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THE NEWTONIAN LIMIT OF RELATIVITY THEORY AND THE RATIONALITY OF THEORY CHANGE

ABSTRACT. The aim of this paper is to elucidate the question of whether Newtonian mechanics can be derived from relativity theory. Physicists agree that classical mechanics constitutes a limiting case of relativity theory. By contrast, philosophers of science like Kuhn and Feyerabend affirm that classical mechanics cannot be deduced from relativity theory because of the incommensurability between both theories; thus what we obtain when we take the limit $c \to \infty$ in relativistic mechanics cannot be Newtonian mechanics *sensu stricto*. In this paper I focus on the alleged change of reference of the term *mass* in the transition from one theory to the other. Contradicting Kuhn and Feyerabend, special relativity theory supports the view that the mass of an object is a characteristic property of the object, that it has the same value in whatever frame of reference it is measured, and that it does not depend on whether the object is in motion or at rest. Thus *mass* preserves the reference through the change of theory, and the existence of a Newtonian limit of relativity theory provides a good example of the rationality of theory change in mathematical physics.

– Does mass really depend on velocity, Dad?
– No! Well, yes. Actually, no, but don't tell your teacher.
Carl Adler, *Am. J. Phys.* **55**, 1987

1. THE *NEWTONIAN LIMIT* OF RELATIVITY THEORY AND THE RATIONALITY OF THEORY CHOICE

According to Albert Einstein (1917, §§22, 29), the most important aim of a scientific theory is to point to the establishment of a more comprehensive one, in which it survives as a limiting case. For instance, Newton's gravitational theory obtains as a first approximation by specializing general relativity theory's equations for the case of weak gravitation fields and low velocities.

Einstein's suggestion of the existence of a *Newtonian limit* is nowadays a universally assumed idea in theoretical physics. Nobel prize winner Lev Landau (1951, §1-1) claims for instance: "The limiting transition from relativistic to classical mechanics can be produced formally by the transition to the limit $c \to \infty$ in the formulas of relativistic mechanics." Steven Weinberg (1972, Ch.7) talks about the *Newtonian limit* of Einstein's field

Synthese **141:** 417–429, 2004.

equations too, and in (1989, pp. 14–15) he repeats that "Einstein's theory of general relativity … reduces to Newton's theory at large distances and small velocities." Also Misner, Thorne and Wheeler (1973, § 17.4) explore the mathematical reduction of general relativity to Newton's theory of gravity "in the 'correspondence limit' of weak gravity and low velocities".

The existence of the Newtonian limit of *special* relativity theory shows that Newton's laws of mechanics can be compared to those of special relativistic mechanics by mathematically deriving the former as a *limiting case* of the latter. Actually, when objects move with velocities v that are small compared with the value of c in empty space, special relativistic phenomena like length contraction and time dilatation disappear, the relativistic expression of momentum agrees with the classical formula, Newton's second law can be restored, the classical expression of kinetic energy reappears, etc.

If we are given two theories, and one of them constitutes a limiting case of the other one, then we are in a privileged situation in order to make a rational choice between them. Indeed the existence of limiting cases in mathematical physics allows one to account for theory change as an intrinsically rational process.

2. INCOMMENSURABILITY AND THE NEWTONIAN LIMIT OF RELATIVITY THEORY

Among philosophers of science, the Popperian view on intertheoretic approximation is very close to that of physicists. Already in the early thirties Karl Popper (1979, pp. 51–52) claimed that old physical theories are coarse approximations of new ones, and in (1935, §79) he asserted that "The old theory, even when it is superseded, often retains its validity as a kind of limiting case of the new theory". Years later Popper (1994, p. 12) affirmed that "the predecessor theory must appear as a good approximation to the new theory". This fact facilitates the comparison of theories; in particular, "Einstein's theory can be compared point by point with Newton's and (…) it preserves Newton's theory as an approximation." In an astonishingly similar way, the structuralist Joseph Sneed (1971, pp. 304–305) claims:

> The new theory must be such that the old theory reduces to (a special case) of the new theory.[1]

Finally Stegmüller (1980, p. 48) claims that the explanation of scientific progress by means of a suitable concept of theory reduction is not logically incompatible with Kuhn's incommensurability thesis.[2]

But can Newtonian mechanics *really* be derived from special relativity theory? This is precisely the question Kuhn (1970, pp.101–102) is concerned with. Kuhn's answer is that the derivation is spurious: the laws derived are not Newton's laws, because "the physical referents of the Einsteinian concepts are by no means identical with those of the Newtonian concepts that bear the same name (Newtonian mass is conserved; Einsteinian is convertible with energy. Only at low relative velocities may the two be measured in the same way and even then they must not be conceived to be the same)". The alleged incomparability of Newtonian mass and Einsteinian mass with respect to their referential properties makes both terms incommensurable in Kuhn's view.

Feyerabend (1978, p.185 and 1981b, p.154) also affirms that even when $v/c \rightarrow 0$ (or $c \rightarrow \infty$), the concepts do not coincide, the 'rest mass' is not the classical mass; according to him 'relativistic' mass gives the measure of the mass of a body relative to a frame of reference, whereas 'classical' mass is an *intrinsic property* of the object under consideration.[3] Since what is measured is not the same in both cases, the derivation of classical mechanics from relativity theory becomes impossible.

Finally Max Jammer (2000, p. 57) who refers to both types of mass, 'classical' and 'relativistic', as "The two most frequently quoted incommensurable terms", still maintains,[4] agreeing with Kuhn and Feyerabend, that in classical physics inertial mass "is an inherent characteristic of a particle and, in particular, is independent of the particle's motion. In contrast, the relativistic mass, ..., is well known to depend on the particle's motion". Moreover he claims that 'rest' or 'proper' mass "is just a particular case of the relativistic mass and there is not yet any cogent reason to identify it with the Newtonian mass of classical physics".

At this point in our discussion it seems perfectly legitimate to pose the question: Who are right, the physicists who affirm the existence of a derivation, as a limiting case, of classical mechanics from relativity theory, or the philosophers who claim that, because of incommensurability, this cannot be the case?

3. THE *ANTI-RELATIVISTIC MASS* POINT OF VIEW

One of Kuhn's favourite reasons for the existence of incommensurability between relativistic and classical mechanics is that Newtonian mass is invariant, whereas Einsteinian mass is not. Feyerabend claims on his side that classical mass is an intrinsic property of bodies, whereas Einsteinian mass is not. Now, they are not right, because an object has the same mass as observed in any inertial frame of reference. Below I show how special relativity accounts for the invariance of mass.

In special relativity theory events occur in four-dimensional Minkowski's spacetime. In this framework the *norm* or *magnitude* of the four-vector *position* x^μ, with time component $x^0 = ct$, and $x^1 = x$, $x^2 = y$, $x^3 = z$ space components, is given by

$$|x^\mu|^2 = (x^0)^2 - [(x^1)^2 + (x^2)^2 + (x^3)^2].$$

This difference measures also the *interval* ds, or distance in spacetime, between events. When events are infinitely close to each other, then

$$ds^2 = c^2dt^2 - [dx^2 + dy^2 + dz^2].$$

Intervals and norms are invariant in relativity theory, i.e., they have the same value in any inertial frame of reference. From *invariance* of intervals, it follows that

$$ds = \gamma^{-1}cdt,$$

where $\gamma = [1 - (v/c)^2]^{-1/2}$.

In spacetime the four-vector *velocity* $u^\mu = dx^\mu/dx$ has time component $u^0 = cdt/ds = \gamma$, and the four-vector *energy-momentum*[5] $p^\mu = mcu^\mu$, has time component $p^0 = mcu^0 = E/c$, i.e., *energy*, whereas its space components are *momentum*:

$$p^1 = mcu^1 = \gamma mv_x,\ p^2 = \gamma mv_y,\ p^3 = \gamma mv_z.$$

$E = \gamma mc^2$ is the total energy of an object with mass m freely moving with velocity v.

Both energy and momentum are *relative* to inertial frames of reference. In fact, from the Lorentz transformations of energy and momentum it follows that the momentum p'_x of the object, measured in an inertial frame of reference S', which moves away from another frame S with constant velocity v along the X axis, is given by

$$p'_x = \gamma^{-1}p_x - v_xE'c^{-2}.$$

If the object is at rest in relation to S', then $p'_x = 0$, and $\gamma = 1$. Thus from $E = \gamma mc^2$, it follows that $E' = mc^2$, and we obtain again

$$p_x = \gamma mv_x.$$

On the other hand, the difference

$$(p^0)^2 - [(p^1)^2 + (p^2)^2 + (p^3)^2] = m^2c^2$$

gives the magnitude of the *energy-momentum* four-vector. Because of the invariance of norms in Minkowski's spacetime, the mass of the object is the *invariant* magnitude of the *energy-momentum* vector (divided by c^2). This amounts to claiming that *any object has the same mass as observed in every inertial frame of reference*. Goldstein (1950, p. 204) affirms that mass m is "a scalar invariant property of the particle unaffected by Lorentz transformation".[6] Lev Okun (1989, pp. 31–32) asserts that "There is only one mass in physics, m, which does not depend on the reference frame", i.e. "mass is a relativistic invariant and is the same in different reference systems". Taylor and Wheeler (1992, p. 246; and "Dialog", pp. 246–251) claim that "mass is the same in *whatever* free-float frame it is figured". And Steven Weinberg (1998, p. 49) in his review of Thomas Kuhn's *Structure of Scientific Revolutions* maintains that "the term 'mass' today is most frequently understood as 'rest mass', an intrinsic property of a body that is not changed by motion".

The invariance of mass, a mathematical result of relativity theory, is devastating to both Kuhn's and Feyerabend's thesis of incommensurability. Contradicting Feyerabend, I claim that in the framework of relativity theory mass is a *characteristic feature* of the object considered. The assumption that, both in classical and relativistic mechanics, mass is a characteristic property of objects, make senseless affirmations like Feyerabend's: 'rest mass' is not the same as 'classical mass'.[7] Contradicting Kuhn I disagree with the view that Newtonian mass is constant, whereas Einsteinian mass is not.

4. MASS TERMS AND *OCKHAM'S RAZOR*

Because of the invariance of mass, terms like 'relative' mass, 'variable' mass, 'proper' mass, 'rest' mass, 'relativistic' mass, etc. should be avoided. They are misleading and redundant.[8] Other uses of *mass* like 'longitudinal' mass and 'transverse' mass are superfluous as well.[9]

Geometrically *energy-momentum* is a four-vector of the *pseudoeuclidean* spacetime. Since the geometry of spacetime is *not* Euclidean, the mass of a body, given by

$$m = (E^2/c^2 - p^2)^{1/2}/c,$$

remains invariant, even if an increasing body's velocity causes its quantity of motion and its energy to increase too.

In Section 3 above I argued that energy and momentum of a given particle are relative to a frame of reference, i.e., they have not the same value in whatever frame they are measured. The difference between both classical and three-dimensional relativistic definitions of momentum lies in that the relativistic value of momentum is proportional to the value of Newtonian momentum, the constant of proportionality being the Einsteinian factor γ. If the particle increases its velocity relative to a reference frame, then γ becomes larger, and the particle's momentum grows. Obviously its energy becomes larger too. But the particle mass m, the intrinsic magnitude of *energy-momentum*, remains constant. In a 1948 letter to Lincoln Barnett Einstein wrote:[10]

> It is not proper to speak of the *mass* $M = m/(1 - v^2/c^2)^{1/2}$ of a moving body, because no clear definition can be given for M. It is preferable to restrict oneself to the 'rest mass' m. Besides, one may well use the expression for momentum and energy when referring to the inertial behavior of rapidly moving bodies.

Often the increase in momentum has been interpreted mistakenly as caused by an increase in mass. For instance Max Born (1962, p. 277) claims, that when $v = c$, "the mass becomes infinitely great". But the view that the mass of a moving object is greater than its mass at rest is not supported by relativity theory, where mass is the same in all frames of reference, independently of whether the object is in motion or at rest. Since we are compelled to revise the assumption of the variability of mass with velocity, we must reject too the claim that 'classical mass' is a limiting case of 'relativistic mass'.[11] This mathematical relation holds only between the classical and the relativistic definitions of momentum and of energy.

According to Okun (1989, p. 35), the doctrine of the dependence of mass on velocity was postulated by Hendrik Lorentz in 1899. For the recent acceptance of this view Okun puts the blame on Wolfgang Pauli, who in the 1921 German edition of his *Theory of Relativity* "gave an undesirably long life to the notorious notion that mass depends on velocity".[12]

Historians of physics have been captivated also by the misconception of the variability of mass with velocity. For instance Max Jammer (1961, pp. 164–165) does not hesitate to identify, as in classical mechanics, the coefficient of velocity in the relativistic momentum expression as the 'relative mass' $m(v)$ of the particle. Moreover, he sees in the application of the calculus of four-vectors a help for the 'definitional character of velocity-dependent mass': "It is the new relation between space and time, ..., that produces the peculiar functional dependence of momentum on velocity and consequently the velocity dependence of mass."[13] Unfortunately Jammer was not able to see that the 'peculiar' functional dependence of momentum on velocity is a consequence of the frame dependence of momentum, and that precisely this dependence of both momentum and energy maintains the particle mass constant.

As to the empirical testing of equation $m(v) = \gamma m$, in his 1961 book Max Jammer has reviewed to some extent the alleged experimental evidence in favour of the velocity dependence of the electron mass.[14] At the end he admits that "an unambiguous direct verification of this all-important velocity dependence of mass is certainly still a matter of serious concern for experimental physics". Moreover his personal attempt to account 'in greater detail' for this experimental verification fails as well, as he confesses that the equation used to measure the curvature radius of an electron beam "could equally be written [...] without ever mentioning the idea of 'variable mass' ".

Finally, among philosophers of science, supporters of the doctrine of incommensurability have been misguided also by the multiplication *praeter necessitatem* of mass concepts. For instance Hartry Field (1973, p. 74), who claimed that in Newtonian mechanics the term *mass* suffers from an indeterminacy of the reference: "Newton's word 'mass' partially denoted proper mass and partially denoted relativistic mass; since it *partially* denoted *each* of them, it didn't *fully* (or *determinately*) denote *either*."[15] More recently Roberto Torretti (1999, pp. 287), who still maintains that 'relativistic mass' is a "function of speed", has argued that the 'relative mass' $m(v)$ of a moving object "is not normally the ratio of force to acceleration and thus not even a measure of inertia in the received sense". Torretti is misled by making use of expressions like 'relativistic mass' and 'proper mass'. Nonetheless, he points to the fact that the classical and relativistic expressions of *force* are different, which is right. Indeed, since force is frame dependent, combining the Lorentz transformations of both force and acceleration, when the particle is at rest in its proper reference frame S', we obtain that in the laboratory frame of reference S the spatial components of the four-vector *force* acting upon the particle are:

$$F_x = \gamma^3.m.a_x$$

$$F_y = \gamma.m.a_y$$

$$F_z = \gamma.m.a_z.$$

But γ is purely a numerical coefficient that has no dimensions. Thus m is always proportional to the ratio of force to acceleration.

Upholders of the *anti-relativistic mass* viewpoint are optimistic about the abandonment of the idea of the velocity dependence of mass.[16] Nonetheless this abandonment is not occurring without resistance. The discussion "Putting to rest mass misconceptions" in section "Letters" in *Physics Today*, May 1990, containing Okun's replies, gives signs of this resistance. Further defences of relativistic mass, like T. R. Sandin's (1991, p.1036) on aesthetic grounds: "Relativistic mass paints a picture of nature that is beautiful in its simplicity", and his avoidance of four-vectors on pedagogic grounds: "instruction with four-vectors make things unnecessarily difficult and obscure the subtle concepts of relativity by moving even further from their [students'] experience" do not support seriously the cause indeed. And Max Jammer (2000, p. 56) feels compelled to bring philosophical considerations into play. The resort to philosophy has of course heuristic value. But the best way to take a decision in a physical dilemma is to go back to experience, not to philosophy. Nonetheless no experiments describing how mass allegedly changes with velocity are referred to in Max Jammer's second book.

5. CONCLUSION

If the mass of a body is the same independently on whether the body is in motion or at rest, i.e., if it has the same value in every frame of reference in which it is measured, then the reference of the term *mass* has not suffered any change in the transition from Newtonian to relativistic mechanics. This is no reason for wondering about. Many other magnitudes like electric charge, spin, baryonic number, etc., are intrinsic properties of each particle as well, i.e., they are invariant scalars, whose values do not depend on any reference frame.

Other relativistic magnitudes, like force and momentum, preserve in three-dimensional notation the form of their homologous classical expressions, except for the fact that they are affected by a non-dimensional coefficient, which points to their frame dependence. Dimensional analysis guarantees that homologous magnitudes are given in the same units, otherwise we should not be measuring *the same*. This is why it is possible to derive mathematically Newtonian mechanics from relativity theory. I completely agree with Dalla Chiara and Toraldo di Francia,[17] that Einstein was talking about *the same magnitudes* as Newton did, although Einstein discovered some new properties of these magnitudes which Newton did not expect them to have.

Furthermore Newtonian mechanics and relativity theory *do share* a number of intended applications, although the first accounts for only a very limited number of phenomena, and fails to explain many others: light deflection by the Sun,[18] the *anomalous* advance of Mercury's perihelion,[19] gravitational redshift and black holes, [20] gravitational radiation, the gravitational lensing effect, the Shapiro time delay, etc., which are predicted by relativity theory. Albert Einstein (1917, § 29) claims explicitly that the observation in astronomy of the first two phenomena mentioned above *represents a failure of classical mechanics*. Thus the *predictive balance* is overwhelmingly favorable to relativity theory, and the decision to choose this theory against Newton's is intrinsically rational.

The comparison of theories, *either* with respect to their mathematical relation *or* to their predictive power, provides the answer to the question of the rationality of theory change.

ACKNOWLEDGEMENTS

This paper is part of a research, of reference BFF2002-01244, supported by the Spanish Ministry of Science and Technology. I thank Prof. Antonio Fernández-Rañada (Faculty of Physics, UCM) for his encouraging comments on an earlier version of this paper. I am very grateful also to two anonymous referees for helpful and precise criticisms.

NOTES

1 Sneed (1977) develops formally an approach to intertheoretic reduction.

2 Nonetheless Kuhn (1977, Section 4) is very sceptical about Sneed-Stegmüller's reduction concept.

3 Feyerabend (1981a, p. 81).

4 Cf. Jammer (2000, p. 41).

5 We obtain the *energy- momentum* four-vector p^{μ} multiplying by mc the velocity four-vector $u^{\mu} = \gamma(1, \vec{v}/c)$.

6 In Goldstein (2002, §7.4) mass is merely a scalar.

7 Any case Feyerabend misunderstood Eddington (1923, §12), who was allegedly the source of his view. Indeed, Eddington relates the 'relative mass' $m(v)$ of a body to its 'invariant mass' m by $m(v) = \gamma m$, so that when $v = 0$, then $\gamma = 1$, and $m(v) = m$. Eddington's conclusion is that "the invariant mass is thus equal to the mass at rest". Following, if Feyerabend's 'classical mass' were the 'invariant mass', Eddington's assertion would refute Feyerabend's claim.

8 This is also Lev B. Okun's (1989, p.31) message. According to Adler (1987, p. 743) the use of 'relativistic' mass is not fundamental to special relativity theory: "Its role in special relativity as developed by Einstein is that of an artifact".

9 Already Herbert Goldstein (1950, p. 205) claims that "In general the use of these various 'masses' (...) is decreasing; they (...) obscure the physics of the situation more than they

reveal". In the 1980 second edition of Goldstein's *Classical Mechanics* any reference to 'longitudinal' and 'transverse' masses nearly disappears. Finally Goldstein et al. (2002) do not mention them at all.

[10] This is an improved translation by Siegfried Ruschin in "Putting to rest mass misconceptions", *Physics Today*, Letters, May 1990, p. 115, of the translation given by Adler (1987, p. 742), and reprinted by Okun (1989, p. 32).

[11] Max Born (1962, p. 273) for instance conceives of 'classical mass' as a limiting case of 'relativistic mass'.

[12] A history of the concept of relativistic mass is given by Jammer (2000, pp. 41–51).

[13] Jammer, *op. cit*., p.165.

[14] Cf. Jammer, *op. cit.*, pp. 166–171.

[15] However he encountered John Earman's (1977, p. 535) answer, that "there is strong evidence in favour of (...) the hypothesis that the Newtonian term 'mass' has the same denotation as the relativistic term 'proper mass' ". Although Earman still makes use of the superfluous expression 'proper mass', his claim is basically correct.

[16] According to Adler (1987, p. 739) "the use of relativistic mass is showing signs of progressive disfavor". In Notes 1–4 Adler shows Sears and Zemanky's dramatic change of view on this subject throughout the different editions of their *University Physics* textbook. And Okun (1990, p. 115) affirms that "the fact is that most leading physics journals, such as *Physical Review*, *Physical Review Letters* and *Physics Letters* don't use relativistic mass. You will not find it in professional books on particle physics".

[17] Cf. Dalla Chiara (1999, Chapter 13, §5).

[18] There can be no doubt that Newton considered the deviation of the trajectory of a photon passing through the Sun's gravitational field as an intended application of his gravitational theory. Indeed, Newton (1704, Book III, Part 1, Query I) asked: "Do not bodies act upon light at distance, and by their action bend its rays; and is not this action (*coeteris paribus*) strongest at the least distance?". This phenomenon had also been foreseen by Kant himself, as appears in *Opus postumum*, volume XXI, p. 404, of Kant's *Gesammelte Schriften*.

In *Newtonian approximation* the bending angle of a photon coming near the Sun in the direction of the Y axis is given by

$$\Delta\alpha \approx \frac{2Gm}{r_s c^2} \text{ radians} = 0''.87.$$

Unfortunately, this is only half of the observed value. On the other hand Einstein's general relativity theory predicts (Cf. L. Landau and E. Lifshitz 1951, §11-8) that the bending of a light ray, given by the formula

$$\Delta\alpha = \frac{4Gm_s}{c^2 r_s},$$

amounts to 1".75, which fits very well with the observed value, as Sir Arthur Stanley Eddington was already able to verify in 1919.

[19] The advance of the perihelion of a planet is a disturbance of its orbit, consisting in the fact that the main axis of the planet's ellipse rotates around the focus where the Sun is placed. If Newtonian mechanics did yield accurate predictions, and the planets of the solar system did not disturb Mercury's motion, its orbit would be a perfect ellipse. Now, according to observations, Mercury's perihelion shows an advance of 43".11 per century which Newtonian mechanics cannot explain, but Einstein's gravitational theory accounts

for as a typical relativistic effect. Given the proximity of Mercury to the Sun, this planet detects the effects of the solar gravitational field most significantly.

Relativity theory predicts an angular advance of any planet's perihelion, given by (Cf. L. Landau and E. Lifshitz 1951, §11-8, and Misner et al. 1973, § 40.5)

$$\Delta\vartheta = \frac{6\pi G m_s}{c^2 a(1-e^2)} \text{ radians per revolutions,}$$

where a is the main half-axis of the ellipse, and e denotes its eccentricity. Since the numerical value corresponding to Mercury is 43".03 per century, it is obvious that Einstein's prediction fits the observations completely.

[20] In Newtonian *approximation* the redshift of a photon trying to escape a star's gravitational field is given by

$$z \approx \left(1 - \frac{1}{2}\frac{r_g}{r}\right)^{-1} - 1,$$

where r_g is a quantity known as *Schwarzschild's radius*. Thus,

$$\lim_{r \to r_g} z = 1.$$

But according to Schwarzschild's metric of general relativity the photon's redshift is

$$z = \left(1 - \frac{r_g}{r}\right)^{-1/2} - 1,$$

whereof it follows that

$$\lim_{r \to r_g} z = \infty.$$

Since the latter result is compatible with the condition for a star with radius r_g to be a black hole, it becomes evident that Newtonian mechanics completely fails to account for black holes.

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