed at the detrended pigouvian level $\bar{\tau}_P^*$, a positive consumption tax rate ($\tau_{C,0} > 0$) and a positive income tax rate ($\tau_{K,0} > 0$). Therefore, the government budget constraint previous to the tax reform is:

$$\begin{aligned} T_{t,0} &= \tau_{K,0}\, r_t K_t + \bar{\tau}_P^*\, K_t^{1-(\chi_1-\chi_2)}\, P_t + \tau_{C,0}\, C_t \\ &= K_0 \cdot \Psi(\tau_{K,0}, \tau_{C,0}, \bar{\tau}_P^*) \cdot [g_{M,0}(\tau_{K,0}, \bar{\tau}_P^*)]^t\,, \quad \text{for } t = 1, 2, 3, \ldots \end{aligned} \tag{22}$$

We explore the effects of both reforms assuming that the initial tax system includes a Pigouvian environmental tax, because we want to keep the welfare gains due to increased efficiency in the allocation of resources apart from those corresponding to increased tax efficiency.

If pollution falls and non-environmental welfare (that due to consumption) rises as a consequence of the reform, then a double dividend is obtained. In section 3.1 we show that the first type of green tax reform only achieves the green dividend. In section 3.2 we show that when debt issuing is allowed, the green tax reform yields a double dividend. We also discuss whether, in our second-best world, the optimal pollution tax deviates from the Pigouvian tax.

3.1 Balanced budget green tax reform

Let us first consider a green tax reform that keeps the whole revenue path constant so that the predetermined public expenditure path is exactly financed and the government budget is balanced every period.

Fernández, Pérez and Ruiz (2002) show in an AK model with a pollution externality in the utility function but without abatement, that the relative change in τ_K and τ_P that keeps tax revenues unchanged at the time the policy reform takes place, also guarantees that the whole path of tax revenues remains unaltered. On the contrary, when the pollution tax is increased in a model with abatement activities, income and consumption tax rates must both be adjusted to guarantee that the whole path of tax revenues remains unaltered.

Proposition 4 *Let* $\{\tau_{K,0},\ \bar{\tau}_{P,0},\ \tau_{C,0}\}$ *be the initial tax mix. Let* $g_{M,0} = g_M(\tau_{K,0},\ \bar{\tau}_{P,0})$ *and* $\Psi_0 = \Psi\left(\tau_{K,0},\ \bar{\tau}_{P,0},\ \tau_{C,0}\right)$ *be the growth rate and the detrended tax revenues before the tax reform, respectively. Let* $\bar{\tau}_{P,1} > \bar{\tau}_{P,0}$ *be the new detrended pollution tax rate. If capital income and consumption tax rates are set as,*

$$\tau_{K,1} = 1 - \frac{\Phi(\tau_{K,0}, \bar{\tau}_{P,0})}{A - \xi\left(\bar{\tau}_{P,1}\right)^{1/(1+\chi_2)}}. \tag{23}$$

$$\tau_{C,1} = \frac{\Psi_0 - A + \Phi(\tau_{K,0}, \bar{\tau}_{P,0}) - \xi\left(\bar{\tau}_{P,1}\right)^{1/(1+\chi_2)}\left(\frac{1-\chi_1}{\chi_1}\right)}{A + 1 - \delta - g_{M,0} - \left(\chi_2\,\bar{\tau}_{P,1}\right)^{\frac{1}{1+\chi_2}}}, \tag{24}$$

then, the new tax mix $\{\tau_{K,1}, \bar{\tau}_{P,1}, \tau_{C,1}\}$ *yields the same path of public revenues than the initial tax mix.*

Proof. See Appendix B. ■

Corollary 5 *The new tax mix described in proposition 4 satisfies: i)* $\tau_{K,1} < \tau_{K,0}$ *and, ii)* $\tau_{C,1} < \tau_{C,0}$ *if* $\tau_{C,0} < \frac{1-\chi_1}{\chi_2}$.

Proof. See Appendix B. ■

Therefore, the new tax mix involves increasing the pollution tax rate and reducing the income tax rate simultaneously, so that (23) holds. The consumption tax rate may increase or decrease. The new tax mix guarantees that the growth rate and the detrended tax revenues do not change.

This green tax reform does not yield double dividend as the following proposition shows.

Proposition 6 *Under the green tax reform described in proposition 4, the new tax mix yields a green dividend but not a blue dividend.*

Proof. From (8):

$$\frac{\partial\left(\frac{Z_t}{K_t}\right)_M}{\partial\bar{\tau}_P} = \frac{\chi_2^{1/(1+\chi_2)}}{1+\chi_2}\left(\bar{\tau}_P\right)^{-\chi_2/(1+\chi_2)} > 0.$$

Taking derivatives in (13) with respect to $\bar{\tau}_P$ and taking into account that the green tax reform keeps growth constant, we find that:

$$\frac{\partial\left(\frac{C_t}{K_t}\right)_M}{\partial\bar{\tau}_P} = -\frac{\chi_2^{1/(1+\chi_2)}}{1+\chi_2}\left(\bar{\tau}_P\right)^{-\chi_2/(1+\chi_2)} < 0$$

Taking both derivatives together we conclude that abatement crowds-out consumption. Since the level of consumption is lower for every period after the reform, non-environmental welfare decreases as a result of the reform and hence the blue dividend is not achieved.

The increase in abatement activities yields an instantaneous fall in pollution, which remains thereafter below the benchmark level:

$$\frac{\partial P_t}{\partial\bar{\tau}_P} = -\frac{\chi_2^{1/(1+\chi_2)}}{1+\chi_2}\left(\bar{\tau}_P\right)^{-\frac{\chi_2}{1+\chi_2}-1} K_0^{\chi_1-\chi_2}\cdot g_M^{(\chi_1-\chi_2)t} < 0,\ \forall t.$$

■

This tax reform is welfare improving for a broad set of parameterizations of the model economy[8]. Since the blue dividend is not present, welfare improvement is only due to the reduction of pollution.

3.2 Tax reform yielding double dividend

Next we design an alternative green tax reform which ensures the same path of public spending, a reduction in pollution and an increase in private consumption leading to higher welfare improvements; therefore, the strong version of the double dividend is present.

Let us suppose that the government considers a permanent increase in the pollution tax rate jointly with a reduction in the income tax rate, while keeping the consumption tax and the same sequence of transfers $\{T_{t,0}\}_{t=0}^{\infty}$ unchanged. In a non-monetary economy, the government will need issuing some debt which might hopefully be retired over time. The new, lower income taxes will increase long-run growth, thereby expanding the tax base and leading to higher revenues at some point. *The new tax-structure is*

[8]It is not possible to determine analytically the effects of fiscal reform on welfare; however, it is possible to evaluate such effects numerically for specific parameter values. The results are avalaible upon request.

feasible if the subsequent increase in the tax base allows the government budget constraint to hold in a present value sense (see constraint 6). That would mean that the deficit in the initial periods after the policy change can be repaid by achieving later on a fiscal surplus in present value that will allow for eventually retiring the initially issued debt, with no need to introduce tax hikes at any point in time.

On the contrary, green tax reforms that substitute pollution tax and debt issuing for consumption taxes, holding income taxes unchanged, are not feasible. The reason is that consumption taxes do not affect growth and hence, the future tax base will not increase under lower consumption taxes and debt would never be retired.

Without loss of generality, we assume $K_0 = 1$. Consider the pollution tax rate is fixed at the detrended pigouvian level $\bar{\tau}_P^*$, a positive consumption tax rate ($\tau_{C,0} > 0$) and a positive income tax rate ($\tau_{K,0} > 0$) satisfying (22). This mix allows the government to exactly finance the predetermined path of public expenditures. Now, consider that, at $t = 0$, the government implements a new tax mix $\{\tau_{K,1}, \bar{\tau}_{P,1}, \tau_{C,0}\}$ with $\tau_{K,1} < \tau_{K,0}$ and $\bar{\tau}_{P,1} \geq \bar{\tau}_P^*$. The new growth rate and the new detrended tax revenues are given by $g_{M,1} = g_M(\tau_{K,1}, \bar{\tau}_{P,1})$ and $\Psi_1 = \Psi(\tau_{K,1}, \bar{\tau}_{P,1}, \tau_{C,0})$, respectively. The revenues path (15) under the new tax structure is given by

$$\Psi_{t,1} = \Psi_1\left(\tau_{K,1},\ \bar{\tau}_{P,1},\ \tau_{C,0}\right) \cdot \left[g_{M,1}\left(\tau_{K,1},\ \bar{\tau}_{P,1}\right)\right]^t, \quad \forall t = 0, 1, 2, \dots . \tag{25}$$

Hence, the goverment budget constraint (4) can be written as

$$\frac{B_{t+1}}{R_1} + \Psi_{t,1} = \Psi_{t,0} + B_t, \quad \text{with} \quad B_0 = 0, \qquad \forall t = 0, 1, 2, \dots . \tag{26}$$

where we maintain the same expenditure path than before the tax cut ($T_{t,0} = \Psi_{t,0}$, $\forall t$) and R_1 is the return on public debt after the tax reform (11).

Using the transversality condition (5) together with the initial condition $B_0 = 0$, (26) can be solved to yield

$$\sum_{t=0}^{\infty} \frac{\Psi_{t,1} - \Psi_{t,0}}{R_1^t} \geq 0,$$

or, equivalently,

$$\frac{\Psi_1\left(\tau_{K,1},\ \bar{\tau}_{P,1},\ \tau_{C,0}\right)}{R_1\left(\tau_{K,1},\ \bar{\tau}_{P,1}\right) - g_{M,1}\left(\tau_{K,1},\ \bar{\tau}_{P,1}\right)} - \frac{\Psi_0\left(\tau_{K,0},\ \bar{\tau}_P^*,\ \tau_{C,0}\right)}{R_1\left(\tau_{K,1},\ \bar{\tau}_{P,1}\right) - g_{M,0}\left(\tau_{K,0},\ \bar{\tau}_P^*\right)} \geq 0. \tag{27}$$

This inequality characterizes feasible green tax reforms (a similar condition is found in Ireland (1994) or Novales and Ruiz (2002)).

It is not possible to determine analytically the range of parameter values for which (27) is satisfied. However, it is possible to evaluate (27) numerically when specific values are chosen for the parameters.

For our simulations, benchmark parameter values are chosen, with one period in the model identified as one year. The detrended tax rate on pollution before the reform is set at the pigouvian level ($\bar{\tau}_P^*$), which is endogenously determined, the income tax rate[9] ($\tau_{K,0}$) is 30% and the consumption tax rate ($\tau_{C,0}$) is assumed to be 10%.

The rate of capital depreciation (δ) is set at 10%, the inverse of the intertemporal elasticity of substitution (σ) is 1.5, and the parameters related to pollution are $\eta = 0.95$, $\chi_1 = \chi_2 = 0.4$ (so the pollution level does not grow along the time). Additionally, A=0.217 and β=0.99 so that the after-tax real rate of interest (R) is 4% and growth is 2% per year, in line with values usually reported for the US economy (see for instance, Ireland, 1994). Given these parameter values, the detrended pigouvian tax ($\bar{\tau}_P^*$) is 0.76%.

We characterize the effects of several feasible reforms, i.e. all of them satisfying (27):

1) If we keep the pollution tax fixed at the pigouvian level ($\bar{\tau}_{P,0} = \bar{\tau}_{P,1} = \bar{\tau}_P^*$ =0.76%) and consider different values for the new income tax, then these reforms do not affect pollution because they do not change the abatement to capital ratio (see (8)) and pollution does not grow because we assume that χ_1 equals χ_2 (see (14)). Therefore, these reforms could yield only one dividend, increasing welfare only through an increase in private consumption.

2) We mainly study tax reforms that increase the pollution tax above its initial level of 0.76%, which is the pigouvian tax[10]. These reforms achieve a 'green dividend' because they increase the abatement to capital ratio and pollution does not grow because $\chi_1 = \chi_2$ (see (8) and (14)). Hence, pollution path after the reform is below the benchmark path. In the section devoted to the sensitivity analysis, we study the green dividend when $\chi_1 \neq \chi_2$.

With regards to the blue dividend, however, the results from the tax reform are not so straightforward. For a given increase in the pollution tax,

[9]Cooley and Hansen (1992) use a capital income tax rate of 50% and a labor income tax rate of 23%. In the paper we report the results for an intermediate rate but the sensitivity analysis for other rates is available upon request. Results remain qualitativaly the same.

[10]The highest pollution tax considered is 2.77% because, as we discuss below, larger increments of the environmental taxation would break the double dividend result under the benchmark parameterization.

the higher the reduction in the income tax rate, the higher the growth rate (see (12)). This induces two opposite effects on the consumption path: consumption will grow at a faster rate, but its initial fall is also more important (see (13)). Hence, the welfare effects of the reform due to private consumption are not obvious, and a numerical solution is needed to see whether the green tax reform yields a blue dividend.

We only need to compute if such a reform leads to an increase in non-environmental welfare, since environmental welfare improvement occurs for any pollution tax above 0.76%, as explained before.

Definition 7 *To measure the non-environmental welfare gain, we assume that pollution is unaffected by the reform (in fact pollution falls for the benchmark parameterization of χ_1 and χ_2), and we compute the change in consumption that an individual would require each period to be as well off under the initial situation as under the new tax structure. Let $C_{t,0}$, $P_{t,0}$ and $Y_{t,0}$ be the path of consumption, pollution and output under the initial policy, and $C_{t,1}$ be the path of consumption under the new policy. When $\sigma \neq 1$, we compute the variable ω from:*

$$\sum_{t=0}^{\infty} \beta^t \frac{\left((1+\omega)C_{t,0}P_{t,0}^{-\eta}\right)^{1-\sigma} - 1}{1-\sigma} = \sum_{t=0}^{\infty} \beta^t \frac{\left(C_{t,1}P_{t,0}^{-\eta}\right)^{1-\sigma} - 1}{1-\sigma}$$

The non-environmental welfare gain of the reform expressed as percentage on output is: $\frac{\omega C_{t,0}}{Y_{t,0}} \times 100$.

A positive value for this measure corresponds to a rise in welfare and, consequently, a blue dividend is achieved. In these cases a double dividend appears.

In addition, solving the model numerically for specific parameter values is necessary to answer questions like: Will the largest cut on capital income tax yield the largest welfare improvement?. Does any welfare-improving tax reform yield a double dividend?

Definition 8 *We measure the welfare effects associated to the green tax reform as the change in consumption that an individual would require each period to be as well off under the initial situation as under the new tax structure. Let $C_{t,0}$, $P_{t,0}$ and $Y_{t,0}$ be the path of consumption, pollution and output under the initial policy, and $C_{t,1}$ and $P_{t,1}$ be the path of consumption and pollution under the new policy. When $\sigma \neq 1$, we compute the variable λ from:*

$$\sum_{t=0}^{\infty} \beta^t \frac{\left((1+\lambda)C_{t,0}P_{t,0}^{-\eta}\right)^{1-\sigma} - 1}{1-\sigma} = \sum_{t=0}^{\infty} \beta^t \frac{\left(C_{t,1}P_{t,1}^{-\eta}\right)^{1-\sigma} - 1}{1-\sigma}$$

The result is expressed as a percentage on output, $\frac{\lambda C_{t,0}}{Y_{t,0}} \times 100$.

Hence, a positive value for this measure corresponds to a rise in welfare, that could be understood as a reduction in the tax system distortion.

For a given increase in the detrended pollution tax, only a certain range of reductions in the income tax rate allows for the condition (27) to hold, so that the tax reform is feasible only when certain income tax cuts are implemented. On the one hand, if the income tax cut is too low growth might decrease as a result of the new $(\tau_{K,1}, \bar{\tau}_{P,1})$-pair, and the initial deficit will not be repaid in the long term. On the other hand, if the income tax cut is too large, the initial deficit will be very large, and condition (27) might also break, even though the increase in growth will also be remarkable. Therefore, there exists a lowest feasible cut and a largest feasible cut in the income tax rate, both of which guaranteing the condition (27) as an equality. Reductions between both of them lead to a present value budget surplus.

3.2.1 Results

Figure 1 shows all feasible $(\tau_{K,1}, \bar{\tau}_{P,1})$-pairs. The top limit of the coloured area corresponds to the lowest income tax cut, for every pollution tax rate. The lowest limit of the area corresponds to the highest income tax cut that yields, for every pollution tax rate, a feasible reform. The top and lowest dark-shaded areas show the $(\tau_{K,1}, \bar{\tau}_{P,1})$-pairs that yield a 'green dividend' but do not increase welfare, that is, they are suboptimal in this second-best world. We will name these areas '*suboptimal*'. The shaded areas next to the suboptimal ones, show the $(\tau_{K,1}, \bar{\tau}_{P,1})$-pairs that yield welfare improvements but do not achieve the '*blue dividend*', that is, welfare gains are only due to environmental improvements. We will name these areas '*green dividend*'. The centered pale area in the graph includes the $(\tau_{K,1}, \bar{\tau}_{P,1})$-pairs that yield a double dividend. Finally the bold solid line shows, for any $\bar{\tau}_{P,1}$, the level of $\tau_{K,1}$ that achieves the highest possible welfare improvement.[11]

[**INSERT FIGURE 1**]

A '*double dividend*' area can be seen to arise for a wide range of green tax reforms. Note that the double dividend area does not include the reforms

[11]Note that the $(\tau_{K,1}, \bar{\tau}_{P,1})$-pairs above the lowest limit of the "*double dividend area*" lead to government budget superavits. Therefore, feasible green tax reforms yielding double dividend could also be found under a benchamrk with initial deficit, although the feasible income tax cuts would be lower.

given by $\bar{\tau}_{P,1} = \bar{\tau}_P^* =$0.76%, and $\bar{\tau}_{K,1} \leq$30%, because such reforms only yield the blue dividend, but not the green one.

Furthermore, this graph shows that the range of income tax cuts yielding a double dividend decreases when $\bar{\tau}_{P,1}$ exceeds a threshold ($\bar{\tau}_{P,1} = 2.03\%$). The explanation for this result is as follows: the higher the pollution tax increase, the higher the enhancement of abatement activities (see (8)), and the higher the necessary income tax cut to guarantee that the reform is feasible (i.e., that (27) holds); as a result of both effects, the instantaneous fall in consumption is larger. This makes more difficult an increase of non-environmental welfare, even though such reform rises the rate of growth. In fact, under the benchmarch parameterization, the double dividend is only obtained for the range $\bar{\tau}_{P,1} \in (0.76\%, 2.77\%)$.

Figure 2 shows the effects on welfare and growth of a revenue-neutral green tax reform. Discontinuous lines show growth and welfare improvements corresponding to the largest feasible income tax cut for any pollution tax. Solid line shows growth and welfare improvements corresponding to the income tax cuts which lead to the highest welfare gain for any $\bar{\tau}_{P,1}$. We only consider the pollution tax ($\bar{\tau}_{P,1}$) range for which the double dividend occurs.

[**INSERT FIGURE 2**]

From the graph we conclude that:

1. The tax reform yielding the highest welfare is different to the reform that achieves the highest growth: relationship between growth and welfare is not monotone. However, any tax reform leading to double dividend increases growth.

2. Largest welfare might not arise from the highest feasible income tax cut for a certain $\bar{\tau}_P$, since a higher cut in τ_K yields a larger consumption growth, but also a larger initial fall in consumption. Hence welfare effects of the reform due to private consumption are not monotonous to income tax cuts. For example, if pollution tax increases up to 2.46% (from 0.76%), and the capital income tax falls to -22.33% (this is the highest tax cut), welfare decreases. However, if the capital income tax falls to 0%, welfare increases by 19.2%.

3. Second-best optimal pollution and income taxes are those leading to the highest welfare improvement; under the benchmark parameterization, the second-best tax mix is: $(\tau_{K,1}, \bar{\tau}_{P,1}) = (3.48\%,\ 0.96\%)$. Hence, the optimal pollution tax rate is higher than the Pigouvian

level (0.76%). Bovenberg and de Mooij (1997) also found this result, although only when the pollution externality on production is powerful enough.

Finally, figure 3 compares the magnitude of the welfare improvement achieved from the balanced budget tax reform, which only achieves a green dividend, and several tax reforms described in this section that yield double dividend (all the reforms included in the figure have been obtained for the same set of parameter values, debt issuing being the only difference). The results are displayed for the range of values of $\bar{\tau}_{P,1}$ for which the double dividend arises (under the benchmark parameterization). The line nearest to zero represents the welfare gains for the balanced budget tax reform, while the top line represents the largest welfare improvement obtained by the tax reform with debt issuing (it is the same solid line displayed in figure 2).

Other curves are included in figure 3. Each line graphs the welfare improvement obtained by a feasible tax reform which leads to a certain level of indebtness as a percentage of output (we refer to the maximum level of debt along the whole debt path). The following debt/output ratios have been considered: 25%, 50%, 100% and 150%. For example, the line corresponding to a debt/output ratio of 25% graphs the welfare gains obtained by different $(\tau_{K,1}, \tau_{P,1})$-pairs that obtain double dividend and lead to a maximum level of indebtness of 25% of output.

[**INSERT FIGURE 3**]

A goverment may care more about the level of deficits and debt than about consumers' welfare. Public debt has political costs in terms of monetary policy credibility, expectations of future tax hikes, and so on. Figure 3 shows that there exists a trade-off between welfare and debt. In one extreme, a government could choose a high level of welfare improvement (31%, corresponding to $\bar{\tau}_{P,1} = 0.761\%$) and a high level of debt (525% on ouput); in the other extreme, lower welfare improvements (16%, corresponding to $\bar{\tau}_{P,1} = 2.76\%$) but also much lower levels of indebtness (1% on output).[12] The more agressive the green tax reform (the higher $\tau_{P,1}$) the lower the level of debt. The latter policy mix could be specially interesting for developing countries, for which a large level of debt is often highly penalized by the international capital markets in terms of currency depreciation, high interest rates on external debt, and so on.

[12]The highest feasible welfare gain is 33%, which is achieved for a debt/output ratio of 490%.

3.2.2 Sensitivity analysis

We study how the double dividend area in figure 1 widens or narrows depending on the values of different parameters that have not been previously calibrated in the literature: the weight of pollution in utility η, and the elasticities of pollution with respect to capital χ_1 and abatement, χ_2. The intertemporal elasticity of substitution σ is also considered since a wide range of calibrated values exists.

In addition, the relative levels of χ_1 and χ_2 are crucial to find a 'green dividend'. If $\chi_1 < \chi_2$ then, from (14), the path of pollution decreases along the time. After the tax reform, the level of pollution is reduced in the first period, the steady-state economy growth rate is increased and, hence, the pollution level decreases at a larger rate than before the reform. Therefore the green dividend will increase along the time. However, if $\chi_1 > \chi_2$, then the growth rate of pollution is $g_M^{\chi_1-\chi_2}$ >1. After the tax reform, the level of pollution drops in the first period (because the reform enhances the abatement to capital ratio), the steady-state growth rate increases and, hence, pollution grows faster than before the reform. Therefore, the pollution path starts with a lower level than before the reform, but after a finite number of periods the level of pollution will exceed the level of pollution corresponding to the initial tax mix. Therefore, the green dividend is achieved only for a finite, although potentially large, number of periods.

Column 1 in table 1, shows the parameter values we have considered in order to study the robustness of tax reforms with debt issuing, keeping constant the remainder of benchmark parameter values. Column 2 shows the pollution tax rate before the reforms (we assume the Pigouvian tax rate) and column 3 shows the pollution tax rate which maximizes welfare (i.e., the second-best optimal pollution tax rate). Fourth column shows the environmental revenues, as a percentage of output, for the pigouvian pollution tax rate (we will name this percentage as 'environmental revenues share'). Finally, column 5 shows how much the government can increase the environmental revenues share, while guaranteing a double dividend.

Analyzing the double dividend with respect to shifts in the detrended pollution tax rate is equivalent to studying changes in the double dividend after shifts in the environmental revenues share. Using the production function together with (9) and (3), it is simple to show that the environmental revenues share depends only on the detrended pollution tax and several structural parameters:

$$\frac{\tau_{P,t}P_t}{Y_t} = \frac{\bar{\tau}_P}{A}\left(\frac{Z_t}{K_t}\right)_M^{-\chi_2} = \frac{\xi}{A\chi_1}\left(\bar{\tau}_P\right)^{\frac{1}{1+\chi_2}}.$$

From table 1, we can conclude that:

1. The Pigouvian pollution tax rate (first-best optimal):

   (a) Is increasing with the weight of pollution in utility (η), the parameter of intertemporal elasticity of substitution (σ) and the elasticity of pollution with respect to capital (χ_1).

   - A higher level of any of these parameters leads to a higher optimal Z_t/K_t ratio (18). A larger weight of pollution in the utility function leads to a lower desired level of aggregate pollution and, therefore, a larger Z_t/K_t ratio is necessary. The larger the intertemporal elasticity of substitution (σ) the lower the rate of growth of the economy and the larger the abatement to capital ratio. A larger χ_1 makes capital stock more dirty, reducing the marginal return on capital investment (net of pollution) relative to the return on abatement activities, and hence increasing the optimal Z_t/K_t ratio.
   - Consequently, a higher pollution tax is necessary in order to enhance firms' abatement activities.

   (b) Is decreasing with the elasticity of pollution with respect to abatement (χ_2). The larger the level for χ_2, the larger the cleaning ability of abatement activities and the lower the optimal abatement to capital ratio.

2. The highest welfare improvement is achieved for a pollution tax rate larger than the pigouvian. That is, the second-best pollution tax rate is always larger than the pigouvian tax in our model. Furthermore, this result does not depend on debt issuing, since it also arises it for the balanced budget reform: in graph 3, the lowest line corresponds to the reform without debt, and the maximum welfare is achieved at $\bar{\tau}_{P,1} = 1.56\%$, well above the pigouvian rate (0.76%)[13].

3. Regarding the double-dividend area (column 5):

   (a) Economies with a low elasticity of intertemporal substitution, which leads to a smoother consumption path, have fewer chances to implement green tax reforms leading to a double dividend result.

[13]The results for the whole sensitivity analysis are available upon request.

(b) Economies with a higher environmental weight (larger η) in the utility function have more chances to achieve double dividend by stablishing green reforms.

(c) With regards to the pollution technology parameters: i) On the one hand, a more dirty productive capital, larger χ_1, (this is more often the case for developing countries), leads to a wider double dividend area. ii) On the other hand, economies with lower χ_2 (abatement is not very effective in lowering the pollution externality of the productive process -also a more frequent pattern for less developed countries-) have a wider 'Double Dividend Area'. Therefore, we find here an interesting result: developing countries, which would obtain greater benefits from the green reform, also have more technological chances to achieve a double dividend.

As a conclusion, suppose two economies with similar weights of pollution in the utility function (η), but one of them with larger χ_1, lower σ and lower χ_2 (the more representative case for a developing country). This economy has more possibilities to implement a green tax reform that yields a double dividend. However, these countries often find it more difficult to commit on a long lasting tax reform, because of the low institutional development and the abrupt political changes it often drives. In addition, since $\chi_1 > \chi_2$ the green dividend arises only for a finite number of periods.

4 Conclusions

This paper has explored the effects of an environmental tax reform on economic growth, pollution and welfare in a second-best framework. In the AK model with a negative pollution externality in utility function and abatement activites, we consider two types of reforms.

The first tax reform consists of increasing the pollution tax and lowering the taxes on consumption and income, in a way such that public debt issuing is not necessary to finance the predetermined path of public expenditure. In this case, abatement activities crowds out private consumption and the growth rate keeps constant. The reform is welfare improving but the double dividend is not present because the non-environmental welfare decreases.

The second tax reform consists of substituing pollution tax for income tax, in such a way that the government budget constraint holds in a present value sense, that is allowing debt issuing.

We show that if we keep the pollution tax fixed at the pigouvian level and consider different values for the new income tax, then these reforms do not affect pollution because they do not change the abatement to capital ratio and the pollution path does not change with respect to the benchmark case. Therefore, these reforms could yield only one dividend, increasing welfare only through an increase in private consumption. On the contrary, we show that increasing the pollution tax above its initial level (the pigouvian tax) together with an income tax cut, yields a 'green dividend' because the abatement to capital ratio is increased and the pollution path after the reform is below the benchmark path. With regards to the blue dividend, the results from the tax reform are not so straightforward. For a given increase in the pollution tax, the higher the reduction in the income tax rate, the higher the growth rate. This induces two opposite effects on the consumption path: consumption will grow at a faster rate, but its initial fall is also more important. Hence, double-dividend will only be present if the first effect dominates. In this case, this reform guarantees the predetermined public expenditure path and enhances growth rate, reduces pollution and increases abatement activities and private consumption. Thus, this green tax reform yields a double dividend; i.e, not only improves environmental quality but also boosts non-environmental welfare by stimulating economic growth. Our sensitivity analysis reveal that the plausibility of double-dividend is high.

We show that developing countries have more possibilities to implement green tax reforms that yield double dividend, althought these economies often find it more difficult to commit on a long lasting tax reform, due to a low institutional development and the abrupt political changes it drives.

Finally, we show that in both types of reforms the second-best environmental tax exceeds the pigouvian tax.

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6 Appendix A

6.1 Characterization of the market equilibrium

From the first-order conditions for the households' problem we obtain an Euler equation, characterizing the consumption intertemporal choice:

$$\left(\frac{C_{t+1}}{C_t}\right)^{\sigma}\left(\frac{K_{t+1}}{K_t}\right)^{\chi_1\,\eta(1-\sigma)}\left(\frac{Z_{t+1}}{Z_t}\right)^{-\chi_2\,\eta(1-\sigma)}=\beta\left[(1-\tau_K)\,r_{t+1}+1-\delta\right], \tag{28}$$

and the equilibrium condition between interest rates on debt and productive capital:

$$R_t=(1-\tau_K)\,r_{t+1}+1-\delta. \tag{29}$$

Equation (29) shows that interest rate on debt is equal to the return on capital renting, net of income capital tax and depreciation.

First-order conditions for the firms' maximization problem are given by:

$$r_t=A-\tau_{P,t}\,\chi_1\frac{P_t}{K_t}, \tag{30}$$

and

$$1=\tau_{P,t}\,\chi_2\,\frac{P_t}{Z_t} \tag{31}$$

Equation (30) shows that firms rent capital up to the point where its marginal costs r_t equals the private marginal product of capital, A, minus the marginal pollution tax payments. Equation (31) shows that the level of abatement activities is chosen so that its marginal costs is equal to the marginal pollution tax payments. That marginal cost is equal to one since we assume a one-to-one technology which allows us to transform output into abatement activities without additional costs.

By combining (31) with (30), the equation for the real interest rate is:

$$r_t=A-\frac{\chi_1}{\chi_2}\frac{Z_t}{K_t}. \tag{32}$$

We get from (30) and (3) the abatement-to-capital ratio:

$$\left(\frac{Z_t}{K_t}\right)_M = \left(\frac{\tau_{P,t}\,\chi_2}{K_t^{1-(\chi_1-\chi_2)}}\right)^{\frac{1}{1+\chi_2}}. \tag{33}$$

Normalizing the optimal pollution tax rule by $K_t^{1-(\chi_1-\chi_2)}$, we obtain the detrended pollution tax ($\bar{\tau}_P$), which is constant along a balanced growth path:

$$\bar{\tau}_P = \frac{\tau_{P,t}}{K_t^{1-(\chi_1-\chi_2)}}, \tag{34}$$

so that equation (33) can be written as:

$$\left(\frac{Z_t}{K_t}\right)_M = (\chi_2\bar{\tau}_P)^{1/(1+\chi_2)}, \tag{35}$$

showing that the pollution tax is the only tax that enhances abatement activities.

By substitution of (8) into (32), we obtain the market return on capital renting (10):

$$r_t = A - \xi\,\bar{\tau}_P^{1/(1+\chi_2)}, \qquad \text{where} \quad \xi = \chi_1 \cdot \chi_2^{-\frac{\chi_2}{1+\chi_2}},$$

which is inversely related to the pollution tax, and unaffected by changes of the other taxes in the economy.

By combining (10) and (28), and imposing the condition for balanced growth rate $\left(\frac{K_{t+1}}{K_t} = \frac{C_{t+1}}{C_t} = \frac{Z_{t+1}}{Z_t} = g_M\right)$, the equation for the market growth rate (g_M) is derived:

$$g_M = \left\{\beta\left[(1-\tau_K)\left(A - \xi\,\bar{\tau}_P^{1/(1+\chi_2)}\right) + 1 - \delta\right]\right\}^{1/(\sigma+(\chi_1-\chi_2)\eta(1-\sigma))}. \tag{36}$$

From (7) and (8) we obtain the market consumption-capital ratio:

$$\left(\frac{C_t}{K_t}\right)_M = A + 1 - \delta - g_M(\bar{\tau}_P, \tau_K) - (\chi_2\bar{\tau}_P)^{1/(1+\chi_2)}, \tag{37}$$

which is increasing with capital income tax, while the response of consumption - capital ratio to a change of pollution tax is indetermined.

Several parameter constraints are necessary for the existence of a balanced growth path. More precisely, a negative consumption-capital ratio and a negative growth rate must be ruled out. Parameters must also satisfy the transversality condition.[14]

7 Appendix B

7.1 Proof of proposition 3

The detrended first-best optimal pollution tax (i.e. pigouvian tax) in equation (21) is obtained by comparison of the market and the efficient abatement/capital ratios (equations (8) and (18) respectively). Using (9), the equation (20) is obtained.

Substitution of the optimal pollution tax into the competitive growth rate equation (12) and by comparison with the planner growth rate (17), the optimal income tax rate is zero $(\tau_K^* = 0)$. Inserting the optimal pollution tax in the equation for $(C_t/K_t)_M$, that is (13), and taking into account the conditions $\tau_K^* = 0$ and $g_M = g_P$, the identity $(C_t/K_t)_P = (C_t/K_t)_M$ is obtained. Since the consumption tax is a lump-sum tax in this setup, it does not distort the intertemporal or intratemporal allocation of consumption and capital accumulation. Therefore, any value of τ_C is optimal.

7.2 Proof of proposition 4

Using (23) in $\Phi(\tau_{K,1}, \bar{\tau}_{P,1}) = (1 - \tau_{K,1}) \left(A - \xi \, \bar{\tau}_{P,1}^{1/(1+\chi_2)} \right)$, we get that $\Phi(\tau_{K,1}, \bar{\tau}_{P,1}) = \Phi(\tau_{K,0}, \bar{\tau}_{P,0})$. So, from (12), the market growth rate remains constant $(g_{M,1} = g_{M,0})$.

[14]From equations (12) and (13) we see that these requirements are fulfilled for:

$$C_t/K_t > 0: \quad A + 1 - \delta - g_M \geq (\chi_2 \bar{\tau}_P)^{\frac{1}{1+\chi_2}} ,$$

$$g_M > 1: \quad \Phi(\tau_K, \bar{\tau}_P) + 1 - \delta \geq \frac{1}{\beta}.$$

$$\text{Transversality condition:} \quad 0 < \beta g_M^{(1-\sigma)(1-\eta(\chi_1 - \chi_2))} < 1 .$$

Combining (8), (13) and (16) and using $\xi = \chi_1 \cdot \chi_2^{-\frac{\chi_2}{1+\chi_2}}$, we obtain:

$$\begin{aligned}\Psi\left(\tau_{K,1},\ \bar{\tau}_{P,1},\ \tau_{C,1}\right) &= \tau_{K,1}\left(A - \xi\left(\bar{\tau}_{P,1}\right)^{1/(1+\chi_2)}\right) + \frac{\xi}{\chi_1}\left(\bar{\tau}_{P,1}\right)^{1/(1+\chi_2)} + \\ &\quad \tau_{C,1}\left(A + 1 - \delta - g_{M,1}\left(\bar{\tau}_{P,1}, \tau_{K,1}\right) - \left(\chi_2 \bar{\tau}_{P,1}\right)^{1/(1+\chi_2)}\right)\end{aligned} \tag{38}$$

By substitution of (23) and (24) in (38), and using the previous result that $g_{M,1} = g_{M,0}$, and $\xi = \chi_1 \cdot \chi_2^{-\frac{\chi_2}{1+\chi_2}}$ we get:

$$\Psi\left(\tau_{K,1},\ \bar{\tau}_{P,1},\ \tau_{C,1}\right) = \Psi_0.$$

It is obvious that since the new tax mix keeps the growth rate (g_M) and the detrended tax revenues (Ψ_0) unchanged, the new tax mix yields for every period the same path of public revenues than the initial tax mix.

7.3 Proof of corollary

From (23), it is obvious that if $\bar{\tau}_{P,1} > \bar{\tau}_{P,0}$, then $\tau_{K,1} > \tau_{K,0}$.

Next we prove that $\tau_{C,1} < \tau_{C,0}$ when $\tau_{C,0} < \frac{1-\chi_1}{\chi_2}$. First, note that detrended tax revenues $\Psi(.)$ are increasing in consumption tax rate because the consumption tax does not affect the intratemporal nor the intertemporal allocation of resources. Second, it can be easily proved that the detrended revenues raised by the tax mix $\left(\tau_{K,1},\ \bar{\tau}_{P,1},\ \tau_{C,0}\right)$ (with $\tau_{K,1}$ given by (23)) are larger than the revenues raised by $\left(\tau_{K,0},\ \bar{\tau}_{P,0},\ \tau_{C,0}\right)$, when $\tau_{C,0} < \frac{1-\chi_1}{\chi_2}$ (that is, $\Psi\left(\tau_{K,1},\ \bar{\tau}_{P,1},\ \tau_{C,0}\right) > \Psi\left(\tau_{K,0},\ \bar{\tau}_{P,0},\ \tau_{C,0}\right)$):

Using (23) in (22), and taking into account that growth rate remains constant:

$$\begin{aligned}\Psi\left(\tau_{K,1},\ \bar{\tau}_{P,1},\ \tau_{C,0}\right) &= A - \Phi(\tau_{K,0}, \bar{\tau}_{P,0}) \\ &\quad + \xi\left(\bar{\tau}_{P,1}\right)^{1/(1+\chi_2)}\left(\frac{1-\chi_1}{\chi_1}\right) \\ &\quad + \tau_{C,0}\left(A + 1 - \delta - g_{M,0} - \left(\chi_2 \bar{\tau}_{P,1}\right)^{1/(1+\chi_2)}\right),\end{aligned}$$

and, therefore,

$$\begin{aligned}&\Psi\left(\tau_{K,1},\ \bar{\tau}_{P,1},\ \tau_{C,0}\right) - \Psi\left(\tau_{K,0},\ \bar{\tau}_{P,0},\ \tau_{C,0}\right) \\ &= \left[\left(\frac{1-\chi_1}{\chi_1}\right)\xi - \tau_{C,0}\left(\chi_2\right)^{1/(1+\chi_2)}\right]\left[\left(\bar{\tau}_{P,1}\right)^{1/(1+\chi_2)} - \left(\bar{\tau}_{P,0}\right)^{1/(1+\chi_2)}\right]\end{aligned}$$

which is positive when $\tau_{C,0} < \frac{1-\chi_1}{\chi_2}$, since $\bar{\tau}_{P,1} > \bar{\tau}_{P,0}$ and $\xi = \chi_1\ \chi_2^{-\frac{\chi_2}{1+\chi_2}}$.

Hence, the green tax reform satisfies $\Psi\left(\tau_{K,1},\ \bar{\tau}_{P,1},\ \tau_{C,1}\right) = \Psi\left(\tau_{K,0},\ \bar{\tau}_{P,0},\ \tau_{C,0}\right)$ only if $\tau_{C,1} < \tau_{C,0}$, when $\tau_{C,0} < \frac{1-\chi_1}{\chi_2}$.

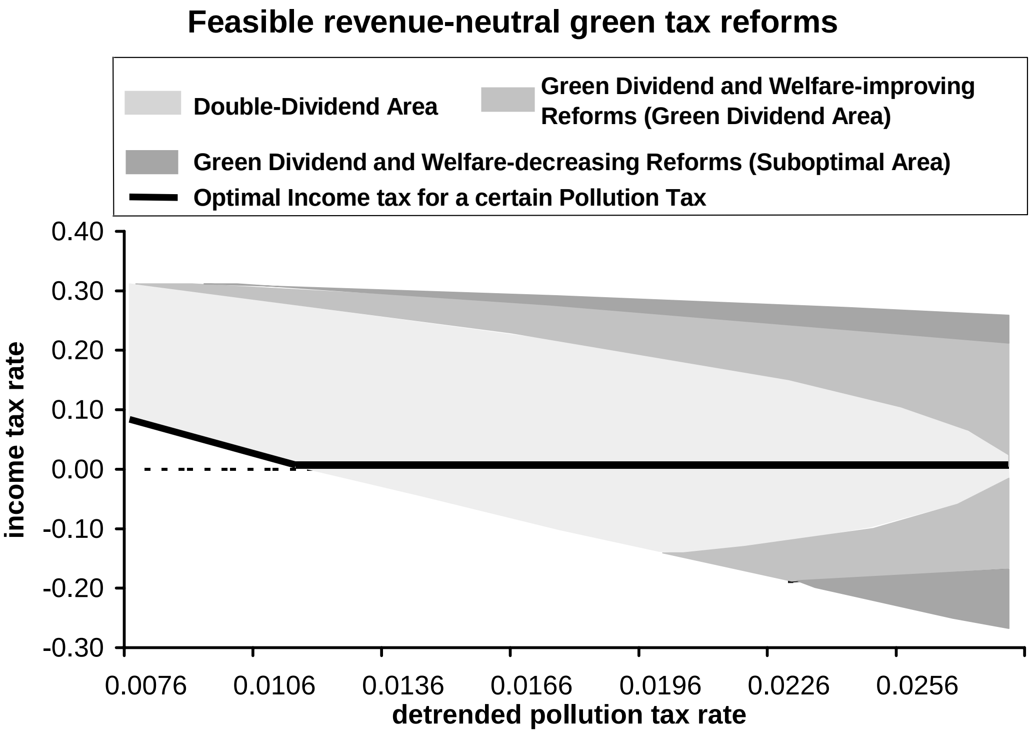

Figure 1. Tax reforms yielding double-dividend

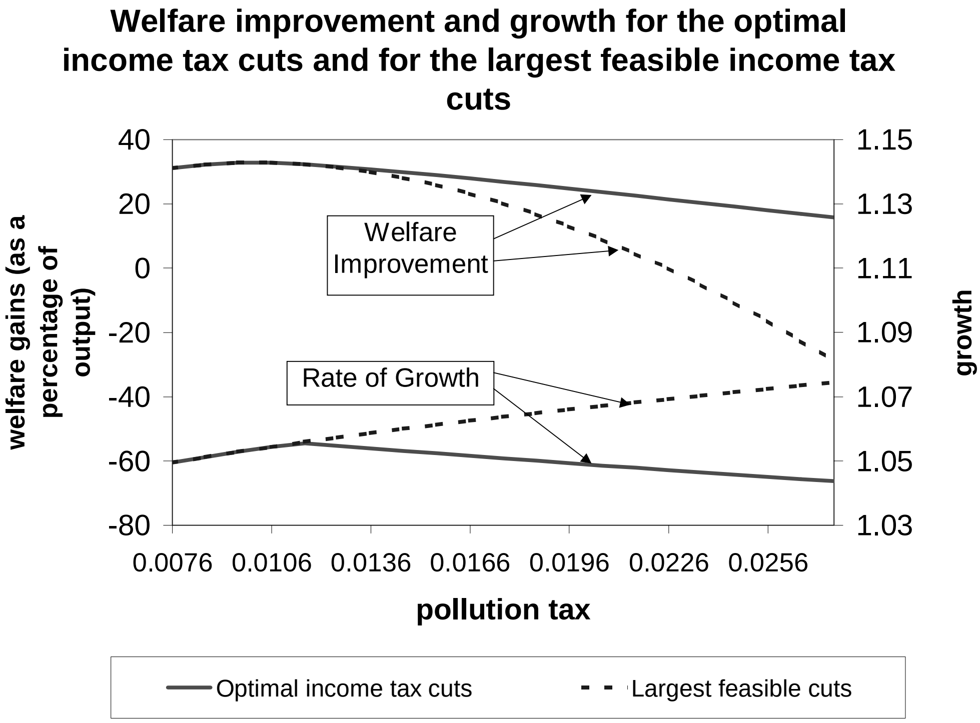

Figure 2. Welfare improvement and growth from optimal tax cuts and largest feasible tax cuts

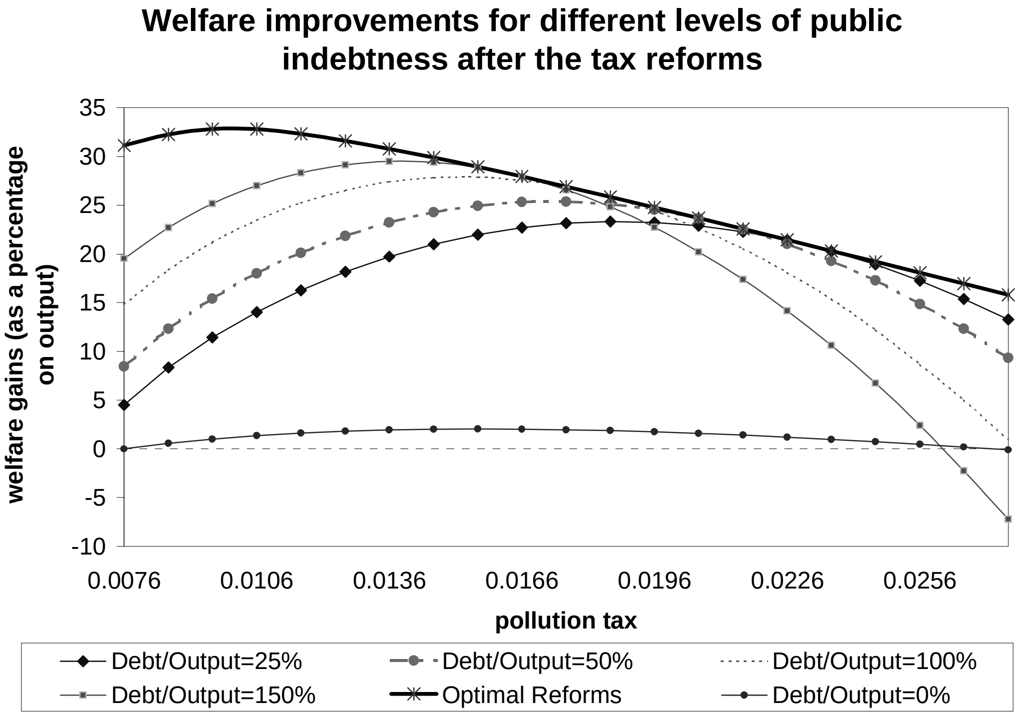

Figure 3. Welfare improvements for different levels of public indebtness after the reform (percentage on output)

	Pigouvian tax (%)	Second-best pollution tax (%)	Ratio environmental tax revenues to output (%) for the initial pollution tax ($\tau_{P,t}P_t/Y_t$)	Percentage Points Increment of the Pollution Tax (revenues/output) satisfying double dividend (X-axis length in Figure 1, %)
$\boldsymbol{\sigma} = \mathbf{1.50}$	**0.76**	**0.96**	**22**	**24**
$\sigma = 2.00$	1.10	1.60	31	13
$\sigma = 3.00$	1.43	2.03	37	5
$\eta = 0.50$	0.34	0.64	16	22
$\boldsymbol{\eta} = \mathbf{0.95}$	**0.76**	**0.96**	**22**	**24**
$\eta = 1.50$	1.31	1.32	27	28
$\chi_1 = 0.10$	0.73	1.13	24	20
$\boldsymbol{\chi}_1 = \mathbf{0.40}$	**0.76**	**0.96**	**22**	**24**
$\chi_1 = 0.80$	0.85	0.90	21	27
$\chi_2 = 0.10$	2.70	2.80	22	115
$\boldsymbol{\chi}_2 = \mathbf{0.40}$	**0.76**	**0.96**	**22**	**24**
$\chi_2 = 0.80$	0.19	0.29	20	9

Table 1: Sensitivity analysis of the "Double Dividend Area" size. We consider three values of each parameter. Benchmark parameterization is emphasized. Second colum shows the pigouvian tax. Third column shows the second-best pollution tax rate. Fourth column shows the tax revenues raised for the pollution tax fixed at its pigouvian level, as a percentage on output. Fifth column shows how much the government can increase tax revenues collected by the pollution tax (as a percentage on output), holding the target of double dividend.