Currency Hedging Strategies Using Dynamic Multivariate GARCH*

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Abstract

This paper examines the effectiveness of using futures contracts as hedging instruments of: (1) alternative models of volatility for estimating conditional variances and covariances; (2) alternative currencies; and (3) alternative maturities of futures contracts. For this purpose, daily data of futures and spot exchange rates of three major international currencies, Euro, British pound and Japanese yen, against the American dollar, are used to analyze hedge ratios and hedging effectiveness resulting from using two different maturity currency contracts, near-month and next-to-near-month contract. Following Chang et al. [17], we estimate four multivariate volatility models (namely CCC, VARMA-AGARCH, DCC and BEKK), and calculate optimal portfolio weights and optimal hedge ratios to identify appropriate currency hedging strategies. The hedging effectiveness index suggests that the best results in terms of reducing the variance of the portfolio are for the USD/GBP exchange rate. The empirical results show that futures hedging strategies are slightly more effective when the near-month future contract is used for the USD/GBP and USD/JPY currencies. Moreover, the CCC and AGARCH models provide similar hedging effectiveness, which suggests that dynamic asymmetry may not be crucial empirically, although some differences appear when the DCC and BEKK models are used.

1.   Introduction

With increasing capital market liberalization and globalization, foreign currency denominated assets circulate rapidly throughout the world and, with increasing internationalization of financial transactions, the foreign exchange market has been profoundly transformed, and became more competitive and volatile. This places the accurate and reliable measurement of market risks in a critical position for both investment decision and hedging strategy designs.

Foreign exchange rate markets are the largest and most liquid of all asset markets. Developments in these markets influence national trade and monetary policies, and the competitiveness of nations. Foreign exchange markets are also important for the increasing number of companies engaged in cross-border trade and investment. Foreign exchange markets are naturally risky as they deal primarily in measuring, pricing, and managing risk. The success of an institution trading in the foreign exchange market depends critically on how well it assesses prices and manages the inherent risk, in its ability to limit losses from particular transactions, and to keep its overall exposure under control.

A key issue in managing currency risk is to control the volatility of the portfolio. The volatility of a portfolio includes variances and covariances of individual positions. Significant losses may arise from holding a portfolio without taking account of the time-varying variance and covariances. If investors can evaluate the interacting dynamics among markets in advance, then adjusting and hedging activities can usefully be implemented.

The aim of hedging is to use derivatives to reduce a particular risk. A relatively inexpensive and reliable strategy for hedging foreign exchange risk involves the use of foreign currency futures markets. Hedging with futures contracts is perhaps the simplest method for managing market risk arising from adverse movements in the foreign exchange market. Hedgers usually short an amount of futures contracts if they hold the long position of the underlying currency, and vice-versa. The question is how many futures contracts should be held for each unit of the underlying currency, as well as the effectiveness measure of that ratio. The hedge ratio provides information on how many

futures contracts should be held, whereas its effectiveness evaluates the hedging performance and the usefulness of the strategy. In addition, hedgers may use the effectiveness measure to compare the benefits of hedging a given position from many alternative contracts.

In general, when the market trend is stable, the hedge ratio will become smaller, whereas if a big fluctuation of the market takes place, it will increase. Several distinct approaches have been developed to estimate the optimal hedge ratio (OHR), also known as the minimum-variance hedge ratio. The static hedging model with futures contracts (Johnson [32], Stein [56], Ederington, [21]) assumes that the joint distribution of spot and futures returns is time-invariant, so that the OHR, defined as the optimal number of futures holdings per unit of spot holdings, is constant over time. The minimum variance OHR use to be derived form the ordinary least squares (OLS) regression of spot price changes on future price changes. There is ample evidence that the simple OLS method is inappropriate to estimate hedge ratios as it suffers from the problem of serial correlation and heteroscedasticity often encountered in spot and futures price series (Herbst et al. [31]).

Therefore, the underlying assumption of the static hedging model of time-invariant asset distributions has been changed. The Autoregressive Conditional Heteroscedastic (ARCH) framework of Engle [23], and its extension to a generalized ARCH (GARCH) structure by Bollerslev [8], have proved to be very successful in modelling asset price second-moment movements. Bollerslev [9], Bailie and Bollerslev [3], and Diebold [19] have shown that the GARCH (1,1) model is effective in explaining the distribution of exchange rate changes. However, Lien et al. [39] compared OLS and constant-correlation vector generalized autoregressive conditional heteroscedasticity (VGARCH), and claimed that the OLS hedge ratio performs better than VGARCH. Chan [16] proposed a dynamic hedging strategy based on a bivariate GARCH jump model augmented with autoregressive jump intensity to manage currency risk. The collective evidence shows that GARCH-based dynamic hedging strategies are empirically appropriate, but the risk-reduction improvements over constant hedges vary across markets, and may also be sensitive to the sample period used.

Regarding foreign currencies, different results seem to hold. Kroner and Sultan [35] demonstrated that GARCH hedge ratios produce better hedging effectiveness than conventional hedge ratios in currency markets. Chakraborty and Barkoulas [15] employed a bivariate GARCH model to estimate the joint distribution of spot and futures currency returns, and constructed a sequence of dynamic (time-varying) OHRs based on the estimated conditional covariances. The empirical evidence strongly supports time-varying OHRs, but the dynamic model provides superior out-of-sample hedging performance compared with the static model, but only for the Canadian dollar. Ku et al. [37] applied the dynamic conditional correlation (DCC) model of Engle [24] with error correction terms to investigate the optimal hedge ratios of British and Japanese currency futures markets, and compared the DCC and OLS estimates. The empirical results show that the DCC model yields the best hedging performance.

Given the distinct theoretical advantages of the dynamic hedging method over the static one, several papers have used the multivariate GARCH framework to examine the hedging performance for various assets. In order to evaluate the impact of model specification on the forecast of conditional correlations, Hakim and McAleer [29] analyze whether multivariate GARCH models incorporating volatility spillovers and asymmetric effects of negative and positive shocks on the conditional variance provide different conditional correlation forecasts. Using three multivariate GARCH models, namely CCC (Bollerslev, [10]), VARMA-GARCH (Ling and McAleer, [41]), and VARMA-AGARCH (McAleer et.al., [45]) they forecast conditional correlations between three classes of international financial assets (stocks, bonds and foreign exchange rates). They suggested that incorporating volatility spillovers and asymmetric effects of negative and positive shocks on the conditional variance does not affect the forecasts of conditional correlations.

In order to estimate time-varying hedge ratios using multivariate conditional volatility models, Chang et.al. [17] examined the performance of four models (namely CCC, VARMA-GARCH, DCC and BEKK) for the crude oil spot and futures returns of two major international crude oil markets (BRENT and WTI). The calculated OHRs from each multivariate conditional volatility model suggested time-varying hedge ratios, which recommended to short in crude oil futures, with a high portion of one dollar long

in crude oil spot. The hedging effectiveness indicated that DCC (BEKK) was the best (worst) model for OHR calculation in terms of the variances of portfolio reduction.

This paper applies the Chang et al. [17] methods to currency hedging. In order to evaluate the impact of model specification on conditional correlations forecasts, this paper calculates and compares the correlations between conditional correlations forecasts arising from four different multivariate models (CCC, VARMA-AGARCH, DCC and BEKK) to estimate the returns on spot and futures (analyzing two sets of futures depending on their maturity) of three currency prices (USD/GBP, USD/EUR and USD/JPY). The purpose is to calculate the optimal portfolio weights and OHRs ratio from the conditional covariance matrices in order to achieve an optimal portfolio design and hedging strategy, and to compare the performance of OHRs from estimated multivariate conditional volatility models by applying the hedging effectiveness index. One of the main contributions of this paper is that it allows a comparison as to whether the results are different depending on the volatility model, currency and maturity of the futures contract selected.

The remainder of the paper is organized as follows. In Section 2 we discuss alternative multivariate GARCH models, and the derivation of the OHR and hedging effective index. In Section 3 the data used for estimation and forecasting, and the descriptive statistics, are presented. Section 4 analyses the empirical estimates, and Section 5 presents the conclusions.

2. Econometric Models

2.1. Multivariate Conditional Volatility Models

This paper closely follows Chang et.al. [17], who analysed dynamic crude oil hedging strategies, and considers the CCC model of Bollerslev [10], VARMA-AGARCH model of McAleer et al. [45], the DCC model of Engle [24], and BEKK model of Engle and Kroner [25] for alternative currencies. Constant conditional correlations are assumed in the first two models, while dynamic conditional correlations are specified in the last two models.

Consider the CCC multivariate GARCH model of Bollerslev [10]:

$$y_1 = E\left(y_t / F_{t-1}\right) + \varepsilon_t, \varepsilon_t = D_t \eta_t \tag{1}$$

$$\mathrm{var}\left(\varepsilon_t / F_{t-1}\right) = D_t \Gamma D_t$$

where $y_t = \left(y_{1t},...,y_{mt}\right)'$, $\eta_t = \left(\eta_{1t},...,\eta_{mt}\right)'$ is a sequence of independent and identically distributed random vectors, F_t is the past information available at time t, $D_t = diag\left(h_1^{1/2},...,h_m^{1/2}\right)$, and m is the number of assets (see, for example, McAleer [43] and Bauwens et al. [6]). As $\Gamma = E\left(\eta_t \eta_t' / F_{t-1}\right) = E\left(\eta_t \eta'\right)$, where $\Gamma = \left\{\rho_{ij}\right\}$ for $i, j = 1,...,$ m, the constant conditional correlation matrix of the unconditional shocks, η_t, is equivalent to the constant conditional covariance matrix of the conditional shocks, ε_t, from (1), $\varepsilon_t \varepsilon_t' = D_t \eta_t \eta_t' D_t, D_t = \left(diag Q_t\right)^{1/2}$, and $E\left(\varepsilon_t \varepsilon_t' / F_{t-1}\right) = Q_t = D_t \Gamma D_t$, where Q_t is the conditional covariance matrix.

The CCC model of Bollerslev [10] assumes that the conditional variance for each return, h_{it}, $i = 1, ..., m$, follows a univariate GARCH process, that is

$$h_{it} = \omega_i + \sum_{j=1}^{r} \alpha_{ij} \varepsilon_{i,t-j}^2 + \sum_{j=1}^{s} \beta_{ij} h_{i,t-j}, \tag{2}$$

where α_{ij} represents the ARCH effect, or short run persistence of shocks to return i, β_{ij} represents the GARCH effect, and $\sum_{j=1}^{r} \alpha_{ij} + \sum_{j=1}^{s} \beta_{ij}$ denotes the long run persistence of the returns shocks.

The CCC model assumes that negative and positive shocks of equal magnitude have identical impacts on the conditional variance. McAleer et al. [45] extended the VARMA-GARCH model to accommodate the asymmetric impacts of the unconditional shocks on the conditional variance, and proposed the VARMA-AGARCH specification of the conditional variance as follows:

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$$H_t = W + \sum_{i=1}^{r} A_i \vec{\mathcal{E}}_{t-i} + \sum_{i=1}^{r} C_i I_{t-i} \vec{\mathcal{E}}_{t-i} + \sum_{j=1}^{s} B_j H_{t-j,} \qquad (3)$$

where C_i are $m \times m$ matrices for $i = 1,...r$ with typical element γ_{ij}, and $I_t = diag(I_{1t},...,I_{mt})$, is an indictor function, given as

$$I(\eta_{it}) = \begin{pmatrix} 0, & \varepsilon_{it} > 0 \\ 1, & \varepsilon_{it} \leq 0 \end{pmatrix} \qquad (4)$$

If $m=1$, (3) collapses to the asymmetric GARCH (or GJR) model of Glosten et al. [28]. If $C_i = 0$ and A_i and B_j are diagonal matrices for all i and j, then VARMA-AGARCH collapses to the CCC model. The structural and statistical properties of the model, including necessary and sufficient conditions for stationarity and ergodicity of VARMA-AGARCH, are explained in detail in McAleer et al. [45]. The parameters of model (1) to (3) are obtained by maximum likelihood estimation (MLE) using joint normal. We also estimate the models using the Student t distribution, for which the appropriate estimator is QMLE.

The assumption that the conditional correlations are constant are not likely to be realistic so, in order to make the conditional correlation matrix time dependent, Engle [24] proposed a dynamic conditional correlation (DCC) model, which is defined as

$$y_t | \mathfrak{I}_{t-1} \sim (0, Q_t), t = 1, 2,..., n \qquad (5)$$

$$Q_t = D_t \Gamma_t D_{t,} \qquad (6)$$

where $D_t = diag(h_1^{1/2},...,h_m^{1/2})$ is a diagonal matrix of conditional variances, and $\mathfrak{I}_t$ is the information set available at time t. The conditional variance, h_{it}, can be defined as a univariate GARCH model, as follows:

$$h_{it} = \omega + \sum_{k=1}^{p} \alpha_{ik} \varepsilon_{i,t-k} + \sum_{l=1}^{q} \beta_{il} h_{i,t-l} \qquad (7)$$

If η_t is a vector of i.i.d. random variables, with zero mean and unit variance, Q_t in (8) is the conditional covariance matrix (after standardization, $\eta_{it} = y_{it} / \sqrt{h_{it}}$). The η_{it} are used to estimate the dynamic conditional correlations, as follows:

$$\Gamma_t = \left\{ (diag(Q_t)^{-1/2} \right\} Q_t \left\{ (diag(Q_t)^{-1/2} \right\} \tag{8}$$

where the $k \times k$ symmetric positive definitive matrix Q_t is given by

$$Q_t = \left(1 - \theta_1 - \theta_2\right)\bar{Q} + \theta_1 \eta_{t-1}\eta'_{t-1} + \theta_2 Q_{t-1}, \tag{9}$$

in which θ_1 and θ_2 are scalar parameters to capture the effects of previous shocks and previous dynamic conditional correlations on the current dynamic conditional correlation, and θ_1 and θ_2 are non-negative scalar parameters. When $\theta_1 = \theta_2 = 0, \bar{Q}$ in (9) is equivalent to CCC. As Q_t is conditional on the vector of standardized residuals, (9) is a conditional covariance matrix, and $\bar{Q}$ is the $k \times k$ unconditional variance matrix of η_t. DCC is not linear, but may be estimated in EViews by using a two-step method based on the likelihood function, the first step being a series of univariate GARCH estimates and the second step being the correlation estimates.

An alternative dynamic conditional model is BEKK, which has the attractive property that the conditional covariance matrices are positive definite. However, BEKK suffers from the so-called "curse of dimensionality" (see McAleer et al. [45] for a comparison of the number of parameters in various multivariate conditional volatility models). The BEKK model for multivariate GARCH (1,1) is given as:

$$H_t = C'_H C'_H + A'\varepsilon_{t-1}\varepsilon'_{t-1}A + B'H_{t-1}B, \tag{10}$$

where the individual element for the matrices C_H, A and B matrices are given as

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}, B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}, C_H = \begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix}$$

The conditional variances are functions of their own lagged values and own lagged returns shocks, while the conditional covariances are functions of the lagged covariances and lagged cross-products of the corresponding returns shocks. The BEKK formulation guarantees H_t to be positive definite almost surely for all t. A critical comparison between BEKK and DCC is given in Caporin and McAleer [12], where the scalar, diagonal and full versions of BEKK are also discussed.

2.2 Optimal Hedge Ratios and Optimal Portfolio Weights

Market participants in futures markets choose a hedging strategy that reflects their attitudes toward risk and their individual goals. Adapting the crude oil hedging strategies discussed in Chang et al. [17] for currencies, consider the case of exchange rates, where the return on the portfolio of spot and futures position can be denoted as:

$$R_{H,t} = R_{S,t} - \gamma R_{F,t,} \qquad \qquad (11)$$

In equation (11), $R_{H,t}$ is the return on holding the portfolio between $t\text{-}1$ and t, $R_{S,t}$ and $R_{F,t}$ are the returns on holding spot and futures positions between t and $t\text{-}1$, and γ is the hedge ratio, that is, the number of futures contracts that the hedger must sell for each unit of spot commodity on which price risk is borne.

According to Johnson [32], the variance of the returns of the hedged portfolio, conditional on the information set available at time $t\text{-}1$ is given by

$$\text{var}\left(R_{H,t}\middle|\Omega_{t-1}\right) = \text{var}\left(R_{S,t}\middle|\Omega_{t-1}\right) - 2\gamma\,\text{cov}\left(R_{S,t}, R_{F,t}\middle|\Omega_{t-1}\right) + \gamma_t^2\,\text{var}\left(R_{F,t}\middle|\Omega_{t-1}\right), \quad (12)$$

where $\text{var}\left(R_{S,t}\middle|\Omega_{t-1}\right)$, $\text{var}\left(R_{F,t}\middle|\Omega_{t-1}\right)$, and $\text{cov}\left(R_{S,t}, R_{F,t}\middle|\Omega_{t-1}\right)$ are the conditional variance and covariance of the spot and futures returns, respectively. The OHRs are defined as the value of γ_t which minimizes the conditional variance (risk) of the hedged portfolio returns, that is $\min_{\gamma_t}\left[\text{var}\left(R_{H,t}\middle|\Omega_{t-1}\right)\right]$. Taking the partial derivate of (12) with respect to γ_t, setting it to zero, and solving for γ_t, yields the OHR_t conditional on the information available at $t\text{-}1$ (see, for example, Baillie and Myers [4]):

$$\gamma_t^* \Big|_{\Omega_{t-1}} = \frac{\text{cov}\left(R_{S,t}, R_{F,t} \Big| \Omega_{t-1}\right)}{\text{var}\left(R_{F,t} \Big| \Omega_{t-1}\right)} \tag{13}$$

where returns are defined as the logarithmic differences of spot and futures prices.

From the multivariate conditional volatility model, the conditional covariance matrix is obtained, such that the OHR is given as:

$$\gamma_t^* \Big|_{\Omega_{t-1}} = \frac{h_{SF,t}}{h_{F,t}}, \tag{14}$$

where $h_{SF,t}$ is the conditional covariance between spot and futures returns, and $h_{F,t}$ is the conditional variance of futures returns.

In order to compare the performance of OHRs obtained from different multivariate conditional volatility models, Ku et al. [37] suggest that a more accurate model of conditional volatility should also be superior in terms of hedging effectiveness, as measured by the variance reduction for any hedged portfolio compared with the unhedged portfolio. Thus, a hedging effective index (HE) is given as:

$$HE = \left[\frac{\text{var}_{unhedged} - \text{var}_{hedged}}{\text{var}_{unhedged}} \right], \tag{15}$$

where the variances of the hedge portfolio are obtained from the variance of the rate of return, $R_{H,t}$, and the variance of the unhedged portfolio is the variance of spot returns (see, for example, Ripple and Moosa [50]). A higher HE indicates a higher hedging effectiveness and larger risk reduction, such that hedging method with a higher HE is regarded as a superior hedging strategy.

Alternatively, in order to construct an optimal portfolio design that minimizes risk without lowering expected returns, and applying the methods of Kroner and Ng [33] and Hammoudeh et al. [30], the optimal portfolio weight of exchange rate spot/futures holding is given by:

$$w_{SF,t} = \frac{h_{F,t} - h_{SF,t}}{h_{S,t} - 2h_{SF,t} + h_{F,t}} \qquad (16)$$

and

$$w^*_{SF,t} = \begin{cases} 0, & if \ w_{SF,t} < 0 \\ w_{SF,t}, & if \ 0 < w_{SF,t} < 1 \\ 1, & if \ w_{SF,t} > 1 \end{cases} \qquad (17)$$

where $w^*_{SF,t} (1-w_{SF,t})$ is the weight of the spot (futures) in a one dollar portfolio of exchange rates spot/futures at time t.

3. Data

We used daily closing prices of spot (S) and futures for three major international foreign exchange rate series, the value of the US dollar to one European Euro (USD/EUR), one British Pound (USD/GBP), or one Japanese Yen (USD/JPY). Futures prices rather than forward prices are used because of the higher data frequency of the former and also because futures data represent two different maturity currency contracts, namely near-month and next-to-near-month contracts.

The 3,006 observations from 3 January 2000 to 11 July 2011 are obtained from the Thomson Reuters-Ecowin Financial Database. The perpetual series of futures prices derived from individual futures contracts. These contracts call for a delivery of a specified quantity of a specified currency, or a cash settlement, during the months of March, June, September and December (the "March quarterly cycle"). Selected contracts are available with two future position continuous series. The futures price series for First Position Future (FUT1) is the price of the near-month delivery contract and the Second Position Future (FUT2) is the price of the next-to-near-month delivery contract. For example, in 1 February 2011, FUT 1 is the price of the contract that expires in March 2011, while FUT 2 is the price of the contract that expires in June 2011.

[Insert Tables 1-3]

The returns of currency i at time t are calculated as $r_{i,t} = \log\left(P_{i,t}/P_{i,t-1}\right)$, where $P_{i,t}$ and $P_{i,t-1}$ are the closing prices of currency i for days t and t-1 respectively. In Tables 1, 2 and 3 we show the descriptive statistics for the return series of EUR, GBP and JPY. The mean is close to zero in all cases. For the EUR and JPY currencies the standard deviation of the futures returns is larger than that of the spot returns, indicating the futures market is more volatile than the spot market for these currencies. The exchange rate return series display high kurtosis and heavy tails. Most of them, except EUR, present negative skewness statistics that signify increased presence of extreme losses than extreme gains (longer left tails). The Jarque-Bera Lagrange Multiplier test rejects the null hypothesis of normally distributed returns for every exchange rate series.

[Insert Figure 1]

Figure 1 presents the plot of spot and futures daily returns for each currency. Extremely high positive and negative returns are evident from September 2008 onward, and have continued well into 2009. Therefore, an increase in volatility during the financial crisis is perceived, however, is lower than in other assets (see, for example, McAleer et al. [46]). In the same way, the plots indicate volatility clustering. Spot and futures returns move in the same pattern suggesting a high correlation (the highest one is between FUT1 and FUT2 for all currencies). Correlations between the returns in European markets (EUR and GBP) are higher than the correlations between these and JPY which is hardly surprising.

[Insert Figure 2]

The volatilities of exchange rate returns are showed in figure 2. These volatilities are calculated as the square of the estimated returns and seem to support the stated above. The plots are similar in all returns and the volatility of the series appears to be high during the early 2000s, followed by a quiet period from 2003 to the beginning of 2007.

Volatility increases dramatically after August 2008, due in large part to the worsening global credit environment.

4.- Empirical Results

4.1 Estimation Results

We estimate four multivariate models (namely CCC, VARMA-AGARCH, DCC and BEKK) for each error distribution, currency and different futures maturities. The estimated parameters are reported in Tables 4-7. Table 4 shows the estimates for the CCC model. The volatility persistence, as measure by the sum of $\alpha + \beta$, in either spot or futures markets for each currency is quite high, ranging from 0.978 to 0.998. All markets satisfy the second moment and log moment condition, which is a sufficient condition for the QMLE to be consistent and asymptotically normal (see McAleer et.al. [44]) The ARCH and GARCH estimates of the conditional variance are statistically significant. The ARCH estimates are generally small (less than 0.04) and the GARCH effects are generally close to one. The estimates are generally smaller for JPY for both spot and futures prices (0.949 and 0.944 against 0.961 and 0.962 for the EUR). There are not large differences among the constant conditional correlation estimates, ranging from 0.799 for EUR to 0.811 for JPY.

[Insert Tables 4-5]

Table 5 reports the estimates of the conditional means and variances for the AGARCH models. The ARCH and GARCH effects are statistically significant in all markets, and are quite similar to the estimates for the CCC model without asymmetric effects. The asymmetric impacts of the unconditional shocks on the conditional variance estimates, γ, are reasonably weak for the three currencies and, in particular, are not statistically significant for JPY.

[Insert Table 6]

The DCC model is used to capture dynamic conditional correlations. Table 6 summaries the results of the DCC estimates for all spot and futures markets. Regarding the conditional variance, the estimates are statistically significant and satisfy the second moment condition. The estimated DCC parameters, $\hat{\theta}_1$ and $\hat{\theta}_2$, are statistically significant for all currencies, which suggests that the conditional correlations are not constant over time. The short run persistence of shocks on the dynamic conditional correlation is greater for JPY at 0.051, although it shows the lower long run persistence of shocks to the conditional correlation 0.913 (0.051+0.862). The EUR shows the lowest short run persistence (0.024) and the greatest long run persistence 0.986 (0.024+0.962). The time-varying conditional correlations between spot and futures returns are given in Figure 3. An apparent change in the conditional correlation appeared upon the bankruptcy of Lehman Brothers in New York on 15 September 2008. Due to an increase in the volatility of spot and futures exchange rates, the conditional correlations seem to change in all the currencies. The GFC caused an apparent decline in the conditional correlations between spot prices and both FUT1 an FUT2.

[Insert Table 7]

Table 7 reports the estimates for the BEKK model. We have restricted the bivariate BEKK model to the reduced form of the diagonal BEKK. The elements of the covariance matrix depend only on past own squared residuals, and the covariances depend only on past own cross products of residuals. The estimates of mean returns are not statistically significant. The elements of the diagonal matrices, A and B, are statistically significant. From the empirical results, we conclude there is time-varying market risk, strong evidence of GARCH effects, and the presence of weak ARCH effects. The results for the covariances are similar, indicating that there is a statistically significant time-varying covariation in shocks, which depends more on own lags than on past innovations. These empirical results show that market shocks are influenced by information which is common to spot and future markets.

4.2 Hedging Performance

With the estimated underlying parameters in the models, we first generate in-sample daily time series of variances and covariances of the spot and futures returns for each currency. Subsequently, we calculate the OHRs and optimal portfolios weights given by equations (14) and (16), respectively.

[Insert Table 8A-8C]

Tables 8A-8C report the average OHR values, the hedge effectiveness, the variance of the portfolio, the hedging effectiveness along with the average value of the optimal portfolio weights for the three currencies using FUT1 and FUT2 contracts when both the Student t and normal error distributions are assumed. We show the results for the four multivariate volatility models.

Tables 8A-8C show that hedging is effective in reducing the risks for every model, currency and maturity. In particular, we find that the average OHR using FUT2 contracts are slightly higher than when FUT1 contracts are used, except for GBP. The highest average OHR value is 0.854 for USD/JPY when FUT2 contracts are used, meaning that, in order to minimize risk, a long (buy) position of one dollar in such a currency should be hedged by a short (sell) position of $0.854 in JPYFUT2 contracts. Additionally, when using the Gaussian error distribution, Tables 8A-8C report lower average OHR values for the three currencies analyzed. The average OHRs from each model are not particularly different, slightly smaller for the DCC and BEKK models when the Student t is used, but larger for GBP and JPY when using the Gaussian distribution. The average OHR values are higher for the USD/JPY exchange rate. On the contrary, hedging effectiveness is higher for the DCC and BEKK models.

For GBP and JPY, we notice that hedging effectiveness is slightly higher when a FUT1 contract is used, as opposed to showing a higher hedging effectiveness when the EURFUT2 contract is used. We find that hedging effectiveness lies between a maximum of 66.3% for USD/GBP, and a minimum of 62.5% for USD/EUR. It seems that hedging effectiveness is slightly higher for the USD/GBP currency.

[Insert Figures 3-4]

Figure 3 shows the DCC estimates between spot and futures exchange rates for both future contracts. The volatility of the dynamic correlations increases during GFC and, as expected, during turbulent periods correlations decreases. This is why OHR volatility increases during the GFC. Figure 4 represents the calculated time-varying OHRs from every multivariate conditional volatility model. There are clearly time-varying ratios. It is interesting to examine the optimal hedging ratios during the GFC for all the models, but DCC optimal hedging ratios seem to increase in average.

As shown in the optimal portfolio weight columns in Tables 8A-8C, there are not big differences among the models. For example, the largest average value corresponds to a portfolio including the JPYFUT1 contract, for which the spot currency weight is calculated using the DCC model assuming a normal error distribution. The value 0.566 would imply that investors should have more spot currency than futures contracts in their portfolio in order to minimize risk without lowering expected returns. In particular, the optimal holding of one USD/JPY spot/future portfolio is 56.6 cents for spot and 43.4 cents for futures. When the Gaussian distribution is used, we find higher optimal portfolio weights. For both USD/EUR and USD/JPY spot/futures portfolios the optimal holding of spot currencies is higher when hedging with FUT1 contracts than when FUT2 are used. This is the opposite of what happens for USD/GBP spot/futures portfolios. The estimates suggest holding spot more than GBPFUT1, whereas they suggest holding spot less than GBPFUT2 on one dollar spot/future portfolio.

[Insert Table 9A-9C]

The correlations of the dynamic patterns in Tables 8A-8C are given in Tables 9A-9C. It is clear that, across all currencies and both error densities, the OHRs are most similar between CCC and VARMA-AGARCH, which suggests that dynamic asymmetry may not be crucial empirically, and also between DCC and BEKK.

In summary, the estimates based on both OHR and optimal weight values recommend holding more FUT2 than FUT1 contracts for USD/EUR and USD/JPY spot/futures portfolios, meaning that we should increase the percentage of futures contracts for longer term portfolios when these currencies are used.

5. Conclusions

This paper sheds light on the importance of measuring conditional variances and covariances when hedging daily currency risk using futures. The findings are of importance to currency hedgers who require taking futures positions in order to adequately reduce the risk. In this paper, we use four multivariate GARCH models, namely CCC, VARMA-AGARCH, DCC and BEKK, to examine the conditional volatilities among spot and two distinct futures maturities, namely near-month and next-to-near-month contracts. The estimated conditional covariances matrices from these models were used to calculate the optimal portfolios weights and optimal hedge ratios.

The empirical results in this paper reveal that there are not big effectiveness differences when either the near-moth or the next-to-near-month contract is used for hedging spot position on currencies. They also reveal that hedging ratios are lower for near-month contract when the USD/EUR and USD/JPY exchange rates are analyzed. This result is explained in terms of the higher correlation between spot prices and the next-to-near-month futures prices than that with near-month contract and additionally because of the lower volatility of the long maturity futures.

Finally, across all currencies and error densities, the CCC and VARMA-AGARCH models provide similar results in terms of hedging ratios, portfolio variance reduction and hedging effectiveness, which suggests that dynamic asymmetry may not be crucial empirically. Some differences appear when the DCC and BEKK models are used. Hedging ratios seem to decrease during the GFC as opposed to increasing ratios when the CCC and VARMA-AGARH models are consider for calculating the conditional covariances. Future research will investigate the effects of the GFC on the conditional correlations between spot and futures contracts, as well as its impact on hedging effectiveness.

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Figure 1. Spot and futures daily returns

USD/EUR	USD/GBP	USD/JPY
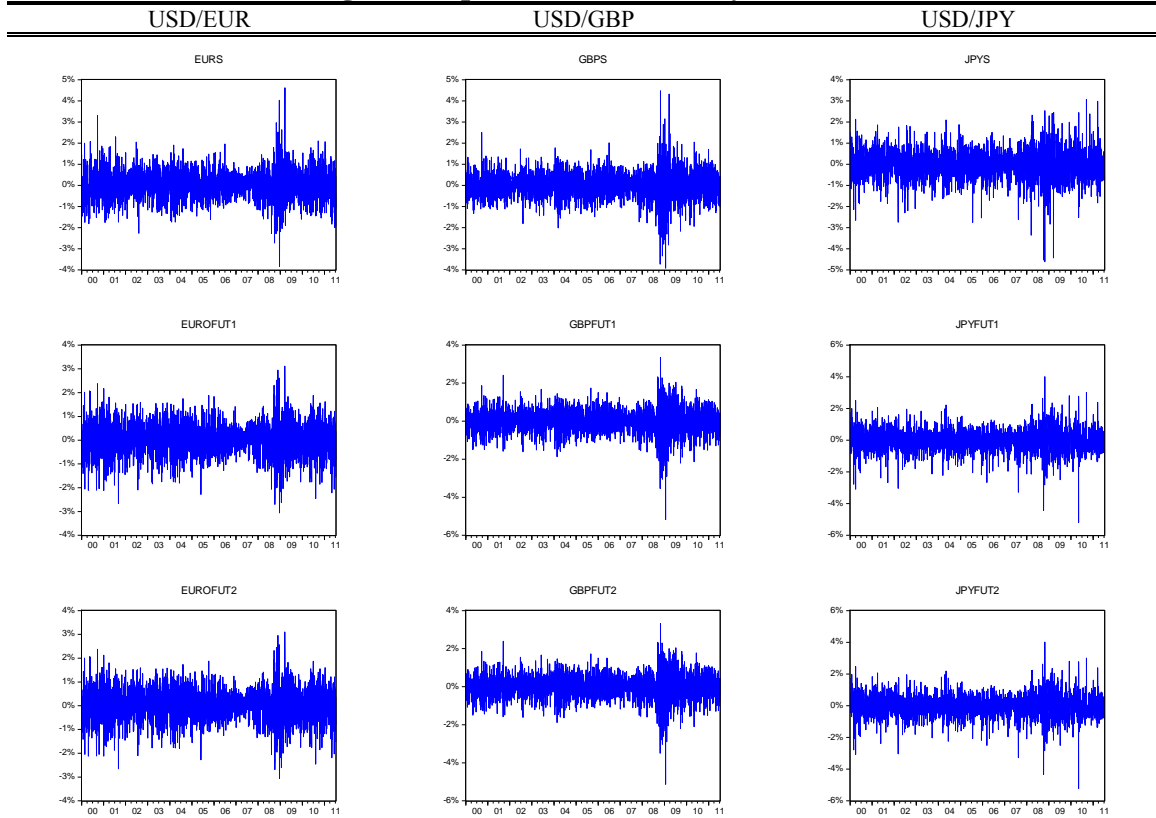
Figure 2. Estimated Conditional Volatilities of Returns

USD/EUR	USD/GBP	USD/JPY
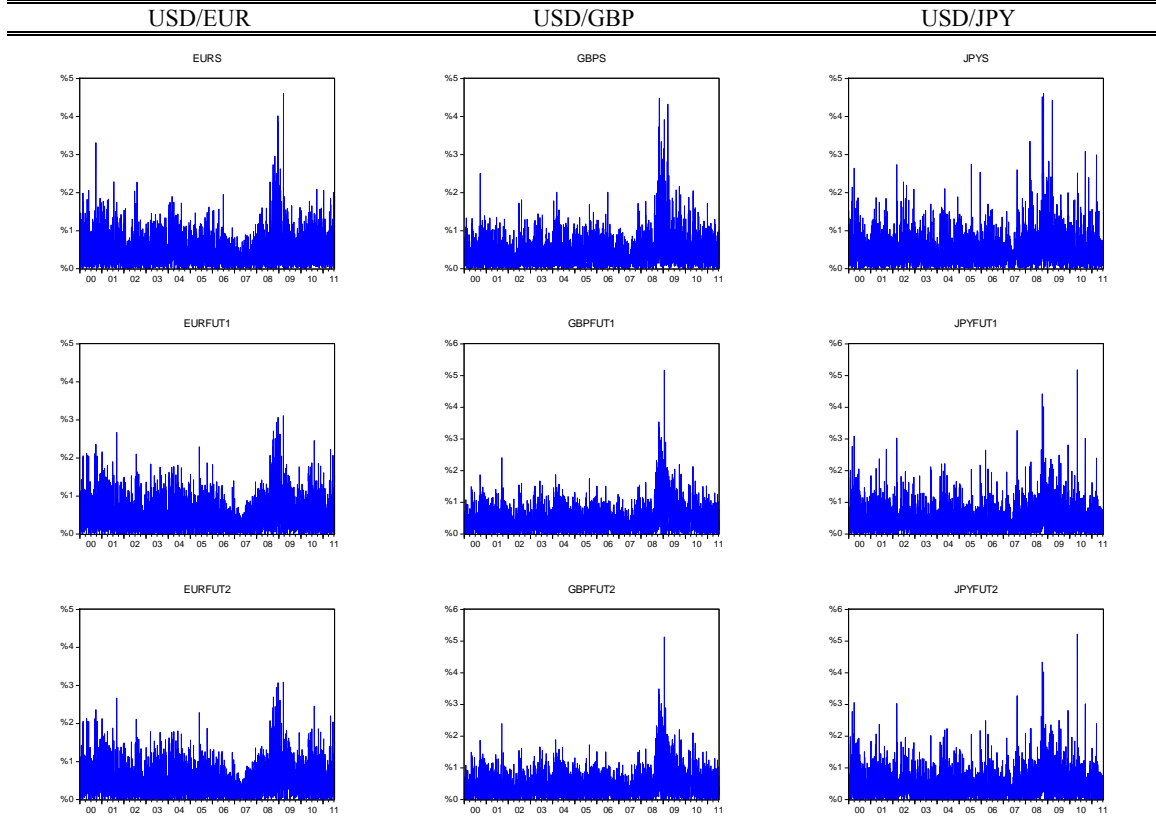
Figure 3. DCC Estimates

USD/EUR	USD/GBP	USD/JPY
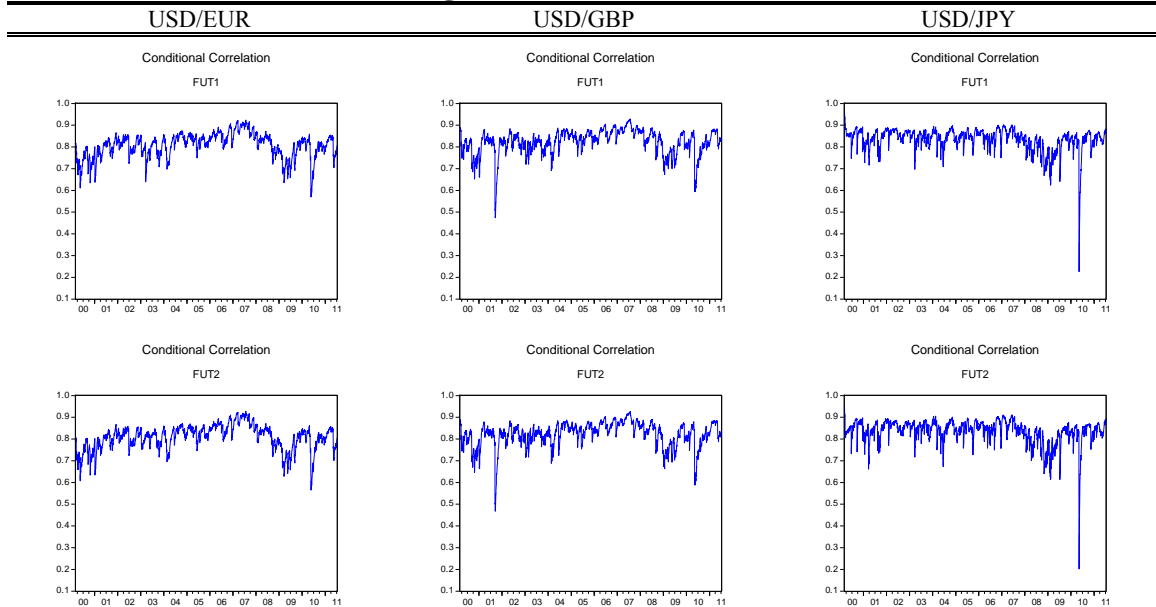
Figure 4. Optimal Hedge Ratios

USD/EUR	USD/GBP	USD/JPY
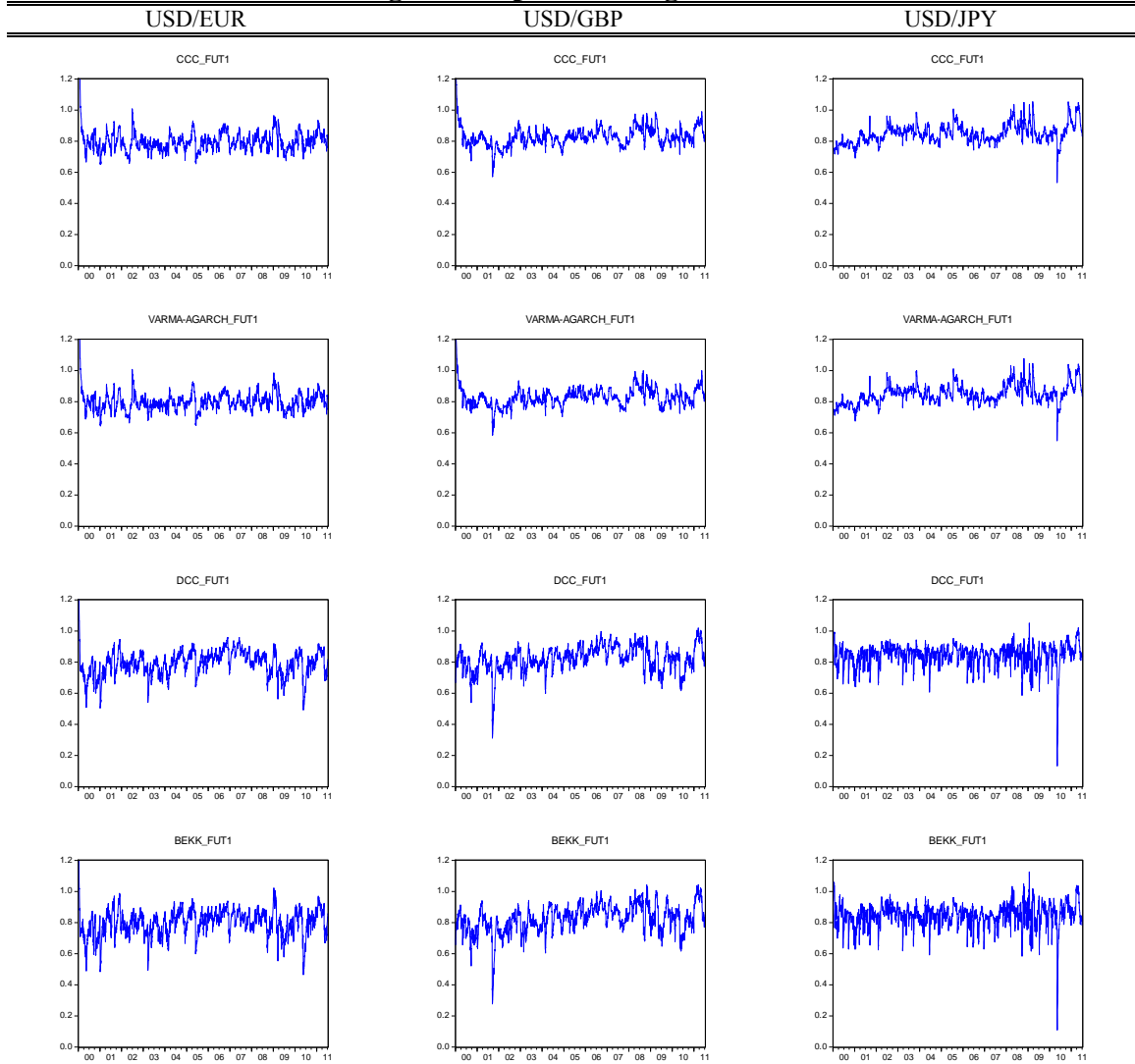
Table 1. EUR Descriptive Statistics

Returns	EUROS	EUROFUT1	EUROFUT2
Mean	0.0107	0.0101	0.0098
Maximum	4.6174	3.1184	3.1007
Minimum	-3.8445	-3.0568	-3.0620
Std. Dev.	0.6485	0.6552	0.6532
Skewness	0.1477	-0.0939	-0.0962
Kurtosis	5.6389	4.3878	4.3556
Jarque-Bera	882.87	245.58	234.73

Table 2. GPB Descriptive Statistics

Returns	GBPS	GBPFUT1	GBPFUT2
Mean	-0.0007	-0.0010	-0.0011
Maximum	4.4745	3.3542	3.3147
Minimum	-3.9182	-5.1703	-5.1326
Std. Dev.	0.6103	0.6010	0.6021
Skewness	-0.0552	-0.3793	-0.3560
Kurtosis	7.3609	6.7628	6.5791
Jarque-Bera	2382.7	1844.9	1667.4

Table 3. JPY Descriptive Statistics

Returns	JPYS	JPYFUT1	JPYFUT2
Mean	-0.0077	-0.0075	-0.0070
Maximum	3.0770	4.0082	4.0187
Minimum	-4.6098	-5.1906	-5.2289
Std. Dev.	0.6594	0.6642	0.6594
Skewness	-0.4246	-0.3285	-0.3020
Kurtosis	6.5503	6.9307	6.9282
Jarque-Bera	1668.5	1988.5	1977.8

Table 4. CCC Estimates

Panel a: EURS_EURFUT1

Returns	C	ω	α	β	α + β	Constant Conditional Correlation	Log-likelihood	AIC
EURS	0.025532 (0.0165)	0.001324 (0.1455)	0.036873 (0.0000)	0.960795 (0.0000)	0.997668	0.798867 (0.0000)	-4105.754	2.738605
EURFUT1	0.022376 (0.0454)	0.001886 (0.0329)	0.034411 0.0000	0.962024 0.0000	0.996435			

Panel b: GBPS_GBPFUT1

Returns	C	ω	α	β	α + β	Constant Conditional Correlation	Log-likelihood	AIC
GBPS	0.014299 (0.1479)	0.002229 (0.0086)	0.030472 (0.0000)	0.962650 (0.0000)	0.993122	0.812791 (0.0000)	-3367.432	2.247209
GBPFUT1	0.012066 (0.2202)	0.002608 (0.0055)	0.028671 (0.0000)	0.963302 (0.0000)	0.991973			

Panel c: JPYS_JPYFUT1

Returns	C	ω	α	β	α + β	Constant Conditional Correlation	Log-likelihood	AIC
JPYS	0.003894 (0.7283)	0.006035 (0.0269)	0.037076 (0.0000)	0.948975 (0.0000)	0.986051	0.811211 (0.0000)	-4239.943	2.827915
JPYFUT1	0.005502 (0.6230)	0.009677 (0.0205)	0.033193 (0.0055)	0.944285 (0.0000)	0.977478			

Note: The p-values are given in parentheses.

Table 5. VARMA-AGARCH Estimates

Panel a: EURS_EURFUT1

Returns	C	ω	α	β	γ	α+β+γ	Constant Conditional Correlation	Log-likelihood	AIC
EURS	0.016007 (0.1350)	0.001446 (0.1021)	0.025002 (0.0027)	0.962974 (0.0000)	0.018713 (0.0489)	1.006689	0.799869 (0.0000)	-4098.227	2.734926
EURFUT1	0.011699 (0.2921)	0.002113 (0.0135)	0.020336 (0.0105)	0.964626 (0.0000)	0.021366 (0.0483)	1.006328			

Panel b: GBPS_GBPFUT1

Returns	C	ω	α	β	γ	α+β+γ	Constant Conditional Correlation	Log-likelihood	AIC
GBPS	0.004260 (0.6588)	0.002587 (0.0021)	0.014752 (0.0446)	0.963269 (0.0000)	0.027465 (0.0117)	1.005486	0.813575 (0.0000)	-3356.194	2.241061
GBPFUT1	0.001990 (0.8368)	0.002791 (0.0021)	0.016439 (0.0301)	0.964716 (0.0000)	0.020035 (0.0644)	1.00119			

Panel c: JPYS_JPYFUT1

Returns	C	ω	α	β	γ	α+β+γ	Constant Conditional Correlation	Log-likelihood	AIC
JPYS	0.004648 (0.6808)	0.006036 (0.0315)	0.038539 (0.0006)	0.949630 (0.0000)	-0.004651 (0.7528)	0.983518	0.811191 (0.0000)	-4239.519	2.828964
JPYFUT1	0.005544 (0.6287)	0.009533 (0.0208)	0.032053 (0.0331)	0.944463 (0.0000)	0.002810 (0.8728)	0.979326			

Note: The p-values are given in parentheses.

Table 6. DCC Estimates

Panel a: EURS_EURFUT1									
Returns	C	ω	α	β	α + β	θ_1	θ_2	Log-likelihood	AIC
EURS	0.020377 (0.0532)	0.004386 (0.0022)	0.027374 (0.0000)	0.961108 (0.0000)	0.988482	0.023709 (0.0000)	0.961618 (0.0000)	-4077.809	2.721337
EURFUT1	0.018240 (0.0910)	0.006484 (0.0002)	0.034551 (0.0000)	0.949876 (0.0000)	0.984427				

Panel b: GBPS_GBPFUT1									
Returns	C	ω	α	β	α + β	θ_1	θ_2	Log-likelihood	AIC
GBPS	0.010971 (0.2487)	0.004841 (0.0001)	0.038697 (0.0000)	0.946627 (0.0000)	0.985324	0.041639 (0.0000)	0.936892 (0.0000)	-3303.009	2.205663
GBPFUT1	0.009160 (0.3353)	0.006684 (0.0001)	0.049814 (0.0000)	0.930739 (0.0000)	0.980553				

Panel c: JPYS_JPYFUT1									
Returns	C	ω	α	β	α + β	θ_1	θ_2	Log-likelihood	AIC
JPYS	0.003990 (0.7237)	0.019263 (0.0000)	0.043614 (0.0000)	0.911911 (0.0000)	0.955525	0.050768 (0.0000)	0.862159 (0.0000)	-4173.424	2.784974
JPYFUT1	0.005513 (0.6364)	0.048472 (0.0000)	0.069140 (0.0001)	0.824529 (0.0000)	0.893669				

Note: The p-values are given in parentheses.

<p style="text-align:center">Table 7. BEKK Estimates</p>

Panel a: EURS_EURFUT1

Returns	C	C$_H$		A		B		Log-likelihood	AIC
EURS	0.020559 (0.0488)	0.003112 (0.0001)		0.163463 (0.0000)	0.000000	0.982880 (0.0000)	0.000000	-4101.786	2.735964
EURFUT1	0.019446 (0.0664)	0.004368 (0.0001)	0.006801 (0.0003)	0.000000	0.213424 (0.0000)	0.000000	0.969448 (0.0000)		

Panel a: GBPS_GBPFUT1

Returns	C	C$_H$		A		B		Log-likelihood	AIC
GBPS	0.009175 (0.3332)	0.003862 (0.0000)		0.195416 (0.0000)	0.000000	0.975276 (0.0000)	0.000000	-3315.017	2.212324
GBPFUT1	0.007995 (0.3992)	0.004889 (0.0000)	0.007320 (0.0001)	0.000000	0.238545 (0.0000)	0.000000	0.960766 (0.0000)		

Panel a: JPYS_JPYFUT1

Returns	C	C$_H$		A		B		Log-likelihood	AIC
JPYS	0.001488 (0.8959)	0.022401 (0.0000)		0.215503 (0.0000)	0.000000	0.950569 (0.0000)	0.000000	-4181.067	2.788730
JPYFUT1	0.001963 (0.8719)	0.046133 (0.0000)	0.083282 (0.0000)	0.000000	0.298570 (0.0000)	0.000000	0.855350 (0.0000)		

Note: The p-values are given in parentheses.

Table 8A. Alternative hedging strategies (USD/EUR)

	MODEL	OHR	Var. PF	HE	Var. UnHed	OPT. W
	Student-t error distribution					
FUT1	**CCC**	0.805	0.158	62.5%	0.420	0.536
	VARMA-AGARCH	0.805	0.157	62.7%	0.420	0.536
	DCC	0.794	0.157	62.7%	0.420	0.542
	BEKK	0.802	0.157	62.6%	0.420	0.542
FUT2	**CCC**	0.808	0.157	62.7%	0.420	0.532
	VARMA-AGARCH	0.808	0.156	62.9%	0.420	0.532
	DCC	0.797	0.156	62.9%	0.420	0.535
	BEKK	0.804	0.156	62.8%	0.420	0.537
	Normal Gaussian error Distribution					
FUT1	**CCC**	0.792	0.158	62.5%	0.420	0.544
	VARMA-AGARCH	0.792	0.157	62.7%	0.420	0.545
	DCC	0.784	0.157	62.7%	0.420	0.554
	BEKK	0.792	0.157	62.6%	0.420	0.550
FUT2	**CCC**	0.799	0.157	62.7%	0.420	0.532
	VARMA-AGARCH	0.799	0.156	62.9%	0.420	0.533
	DCC	0.791	0.156	62.9%	0.420	0.538
	BEKK	0.798	0.156	62.8%	0.420	0.537

Notes: Optimal Hedging Ratio (**OHR**), Variance of Portfolios (**Var. PF**), Hedging Effective Index (**HE**), Variance of ungedged portfolio (**Var. UnHed)** and Optimal Portfolio Weights (**OPT. W**). FUT1 is when the near-month delivery contract is used for hedging and FUT2 implies that is the next-to-near-month delivery contract the one used for hedging. For each error distribution and future contract, results for the four multivariate variance models, CCC, VARMA-AGARCH, DCC and BEKK are shown.

Table 8B. Alternative hedging strategies (USD/GBP)

	MODEL	OHR	Var. PF	HE	Var. UnHed	OPT. W
	Student-t error distribution					
FUT1	CCC	0.829	0.126	66.2%	0.372	0.496
	VARMA-AGARCH	0.830	0.125	66.3%	0.372	0.498
	DCC	0.822	0.126	66.2%	0.372	0.497
	BEKK	0.826	0.126	66.2%	0.372	0.490
FUT2	CCC	0.826	0.127	65.9%	0.372	0.510
	VARMA-AGARCH	0.826	0.126	66.1%	0.372	0.512
	DCC	0.817	0.127	65.9%	0.372	0.511
	BEKK	0.821	0.127	65.9%	0.372	0.505
	Normal Gaussian error Distribution					
FUT1	CCC	0.816	0.126	66.2%	0.372	0.499
	VARMA-AGARCH	0.815	0.125	66.3%	0.372	0.503
	DCC	0.818	0.126	66.1%	0.372	0.500
	BEKK	0.822	0.127	66.0%	0.372	0.495
FUT2	CCC	0.812	0.127	65.9%	0.372	0.510
	VARMA-AGARCH	0.812	0.126	66.1%	0.372	0.513
	DCC	0.813	0.127	65.8%	0.372	0.513
	BEKK	0.817	0.128	65.7%	0.372	0.508

Notes: Optimal Hedging Ratio (**OHR**), Variance of Portfolios (**Var. PF**), Hedging Effective Index (**HE**), Variance of ungedged portfolio (**Var. UnHed**) and Optimal Portfolio Weights (**OPT. W**). FUT1 is when the near-month delivery contract is used for hedging and FUT2 implies that is the next-to-near-month delivery contract the one used for hedging. For each error distribution and future contract, results for the four multivariate variance models, CCC, VARMA-AGARCH, DCC and BEKK are shown.

Table 8C. Alternative hedging strategies (USD/JPY)

	MODEL	OHR	Var. PF	HE	Var. UnHed	OPT. W
	Student-t error distribution					
FUT1	**CCC**	0.849	0.153	64.8%	0.435	0.463
	VARMA-AGARCH	0.849	0.153	64.8%	0.435	0.464
	DCC	0.845	0.153	64.8%	0.435	0.475
	BEKK	0.849	0.154	64.7%	0.435	0.474
FUT2	**CCC**	0.853	0.154	64.6%	0.435	0.450
	VARMA-AGARCH	0.853	0.154	64.6%	0.435	0.450
	DCC	0.850	0.154	64.7%	0.435	0.464
	BEKK	0.854	0.154	64.6%	0.435	0.468
	Normal Gaussian error Distribution					
FUT1	**CCC**	0.803	0.152	65.0%	0.435	0.535
	VARMA-AGARCH	0.802	0.152	65.0%	0.435	0.537
	DCC	0.812	0.153	64.8%	0.435	0.566
	BEKK	0.817	0.153	64.7%	0.435	0.570
FUT2	**CCC**	0.810	0.153	64.8%	0.435	0.514
	VARMA-AGARCH	0.809	0.153	64.8%	0.435	0.515
	DCC	0.818	0.154	64.6%	0.435	0.549
	BEKK	0.823	0.154	64.5%	0.435	0.555

Notes: Optimal Hedging Ratio (**OHR**), Variance of Portfolios (**Var. PF**), Hedging Effective Index (**HE**), Variance of ungedged portfolio (**Var. UnHed)** and Optimal Portfolio Weights (**OPT. W**). FUT1 is when the near-month delivery contract is used for hedging and FUT2 implies that is the next-to-near-month delivery contract the one used for hedging. For each error distribution and future contract, results for the four multivariate variance models, CCC, VARMA-AGARCH, DCC and BEKK are shown.

Table 9A. Correlations between OHRs (USD/EUR)

	CCC	VARMA-AGARCH	DCC	BEKK
Student-t error distribution				
CCC	1.00			
VARMA-AGARCH	0.99	1.00		
DCC	0.66	0.65	1.00	
BEKK	0.65	0.65	0.96	1.00
Normal Gaussian error Distribution				
CCC	1.00			
VARMA-AGARCH	0.99	1.00		
DCC	0.66	0.65	1.00	
BEKK	0.67	0.67	0.96	1.00

Table 9B. Correlations between OHRs (USD/GBP)

	CCC	VARMA-AGARCH	DCC	BEKK
Student-t error distribution				
CCC	1.00	0.97	0.59	0.64
VARMA-AGARCH	0.97	1.00	0.59	0.63
DCC	0.59	0.59	1.00	0.97
BEKK	0.64	0.63	0.97	1.00
Normal Gaussian error Distribution				
CCC	1.00			
VARMA-AGARCH	0.97	1.00		
DCC	0.59	0.59	1.00	
BEKK	0.64	0.62	0.97	1.00

Table 9C. Correlations between OHRs (JPY/USD)

	CCC	VARMA-AGARCH	DCC	BEKK
Student-t error distribution				
CCC	1.00			
VARMA-AGARCH	0.99	1.00		
DCC	0.55	0.55	1.00	
BEKK	0.55	0.55	0.96	1.00
Normal Gaussian error Distribution				
CCC	1.00			
VARMA-AGARCH	0.99	1.00		
DCC	0.59	0.59	1.00	
BEKK	0.54	0.54	0.93	1.00