THE STOCHASTIC SEASONAL BEHAVIOR OF ENERGY COMMODITY CONVENIENCE YIELDS (*)

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Abstract

This paper contributes to the commodity pricing literature by consistently modeling the convenience yield with its empirically observed properties. Specifically, in this paper, we show how a four-factor model for the stochastic behavior of commodity prices, with two long- and short-term factors and two additional seasonal factors, may accommodate some of the most important empirically observed characteristics of commodity convenience yields, such as the mean reversion and stochastic seasonality. Based on this evidence, a theoretical model is presented and estimated to characterize the commodity convenience yield dynamics that are consistent with previous findings. We also show that commodity price seasonality is better estimated through convenience yields than through futures prices.

Key Words: Stochastic Calculus, Commodity Prices, Convenience Yield, Seasonality, Kalman Filter.

JEL Codes: C32, C51, C60, G13.

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1. Introduction

In the case of consumption commodities (commodities that are consumption assets rather than investment assets), the benefit from holding the physical asset net of the storage cost is sometimes referred to as the "convenience yield" provided by the commodity (see for example Hull, 2003).

In other words, if we denote by F_t and S_t the futures and spot prices, respectively, in the case of consumption commodities, we do not necessarily have equality in $F_t \leq S_t \cdot e^{(r+u)\cdot(T-t)}$ (where r and u represent the risk-free rate and storage costs, respectively, and $T–t$ is the time to maturity) because users of a consumption commodity may feel that ownership of the physical commodity provides benefits that are not obtained by holders of futures contracts. For example, an oil refiner is unlikely to regard a futures contract on crude oil as equivalent to crude oil held in inventory. The crude oil in inventory can be an input to the refining process, whereas a futures contract cannot be used for this purpose. In general, ownership of the physical asset enables a manufacturer to keep a production process running and perhaps profit from temporary local shortages. A futures contract does not do the same (see, for example, Brennan and Schwartz, 1985). Therefore, the convenience yield net of storage costs, denoted by δ^*, is defined such that $F_t \cdot e^{\delta^* \cdot (T-t)} = S_t \cdot e^{r \cdot (T-t)}$.

Previous studies have considered the convenience yield as a deterministic function of time, such as those by Brennan and Schwartz (1985), or as a stochastic process, such as those by Gibson and Schwartz (1990) and Schwartz (1987). Specifically, Gibson and Schwartz (1990) allow for stochastic convenience yield of crude oil to develop a two-factor oil contingent claims price model. Moreover, Gibson and Schwartz (1990) show that convenience yields exhibit mean reversion, which is consistent with the theory of storage (see, for example, Brennan, 1985) in which an inverse relationship is established between the net convenience yield and the inventory level. Schwartz (1997) presents and empirically compares several factor models in which the convenience yield is assumed to be a stochastic factor. Hilliard and Reis (1998) and Miltersen and Schwartz (1998) use models with stochastic convenience yield to value commodity derivatives (futures and options). More recently, Casassus and Collin-Dufresne (2005) characterize a three-factor model, "maximal" in a sense of Dai and Singleton (2000), of commodity spot prices, convenience yields and interest rates, which nests many existing specifications.

Wei and Zhu (2006) investigate the empirical properties of convenience yields in the US natural gas market, finding that convenience yields are highly variable and economically significant, with their variability depending on the spot price level, the spot price variability and the variability of lagged convenience yields.

Although there have been many papers analyzing the seasonal behavior of some commodity prices (Lucia and Schwartz, 2002, Sorensen, 2002, Manoliu and Tompaidis, 2002, Garcia et al., 2012, among others), considerably less attention has been paid to the seasonal behavior of convenience yields. Based on the finding of seasonality in the convenience yield made by Fama and French (1987), Amin et al. (1994) propose a one-factor model for the spot price with a deterministic seasonal convenience yield. More recently, Borovkova and Geman (2006) present a two-factor model in which the first factor is the average forward price, instead of the commodity spot price, and the second factor is the stochastic convenience yield. These authors allow for a deterministic seasonal premium within the convenience yield.

In this paper, we go further by presenting a factor model in which the (stochastic) convenience yield exhibits stochastic seasonality. Specifically, we show that the four-factor model presented by Garcia et al. (2012), with two long- and short-term factors and two additional trigonometric seasonal factors, may generate stochastic seasonal convenience yields. An expression for the instantaneous convenience yield within this model is obtained, showing that the instantaneous convenience yield exhibits mean reversion and stochastic seasonality. Moreover, a $\pi/2$ lag is found in the convenience yield seasonality with respect to spot price seasonality.

Based on this evidence, the next step is to present a theoretical model to characterize the commodity convenience yield dynamics consistent with previous findings. Specifically, the model considers mean reversion and stochastic seasonal effects in the convenience yield. The model is estimated using data from a variety of energy commodity futures prices: crude oil, heating oil, gasoline and natural gas. We also show that commodity price seasonality is better estimated through convenience yields rather than through futures prices. The reason is that futures prices are driven for many things, such as supply, demand, political aspects, speculation, weather conditions, etc. Therefore, it may sometimes be difficult to extract the seasonal component from futures prices. However, as we will show in Section 2, the convenience yield is estimated though a ratio of two futures prices, so many of these non-seasonal factors tend to disappear, facilitating the estimation of the seasonal component.

The remainder of this paper is organized as follows. Section 2 presents the data and some preliminary findings regarding seasonality in convenience yields. We show that convenience yields exhibit mean reversion and stochastic seasonality, using data from crude oil, heating oil, gasoline and natural gas futures markets. In Section 3, we present the four-factor model accounting for stochastic seasonality in commodities and the expression for the instantaneous convenience yield derived from this four-factor model. In Section 3, we also discuss the properties of the model-estimated convenience yields for the four commodities under study, showing that they in fact exhibit mean reversion, stochastic seasonality and a $\pi/2$ lag with respect to spot price seasonality. Based on this empirical evidence, in Section 4, a factor model is proposed and estimated characterizing the commodity convenience yield dynamics, considering mean reversion and stochastic seasonal effects in the convenience yield. Finally, Section 5 concludes with a summary and discussion.

2. Data and Preliminary Findings

In this section, we present a data description of the futures prices for the four commodities used in the paper, i.e., WTI crude oil, heating oil, RBOB gasoline and Henry Hub natural gas. In addition, the procedure presented by Gibson and Schwartz (1990) is described to obtain the convenience yield data. The section concludes by analyzing the main empirically observed characteristics of the convenience yield data.

2.1. Data Description

2.1.1. Futures Prices

The data set used in this paper consists of weekly observations of WTI (light sweet) crude oil, heating oil, unleaded gasoline (RBOB) and natural gas futures prices traded on NYMEX during the period 9/27/1999 to 7/4/2011 (615 weekly observations).

Futures are traded on NYMEX with maturities from one month up to seven years for WTI crude oil, from one to eighteen months for heating oil, from one to twelve months for RBOB gasoline and from one month to six years for Henry Hub natural gas. However, liquidity is scarce for the futures with longer maturities, mostly in the case of gasoline.

In the estimation of the models presented below, a representative set of maturities has been used for each commodity. Therefore, in the case of WTI crude oil, the data set comprises contracts F1, F4, F7, F11, F14, F17, F20, F24 and F27, where F1 is the contract for the month closest to maturity, F2 is the contract for the second-closest month to maturity, and so on. In the case of heating oil, the data set comprises contracts F1, F3, F5, F7, F10, F12, F14, F16 and F18. In the case of RBOB gasoline, the data set contains contracts F1, F3, F5, F7, F9 and F12. Finally, in the case of Henry Hub natural gas, the data set contains contracts F1, F5, F9, F14, F18, F22, F27, F31 and F35. The main descriptive statistics of these variables are contained in Table 1.

2.1.2. Convenience Yield

The estimation of the convenience yield series is conducted using the procedure defined by Gibson and Schwartz (1990). Based on the convenience yield definition, $F_t \cdot e^{\delta^* \cdot (T-t)} = S_t \cdot e^{r \cdot (T-t)}$, we have the following:

$$F(S_t,T)=S_t\cdot\exp\{(r_{t,T}-\delta^*{}_{t,T})\cdot(T-t)\}$$

where $r_{t,T}$ is the interest rate of a zero-coupon bond with $T–t$ years to maturity and $\delta^*{}_{t,T}$ is the cumulate convenience yield in the following $T–t$ years for this commodity. Analogously

$$F(S_t,T+\Delta T)=S_t\cdot\exp\{(r_{t,T+\Delta T}-\delta^*{}_{t,T+\Delta T})\cdot(T+\Delta T-t)\}$$

where $r_{t,T+\Delta T}$ is the interest rate of a zero coupon bond with $T+\Delta T–t$ years to maturity and $\delta^*{}_{t,T+\Delta T}$ is the cumulate convenience yield in the following $T+\Delta T–t$ years for this commodity.

From these expressions we have

$$\frac{F(S_t,T+\Delta T)}{F(S_t,T)}=\exp\Big\{\big((T+\Delta T-t)\cdot r_{t,T+\Delta T}-(T-t)\cdot r_{t,T}\big)-\big((T+\Delta T-t)\cdot\delta^*{}_{t,T+\Delta T}-(T-t)\cdot\delta^*{}_{t,T}\big)\Big\} \quad (1)$$

On the other hand, by definition,

$$\exp\Big\{(T+\Delta T-t)\cdot r_{t,T+\Delta T}-(T-t)\cdot r_{t,T}\Big\}=\exp\{r_{implicit\_T\_to\_T+\Delta T}\}$$

where $r_{implicit\_T\_to\_T+\Delta T}$ is the implicit interest rate from T to $T+\Delta T$, and

$$\exp\Big\{(T+\Delta T-t)\cdot\delta^*{}_{t,T+\Delta T}-(T-t)\cdot\delta^*{}_{t,T}\Big\}=\exp\{\delta_{implicit\_T\_to\_T+\Delta T}\}$$

where $\delta_{implicit\_T\_to\_T+\Delta T}$ is the implicit convenience yield from T to $T+\Delta T$.

Taking into account these definitions, expression (1) can be written as

$$\frac{F(S_t,T+\Delta T)}{F(S_t,T)}=\exp\{r_{implicit\_T\_to\_T+\Delta T}-\delta_{implicit\_T\_to\_T+\Delta T}\}$$

or, equivalently

$$\delta_{implicit\_T\_to\_T+\Delta T}=r_{implicit\_T\_to\_T+\Delta T}-\ln\left\{\frac{F(S_t,T+\Delta T)}{F(S_t,T)}\right\}$$

$\delta_{implicit\_T\_to\_T+\Delta T}$ may be used as a proxy for the instantaneous forward convenience yield $\delta_{t,T}$, where T = 1/12, 2/12, … When $T = t$, this instantaneous forward convenience yield became the instantaneous spot convenience yield: $\delta_{t,t}\equiv\delta_t$. This instantaneous forward convenience yield is the one that is modeled in the related literature (see, for example, Schwartz, 1997 or Casassus and Collin-Dufresne, 2005) and the one that will be modeled in the following sections.

Following this procedure, we have estimated the instantaneous forward convenience yield series for the four commodity futures prices series described above. Therefore, convenience yield series have been estimated from one to twenty seven months in the case of WTI, from one to eighteen months in the case of heating oil, from one to twelve months in the case of RBOB gasoline and from one to thirty five months in the case of Henry Hub natural gas. δ_1 denotes the estimated convenience yield corresponding to month 1, δ_2 the estimated convenience yield corresponding to month 2, and so on. The main descriptive statistics of some of these convenience yield series are summarized in Table 2. In Figure 1, we plot the time series evolution of some of the estimated convenience yields together with the corresponding futures prices for the four commodities under study. The figures depict the mean-reverting and seasonality effects, which are much clearer in the convenience yield than in the spot/futures prices. However, because crude oil is not a clearly seasonal commodity, the pattern is less clear in the case of WTI crude oil. These issues are further discussed below.

2.2. Preliminary Findings

Previous studies have found evidence of mean reversion in the convenience yield dynamics. From convenience yield data obtained as in the previous sub-section, Gibson and Schwartz (1990) show a strong mean-reverting tendency in the convenience yield, which is consistent with the theory of storage (see, for example, Brennan, 1985) in which an inverse relationship is established between the level of inventories and the relative net convenience yield.

Fama and French (1987) note that seasonality in production or demand may generate seasonality in inventories. Under the theory of storage, inventory seasonality generates seasonality in the marginal convenience yield. Following this reasoning, Borovkova and Geman (2006) present a model allowing for a deterministic seasonal premium within the convenience yield.

In this study, using the estimated instantaneous spot and forward convenience yield series from the previous sub-section for the four commodities under study, we will investigate the existence of mean reverting and seasonal effects in the convenience yield.

Table 3 presents the results of the unit root tests for WTI, heating oil, gasoline and Henry Hub natural gas convenience yield series. The empirical evidence from previous studies of mean reversion is confirmed in the present work using the standard Augmented Dickey-Fuller test. Specifically, we are able to reject the null hypothesis of a unit root in all the cases, with the only exception being WTI crude oil (mostly as we go further in time). These results are consistent with the time evolution of the series shown in Figure 1.

The presence of seasonality in the estimated convenience yield series is assessed through the Kurskal-Wallis test. To perform the test, we have computed monthly averages from the weekly estimated convenience yield series. The null hypothesis of the test is that there are no monthly seasonal effects. The test was also performed for the futures prices series. The results of the test are shown in Table 4. The results for the convenience yield series indicate the rejection of the null hypothesis of no seasonal effects in all cases, except for WTI crude oil. The seasonal effects are even clearer in the cases of RBOB gasoline and Henry Hub natural gas convenience yield series. The test does not detect seasonal effects in the case of the futures prices series. These seasonal effects are evident in Figure 1.

As explained above, Borovkova and Geman (2006) allow for a deterministic seasonal premium within the convenience yield. However, it may be possible that seasonal effects in the convenience yield are stochastic rather than deterministic. Garcia et al. (2012) present a model of the stochastic behavior of commodity prices allowing for stochastic seasonality in commodity prices. Following this idea, we will test for the existence of stochastic seasonal effects in the convenience yield series.

The RBOB gasoline[1] convenience yield spectrum and its first differences are depicted in Figure 2, assuming that the series follows an AR(1) process with yearly seasonality, following the procedure described in Garcia et al. (2012). As explained by Garcia et al. (2012), sharp spikes in the spectrum are likely to indicate a deterministic cyclical component, whereas broad peaks often indicate a nondeterministic seasonal component. The asterisk (*) shown in the Figure denote harmonic points, calculated as $2\pi k/12$ (peaks) and $\pi(2k\text{-}1)/12$ (troughs), where k = 1, 2, 3, 4, 5 and 6.

Examining Figure 2, it appears that, more or less, the spectrum exhibits broad peaks and thoughts, suggesting that seasonality in convenience yields is stochastic rather than deterministic. However, these results must be taken with care, as aliasing effects and estimation errors may confuse deterministic and stochastic patterns.

In Figure 3, we plot the forward curves for the futures prices and the estimated convenience yield series on a representative date (July 4, 2011) in the case of Henry Hub natural gas prices[2]. Examining the figure, it can be appreciated that both futures and convenience yield series present an evident seasonal pattern; however, this evidence of seasonal pattern appears clearer in the convenience yield data than in futures data. Moreover, it is interesting to observe how the seasonal picks in the convenience yield series are delayed by three months compared to those observed in the futures series.

3. The Price Model

In this section, we show that a four-factor model for the stochastic behavior of commodity prices, with two long- and short-term factors and two additional seasonal factors, may accommodate some of the most important empirically observed characteristics of commodity convenience yields described in Section 2, such as the mean reversion, stochastic seasonality and a three-month delay in the convenience yield seasonality with respect to the spot price seasonality.

[1] The pattern for the rest of commodities is very similar.
[2] For short only the figure for Henry Hub natural gas is presented. The pattern is similar in the rest of the cases.

3.1. General Considerations

Based on the convenience yield definition, $F \cdot e^{\delta \cdot T} = S \cdot e^{r \cdot T}$, considering that the spot price (S_t) and the convenience yield (δ_t) are stochastic if $T > 0$, the previous equation may be expressed as an SDE in the following way:

$$dS_t = S_t(r - \delta_t)dt + \sigma dW_t^* \tag{2}$$

which is the classical definition of the convenience yield under the Q-measure (see, for example, Schwartz, 1997, or Casassus and Collin-Dufresne, 2005). Under the P-measure, the SDE may be expressed in the following way:

$$dS_t = S_t(\mu - \delta_t)dt + \sigma dW_t \tag{3}$$

To characterize the convenience yield dynamics, let $X_t = \log(S_t)$ be the log of the spot price. If we assume a linear model, as in the studies listed above, the general dynamics is given by

$$\begin{cases} dX_t = (m + AX_t)dt + RdW_t \\ S_t = \exp(\phi_0 + CX_t) \end{cases} \tag{4}$$

As shown by Garcia et al. (2008), the model above has an explicit (unique) solution (note that it is enough to solve for X_t):

$$X_t = e^{At}\left[X_0 + \int_0^t e^{-As} m ds + \int_0^t e^{-As} RdW_s\right]$$

Note that $S_t = \exp(\phi_0 + CX_t)$, and we would like to establish a stochastic differential equation for S_t. Taking differentials and using Ito's lemma

$$dS_t = \exp(\phi_0 + CX_t)CdX_t + \frac{1}{2}\exp(\phi_0 + CX_t)C(dX_t)(dX_t)'C = S_t\left[CdX_t + \frac{1}{2}C(dX_t)(dX_t)'C'\right]$$

Using the fact that $dtdt = dtdW_t = 0$ and $(dW_t)(dW_t)' = Idt$, we obtain

$$dS_t = S_t\left[C(m + AX_t)dt + CRdW_t + \frac{1}{2}CRR'C'dt\right]$$

and finally

$$dS_t = S_t\left[C\left(m + \frac{1}{2}RR'C' + AX_t\right)dt + CRdW_t\right] \tag{5}$$

If m is defined as $m = \begin{pmatrix} \mu \\ 0 \\ \vdots \\ 0 \end{pmatrix}$, which is necessary for the model to be maximal (or globally identifiable), we obtain that $Cm = \mu$ and from (4)

$$\delta_t = -C\left(\frac{1}{2}RR'C' + AX_t\right) \tag{6}$$

Therefore, with (6) we obtain the instantaneous convenience yield dynamics from the model factor dynamics.

3.2. Theoretical Model

In this subsection, we present a model to characterize the commodity price dynamics that considers seasonal effects and which is consistent with previous findings.

In the four-factor model by Garcia et al. (2012), the log spot price (X_t) is the sum of three stochastic factors, a long-term component (ξ_t), a short-term component (χ_t) and a seasonal component (α_t).

$$X_t = \xi_t + \chi_t + \alpha_t \tag{7}$$

The fourth stochastic factor is the other seasonal factor (α_t^*), which complements α_t. The SDEs of these factors are

$$d\xi_t = \mu_\xi dt + \sigma_\xi dW_{\xi t} \tag{8}$$

$$d\chi_t = -\kappa\chi_t dt + \sigma_\chi dW_{\chi t} \tag{9}$$

$$d\alpha_t = 2\pi\varphi\alpha_t^* dt + \sigma_\alpha dW_{\alpha t} \tag{10}$$

$$d\alpha_t^* = -2\pi\varphi\alpha_t dt + \sigma_\alpha dW_{\alpha^* t} \tag{11}$$

Equations (8) and (9) are identical to equations (2) and (1), respectively, reported by Schwartz and Smith (2000).

This model is "maximal" as described by Dai and Singleton (2000). Moreover, this model follows the Dai-Singleton $A_0(4)$ model, as shown in Appendix A. To see this, note that in the canonical form, given by expressions (4)

$$A = \begin{pmatrix} a & 0 & 0 & 0 \\ 0 & -\alpha & 0 & 0 \\ 0 & 0 & k & 2\pi\varphi \\ 0 & 0 & -2\pi\varphi & k \end{pmatrix}$$

and the model is globally identifiable. Garcia et al.'s (2012) model imposes the restriction $a = k = 0$ and $\alpha > 0$. In addition, as a restriction of a globally identifiable model imposing concrete values and intervals to the parameters, it is also globally identifiable and maximal. As stated above, in Garcia et al.'s (2012) model, we have[3]

[3] As seen by Garcia et al. (2012), $\rho_{\alpha\alpha^*} = 0$ and $\sigma_\alpha = \sigma_{\alpha^*}$.

$$A = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -k & 0 & 0 \\ 0 & 0 & 0 & 2\pi\varphi \\ 0 & 0 & -2\pi\varphi & 0 \end{pmatrix}, \; C = \begin{pmatrix} 1 & 1 & 1 & 0 \end{pmatrix}, \; m = \begin{pmatrix} \mu \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

And

$$RR' = \begin{pmatrix} \sigma_\xi^2 & - & - & - \\ \sigma_\xi \sigma_\chi \rho_{\xi\chi} & \sigma_\chi^2 & - & - \\ \sigma_\xi \sigma_\alpha \rho_{\chi\alpha} & \sigma_\chi \sigma_\alpha \rho_{\chi\alpha} & \sigma_\alpha^2 & - \\ \sigma_\xi \sigma_\alpha \rho_{\chi\alpha*} & \sigma_\chi \sigma_\alpha \rho_{\chi\alpha*} & 0 & \sigma_\alpha^2 \end{pmatrix}$$

Under this model, using expression (6), the instantaneous convenience yield may be written in the following way[4]:

$$\delta_t = -\frac{1}{2}(\sigma_\xi^2 + \sigma_\chi^2 + 2\sigma_\alpha^2 + 2\sigma_\xi\sigma_\chi\rho_{\xi\chi} + 2\sigma_\xi\sigma_\alpha\rho_{\chi\alpha} + 2\sigma_\xi\sigma_\alpha\rho_{\chi\alpha*} + 2\sigma_\chi\sigma_\alpha\rho_{\chi\alpha} + 2\sigma_\chi\sigma_\alpha\rho_{\chi\alpha*}) + k\chi_t - 2\pi\varphi\alpha_t^* \quad (12)$$

As can be appreciated in the previous expression, δ_t does not depend on the long-term factor, ξ_t, or the seasonal factor, α_t. However, it depends on the sum of factor variances, the short-term factor, χ_t, (times the speed of mean reversion) and the seasonal factor, which complements the one defined in the spot price, α_t^*, (times the seasonal frequency). In other words, the convenience yield is the sum of a constant term plus a short-term factor plus a seasonal factor.

The fact that δ_t is stationary (does not depend on the long-term factor and depends on the short-term one) in the previous expression is consistent with our previous findings and with the fact that the two-factor model defined by Schwartz-Smith (2000) is equivalent to that defined in Schwartz (1997), in which δ_t follows an Ornstein-Uhlenbeck process, which is a mean-reverting one. It is clear, therefore, that δ_t should depend on χ_t instead of ξ_t. It is also clear that the dependency should be modulated by *k* because a higher mean-reverting speed is associated with a greater benefit of holding the physical asset. Consider the following example: in a shortage, if the price reverts to its equilibrium level in a short-term period (high mean-reverting speed), the owner of the physical asset may sell the commodity and buy it again in a short period (consequently with a low cost), obtaining the benefit. In the other hand, if there is a delay in the reversion of the price to the equilibrium level (low mean-reverting speed), the owner of the physical asset must buy the commodity again at a higher price or else will be unable to keep the production process running.

[4] This is an expression for the instantaneous spot convenience yield. Taking expectations in (12) under the risk-neutral measure, it is easy to obtain an expression for the instantaneous forward convenience yield, which is estimated in Section 2.

Considering expression (2) and getting around the stochastic part of it, the following is clear: $\frac{dS_t}{S_t dt} \propto -\delta_t$. As $\frac{d\alpha_t}{dt} = 2\pi\varphi\alpha_t^*$, it is not suppressive that δ_t depends on α_t^* instead of α_t, which implies a $\pi/2$ lag in the convenience yield seasonality with respect spot price seasonality, which is consistent with our previous findings. As in the previous case, the dependency should be modulated by φ because a higher seasonal frequency is associated with a higher benefit of holding the physical asset.

The same can be said about the sum of factor variances; a higher variance is associated with a higher convenience yield (in absolute value) because the benefit of holding the physical asset is higher. It is interesting to note that the convenience yield depends on the sum of the factor variances instead of the spot price variance; that is, it depends on the whole system variance and not only the variance of the factors composing the spot price.

Finally, as stated above, it is worth noting that expression (12) for the instantaneous convenience yield is consistent with the empirical facts observed for the convenience yield in Section 2.2: mean reversion, (stochastic) seasonality and a three-month ($\pi/2$) lag in the convenience yield seasonality with respect to that of the spot price.

3.3. Estimation Results

Here, we present the results of the estimation of the four-factor model for the four commodities presented above. The model presented in Section 3.2 was estimated using the Kalman filter methodology (see, for example, Harvey 1989). The results are shown in Table 5.

It is found that in all cases, the seasonal factor volatility (σ_α) is significantly different from zero, and the seasonal period (φ) is more or less one year, implying that seasonality in all four commodity prices is stochastic with a period of one year, which is consistent with the results obtained by Garcia et al. (2012). Moreover, the speed of adjustment (k) is highly significant, implying, mean reversion in commodity prices, which is consistent with the results obtained by Schwartz (1997). It is also found that the long-term trend (μ_ξ) is positive and significantly different from zero in all cases, implying long-term growth in commodity prices, especially in the cases of RBOB gasoline, heating oil and WTI crude oil.

It is also interesting to note that short-term volatility (σ_χ) is higher than long-term volatility (σ_ξ) in all cases, which is consistent with the results found by Schwartz (1997) and Garcia et al. (2012).

Concerning the market prices if risk, it is found that the risk premium associated with the long-term factor (λ_ξ) is significantly different from zero in all cases, whereas the risk premium associated with the short-term one (λ_χ) is not, suggesting that the risk associated with the long-term factor is more difficult to diversify than the risk associated with the short-term one.

Moreover, the market prices of risk associated with the real and complex parts of the seasonal component (λ_α and λ_{α^*}, respectively) are not significantly different from zero in most of the cases, suggesting that the risk associated with the seasonal component may be diversified in most cases.

However, from the point of view of the goal of this paper, it is interesting to analyze the influence of the estimated parameters for each commodity on its convenience yield. As stated above, the speed of adjustment (k) is relatively high and significantly different from zero in all cases, implying a high convenience yield, especially in the case of RBOB gasoline, followed by Henry Hub natural gas. It is also found that the highest value of the seasonal period (φ) is found in the case of Henry Hub natural gas, followed by RBOB gasoline, heating oil and WTI crude oil, implying a higher convenience yield for Henry Hub natural gas and a lower one for WTI crude oil (in absolute value). Finally, from the estimated values shown in Table 5, it is easy to compute the term in parenthesis in expression (12), involving the standard deviations and the correlations among the model factors. It is found that the highest value for this term, and therefore the highest absolute value for the convenience yield, corresponds to Henry Hub natural gas (with a value of 0.2336), followed by RBOB gasoline (0.1296), WTI (0.1128) and heating oil (0.0993). Therefore, we can conclude that the highest estimated values of the convenience yield are found in the cases of Henry Hub natural gas and RBOB gasoline.

Finally, Figure 4 shows the time series evolution of the estimated seasonal components and the estimated convenience yield, both obtained with the four-factor model. A three-month delay of convenience yield seasonality (solid grey line) with respect to the commodity price seasonality (dotted grey line) is observed, although the pattern is less clear in the case of WTI crude oil because, as stated above, the seasonality of this commodity is less clear[5]. It is also interesting to observe how convenience yield seasonality is associated with the commodity price's complementary seasonal factor (black line). Nevertheless, the seasonal pattern is less clear in the case of WTI, which is consistent with the results found in Section 2.

4. The Convenience Yield Model

Here we present a model for the stochastic behavior of convenience yields. This model will account for stochastic seasonality. Moreover, it could be that in certain commodities such as crude oil, in which seasonality is not observed, it is possible that there is a weak seasonal component that is hidden by other factors, and this seasonal component may be estimated through the convenience yield.

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[5] This finding is due to crude oil's having a more or less regular demand during the year in contrast to heating oil and gasoline, which have more seasonal demand through the year.

Specifically, the proposed model for the convenience yield is the three-factor model of Garcia et al. (2012). This model will allow us to estimate crude oil seasonality through its convenience yield and to compare the spot price and convenience yield seasonality.

4.1. Theoretical Model

Here, we present a model characterizing the commodity convenience yield dynamics, which consider the seasonal effects and which are consistent with previous findings.

The proposed model for the stochastic behavior of convenience yields is the three-factor model of Garcia et al. (2012)[6]. In this three-factor model, the spot convenience yield (X_t) is the sum of a deterministic long-term factor (ξ_t) and two stochastic factors[7], a short-term component (χ_t) and a seasonal component (α_t):

$$X_t = \xi_t + \chi_t + \alpha_t \tag{13}$$

The third stochastic factor is the other seasonal factor (α_t^*), which complements α_t. The SDEs of these factors are as follows:

$$d\xi_t = \mu_\xi dt \tag{14}$$

$$d\chi_t = -\kappa\chi_t dt + \sigma_\chi dW_{\chi t} \tag{15}$$

$$d\alpha_t = 2\pi\varphi\alpha_t^* dt + \sigma_\alpha dW_{\alpha t} \tag{16}$$

$$d\alpha_t^* = -2\pi\varphi\alpha_t dt + \sigma_\alpha dW_{\alpha^* t} \tag{17}$$

As shown in the case of the four-factor model, this model is “maximal” according to Dai and Singleton (2000). Moreover, this model is of the Dai-Singleton (2000) $A_0(3)$ type, as shown in Appendix A.

4.2. Estimation Results

The three-factor model presented above has been estimated though the Kalman filter methodology using the convenience yield data estimated in Section 2. The results of the model estimation are shown in Table 6. The results indicate a high degree of mean reversion (high value of κ), mostly in the case of Henry Hub natural gas, which is consistent with the preliminary results obtained in Section 2.

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[6] A four-factor model like the one presented in Section 3 has been estimated for the convenience yield; however, the stochastic parameters related to the long-term factor were not significant, confirming previous evidence regarding the strong mean-reverting behavior of convenience yield series.

[7] It should be noted that in the original three-factor model by Garcia et al. (2012), the log-spot price is the sum of three stochastic factors. However, here we model the convenience yield price directly instead of its log, given that the convenience yield can take negative values.

However, the most important issue in Table 6, from the point of view of this paper goal, is that the standard deviation of the seasonal factor (σ_α) is significantly different from zero for all four commodities. This result suggests not only that convenience yields show seasonality but also that this seasonality is stochastic rather than deterministic. Moreover, the values of the standard deviation of the seasonal factor obtained in Table 6 for the convenience yield series are considerably higher than those obtained in Table 5 for the commodity price series. This result suggests that seasonality is even clearer in the convenience yield series than in the commodity price ones, as stated above. It is interesting to observe the high values of σ_α obtained in the cases of RBOB gasoline and Henry Hub natural gas convenience yield series, which is consistent with the results shown in Figure 1. It is also very interesting to observe that the WTI convenience yield series (and the WTI futures prices series in Table 5) also shows evidence of stochastic seasonality, although the tests in Section 2 did not detect evidence of seasonality in the case of WTI crude oil convenience yield series.

Examining expression (12), it is clear that the short-term component in the convenience yield is equal to the short-term component in the spot price multiplied by the speed of adjustment in the four-factor model (κ). Given that the estimated values of κ in the four-factor model (Table 5) do not differ significantly from one, the standard deviations of the short-term components in the convenience yield and the spot price series should be similar. This is the result found in the cases of RBOB gasoline and heating oil. The values of the standard deviations of the short-term component in the WTI and Henry Hub natural gas convenience yield series (Table 6) are higher than the corresponding values in the spot price series (Table 5) due to the high variability found in these convenience yield series, as shown in Figure 1.

Moreover, from expression (12), we can conclude that the seasonal component in the convenience yield is equal (in absolute value) to the complementary seasonal component in the spot price multiplied by $2\pi\varphi$. Given that the estimated values of the seasonal period (φ) in Table 5 are very close to one, the standard deviation of the spot price complementary factor[8] should be similar to the standard deviation of the convenience yield divided by 2π. In the case of WTI crude oil, the standard deviation of the complementary seasonal factor in the spot price model is 0.0106, whereas the standard deviation of the seasonal factor in the convenience yield model (divided by 2π) is 0.00844. The figures in the case of heating oil are 0.0118 and 0.0115, respectively. In the case of RBOB gasoline these figures are 0.0425 and 0.0760, respectively. Finally, the figures in the case of Henry Hub natural gas are 0.0385 and 0.0600, respectively.

This result can be corroborated by examining Figure 4. In this figure, the estimated convenience yield (solid gray line) shows a very similar pattern to that of the complementary

[8] Remember that in the four-factor model $\sigma_\alpha = \sigma_{\alpha^*}$.

seasonal factor (α^*) in the four-factor model (black line), although as before, the pattern is less clear in the case of WTI crude oil.

Table 7 presents a summary of the influence of the seasonal components on the commodity price (four-factor model for commodity spot prices) and on the convenience yield (three-factor model for convenience yields). Specifically, the table shows the average weights of the seasonal factors (α and α^*) in the log price of the commodity (Panel A) and in the convenience yield (Panel B)[9]. It is quite striking to observe how the weights of the seasonal components are considerable higher in the model for the convenience yield (Panel B). In both panels, the highest weights are achieved in the cases of RBOB, heating oil and Henry Hub natural gas. Finally, it is also interesting to observe the relatively high weight of the seasonal pattern on the convenience yield in the case of WTI crude oil, suggesting that in commodities such as crude oil, in which seasonality is not observed, there is a weak seasonal component, and this seasonal component may be estimated through the convenience yield.

In summary, we may conclude that the estimated convenience yield series show evidence of stochastic seasonality and that this seasonality is even clearer than in the case of commodity spot price series. This result suggests that commodity price seasonality may be better estimated through convenience yields rather than through futures prices. The reason is that futures prices are driven by many factors, such as supply, demand, political aspects, speculation, weather conditions, etc. Therefore, sometimes it may be difficult to extract the seasonal component from futures prices. However, as shown in Section 2, the convenience yield is estimated though a ratio of two futures prices, so many of these non-seasonal factors tend to disappear, facilitating the estimation of the seasonal component.

5. Conclusion

This paper focuses on commodity convenience yields, and, in this sense, convenience yields for four energy commodities (WTI crude oil, heating oil, RBOB gasoline and Henry Hub natural gas) are estimated using the procedure defined by Gibson and Schwartz (1990), finding, as in previous studies, that convenience yields exhibit seasonality and mean reversion and that convenience yield seasonality presents a three-month lag with respect to spot price seasonality. Based on this empirical evidence, we present a factor model in which the convenience yield exhibits mean reversion and stochastic seasonality. Specifically, we show that the four-factor model presented by Garcia et al. (2012), with two long- and short-term factors and two additional trigonometric seasonal factors, may generate stochastic seasonal mean-reverting

[9] The weight of the sum of the two seasonal factors (α and α^*) over the convenience yield price in Panel B of Table 7 is greater than 100%. This is because in the three-factor model, the convenience yield is the sum of a long-term (ξ, deterministic) component, a short-term (χ, stochastic) component and a seasonal (α, stochastic) component. The other seasonal component, α^*, does not influence the convenience yield price.

convenience yields. Moreover, a π/2 lag is found in the convenience yield seasonality with respect to spot price seasonality, which is also consistent with empirical findings.

Based on this evidence, the next step is to present a theoretical model to characterize the commodity convenience yield dynamics, which is consistent with previous findings. Specifically, the model considers mean reversion and stochastic seasonal effects in the convenience yield. We also show that commodity price seasonality may be better estimated through convenience yields rather than through commodity prices. The reason is that commodity prices are driven by several factors, such as supply, demand, political aspects, speculation, weather conditions, etc. Therefore, sometimes it may be difficult to extract the seasonal component from commodity prices. However, the convenience yield is estimated though a ratio of two commodity prices, so many of these non-seasonal factors tend to disappear, facilitating the estimation of the seasonal component.

APPENDIX A. CANONICAL REPRESENTATION

Introduction

In this appendix, we observe how our models may be related to Dai-Singleton (2000) $A_0(n)$ class, with the important distinction of allowing complex eigenvalues. We will then show global identification properties.

General setup

Let $Z_t = \log(S_t)$ be the log of the spot price. If we assume a linear model, its real dynamics is given by

$$\begin{cases} dX_t = (m + AX_t)dt + RdW_t \\ S_t = \exp(\phi_0 + CX_t) \\ X_0 \ \text{given} \end{cases} \qquad \text{(F)}$$

whereas its risk neutral dynamics is given by

$$\begin{cases} dX_t = (m - \lambda + AX_t)dt + RdW_t \\ S_t = \exp(\phi_0 + CX_t) \\ X_0 \ \text{given} \end{cases} \qquad \text{(FN)}$$

where R is the full rank lower triangular. We would like to know how this general setup may be reduced to a model that is maximal, i.e., cannot be reduced to an equivalent model with fewer states and parameters (another way to see this is saying that has the maximum number of identifiable parameters). We shall concentrate first on (F).

First (see, for example, Sontag 1990), a model has the minimal number of states if and only if is observable and controllable, i.e., $rank\begin{pmatrix} C \\ CA \\ \dots \\ CA^{n-1} \end{pmatrix} = n$ (observability condition) and $rank\left(R \;\; AR \;\; A^2R \dots A^{n-1}R\right) = n$ (controllability condition). Because the latter is always satisfied if R is full rank, we just impose the former. Moreover, in the context of stochastic systems, controllability plays a small role, as it means that some states are unaffected by noise; thus, whether they are observationally equivalent to other system depends only on initial states.

Invariant transformations

Following Dai and Singleton (2000), we allow for the following transformations

1. Affine transformations of states: $\widetilde{X}_t = v + GX_t$ where G is non-singular and v is an arbitrary vector. Note the important role of constants ϕ_0 and ϕ_1. If they were not present and output equation were CX_t, v could not be arbitrary but instead would have to accomplish $Cv = 0$.
2. Rotations of Brownian motions. $\widetilde{W}_t = UW_t$ where $UU^T = I$, as Brownian motion is unobserved.

Note that these transformations preserve the observability and rank of R.

Relationship with $A_0(n)$

We will first show how to relate our model to the Dai-Singleton (2000) $A_0(n)$ class, i.e., a system like (DS) $\begin{cases} dY_t = -KY_t dt + \Sigma d\widetilde{W}_t \\ S_t = \exp\left(\delta_0 + \widetilde{C}Y_t\right) \end{cases}$ where $R = I$, and K is lower triangular with all their diagonal elements strictly positive, i.e., $K_{ii} > 0$.

This indicates several restrictions within the system:

1. The dynamics matrix $-K$ is full rank, and all eigenvalues are real and negative.
2. The noise matrix is also full rank.

All of these properties are preserved through invariant transformations, so we would have to impose them on our system. However, we have complex eigenvalues, so we must use a different, although similar, canonical form. To summarize, we replace Dai-Singleton's (2000) restrictions with others, so our approaches are similar but not directly comparable.

First canonical form

If all eigenvalues are different, then the pair (F) can be reduced to

(F1) $\begin{cases} d\tilde{X}_t = \left(\tilde{m} + \tilde{A}\tilde{X}_t\right)dt + \tilde{R}d\tilde{W}_t \\ S_t = \exp\left(\tilde{C}\tilde{X}_t\right) \end{cases}$, where

1. $\tilde{A}$ is diagonal (real only if there are no complex eigenvalues)
2. $C = \left(1\ 1\ ...1\right)$.
3. $\tilde{R}$ is lower triangular, and all its diagonal elements are strictly positive.
4. $\tilde{m} = \begin{pmatrix} m_0 \\ 0 \end{pmatrix}$ with $m_0 \in \Re$.

Moreover, if we start with a canonical form (F1), the system is observable and controllable (therefore has the minimal number possible of states).

Proof

If all the eigenvalues are different, then A is diagonalizable. Therefore, changing the base, we have a representation where $\tilde{A}$ is diagonal. We shall see now that all elements in C are not null.

Let $\tilde{A} = diag(d_1 ... d_n)$. By the observability condition, the matrix $\begin{pmatrix} \tilde{C} \\ \tilde{C}\tilde{A} \\ ... \\ \tilde{C}\tilde{A}^{n-1} \end{pmatrix}$ is full rank.

However, this matrix equals $\begin{pmatrix} c_1 & c_2 & ... & c_n \\ c_1 d_1 & c_2 d_2 & ... & c_n d_n \\ ... & ... & ... & ... \\ c_1 d_1^{n-1} & c_2 d_2^{n-1} & ... & c_n d_n^{n-1} \end{pmatrix}$. Should any of the c_i be null, then its full column would be null, and therefore the system would not be observable. This also proves that, starting from canonical form (F1), the system is observable.

As a result, we may define the transformation $L_0 = diag\left(\frac{1}{c_1}, ..., \frac{1}{c_n}\right)$. Under this change of variables, $\tilde{C} = \left(1 ... 1\right)$ and $\tilde{A}$ is diagonal. Using a suitable orthogonal transformation of the noise, we may also impose the conditions on $\tilde{R}$ via a Choleski decomposition (thus proving also that the system is controllable because the noise matrix is full rank).

Now for the form of $\tilde{m}$, we define the new state as follows:

$\tilde{\tilde{X}}_t = \tilde{X}_t - \begin{pmatrix} \phi_0 + \mu_2 / d_2 + ... + \mu_n / d_n \\ -\mu_2 / d_2 \\ ... \\ -\mu_n / d_n \end{pmatrix}$. Clearly, it verifies the conditions.

Complex eigenvalues

It is now time to consider complex eigenvalues. The results are essentially the same, but the canonical form is slightly different. However, the two are perfectly equivalent. The proof of these results is a bit more involved but similar to the one above (see, for example, Sontag (1990)).

Second canonical form

If all eigenvalues are different, then (F) can be reduced to

(F2) $\begin{cases} d\widetilde{X}_t = \left(\widetilde{m} + \widetilde{A}X_t\right)dt + \widetilde{R}d\widetilde{W}_t \\ S_t = \exp\left(\widetilde{C}X_t\right) \end{cases}$, where all matrices are real and

1. $\widetilde{A} = \begin{pmatrix} A_1 & 0 & \dots & 0 \\ 0 & A_2 & \dots & 0 \\ .. & \dots & \dots & \dots \\ 0 & 0 & \dots & A_r \end{pmatrix}$ and either $A_i = \lambda_i \in \Re$ or $A_i = \begin{pmatrix} k & \varphi \\ -\varphi & k \end{pmatrix}$
2. $C = \left(C_1 \ldots C_r\right)$ each corresponding to A_i and $C_i = 1$ if $A_i = \lambda_i \in \Re$ or $C_i = \left(1\ 0\right)$ otherwise.
3. $\widetilde{R}$ is lower triangular and all its diagonal elements are strictly positive.
4. $\widetilde{m} = \begin{pmatrix} m_0 \\ 0 \end{pmatrix}$ with $m_0 \in \Re$

Maximality

To show that the model set is maximal, we observe that the model is globally identifiable, as in general, the latter implies the former if all parameters are admissible. To see this, remember that in a globally identifiable model, different parameters yield different realizations. Suppose that a model has *n* parameters and is not maximal but admits a representation with *k*<*n* parameters. By redefining the parameter space (under some conditions), we find that the last parameters are functions of the first; formally, $\theta = \left(\phi, \varphi(\phi)\right)$.

However, for a value ϕ^*, we may take a different value $\left(\phi^*, \varphi^*\right) \neq \left(\phi^*, \varphi\left(\phi^*\right)\right)$, thus obtaining a different admissible value. The only way to avoid contradiction would be that $\left(\phi^*, \varphi^*\right) \neq \left(\phi^*, \varphi\left(\phi^*\right)\right)$ achieves the same realization, but this is impossible, as the model is globally identifiable. We thus must conclude that the model is not maximal.

Two results give us the globally identification property.

Proposition

If S_t is observable, model (F3) is globally identifiable (including the initial state X_0)

Risk premia

It is now time to consider whether risk premia may be identified. If we start with model

$$(F2)\begin{cases} d\widetilde{X}_t = \left(\widetilde{m} + \widetilde{A}X_t\right)dt + \widetilde{R}d\widetilde{W}_t \\ S_t = \exp\left(\widetilde{C}X_t\right) \end{cases}$$

, its risk neutral version is given by

$$(F2N)\begin{cases} d\widetilde{X}_t = \left(\widetilde{m} - \lambda + \widetilde{A}X_t\right)dt + \widetilde{R}d\widetilde{W}_t \\ S_t = \exp\left(\widetilde{C}X_t\right) \end{cases}$$

If all futures are observable, then the system with the risk-neutral dynamics is also globally identifiable.

Proposition

In the above conditions, if $F_{t,T} = E^Q\left[S_{t+T} / I_t\right]$ is observable, then model (F3N) is globally identifiable.

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TABLE 1

DESCRIPTIVE STATISTICS. FUTURES PRICES

The table shows the mean and volatility of the four commodity futures prices series. The sample period is 9/27/1999 to 7/4/2011 (615 weekly observations). F1 is the futures contract closest to maturity, F2 is the contract second-closest to maturity and so on.

WTI Crude Oil			Heating Oil			Gasoline			Henry Hub		
	Mean	Volatility		Mean	Volatility		Mean	Volatility		Mean	Volatility
F1	55.06	31.30%	F1	64.46	31.73%	F1	64.59	36.81%	F1	5.68	46.80%
F4	55.59	26.49%	F3	64.96	28.08%	F3	64.19	30.13%	F5	6.04	32.53%
F7	55.57	23.83%	F5	65.17	26.04%	F5	63.73	26.26%	F9	6.17	26.91%
F11	55.36	21.69%	F7	65.27	24.08%	F7	63.37	24.53%	F14	6.15	22.48%
F14	55.17	20.57%	F10	65.23	21.59%	F9	63.24	24.31%	F18	6.13	20.80%
F17	54.98	19.72%	F12	65.13	20.61%	F12	63.00	23.77%	F22	6.06	21.55%
F20	54.80	19.05%	F14	65.07	20.10%	-	-	-	F27	5.99	19.57%
F24	54.60	18.44%	F16	65.04	20.04%	-	-	-	F31	5.96	20.05%
F27	54.48	18.13%	F18	65.02	19.95%	-	-	-	F35	5.89	19.17%

TABLE 2

DESCRIPTIVE STATISTICS. CONVENIENCE YIELD

The table shows the mean and volatility of the commodity convenience yield estimated prices series for the four commodities under study. The sample period is 9/27/1999 to 7/4/2011 (615 weekly observations). δ_i denotes the implicit convenience yield from month "*i*" to month to "*i*+1".

	WTI Crude Oil			Heating Oil			Gasoline			Henry Hub	
	Mean	Stand. Dev.		Mean	Stand. Dev.		Mean	Stand.Dev.		Mean	Stand. Dev.
δ_1	-0.01	0.29	δ_1	0.01	0.28	δ_1	0.08	0.41	δ_1	-0.30	0.58
δ_4	0.06	0.15	δ_3	0.04	0.24	δ_3	0.07	0.40	δ_5	-0.06	0.52
δ_7	0.08	0.11	δ_5	0.06	0.21	δ_5	0.09	0.34	δ_9	0.03	0.52
δ_{11}	0.08	0.09	δ_7	0.06	0.19	δ_7	0.08	0.33	δ_{14}	0.06	0.48
δ_{14}	0.07	0.08	δ_{10}	0.07	0.18	δ_9	0.07	0.35	δ_{18}	0.05	0.48
δ_{17}	0.07	0.07	δ_{12}	0.06	0.17	δ_{12}	-1.76	3.20	δ_{22}	0.07	0.50
δ_{20}	0.06	0.06	δ_{14}	0.06	0.16	-	-	-	δ_{27}	0.08	0.47
δ_{24}	0.06	0.05	δ_{16}	0.06	0.15	-	-	-	δ_{31}	0.06	0.51
δ_{27}	0.06	0.04	-	-	-	-	-	-	-	-	-

TABLE 3

UNIT ROOT TEST

The table shows the statistic of the Augmented Dickey-Fuller (ADF) test. The MacKinnon critical values for the rejection of the null hypothesis of a unit root tests are -3.4408 (1%), -2.8661 (5%) and -2.5692 (10%). δ_i denotes the implicit convenience yield from month "*i*" to month to "*i*+1".

	δ_1	δ_2	δ_3	δ_5	δ_8	δ_9	δ_{13}	δ_{15}	δ_{19}
WTI	-3.9166	-2.7213	-2.8424	-2.8563	-2.4789	-2.5284	-2.3123	-2.0427	-2.2670
Heating Oil	-4.6782	-3.5201	-5.3899	-5.1484	-5.4680	-6.0645	-5.4280	-5.5776	-
RBOB	-7.7077	-6.7132	-6.4348	-6.8391	-5.7703	-5.4273	-5.8969	-	-
Henry Hub	-5.8121	-5.8404	-6.2003	-6.8154	-6.7486	-7.5143	-6.8356	-7.4454	-7.7367

TABLE 4

SEASONALITY TEST

The table shows the statistic of the Kruskal-Wallis test for the presence of monthly seasonal effects in the convenience yield and futures prices series. The test statistic is distributed, under the null hypothesis of no seasonal effects, as a χ^2 with 11 degrees of freedom. The critical value for the rejection of the null hypothesis at 99% is 24.725. δ_i denotes the implicit convenience yield from month "*i*" to month to "*i*+1".

PANEL A: CONVENIENCE YIELD SERIES									
	δ_1	δ_2	δ_3	δ_4	δ_5	δ_6	δ_7	δ_8	δ_9
WTI	3.1077	1.9255	1.3683	1.2326	1.1293	0.8077	1.1751	1.2313	1.5310
H. Oil	44.1184	43.2305	49.3397	55.8876	65.7434	67.7825	72.0962	79.1446	82.5958
RBOB	74.5228	85.3857	91.9193	92.2936	94.8757	99.9759	99.1085	99.1205	101.5181
H. Hub	80.7324	82.6555	88.6157	101.8594	106.3644	107.5883	112.3180	113.4766	115.0153

PANEL B: FUTURES PRICES SERIES									
	F1	F2	F3	F4	F5	F6	F7	F8	F9
WTI	1.6030	1.6185	1.7030	1.8211	1.8778	1.9701	2.0292	2.0191	2.0056
H. Oil	0.8011	1.2526	1.8597	2.6570	3.4363	3.9374	4.0792	3.6899	3.0784
RBOB	7.2394	6.3069	4.9887	2.9503	1.3336	0.6160	0.7252	1.9820	3.9080
H. Hub	2.5974	1.5667	1.2366	1.3626	2.5048	3.5382	4.5504	4.8063	4.5406

TABLE 5

ESTIMATION RESULTS. FOUR-FACTOR MODEL

The table presents the results for the four-factor model applied to the four commodities under study: WTI crude oil, heating oil, RBOB gasoline and Henry Hub natural gas. Standard errors are in parentheses. The estimated values are reported with * denoting significance at 10%, ** denoting significance at 5%, and *** denoting significance at 1%.

	WTI	Heating Oil	RBOB	Henry Hub
Contracts	F1, F4, F7, F11, F14, F17, F20, F24, F27	F1, F3, F5, F7, F10, F12, F14, F16, F18	F1, F3, F5, F7, F9, F12	F1, F5, F9, F14, F18, F22, F27, F31, F35
μ_ξ	0.1132*** (0.0409)	0.1158*** (0.0395)	0.1084** (0.0474)	0.0655** (0.0302)
κ	1.0225*** (0.0101)	1.0301*** (0.0143)	1.9649*** (0.1691)	1.1323*** (0.0206)
φ	0.9566*** (0.0051)	0.9978*** (0.0002)	1.0029*** (0.0009)	1.0088*** (0.0002)
σ_ξ	0.1626*** (0.0045)	0.1573*** (0.0044)	0.1885*** (0.0058)	0.1201*** (0.0049)
σ_χ	0.2752*** (0.0090)	0.2458*** (0.0072)	0.3051*** (0.0119)	0.4367*** (0.0165)
σ_α	0.0106*** (0.0005)	0.0118*** (0.0006)	0.0425*** (0.0020)	0.0385*** (0.0022)
$\rho_{\xi\chi}$	0.0518 (0.0429)	0.1311*** (0.0409)	0.0573 (0.0722)	0.0117 (0.0603)
$\rho_{\xi\alpha}$	-0.2794*** (0.0719)	-0.1600** (0.0650)	-0.1050* (0.0556)	-0.0892 (0.0845)
$\rho_{\xi\alpha*}$	-0.2488*** (0.0695)	-0.1357* (0.0693)	0.2353*** (0.0620)	-0.0067 (0.0797)
$\rho_{\chi\alpha}$	0.3073*** (0.0759)	0.0994 (0.0685)	0.1760*** (0.0655)	0.2518*** (0.0812)
$\rho_{\chi\alpha*}$	0.3166*** (0.0727)	0.2957*** (0.0722)	-0.3956*** (0.0549)	0.3145*** (0.0740)
λ_ξ	0.1372*** (0.0409)	0.1532*** (0.0396)	0.1515*** (0.0492)	0.1025*** (0.0303)
λ_χ	0.0503 (0.0692)	-0.0011 (0.0619)	-0.0651 (0.0825)	-0.0869 (0.1101)
λ_α	-0.0017 (0.0029)	-0.0014 (0.0032)	-0.0062 (0.0112)	0.0111 (0.0105)
$\lambda_{\alpha*}$	-0.0050* (0.0029)	-0.0077** (0.0031)	0.0027 (0.0130)	-0.0138 (0.0106)
σ_η	0.0112*** (0.0001)	0.0094*** (0.0001)	0.0117*** (0.0002)	0.0376*** (0.0003)
Log-likelihood	27057.14	28139.46	16835.78	19318.02
AIC	27025.14	28107.46	16803.78	19286.02
SIC	26949.69	28032.02	16728.33	19210.58

TABLE 6

ESTIMATION RESULTS. THREE-FACTOR MODEL FOR THE CONVENIENCE YIELD

The table presents the results for the three-factor model applied to the four commodity convenience yield series under study: WTI crude oil, heating oil, RBOB gasoline and Henry Hub natural gas. Standard errors are in parentheses. The estimated values are reported with * denoting significance at 10%, ** denoting significance at 5%, and *** denoting significance at 1%. δ_i denotes the implicit convenience yield from month "*i*" to month to "*i*+1".

	WTI	Heating Oil	RBOB	Henry Hub
Convenience Yield Series	$\delta_1, \delta_4, \delta_7, \delta_{11}, \delta_{14}, \delta_{17}, \delta_{20}, \delta_{24}, \delta_{27}$	$\delta_1, \delta_3, \delta_5, \delta_7, \delta_{10}, \delta_{12}, \delta_{14}, \delta_{16}$	$\delta_1, \delta_3, \delta_5, \delta_7, \delta_9, \delta_{12}$	$\delta_1, \delta_5, \delta_9, \delta_{14}, \delta_{18}, \delta_{22}, \delta_{27}, \delta_{31}$
μ_ξ	0.7882*** (0.1823)	-0.0069 (0.0800)	0.0304 (0.0990)	-2.5086*** (0.7047)
κ	1.2705*** (0.0002)	0.9639*** (0.0294)	0.8112*** (0.2255)	5.8064*** (0.0003)
φ	0.7906*** (0.0000)	1.0114*** (0.0031)	1.0080*** (0.0103)	1.0012*** (0.0000)
σ_χ	0.6205*** (0.0002)	0.2727*** (0.0168)	0.3000*** (0.0483)	2.2847*** (0.0003)
σ_α	0.0530*** (0.0002)	0.0725*** (0.0054)	0.4773*** (0.0407)	0.3772*** (0.0003)
$\rho_{\chi\alpha}$	0.7771*** (0.0002)	0.6338*** (0.0806)	0.4382*** (0.1263)	-0.3040*** (0.0003)
$\rho_{\chi\alpha^*}$	0.3725*** (0.0002)	0.1087 (0.01055)	0.4922*** (0.1160)	0.1639*** (0.0003)
λ_χ	0.8087*** (0.1809)	-0.0615 (0.0798)	0.0941 (0.0983)	-1.0521 (0.6875)
λ_α	0.0602*** (0.0163)	0.0660*** (0.0240)	-0.0660 (0.1510)	-0.0038 (0.1186)
λ_{α^*}	-0.0129 (0.0164)	-0.0427** (0.0235)	0.1003 (0.1437)	0.1375 (0.1219)
σ_η	0.0517*** (0.0002)	0.0779*** (0.0008)	0.2170*** (0.0031)	0.3772*** (0.0003)
Log-likelihood	13008.48	9719.43	3386.01	2501.60
AIC	12986.48	9697.43	3364.01	2479.60
SIC	12937.85	9648.79	3315.37	2430.97

TABLE 7

WEIGHTS OF SEASONAL COMPONENTS

The table presents the average weights of the estimated seasonal factors in the spot price (four-factor model for spot commodity prices) and in the convenience yield (three-factor for convenience yields), for the four commodities under study: WTI crude oil, heating oil, RBOB gasoline and Henry Hub.

PANEL A: FOUR FACTOR MODEL, COMMODITY SPOT PRICES		
	$\|\alpha\|/\log(S)$	$(\|\alpha\| + \|\alpha^*\|)/\log(S)$
Henry Hub	2.8866%	5.7299%
Heating Oil	0.5270%	1.0625%
RBOB	0.8295%	1.6809%
WTI	0.0826%	0.1439%

PANEL B: THREE-FACTOR MODEL, CONVENIENCE YIELDS		
	$\|\alpha\|/\log(S)$	$(\|\alpha\| + \|\alpha^*\|)/\log(S)$
Henry Hub	34.5652%	94.0058%
Heating Oil	46.6621%	128.1067%
RBOB	63.6727%	180.6800%
WTI	9.8590%	21.0083%

FIGURE 1

TIME SERIES EVOLUTION OF ESTMATED CONVENIENCE YIELDS

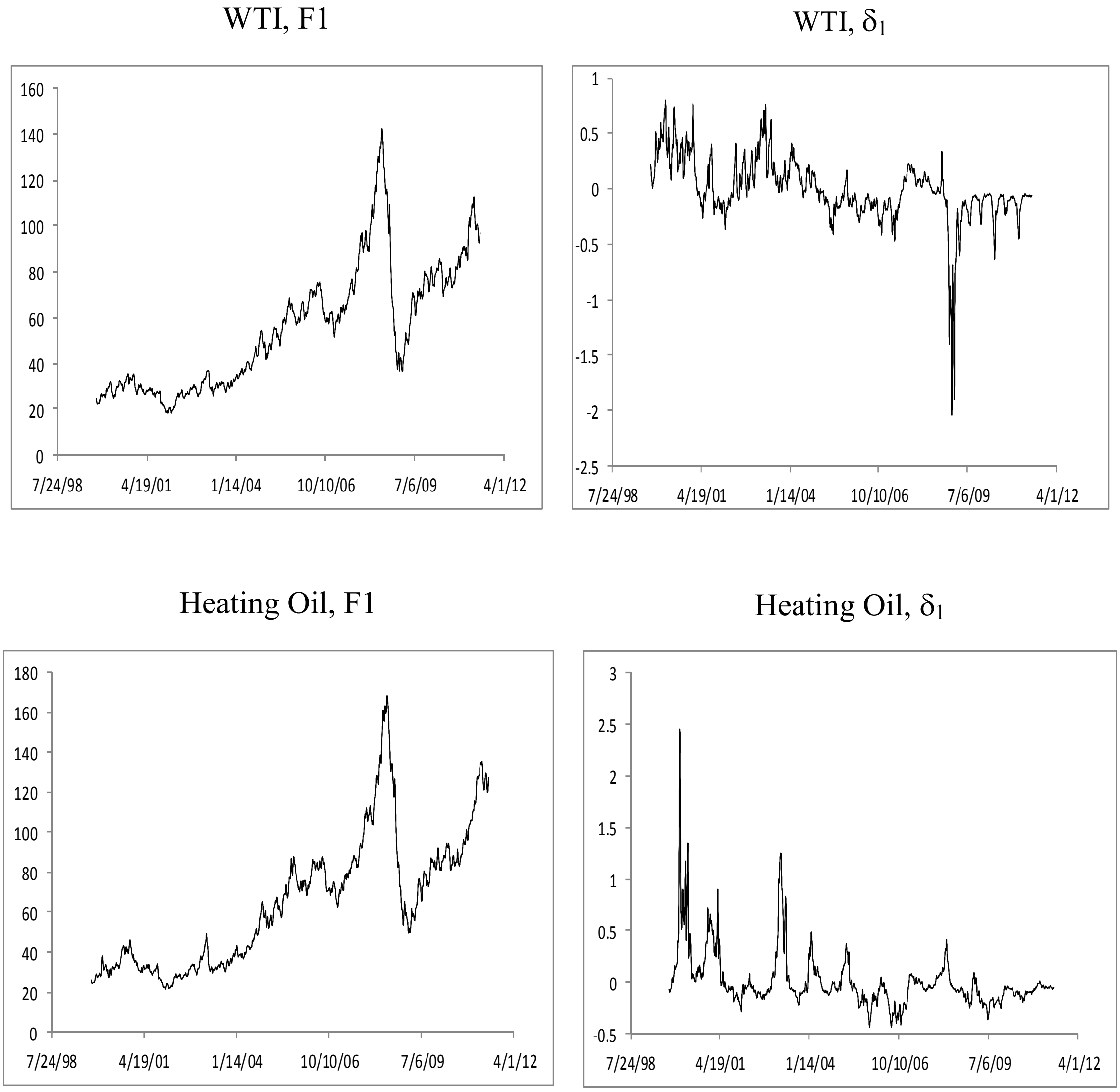
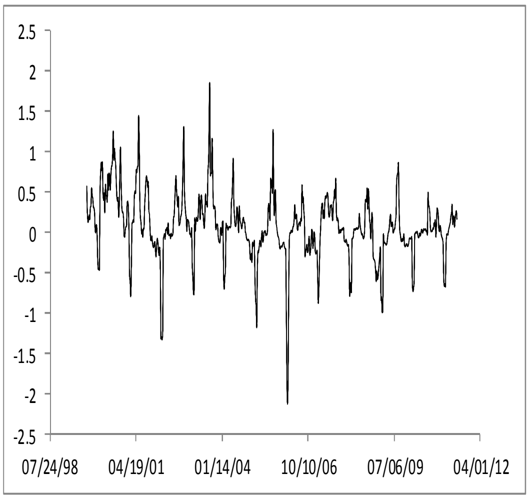


FIGURE 1

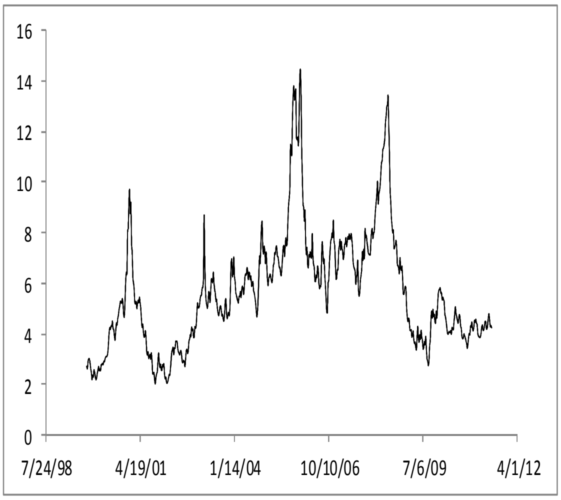
TIME SERIES EVOLUTION OF ESTMATED CONVENIENCE YIELDS (CONT.)

RBOB, F1

RBOB, δ_1



Henry Hub, F1



Henry Hub, δ_1

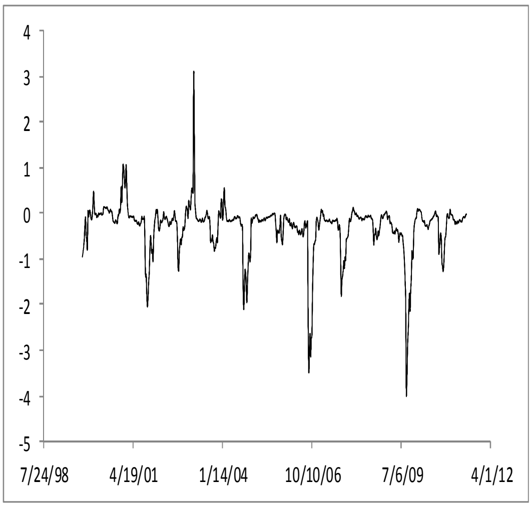
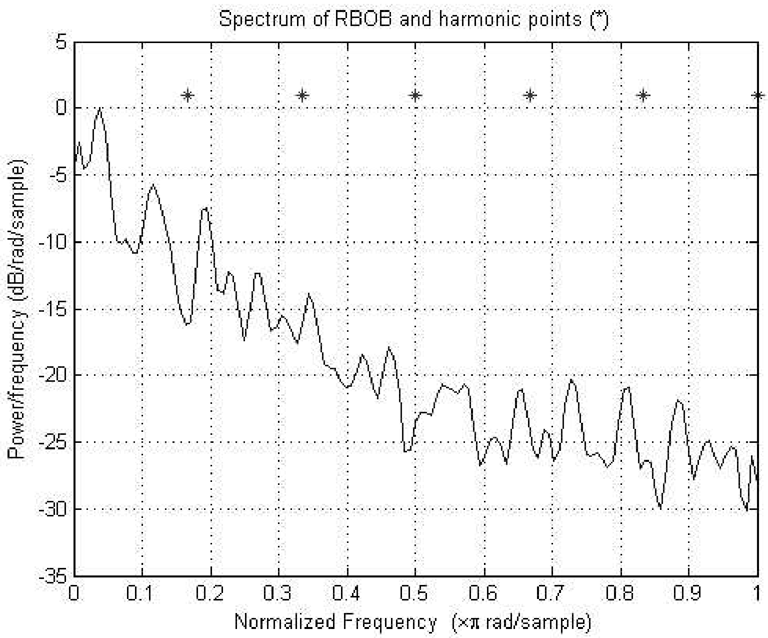


FIGURE 2

RBOBO GASOLINE CONVENIENCE YIELD SPECTRUM



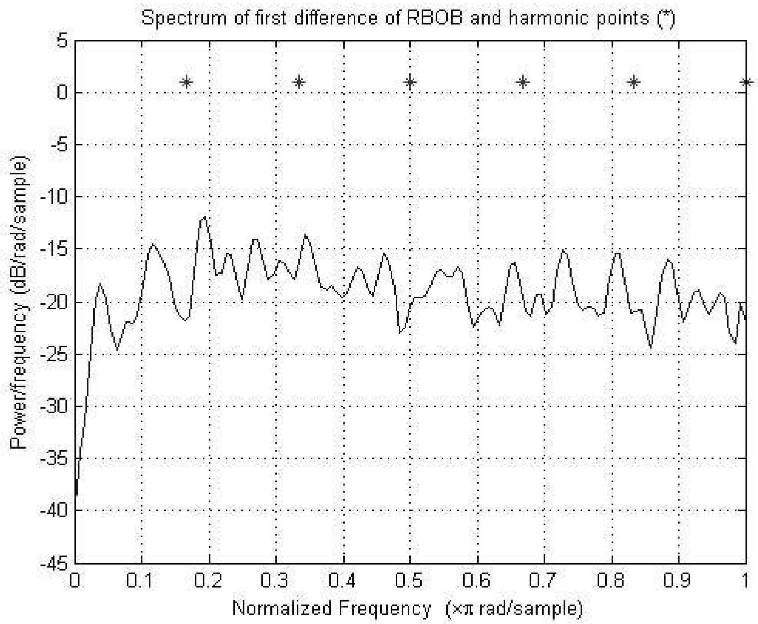


FIGURE 3

FORWARD CURVES FUTURES AND CONVENIENCE YIELD

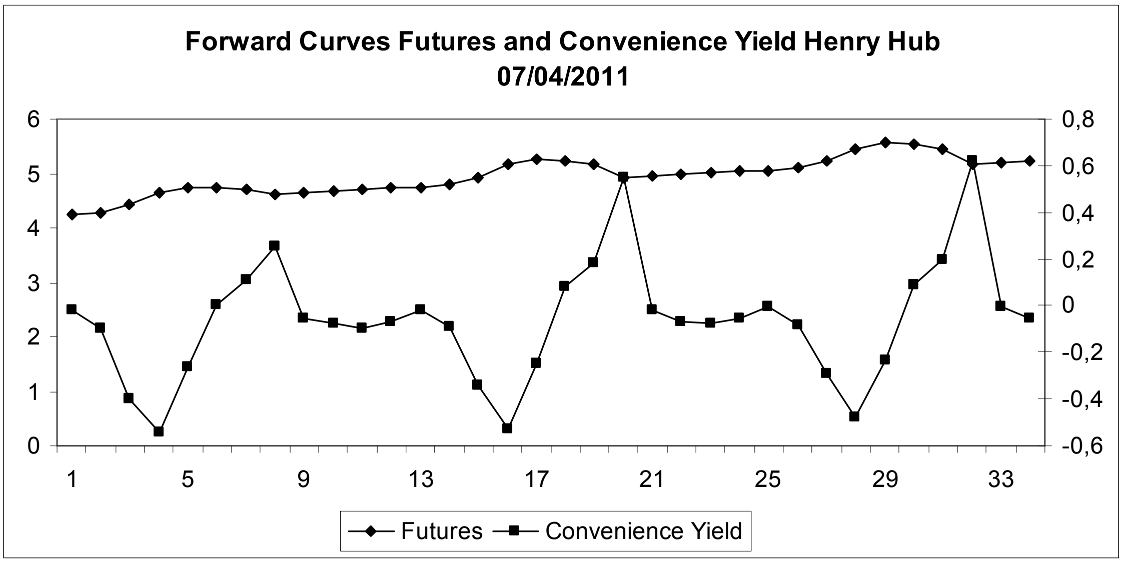
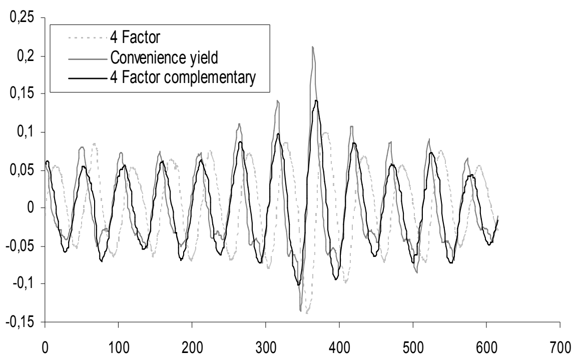


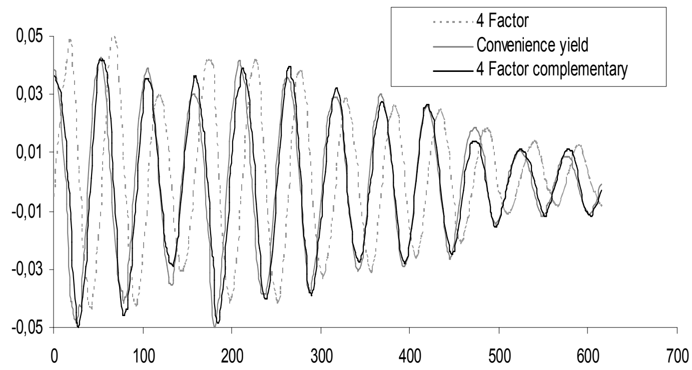
FIGURE 4

COMMODITY SEASONAL COMPONENTS AND CONVENIENCE YIELD

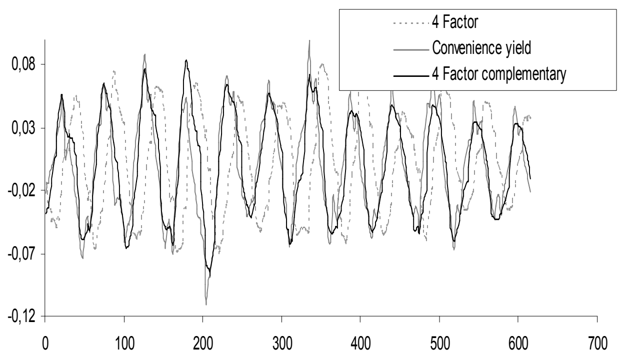
Seasonal Components of Henry Hub



Seasonal Components of Heating Oil



Seasonal Components of RBOB



Seasonal Components of WTI