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**Is the Forward Rate a True Unbiased Predictor of the Future Spot  
Exchange Rate?**

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## **ABSTRACT**

In the past decades, there have been many empirical studies both in support of or opposing the Forward Rate Unbiasedness Hypothesis. This hypothesis argues that the forward rate fully reflects the information regarding exchange rate expectations and so, forward premiums predicts the direction change in future spot rates. In this paper we examine monthly data on spot and one-month forward prices for the yen, the euro and the sterling pound, all relative to the USD. Our purpose is to study the relationship between forward rates and future spot rates before and after the beginning of the Global Financial Crisis of September 2008, by testing if the forward rate is an unbiased estimator of the future spot rate. To test this hypothesis the conventional method is followed, by using an OLS regression with the change in spot exchange rate as the dependent variable, while the forward premium as the independent variable. To support this hypothesis, the constant term would not differ from zero, the coefficient of the forward premium would not significantly differ from one and the error term would not exhibit any serial correlation. At the end, we conclude that forward exchange rates have little effect as forecasts of future spot exchange rates since the Forward Rate Unbiasedness Hypothesis is rejected.

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## 1. INTRODUCTION

This paper reviews the Forward Rate Unbiasedness Hypothesis which states that forward rate is equal to the conditional expectation of the future spot exchange rate under the assumptions of rational expectations and risk neutrality. The main purpose of this study is to conclude if the former can be considered as an unbiased predictor for the latter or on the contrary, the Forward Rate Unbiasedness Hypothesis must be rejected.

The study of the relationship between the forward and the corresponding future spot rate and how exchange rates are determined are of great concern for individual investors and policy makers. In fact, exchange rate is one of the most determinant factors in a country because it links the domestic economy to the rest-of-world economy. Hence, the exchange rate strongly influences the competitiveness of commodities' markets and final allocation of resources.

An appreciation of a currency raises the price of domestic goods relative to the price of foreign goods. As a result, domestic exports became less competitive in world markets, and import substitution goods became less competitive in the national country. Alternatively, home currency depreciation results in a more competitive traded goods sector, stimulating domestic employment and inducing a shift in resources from the non traded-to the traded-goods sector. The bad part is that currency weakness also results in higher prices for imported goods and services, worsening living standards and domestic inflation.

In that sense, it is important to analyze the movements experienced by the exchanges rates of the main currencies and their effects on the current Global Financial Crisis. One of the main characteristics showed by the World Economy with the beginning of the crisis and the fall of Lehman Brothers in September 2008, was the existence of large external deficits in some countries, like the U.S., and surpluses in others, as in China. This global disequilibrium is the result of several factors and one of the fundamental ones are the exchange rates.

There has always been external disequilibrium, but never in history with the magnitude of recent years. So, the evolution of exchange rates and the currency prices will determine the adjustment of these global economic imbalances and to a large extent, the recovery of production and the most important consequence, the creation employment.

To test the Forward Rate Unbiasedness Hypothesis and analyze the fluctuations produced in the exchange rates, the conventional method was followed, which implies the use of an OLS regression, with the variation in spot

exchange rate ( $S_{t+1} - S_t$ ) as the explained variable, while the forward premium ( $F_t - S_t$ ) as explanatory one. We conclude that the Unbiasedness hypothesis cannot be demonstrated and so, forward rates have little effect as forecasts of future spot exchange rates.

The present paper is organized as follows. Section 2 provides information about the main stylized facts of the Foreign Exchange Market. Then, section 3 addresses a theoretical background about exchange rates determination and it reviews previous studies regarding the possible validity of the forward rate as an unbiased predictor. Section 4 deals with the methodology of the regression model used for this research. Next, section 5 reports the results from the regression analysis, after the model was corrected for serial correlation and heteroscedasticity. Finally, section 6 summarizes all the conclusions derived from our study.

## **2. SOME FACTS ABOUT THE FOREIGN EXCHANGE MARKET**

The Foreign Exchange Market, also called the “Forex” is the most liquid financial market in the world. Nowadays, since currencies are the main regulation mechanism for individuals’ interactions in an economy, it is important to understand how their values are determined.

Currencies have increasingly become one of the more actively traded assets and so, the volume and speed of their flows are just amazing. Approximately, average daily turnover in global foreign exchange markets is estimated at \$3.98 trillion. The \$3.98 trillion break-down is as follows. Approximately, \$1.490 trillion in spot transactions, \$475 billion in outright forwards and \$1.765 trillion in foreign exchange swaps. The rest, around \$250 billion are divided into currency swaps, options and other products. Besides, the major currencies which are traded in the Forex are U.S dollar, the Euro, the Japanese Yen and also, the Sterling Pound.

There is not just a unified or centrally established market for the majority of trades. Due to the over-the-counter (OTC) nature of currency markets, there are rather a many interconnected marketplaces, where different currencies instruments are negotiated. For that reason, there is not a single exchange rate but rather a number of different rates or prices depending on which bank or investor is trading and the location of this one.

In that sense, the main trading centers are New York and London, although Tokyo, Hong Kong and Singapore are important as well. Currency trading happens continuously throughout the day, so when the Asian trading session ends, the European session begins, followed by the North American session and then coming back again to the Asian session.

### 3. THEORETICAL REVIEW

This section provides a theoretical literature review in order to understand how the forecasting of future spot exchange rates works and also, the conditions under which the Forward Rate Unbiasedness Hypothesis is satisfied.

Our purpose is to derive the Unbiasedness Hypothesis by following both the theoretical reasoning of the Covered and Uncovered Rate Parity Conditions.

The Covered Interest Parity is considered as the no-arbitrage condition in foreign exchange markets. Concretely, it implies a situation in where the relationship between interest rates and the spot and forward currency values of two countries are in equilibrium. As a result, there are no interest rate arbitrage opportunities between those two currencies.

In this sense, the Covered Interest Parity (CIP) represents a condition under which investors are not exposed to a foreign exchange risk by means of the use of a forward contract, so the exchange rate risk is effectively covered. Under this condition, a domestic investor would earn equal returns from investing in domestic assets or converting currency at the spot exchange rate, investing in foreign currency assets, and exchanging the foreign currency for domestic currency at the negotiated forward rate. Investors will be indifferent to the interest rates on deposits in these countries due to the equilibrium resulting from the forward exchange rate. The condition allows for no arbitrage opportunities because the return on domestic deposits  $(1 + i_d)$  is equal to the return on foreign deposits  $\frac{F}{S}(1 + i_f)$ .

The following equation reflects the concept of this CIP:

$$(1 + i_d) = \frac{F}{S}(1 + i_f) \quad (1)$$

Rearranging the previous equation and solving for  $F$ , what we obtain is:

$$F = S \frac{(1+i_d)}{(1+i_f)} \quad (2)$$

Equation (2) which results from the relationship between forward and spot exchange rates within the context of CIP is responsible for avoiding arbitrage strategies and so, potential opportunities to obtain profits. However, in order for this equilibrium to hold under differences in interest rates between two countries, the forward exchange rate must generally differ from the spot

exchange rate, such that a no-arbitrage condition is sustained. Therefore, the forward rate is said to contain a premium or discount, reflecting the interest rate differential between two countries.

The forward exchange rate differs by a premium or discount of the spot exchange rate:

$$F = S(1 + P) \quad (3)$$

Where  $P$  is the premium or discount.

Equation (3) can be rearranged as follows in order to solve for the forward premium/discount:

$$P = \frac{F}{S} - 1 \quad (4)$$

On the contrary, when the no-arbitrage condition is satisfied without the use of a forward contract to hedge against exposure to exchange rate risk, interest rate parity is said to be uncovered and so, we arrive to the Uncovered Interest Parity (UIP). In this situation, risk-neutral investors will be indifferent among the available interest rates in two countries because the exchange rate between those countries is expected to adjust such that the domestic return on domestic deposits is equal to the domestic return on foreign deposits, thereby eliminating the potential for uncovered interest arbitrage profits.

The following equation represents the Uncovered Interest Parity condition:

$$(1 + i_d) = [E_t(S_{t+k})/S_t](1 + i_f) \quad (5)$$

Now, we are going to demonstrate that if we combine both conditions, so that both Covered and Uncovered Interest Parity hold, we can derive an important relationship between the forward and expected future spot exchange rates:

$$\text{CIP: } (1 + i_d) = \frac{F}{S} (1 + i_f) \quad (6)$$

$$\text{UIP: } (1 + i_d) = [E_t(S_{t+k})/S_t](1 + i_f) \quad (7)$$

Dividing UIP between CIP yields the following equation:



$$1 = E_t(S_{t+k})/F_t \quad (8)$$

Equation (8) can be rewritten and solved for  $F_t$ , so that:

$$F_t = E_t(S_{t+k}) \quad (9)$$

$F_t$  is the forward exchange rate at  $t$

$E_t(S_{t+k})$  is the expected future spot rate at  $t+k$ , where  $k$  is the number of periods into the future from time  $t$

This last expression represents the Unbiasedness Forward Rate Hypothesis, suggesting that the forward rate it can be assumed as an unbiased predictor of the future spot rate.

Up to our days, economists have found empirical evidence that CIP generally holds, although not in a completely accurate way. In that sense, the Forward Rate Unbiasedness Hypothesis can serve as a test to determine whether UIP holds, so in order for the forward rate to reflect the true spot rate value, both CIP and UIP conditions must hold.

The Unbiasedness Hypothesis states that under conditions of rational expectations and risk neutrality, the forward exchange rate is an unbiased estimator of the future spot exchange rate. This Unbiasedness Hypothesis is a key puzzle among economists and financial researchers. In general, the majority of recent studies regarding the Unbiasedness Forward Rate Hypothesis have empirically demonstrated the inability of the forward rate to be an unbiased and good predictor for the future spot rate.

Nowadays, there exists an enormous literature available on whether the forward exchange rate is an unbiased predictor of the future spot exchange rate. Due to the vast nature of the literature present in the field, we only refer to some important works in this paper.

Eugene Fama (1984) considered that forward rate could be interpreted as the sum of a premium and the expected future spot rate. More precisely, "The forward exchange rate  $f_t$  observed for an exchange at time  $t+1$  is the market determined certainty equivalent of the future exchange rate  $s_{t+1}$ "<sup>1</sup>

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<sup>1</sup> Fama, Eugene. (1984). *Forward and Spot Exchanges Rates*.

Fama conducted a study testing a model for measurement of both variation in the premium and the expected future spot rate components of forward rates. Assuming that the forward market is efficient or rational, the study found evidence that both components of forward rates vary through time. In fact, the study has two important conclusions. The first is that the premium and the expected future spot rate components of forward rates are negatively correlated. The second one it was that most of the variation in forward rates is due to the variation in the premiums.

Besides, Thomas Chiang (1988) conducted a study developing a stochastic coefficient model to examine the unbiased forward rate hypothesis proposing that “with effective use of information underlying the stochastic pattern of the estimated parameters in forecasting, it is possible to improve the accuracy of the exchange rate predictions”<sup>2</sup>.

However, his study also considers that through the use of Brown-Durbin-Evans test and the Chow test, the constant coefficient hypothesis cannot be supported. He found that the constant term and the coefficient for the one-period lagged forward rate are subject to newly available information and vary through the sub-sample periods that he tested. Specifically, he realized that when he tested sub-samples, in many cases, the constant term was significantly different from zero and the coefficient of one-period lagged forward rate was significantly different from one. Another interesting aspect of Chiang’s study is that he added the two-period lagged forward rate as independent variable in predicting the spot rate and this variable was not found to be significant at the 5% level, suggesting that it contains no significant contribution to the explanation for the spot rate.

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<sup>2</sup> Chiang, Thomas C. (1988). *The Forward Rate as a Predictor of the Future Spot Rate. A Stochastic Coefficient Approach*.

## 4. METHODOLOGY

In this section, we provide the methodology that leads us to conclude that regression (10) is the better one we can use to analyze the validity or invalidity of the Forward Rate Unbiasedness Hypothesis.

$$S_{t+k} - S_t = \alpha_i + \beta_i(F_t - S_t) + U_{i,t+k} \quad (10)$$

where the change in spot exchange rate is the dependent variable and the forward premium is independent variable.

Under conditions of risk neutrality and rational expectations on the part of market agents, the forward rate is an unbiased predictor of the corresponding future spot rate. Assuming the absence of risk premium in the foreign exchange market it must hold true that

$$E_t(S_{t+k}) = F_t \quad (11)$$

Where  $F_t$  is the log forward rate at time  $t$  for delivery  $k$  periods later,  $S_{t+k}$  is the corresponding log spot rate at time  $t+k$ , and  $E_t(S_{t+k})$  is the mathematical expectations operator conditioned on the information set available at time  $t$ .

Assuming the formation of rational expectations, as Muth (1960) stated, "expectations are essentially the same as the predictions of the relevant economic theory."<sup>3</sup>

$$S_{t+k} = E_t(S_{t+k}) + u_{t+k} \quad (12)$$

Where  $u_{t+k}$ , the rational expectations realized forecast error, must have a conditional expected value of zero and be uncorrelated with any information available at time  $t$ .

However, as time went on Fama (1984) deepened in this analysis regarding the price determination on future markets. He stated that correct analysis will be the one in which  $F_t$  observed at time  $t$  for an exchange at time  $t+k$ , reflects the future variation of the future spot exchange rate  $S_{t+k}$ . Besides, Fama considered that the forward rate could be divided into an expected future spot rate  $E_t(S_{t+k})$  and premium ( $P_t$ ) so that,

$$F_t = E(S_{t+k}) + P_t \quad (13)$$

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<sup>3</sup> Muth, John F.(1961). *Rational Expectations and the Theory of Price Movements*.

Note that  $F_t = \ln F_t$ ,  $S_{t+k} = \ln S_{t+k}$  and that, the expected future spot rate  $E(S_{t+k})$  is the rational forecast, derive from all the information of period  $t$ . In that sense, the idea behind using logs is basically in order to make easier our analysis. This implies that our analysis is independent on whether we use one or another currency.

Following the methodology established by Fama, from (13) if we subtract  $S_t$  on both sides of the equation, we observe that the difference between the forward rate and the current spot rate is defined as:

$$F_t - S_t = P_t + E(S_{t+k} - S_t) \quad (14)$$

Which reflects that forward premium ( $F_t - S_t$ ) will be determined by the risk premium of the market ( $P_t$ ) and the expected variation in spot exchanges rates  $E(S_{t+k} - S_t)$ .

Then, focusing on the main target, the study of the Forward Rate Unbiasedness Hypothesis, that is, trying to figure out if the forward premium really predict the future spot rate, Fama proposed two different regression models. What changes in each one of them is the dependent variable, which are  $F_t - S_{t+k}$  and  $S_{t+k} - S_t$  (both observed at  $t+k$ ), while the independent variable is the same for the both of them,  $F_t - S_t$  (observed at  $t$ ). What we obtain are the following equations, (15) and (16):

$$F_t - S_{t+k} = \alpha_1 + \beta_1(F_t - S_t) + u_{1,t+k} \quad (15)$$

$$S_{t+k} - S_t = \alpha_2 + \beta_2(F_t - S_t) + u_{2,t+k} \quad (16)$$

In this way, estimates of (16) tell us whether the current forward-spot differential or forward premium,  $F_t - S_{t+k}$ , has power to predict the future change in spot rate,  $S_{t+k} - S_t$ . Evidence that  $\beta_2$  is reliably non-zero means that the forward rate observed at  $t$  has information about the spot rate to be observed at  $t+k$ . Likewise, since  $F_t - S_{t+k}$  is the premium  $P_t$  plus  $E(S_{t+k} - S_t)$  (see equation 14), the random error of the rational forecast  $E(S_{t+k})$ , evidence that  $\beta_1$  in (15) is reliably non-zero means that the premium component of  $F_t - S_t$  has variation that shows up reliably in  $F_t - S_{t+k}$ .

Since  $F_t - S_{t+k}$  and  $S_{t+k} - S_t$  sum to  $F_t - S_t$ , the sum of the intercepts in (15) and (16) must be zero, the sum of the slopes must be 1 and the disturbances period by period must sum 0. In other words, what Fama said is that both regressions (15)

and (16) contain identical information about the variation of the  $P_t$  and  $E(S_{t+k}-S_t)$  components of  $F_t-S_t$ .

In this sense, Fama suggested that considering a joint analysis of the two regressions is what will reflect clearly the information that either contains.

However, for our analysis we are going to choose the second, equation (16). In fact, as Fama concludes regression (16) become more common in financial literature and so, it has seemed to be more efficient in telling us whether the current forward-spot differential or forward premium,  $F_t-S_t$  has a power to predict the future change in the spot rate,  $S_{t+k} - S_t$ .

Finally, just to be as much precise as possible, it is important to know that earliest studies in the 1970s dealt with a simple regression of the future spot exchange rate ( $S_{t+k}$ ) on the current forward exchange rate ( $F_t$ ) with an error term with a conditional mean of zero [ $E_t(u_{t+k})$ ].

$$S_{t+k} = \alpha_i + \beta_i F_t + u_{i,t+k} \quad (17)$$

However, this regression model was found to be incorrect as both the forward and the spot rates were shown to be non-stationary series, they were integrated of order one. Subsequently, to resolve this problem, the model was modified and the Forward Rate Unbiasedness Hypothesis was tested by running the regression of the change in the future spot exchange rate ( $S_{t+k} - S_t$ ) on the forward premium ( $F_t - S_t$ ).

In such a way, as we announced previously, the standard test to determine whether the forward rate is an unbiased efficient predictor of future spot rates, has become to run a regression as (18):

$$S_{t+k} - S_t = \alpha_i + \beta_i (F_t - S_t) + U_{i,t+k} \quad (18)$$

If  $F_t$  was an unbiased, efficient predictor of  $S_{t+k}$ , then :

$$\alpha_i = 0 \text{ and } \beta_i = 1$$

## 5. RESULTS

### 5.1. DESCRIPTIVE STATISTICS

The dataset used in this study consists of monthly frequency observations obtained from EcoWin database of the spot and one month forward exchange rates for three major currencies, the Sterling Pound, the Euro and the Yen. The range of observations it goes from the January, 1990 to May, 2013 and all the rates are U.S. dollars per unit of foreign currency.

Table 1 shows the descriptive statistics for the full sample of the spot and one month forward rates of each one of the currencies in relation to the dollar. Then, Tables 2 and 3 report the same descriptive statistics for the data used from the pre-crisis and crisis period.

**Table 1. Full Sample Period (January 1990- May 2013)**

Descriptive Statistics	Spot Rate			Forward Rate (1 month)		
	£/\$	€/€	¥/\$	£/\$	€/€	¥/\$
Number of observations	281	281	149	281	281	149
Mean	0.608258	0.837369	104.4834	0.610334	0.840683	104.2799
Median	0.617971	0.811400	108.5000	0.618720	0.817220	107.6530
Minimum	0.480469	0.634000	76.19000	0.480880	0.635098	76.1645
Maximum	0.709019	1.182600	134.5600	0.711319	1.182992	133.6475
Standard Deviation	0.055799	0.119799	15.82061	0.054860	0.119531	15.70142
Coefficient of Variation	0.091736	0.143066	0.151417	0.089885	0.142183	0.150570
Skewness	-0,400046	1.079809	-0.318687	-0.404786	1.008286	-0.324148
Kurtosis	2.358889	3.737178	1.894767	2.456863	3.629032	1.826046

**Table 2. Pre-crisis Period (January 1990- August 2008)**

Descriptive Statistics	Spot Rate			Forward Rate (1 month)		
	£/\$	€/€	¥/\$	£/\$	€/€	¥/\$
Number of observations	224	224	92	224	224	92
Mean	0.601416	0.860975	115.4004	0.603999	0.865129	115.4517
Median	0.609663	0.829000	116.6350	0.611076	0.835596	116.4517
Minimum	0.480469	0.634000	99.81000	0.480880	0.635098	99.54430
Maximum	0.709019	1.182600	134.5600	0.711319	1.182992	133.6475
Standard Deviation	0.059184	0.121991	7.441914	0.058339	0.120839	7.371949
Coefficient of Variation	0.098408	0.141689	0.064788	0.096588	0.139677	0.064038
Skewness	-0.173537	0.867127	0.205585	-0.190320	0.800594	0.155371
Kurtosis	2.055174	3.286178	2.810310	2.139034	3.259795	2.739126

**Table 3. Crisis Period (September 2008- May2013)**

Descriptive Statistics	Spot Rate			Forward Rate (1 month)		
	£/\$	€/€	¥/\$	£/\$	€/€	¥/\$
Number of observations	57	57	57	57	57	57
Mean	0.635147	0.744602	86.86281	0.635280	0.744615	86.78592
Median	0.630358	0.749500	86.34000	0.630354	0.749372	86.32470
Minimum	0.560150	0.666400	76.19000	0.559616	0.666427	76.16450
Maximum	0.698861	0.817400	106.0300	0.699012	0.817220	105.4406
Standard Deviation	0.026383	0.038551	7.724072	0.026471	0.038801	7.664199
Coefficient of Variation	0.041538	0.051774	0.088923	0.041668	0.052109	0.883115
Skewness	0.423658	0.090530	0.386450	0.405172	-0.085193	0.365928
Kurtosis	3.669874	2.0993539	2.073365	3.731931	2.088576	2.026869

It can be appreciated that the three currencies have experienced a depreciation trend from the beginning of the crisis in September 2008 with the fall of Lehman Brothers. The mean reflected in table 3 for both spot and forward rates are lower than those in table 2, except for the case of the British Pound. Both the euro/dollar and yen/dollar exchange rate parity have declined during the crisis, whereas the British Pound showed a higher value of the mean for the Crisis period than for the Pre-Crisis one. This fact could reflect that both the Euro and the Yen are increasingly losing their value and so, they are weaker now than before.

Besides, there is another interesting point here. The volatility of exchanges rates which are measured by the standard deviation and the coefficient of variation are lower in table 3 than in table 2. Initially, we could expect just the opposite.

However, one reason which could explain the higher volatility in table 2 is that for this Pre-Crisis period we have included year 2007 and 2008 up to September. In fact, although for this paper we consider the beginning of the Crisis with the fall of Lehman Brothers in September 2008, all the studies and economic research suggest that crisis effects appeared in the summer of 2007.



In such a way, the contrast between the higher values of the exchange rates in the 90's with those of 2007 and 2008 which are lower for all the currencies, could explain this greater volatility in table 2 than in table 3.

Regarding table 1, which included the full sample period (1990-2013), we can observe that all values are intermediate with respect to those showed in tables 2 and 3 for the Pre-Crisis and Crisis periods, which seems to be very reasonable because it includes both subsamples.

## 5.2. UNIT ROOT TESTS

One important assumption of the Unbiased Forward Hypothesis is that the forward and spot rates are stationary. More sophisticated techniques in econometrics have shown that macroeconomic time series in their levels are non-stationary and hence their variances tend to increase with time.

In fact, non-stationarity of time series is regarded as a problem in econometric analysis. A series is said to be (weakly or covariance) stationary if the mean and the autocovariances of the series do not depend on time.

A common example of non-stationary series is the random walk:

$$y_t = y_{t-1} + \epsilon_t \quad (19)$$

Where  $\epsilon$  is a stationary random disturbance term. The series  $y$  has a constant forecast value, conditional on  $t$ , and the variance is increasing over the time. The random walk is a difference stationary series since the first difference of  $y$  is stationary.

$$\Delta y_t = y_t - y_{t-1} = \epsilon_t \quad (20)$$

A difference stationary series is said to be integrated and is denoted as  $I(d)$  where  $d$  is the order of integration. The order of integration is the number of unit roots contained in the series, or the number of differencing operations it takes to makes the series stationary. For the random walk above, there is one unit root test, so it is an  $I(1)$  series. Similarly, a stationary series is  $I(0)$ .

Standard inference procedures do not apply to regressions which contain an integrated dependent variable or integrated regressors. Therefore, it is important to check whether a series is stationary or not before using it in a regression. The formal method to test the stationarity of a series is the unit root test.

To analyze the unit root test, the Dickey-Fuller method is applied. If we consider a regression equation as the following:

$$y_t = \alpha + \beta y_{t-1} + \epsilon_t \quad (21)$$

And then, we take the first difference:

$$y_t - y_{t-1} = \alpha + \beta y_{t-1} + \epsilon_t \quad (22)$$

Which can be rewritten as follows:

$$\nabla y_t = \alpha + (\beta - 1) y_{t-1} + \epsilon_t \quad (23)$$

And finally, considering  $(\beta - 1)$  as  $\rho$ , then to illustrate the use of Dickey Fuller tests, we can consider an AR(1) process:

$$y_t = \alpha + \rho y_{t-1} + \epsilon_t \quad (24)$$

Where  $\alpha$  and  $\rho$  are parameters and  $\epsilon_t$  is assumed to be white noise.

Besides,  $y_t$  is a stationary series if  $-1 < \beta < 1$ .  $y_t$  is a non-stationary series if the process it started at some point, the variance of  $y$  increases steadily with time and goes to infinity. If the absolute value of  $\rho$  is greater than one, the series is explosive. Therefore, the hypothesis of a stationary series can be evaluated by testing whether the absolute value of  $\beta$  is strictly less than one. In that sense, the Dickey Fuller test takes the unit root test as the null hypothesis  $H_0: \beta = 1$ . Since explosive series do not take much economic sense, this null hypothesis is tested against the one-sided alternative  $H_1: \beta < 1$ .

Since  $\rho = \beta - 1$ , at the end, the null and alternative hypothesis are:

**$H_0: \rho = 0$  (i.e. the data needs to be differenced to make it stationary)**

**$H_1: \rho < 0$  (i.e. the data is stationary and doesn't need to be differenced)**

The null hypothesis of a unit root is rejected if the t-statistic is less than the critical value. The Augmented Dickey-Fuller t-statistic does not follow a standard t-distribution because is skewed to the left with a long, left-hand-tail.

The following tables 4, 5 and 6 show the ADF test for the change in spot rate and the forward premium for the three bilateral currency relationships. Table 4 used the data for the full sample period, table 5 refers to the data used from the Pre-Crisis period and table 6 shows the ADF test for the Crisis period.

**Table 4. Full Sample Period (January 1990- May 2013)**

	ADF Test			
<b>S(t+1) –S(t) £/\$</b>	<b>Test statistic</b>	<b>-6.961614</b>	<b>F(t)-S(t) £/\$</b>	<b>-10.29146</b>
	5% Critical value	-2.8748		-2.8748
<b>S(t+1) –S(t) €/€</b>	<b>Test statistic</b>	<b>-7.223570</b>	<b>F(t)-S(t) €/€</b>	<b>-4.780750</b>
	5% Critical value	-2.8748		-2.8748
<b>S(t+1) –S(t) ¥/\$</b>	<b>Test statistic</b>	<b>-5.862462</b>	<b>F(t)-S(t) ¥/\$</b>	<b>-4.527726</b>
	5% Critical value	-2.8815		-2.8815

**Table 5. Pre-crisis Period (January 1990- August 2008)**

	ADF Test			
<b>S(t+1) –S(t) £/\$</b>	<b>Test statistic</b>	<b>-4.517097</b>	<b>F(t)-S(t) £/\$</b>	<b>-10.81014</b>
	5% Critical value	-2.8748		-2.8748
<b>S(t+1) –S(t) €/€</b>	<b>Test statistic</b>	<b>-5.050718</b>	<b>F(t)-S(t) €/€</b>	<b>-4.707195</b>
	5% Critical value	-2.8748		-2.8748
<b>S(t+1) –S(t) ¥/\$</b>	<b>Test statistic</b>	<b>-7.587347</b>	<b>F(t)-S(t) ¥/\$</b>	<b>-5.704846</b>
	5% Critical value	-2.8939		-2.8939

**Table 6. Crisis Period (September 2008- May 2013)**

	ADF Test			
<b>S(t+1) –S(t) £/\$</b>	<b>Test statistic</b>	<b>-5.230318</b>	<b>F(t)-S(t) £/\$</b>	<b>-3.744083</b>
	5% Critical value	-2.9146		-2.9146
<b>S(t+1) –S(t) €/€</b>	<b>Test statistic</b>	<b>-7.268955</b>	<b>F(t)-S(t) €/€</b>	<b>-3.602781</b>
	5% Critical value	-2.9146		-2.9146
<b>S(t+1) –S(t) ¥/\$</b>	<b>Test statistic</b>	<b>-4.738967</b>	<b>F(t)-S(t) ¥/\$</b>	<b>-5.341399</b>
	5% Critical value	-2.9146		-2.9146

### 5.3. AUTOCORRELATION AND HETEROCEDASTICITY ANALYSIS

Before analyzing the results obtained for the OLS regression test, it is important to check that our variables do not present serial correlation. In regression analysis for time series data, autocorrelation of the errors is a problem. Autocorrelation of the errors, which themselves are unobserved, can generally be detected since it produces autocorrelation in the observed residuals. In fact, autocorrelation violates the OLS assumption that the error terms are uncorrelated. While it does not bias the OLS coefficient estimates, the standard errors tend to be underestimated when the autocorrelations of the errors at low lags are positive.

The traditional test for the presence of first-order autocorrelation is the Durbin–Watson statistic, which if lower than 2, it implies a positive autocorrelation between variables.

In this case, when we initially compute the regression for each one of the currencies with respect to the dollar, for the Full Sample, the Pre-Crisis and the Crisis periods, all the Durbin Watson obtained were lower than 2. In order to fix the serial correlation, an autoregressive term AR(1) is going to be included for all the regression equations. In fact, the AR(1) term is going to be significant and the Durbin Watson statistic closed to 2.

Otherwise, we could have used another method to detect the existence of autocorrelation just by looking at the residual graph. In this sense, if positive errors are followed by positive and negative errors of the same size, then we are in the presence of positive autocorrelation.

Regarding the variability of the variables it is important to detect if they exhibit a constant variance (homoscedasticity) or not (heteroscedasticity).

This fact is also a major concern for the latter application of regression analysis because the presence of heterocedasticity can invalidate statistical test of significance which assumes that the modeling errors are uncorrelated, normally distributed and their variance does not vary with the effects being modeled.

For all of the cases, it can be observed the existence of heterocedasticity. In that sense, we apply HAC Consistent Covariances (Newey-West) that is consistent in the presence of heterocedasticity and autocorrelation.

Finally, in order to determinate if the unbiased forward rate hypothesis is fulfilled or not, we will proceed to the regression analysis of the equations considered in this study. We will try to answer to the crucial question focusing on whether or not the forward premiums it contains so valuable information that can be used to predict the future fluctuation of the spot exchange rates. Besides, we will

analyze the results from the following regressions in order to determinate how exchanges rates have behaved before and after the beginning of the Great Financial Crisis.

#### 5.4. REGRESSION TEST

As we already mention, this study examines the effectiveness of the forward premium ( $F_t - S_t$ ) in determining the future spot exchange rate ( $S_{t+1} - S_t$ ) in order to conclude whether the Forward Rate Unbiasedness hypothesis is fulfilled or not. The regression used is:

$$S_{t+1} - S_t = \alpha + \beta(F_t - S_t) + U_{1,t+1} \quad (25)$$

Ordinary Least Squares Method (OLS) is used, which ensures that the coefficients will be best linear unbiased estimators (BLUE).

Tables 7, 8 and 9 summarize all the regressions considered for this study, which are computed for the three different exchange rates relationships (£/\$, €/€ and ¥/\$) for the Full Sample, the Pre-Crisis and Crisis periods.

**Table 7. Full Sample Period (January 1990- May 2013)**

Equation	Source	Exchange Rate	Horizon	Frequency	Time Period	No. Of observations
1	EcoWin	£/\$	1 month	Monthly	1990-2013	281
2	EcoWin	€/€	1 month	Monthly	1990-2013	281
3	EcoWin	¥/\$	1 month	Monthly	2001-2013	149

**Table 8. Pre-Crisis Period (January 1990- August 2008)**

<b>Equation</b>	<b>Source</b>	<b>Exchange Rate</b>	<b>Horizon</b>	<b>Frequency</b>	<b>Time Period</b>	<b>No. Of observations</b>
1	EcoWin	£/\$	1 month	Monthly	1990-2008	224
2	EcoWin	€/€	1 month	Monthly	1990-2008	224
3	EcoWin	¥/\$	1 month	Monthly	2001-2008	92

**Table 9. Crisis Period (September 2008- May 2013)**

<b>Equation</b>	<b>Source</b>	<b>Exchange Rate</b>	<b>Horizon</b>	<b>Frequency</b>	<b>Time Period</b>	<b>No. Of observations</b>
1	EcoWin	£/\$	1 month	Monthly	2008-2013	57
2	EcoWin	€/€	1 month	Monthly	2008-2013	57
3	EcoWin	¥/\$	1 month	Monthly	2008-2013	57

The results of these OLS regressions are contained in tables 10, 11 and 12. Since we are analyzing the unbiasedness of the one-month forward exchange rate, the value of  $k$  in the regression model is equal to 1. The following tables

show the results obtained for the  $\beta$  coefficients; the constant terms ( $\alpha$ ); the adjusted determination coefficient ( $R_2$ ) and the Durbin Watson Statistic (DW).

**Table 10. Full Sample Period (January 1990- May 2013)**

Exchange Rate Parity	$\alpha$	$\beta$	$R_2$	DW
£/\$	-0.001633 (0.231808)	-0.677551* (0.287256)	0.071049	1.719260
€/\$	-0.000732 (0.221849)	-0.770551** (0.302178)	0.054020	1.864846
¥/\$	0.000684 (0.296368)	-1.017553* (0.312556)	0.039576	1.801432

\*\* Significant at 1%

\* Significant at 5%

Standard errors are indicates in parentheses

**Table 11. Pre-Crisis Period (January 1990- August 2013)**

Exchange Rate Parity	$\alpha$	$\beta$	$R_2$	DW
£/\$	-0.003601* (0.221510)	-0.747418* (0.237407)	0.050952	1.928035
€/\$	-0.001609* (0.212125)	-0.767829** (0.287349)	0.036034	1.957674
¥/\$	-0.002296* (0.283023)	1.262796** (0.294326)	0.075913	1.943915

\*\* Significant at 1%

\* Significant at 5%

Standard errors are indicates in parentheses

**Table 12. Crisis (2008-2013)**

Exchange Rate Parity	$\alpha$	$\beta$	$R_2$	DW
£/\$	-0.003261* (0.243631)	-0.671510** (0.334778)	0.063716	2.089181
€/€	0.002541* (0.254044)	-0.758909** (0.348952)	0.043726	2.013084
¥/\$	-0.004787* (0.314895)	-1.040013** (0.324263)	0.071568	1.958555

\*\* Significant at 1%

\* Significant at 5%

Standard errors are indicates in parentheses

As we can appreciate the majority of constant terms and  $\beta$  coefficients present negative values. Besides, all of the constant terms are significant at 5% whereas the majority of beta coefficients are significant at 1%.

Moreover, since the regressor  $F_t - S_t$  has a low variation relative to  $S_{t+1} - S_t$ , the coefficients of determination ( $R^2$ ) for the regressions are small.

In terms of observed variability, the analysis here shows that exchange rate fluctuations have increased during the crisis period and so, we can see how the standard errors of the coefficients are higher in table 12 than in table 11. In such a way, table 10 for the full sample period presents intermediate values in terms of volatility between those numbers showed in the pre-crisis and crisis tables.

One of the main causes of this greater exchange rate volatility could have been the differences between interest rates which influenced even more the exchange rate fluctuations arising from the crisis.

Moreover, it can be observed that the Unbiasedness Rate Hypothesis is not satisfied. As we stated before, in order for the hypothesis to be accepted, constant terms must be 0 and beta coefficient have to be 1, and this is not met in any of the cases. For that reason, we can conclude that forward premium is not an unbiased predictor of the future spot exchange rate.

If we come back again to the theoretical model developed by Fama(1984), it will be possible to provide an explanation for the preponderance of negative  $\beta$  estimates in tables 10, 11 and 12.



To explain why the vast majority of  $\beta$  coefficients are negative we have to look back again to the methodology introduced by Fama, we explained in the fourth section of this paper. In such a way, as we previously stated, assuming rational expectations, the forward premium can be given as:

$$F_t - S_t = P_t + E(S_{t+k} - S_t) \quad (26)$$

So, based on this equation the OLS estimate of  $\beta$  coefficient can be explained as follows:

$$\begin{aligned} \beta &= \frac{cov(S_{t+1} - S_t, F_t - S_t)}{var(F_t - S_t)} \\ &= \frac{var(E(S_{t+1} - S_t)) + cov(P_t, E(S_{t+1} - S_t))}{var(P_t) + var(E(S_{t+1} - S_t)) + 2cov(P_t, E(S_{t+1} - S_t))}. \end{aligned} \quad (27)$$

Hence for  $\hat{\beta}$  to be negative, it has to be satisfied that

$$var(P_t) > cov(S_{t+1} - S_t, F_t - S_t) > var(E(S_{t+1} - S_t)) \quad (28)$$

Since  $var(E(S_{t+1} - S_t))$  in 27 must be non-negative, a negative estimate of  $\beta$  implies that  $cov(P_t, E(S_{t+1} - S_t))$  is negative and larger in magnitude than  $var(E(S_{t+1} - S_t))$ . Besides a negative covariation between  $P_t$  and  $E(S_{t+1} - S_t)$  attenuates the variability of  $(F_t - S_t)$  and the interpretation of the regression slope coefficient of expression 27. Nevertheless, the regression coefficients provide interesting information that both the premium  $P_t$  and the expected change in the spot rate,  $E(S_{t+1} - S_t)$  in  $(F_t - S_t)$  vary through time and  $var(P_t)$  is large relative to the  $var(E(S_{t+1} - S_t))$ .

For example in the PPP model for the exchange rate, the dollar is expected to appreciate relative to a foreign currency, that is,  $(E(S_{t+1} - S_t))$  is negative when the expected inflation rate in the U.S is lower than in the foreign country. A negative  $cov(P_t, E(S_{t+1} - S_t))$  then implies a higher purchasing power risk premium in the expected real returns on dollar denominated bonds relative to foreign currency bonds when the anticipated US inflation rate is low relative to the anticipated foreign inflation rate.

In view of the results above we could say that the forward premium will have no power to predict the direction of future spot exchange rates and the Unbiasedness Forward Rate Hypothesis must be rejected.

## 6. CONCLUSION

This study examines the Unbiasedness Forward Rate Hypothesis both theoretically and empirically. First, we show the forward rate determination and the rationale of the Forward Rate Unbiasedness Hypothesis derived from the Covered and Uncovered Interest Parity Conditions. Following the empirical methodology of the regression model developed by Fama (1984), we analyzed if variations in the spot exchange rate can be explained by the forward premium.

This regression model is applied for monthly data on spot and one-month forward exchanges rates for the yen, the euro and the sterling pound, all relative to the USD. If the results obtained from the OLS regressions show constant terms equal zero and beta coefficients of one, then the hypothesis must not be rejected. However, once we treat our regression model for serial correlation and heteroscedasticity, none of the constant terms were zero and the majority of beta estimates were significant at one percent level, negative and different from one.

In this sense, the results obtained from that model provide us two main different conclusions. The first and more fundamental one is that indeed, forward premium does not fully reflect the expected variation in the future spot rate and so, the Forward Rate Unbiasedness Hypothesis is strongly rejected.

Secondly, in terms of variability, the regression analysis shows that exchange rate fluctuations have increased during the Global Financial Crisis of 2008. In fact, one of the main causes of this higher exchange rate volatility could have been the differences between interest rates which influenced even more the exchange rate movements derived from the crisis.

## 6. REFERENCES

Chiang, Thomas C. (1988). *The Forward Rate as a Predictor of the Future Spot Rate. A Stochastic Coefficient Approach*. Journal of Money, Credit and Banking . 20 (2), 212-232.

Dickey, D.A. and W.A. Fuller (1979). *Distribution of the Estimators for Autoregressive Time Series with a Unit Root*. Journal of the American Statistical Association, 74, p. 427–431.

De Castro, Francisco y Novales, Alfonso. (1997). *The Joint Dynamics of Spot and Forward Exchange Rates*. Banco de España - Servicio de Estudios. Documento de Trabajo nº 9715.

Fama, Eugene. (1984). *Forward and Spot Exchanges Rates*. Journal of Monetary Economics . 14, 319-338.

Muth, John F.(1961). *Rational Expectations and the Theory of Price Movements*. Econometrica. Journal of Econometric Society. 29, 315-335.