ABDUCTIVE REASONING, THEORETICAL PREDUCTION, AND THE PHYSICAL WAY OF DEALING FALLIBLY WITH NATURE[1]

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1. Introduction

Many philosophers of science think that science does harbour some privileged method of discovery. Were this right, then it would be possible for scientists to discover both truths about facts and truths about theories. Were science successful with both forms of dealing with Nature: the discovery of facts and the postulation of true theories about the world, it would be justified to claim that description, explanation, understanding and prediction are the aims of science.

The question is whether it really exists a privileged form of discovery in scientific methodology. From its very beginning nearly 2400 years ago, Western science has been concerned with two main forms of dealing scientifically with Nature:

1. According to Plato's *astronomer method* -as defined by Duhem (1908)- the aim of science is to propose hypotheses intended to save the appearances. The task of hypothesis building is provided by *abduction*.

2. According to Aristotle, the aim of demonstrative science is to discover the principles which would serve as explanations for the observed phenomena. The task of discovery is provided by *induction.*

In the following sections I dwell upon induction and abduction, and conclude that they are heuristics providing hypotheses intended to deal fallibly with Nature. In the last section I introduce preduction, an implementation of the hypothetical-deductive method very commonly applied in theoretical physics. I will also argue that preduction is a reasoning strategy that merely allows us to deal fallibly with Nature. Fallibly dealing with Nature is all what science can do.

2. Induction

Let us assume *induction* to be a kind of ampliative (and truth conservative) inference. Then *inductive reasoning* would be a method of scientific discovery. In order to see if this holds true, I am going on to scrutinize Aristotle's view on induction.

In the *Prior Analytics,* Book II, 68^b15-29: Aristotle proceeds in the following way:

Let *A* stands for long-liver,
B for bileless,
C for the particular long-lived animals, e.g. man, horse, mule.

And let "Induction (…) consists in proving through *C* that *A* belongs to *B*. For this is the manner in which we make inductions", then Aristotle's *inductive syllogism* looks like this:

[1] This paper is part of a research project on *Theoretical Models in Physics* supported by the Spanish Ministry of Education and Science.

All *C* is *A* (*A* belongs to the whole of *C*)
All *C* is *B* (*B* also belongs to every *C*)
Hence: All *B* is *A* (*A* belongs to *B*)

Now, were this way of reasoning formally correct, then *after a finite enumeration of individuals sharing properties A and B*, it would be logically legitimate to conclude that *A* belongs to *B*. Thus Aristotle seems to assume that induction consists in proving *through* a finite class *C* of individuals, that *A* belongs to *B*.

Two problems arise immediately: The first one is that Aristotle's inductive syllogism is not a syllogism of the first figure (i. e. it is not like *Barbara*). The second is that Aristotle's inductive syllogism is not a valid syllogism of the third figure, i. e. it is not of the kind of *Darapti, Felapton, Disamis, Datisi, Bocardo, Ferison*. In conclusion: Aristotle's inductive syllogism is not any syllogism at all.

Aristotle was perfectly aware of this logical problem. Therefore he offered the following solution to it: "If then *C* is convertible with *B*, and the middle term is not wider in extension, it is necessary that *A* should belong to *B*." According to this solution, Aristotle's inductive syllogism would look like a syllogism in *modus Barbara*, and would be formally correct:

All *C* is *A*	(man, horse and mule are long-lived animals)
All *B* is *C*	(All bileless animals are man, horse and mule, i.e. the only bileless animals are man, horse and mule)
Hence: All *B* is *A*	(All bileless animals are long-liver)

Unfortunately, Aristotle's purported solution brought a new problem into light, since in order to affirm that *All B is C*, it is necessary to make a complete enumeration of all (species of) animals. Thus *Aristotle's cost for the intended conversion is complete enumeration*. Surprisingly Aristotle justifies his step by claiming (*Prior Analytics*, Book II, 68^b15-29) that "we must apprehend *C* as made up of all the particulars. For induction proceeds through an enumeration of all the cases."

Were Aristotle prepared to accept that induction proceeds by complete enumeration of all cases, the cost would be to transform induction into deduction, thus ceasing to be a peculiar form of scientific discovery. The alternative could be to maintain induction as a logically illegitimate way of reasoning (thus there would not be any logically legitimate *inductive syllogism* at all). In this case inductive conclusions would not necessarily be true, and induction would become only *a fallible way of reasoning about Nature*.

Can abduction replace induction as a way of scientific discovery?

3. Abduction

Abduction is generally conceived of as a kind of reasoning by means of which surprising phenomena suggest hypotheses that may be used as premises for further predictions. The very well-known Peircian scheme of abduction is:

Some phenomenon *F* has been observed
Were *H* a true explanation of *F*, then *F* would be a matter of fact
Hence: There is reason to believe that *H* is true.

There are lots of examples of abduction in the history of science, of which a few ones are:

- The theoretical models accounting for planetary motions in ancient astronomy until Copernicus.
- The celebrated abduction by Kepler of Mars's elliptic orbit.
- Charles Darwin's hypothesis of natural selection in *On the Origin of Species by Means of Natural Selection*, 1859, abductively inferred on the basis of abundant available biological and fossil evidence.
- Ernest Rutherford's postulation of the *atomic planetary model* in 1911 on the basis of the scattering of alpha-particles by a thin gold plate in an experiment conducted by Geiger and Marsden in 1910.
- Alfred Lothar Wegener's (1880-1930) postulation of the hypothesis of *continental drift* in his *Die Entstehung der Kontinente und Ozeane* (*The Origin of the Continents and Oceans*), 1915. Evidences supporting Wegener's abduction of the *continental drift hypothesis* were:
  1) Measures of the continuous separation of Europe and America;
  2) Geographic-morphological coincidences between South-America and Africa, and between Labrador, Greenland and Scandinavia.
  3) Geological affinities of the cordilleras in Norway, Scotland and Canada; of the plateaus of Africa and Brazil; and between Patagonia and the Cape region.
  4) Paleontological discoveries of fossils of pre-historical identical organisms in distant regions of South-America, India and Australia
  5) Paleoclimatic signs: In Pangea the Poles didn't coincide with the actual ones. Some tropical and subtropical regions today were covered with ice 300 millions years ago. And vice-versa, tropical regions then are situated today in very cold zones, as fossil petrol and coal deposits show.

No wonder that abduction has attracted the interest of so many philosophers of science, who have seen in it a form of *scientific discovery*, an *inference to the best explanation*, or merely a tool for *scientific creativity*. According to Lorenzo Magnani (2001, 17-18)

> Theoretical abduction is the process of *inferring* certain facts and/or laws and hypotheses that render some sentences plausible, that *explain* or *discover* some (eventually new) phenomenon or observation; it is the process of reasoning in which explanatory hypotheses are formed and evaluated.

And for John R. Josephson (1994, p. 5):

> *Abduction*, or *inference to the best explanation*, is a form of inference that goes from data describing something to a hypothesis that best explains or accounts for the data. Thus abduction is a kind of theory-forming or interpretative inference.

Abductions share with inductive generalizations (Josephson, p. 13), that they

> are *ampliative inferences*; that is, at the end of an abductive process, having accepted a best explanation, we may have more information than we had before. The abduction

> transcends the information of its premises and generates new information that was not previously encoded there at all.

But abductions disagree with deductions that

> Deductions are *truth preserving*, whereas successful abductions may be said to be *truth producing*.

My question is: *Is abduction really an inference to the best explanation?* The first method of abduction in the history of science goes back to Plato. Plato's *astronomer method* is implicit in the following question (Cf. Duhem 1908, 5):

> What circular motions, uniform and perfectly regular, are to be admitted as hypotheses so that it might be possible to save the appearances presented by the planets?

According to Duhem's (1908, 5-6) interpretation of Plato's methodological proposal,

> 1. Astronomy is the science that so combines circular and uniform motions as to yield a resultant motion like that of stars. When its geometric constructions have assigned each planet a path which conforms to its visible path, astronomy has attained its goal, because *its hypotheses have then saved the appearances.*
>
> 2. The astronomer must declare himself fully satisfied when the hypotheses he has combined succeed in saving the appearances[2].

Duhem's interpretation thus sheds some doubts on the view of Platonic abduction as inference to the best explanation. (And Plato's *abductive reasoning* taken by and large provided the way for the construction of geometrical models in ancient astronomy until Copernicus.) Moreover, according to Peirce (C.P. 2.776 and 2.777)

> the hypothesis which [presumption, abduction, A.R.] problematically concludes is frequently utterly wrong itself, and even the method need not ever lead to the truth.

Since Peircian abduction shares with induction that any abduced conclusion can be wrong, should abduction be conceived of as a method of discovery, or merely as a kind of heuristics providing hypotheses intended to deal fallibly with Nature?

4. Modeling abduction from the viewpoint of Bayesian probabilistic inference

According to Josephson's conceptual analysis (*op. cit.*, p. 5), abduction is a distinctive kind of inference that follows this pattern:

> D is a collection of data (facts, observations, givens).
> H explains D (would, if true, explain D).
> No other hypothesis can explain D as well as H does.
> Therefore, H is probably true.

[2] This sentence reminds Paul Dirac's (1963, 48) view that "If the physicist knows how to calculate results and compare them with experiment, he is quite happy if the results agree with his experiments, and this is all he needs."

Now, if abduction is inference to the best, i. e., the most probably true explanation, then there are to ways, both of them put forward by Josephson (p. 14), by which abduction could be strengthen:

1. an abductive process might seek further information beyond that given in the data initially to be explained.

2. there may be a need to distinguish between explanatory alternatives; for help in forming hypotheses; or for help in evaluating them.

Josephson's case 1. above amounts to determining whether or not some growing evidence on behalf of a hypothesis -further information beyond that given in the data initially to be explained- contributes to increase its probability to be true.

Let e_i, e_j, e_k, e_l, be different evidences favorable to a given hypothesis *h*. And let assume that each one of these evidences satisfy the following conditions: 1) $h \rightarrow e_i$ and 2) $0<p(h)<p(e_i)<1$. If the evidence supporting *h* growths, Bayes' Theorem looks like:

$$p(h \mid e_j \wedge e_i) = \frac{p(h).p(e_j \wedge e_i \mid h)}{p(e_j \wedge e_i)} .$$

Well, it happens that

$$p(h \mid e_j \wedge e_i) > p(h \mid e_i) .$$

And in general:

$$\ldots > p(h \mid e_j \wedge e_i \wedge e_k) > p(h \mid e_j \wedge e_i) > p(h \mid e_i) > p(h)$$

Now, two problems arise now here:

1. When either novel evidence or even predicted evidence continues to support the hypothesis, then, according to Bayes' theorem, the probability for the hypothesis to be true would increase continuously. Thus after a considerable amount of favorable evidence the probability of the hypothesis to be true would approach 1. Nevertheless, if *surprisingly new evidence or a new prediction of the hypothesis fails, then a highly probably true hypothesis suddenly would become false.*
2. Newtonian mechanics was a highly confirmed hypothesis. But let's write its history differently: Let us assume that Newton's *Query I* of his *Optics*, 1704: "Do not bodies act upon light at distance, and by their action bend its rays; and is not this action (*coeteris paribus*) strongest at the least distance?" had been answered earlier: the bending angle amounts to 0".87 seconds of arc[3]. And let assume that Newton himself had given the conditions for the confirmation of this prediction, and that astrophysical and technical conditions would have been given for the empirical proof of the hypothesis. *Then Newton himself would have been forced to recognize that his gravitational hypothesis was simply wrong*.

[3] Cf. A. Rivadulla (2004*a*).

Josephson's case 2. above amounts to determining whether an available evidence can contribute to discriminate among many competing hypotheses: "there may be a need to distinguish between explanatory alternatives".

It seems here that abduction is a form of Baconian eliminative induction, and Bayes' Theorem can be used as well as a model for it. A Bayesian model for Case 2. might be the following: Let the empirical evidence *e* be the angular deviation $\Delta\vartheta = 1''.75$ of light by the Sun. And let the alternative explanatory hypotheses be relativity theory (*TR*), Newtonian mechanics (*MN*), and a third hypothesis h_3, which is assumed to be theoretically incompatible, but empirically equivalent, with *TR*.

Bayes' Theorem takes the form

$$p(h_i \mid e) = \frac{p(h_i).p(e \mid h_i)}{\sum_{i=1}^{3} p(h_i).p(e \mid h_i)},$$

and its applicability is restricted by conditions: $p(h_1)+...+ p(h_n)=1$, and $p(h_1|e)+...+ p(h_n|e)=1$.

We know that according to *MN* the value of the light deflection should amount to 0''.87, whereas *TR* predicts a deviation of 1''.75, and the same does h_3, which is assumed to be empirically equivalent to *TR*. Thus, given that the available evidence is $\Delta\vartheta = 1''.75$, then $p(\vartheta = 1''.75 \mid MN) = 0$ and $p(e \mid h_3) = p(\vartheta = 1''.75 \mid TR) = 1$.

Let's now distribute the initial probabilities of these hypotheses in the following way (These values are arbitrary, but irrelevant): $p(MN)=0.5$, $p(TR)=0.35$, $p(h_3)=0.15$. Applying Bayes' Theorem it results that $p(MN|e) = 0$, $p(TR|e)=0.7$, and $p(h_3|e)=0.3$.

At the first look Bayes' Theorem seems to accomplish its task: The probability of Newtonian Mechanics to be true becomes 0, since it has been refuted by experience, whereas the probability of both relativity theory and h_3 has grown, since they have both been confirmed by experience.

But this last claim is misleading, since as following equivalence shows

$$\frac{0.35}{0.15} = \frac{0.7}{0.3}$$

the reason between a priori probabilities holds also a posteriori[4].

Thus Bayes' Theorem is of no help in modeling abduction as an inference to the best, i. e. to the most probably true hypothesis. Abduction remains a kind of heuristics providing ways of reasoning that merely allow us to infer theoretical constructs intended to deal fallibly with Nature.

5. Theoretical preduction

Induction and abduction clearly fail as legitimate logic forms of scientific discovery. Thus there seems to be *stricto sensu* no *ars inveniendi* in scientific methodology capable to lead to true (or even probably true) theories, theoretical models or theoretical explanations.

[4] This example has been taken from Rivadulla (2004*b*, Chapter 1)

Nonetheless induction and abduction are not the only ways of reasoning capable to produce hypotheses usable for further fallible inferences. Astonishingly theoretical *preduction* has escaped notice in the philosophy of science[5], although it is a methodological strategy theoretical physicists commonly apply in order to postulate theoretical models, hypotheses, laws and principles.

I propose here to call *preduction* a form of reasoning that starts *ab initio* from first principles, *methodologically* postulated as premises of an inferential procedure. Predictive reasoning differs from abduction in that the preduced products are not suggested by data, but constructed on the basis of the available theoretical background. (This is the reason why I call it *theoretical preduction.*) Thus they depend more on the theoretical framework than on the empirical data. Preduction implements the hypothetical-deductive method, but it should not be confused with the axiomatic-deductive method.

The reason for the differentiation of this particular implementation of the hypothetical-deductive method, which I call preduction, is that the premises used for further inferences in a preductive argument very usually proceed from different theories. This means that physicists resort to the available theoretical background *as a whole* in order to achieve their inferential aims. Substitution *salva veritate* and dimensional analysis are important means to reach their aims. Only one strategy is forbidden: to use theories in parts where they contradict each other.

Preduction provides indeed the way of reasoning by which most theoretical models are postulated in science, mainly when we are concerned with domains orphan of theories. This is the case of astrophysics. In astrophysics scientists make use of the whole of physics, in order to accommodate the scarce data provided by the light of the celestial objects astrophysicists are concerned with. But preduction does not only provides us with theoretical models; factual hypotheses and theoretical laws are also theoretical products achieved by the application of preductive reasoning. Factual hypotheses like the temperature of the solar surface, Jeans mass critical value for the formation of protostars, or the critical density of the Universe; theoretical laws like Rayleigh-Jeans law, Planck's own radiation law, or Schrödinger's equation; and theoretical models, like Helmholtz-Kelvin gravitational collapse model for the energy of stars, Bohr's atomic planetary model, or the stars interior models in astrophysics, are some examples of theoretical products achieved by preduction in physics.

Nevertheless a last important point needs to be made clear. Since preduced theoretical constructs depend on the available theoretical background, and this cannot be known to be true, restrictions do frequently occur in the domain of their intended applications, and refutations sometimes happen as well. We learn indeed from the methodology and the history of theoretical models in physics, for instance, that no matter how a given theoretical model has been achieved: by induction, by abduction or even by *preduction*, we cannot a priori know how many phenomena will belong to its application domain, nor how many of them will have to be removed from its domain of intended applications[6]. *Thus preduction only supplies a fallible strategy in order to deal predictably with Nature.*

Induction, abduction and preduction are merely fallible ways of dealing scientifically with Nature. This is all what theoretical science can do.

References

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[5] As I have been told, the word *preduction* has been used previously in artificial intelligence by Jum Arima and by Allen Courtney and Norman Foo.
[6] Cfr. Rivadulla (2006, Sections 3 and 4)

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