## Moral hazard and tradeable pollution emission permits Francisco Álvarez

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#### Resumen

Analizamos el mercado de derechos de emisión en un entorno en el que la contaminación, generada a partir de la actividad productiva de las empresas, se determina como la suma de un shock específico de cada empresa y la decisión de las mismas sobre el esfuerzo que dedican a reducir la contaminación. En este escenario, una sociedad mazimizadora de la utilidad desea inducir el esfuerzo óptimo en cada empresa. Dado que este esfuerzo lo decide cada empresa y no lo observa el regulador medioambiental, nos encontramos con un problema de riesgo moral en el que: (i) las empresas (agentes) tienen una ventaja informacional con respecto al regulador (principal) y (ii) el único enlace entre las empresas contaminantes es el mercado de derechos de emisión, que es una manera de intercambiar derechos (contratos) una vez han sido asignados por el regulador. Nuestro objetivo principal es analizar las consecuencias de la existencia de este mercado competitivo puesto que aumenta el conjunto de estrategias de las empresas. Desde un punto de vista teórico, caracterizamos las condiciones bajo las cuales el mercado mejora (o empeora) a las empresas con respecto a la situación sin mercado de derechos.

Palabras clave: Riesgo Moral, mercado de derechos de emisión.

Clasificación JEL: D21, D82.

#### **Abstract**

We consider a market for pollution emission permits in a setting in which pollution, generated as by-product of firms' activity, is determined as the sum of firm-specific random shocks and each firm's abatement effort. In such a setting, an expected utility maximizer society demands an optimal abatement effort from each firm. As long as the abatement effort is decided by each firm and not observed by the environmental regulator, a moral hazard problem arises in which: (i) firms (agents) have informational advantage with respect to the regulator (principal) and (ii) the only link among firms is precisely the market for permits, which is nothing but a chance to trade permits (contracts) once they have been assigned by the regulator. Our main point is to raise doubts on the social desirability of the -competitive- market, since it enlarges the firms' strategy space. We theoretically characterize conditions under which the market improves (or worsens) the firms with respect to an autarchy scenario.

**Key Words**: Moral Hazard, emission permits market.

JEL Classification: D21, D82.

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#### 1 Introduction

The use of tradeable emission permits as a mechanism to comply with environmental standards has steadly increased over the recent years to become nowadays a central issue at the design of environmental policies worldwide. One of the leading examples is the Carbon Dioxide emission permits market in the EU zone. It was first introduced at the celebrated Kyoto protocol, in 1988, and later developed and implemented for the EU member states in different Directives, starting at 2003/87/EC. Periodical Technical Reports by the European Environmental Agency indicate that those emission permits markets are already operating in nine EC countries, and all predictions point out that the number of markets of this kind will keep on growing.

The supposed efficiency-improving character of a market for emission permits is clear-cut. If all that matters is the aggregate level of pollution and not its share among polluting firms (as it is the case of Carbon Dioxide), a competitive market for permits seems to be the more natural assignment mechanism to attain the aggregate goal with a firm-wise optimal share. Of course, any departure from the competitive behavior at the marketplace, say, due to the existence of market power, raises doubts on the overall efficiency of the market allocation, and the list of possible market failures is much larger. We do not deal with these problems in this paper.

The aim of our study is to pose a rather striking question on the desirability of the mere existence of a market, even a full-fledged competitive market. We analyze the effects of the introduction of a pollution emission permits market on the cost for the polluting agents of undertaking socially optimal abatement effort. Consider, for instance, the referred case of the Carbon Dioxide within the EU: an environmental regulator (ER) assigns emission permits among polluting firms and then firms are allowed to trade permits among themselves. This trade might diminish the ER's capability to enforce firms to exert the socially optimal abatement effort. Roughly, buying permits might be cheaper than undertaking such effort, at least for some firms. The essence of this argument remains valid if we assume a price-taking behavior across firms. Thus, might the society, as a whole, be better off if the permits assigned by the ER were not tradeable?

The crucial point of the above argument is that the control of emissions is not the ultimate target but an instrument to induce abatement effort. This seems particularly realistic in the air pollution scenario. The observed air pollution levels do depend on factors that cannot be controlled by economic agents, not at least in the short run, such as climate conditions or simply good luck in adopting the right abatement technology. Those factors can be viewed as random shocks which, if firms are spaced out, differ across firms. Then, each firm emission level is the sum of the abatement effort exerted and a firm-specific random shock. In this setting, a (von Neumann-Morgenstern) expected utility maximizer society demands from each firm an optimal level of abatement effort, instead of an optimal level of emissions.<sup>1</sup> To complete the scene, we postulate that whereas firm's emission level is publicly observable, his abatement effort is not.<sup>2</sup>

<sup>&</sup>lt;sup>1</sup>Informally, as long as the emissions are subject to random elements, a low realized level of emissions at a given point in time does not make the society *happy*, since it is discounted that it might be due to favorable realizations of the shock. What the society seeks are low emissions to be persistent in time, and that is equivalent to an appropriate abatement effort.

<sup>&</sup>lt;sup>2</sup>We might argue that in fact only aggregate emissions are observable, this is the so called non-point pollution problem. Clearly, in such a setting, an assignment of permits among individual firms makes no sense and we must define precisely what do we mean then by emission permits. Since this additional complication adds nothing to our analysis, we adopt the simplest approach: each firm's emission is observable.

We adopt a theoretical point of view to study a moral hazard problem with a single principal, the ER, and multiple agents, the polluting firms. The ER designs contracts to induce a Pareto Optimal level of abatement effort on firms. Each contract specifies an assignment of permits and a system of fines for over-polluting agents. Since individual emissions are observable, contracts are contigent on them. The timing is as follows. First, the ER sets up the contracts. Second, firms trade permits in a competitive market (firms are price-takers) and decide the abatement effort. Finally shocks are realized, emissions are observed and payoffs take place. The only connection among firms is the market for permits: the agents are allowed to trade permits, that is, part of their contracts, once permits are assigned by the principal. In this setting, it is not straightforward to know whether society is better off with or without a (competitive) market for emission permits. To the best of our knowledge, this connection among agents is novel within the moral hazard literature, and constitutes the main interest of this paper from a purely theoretical point of view.

Previous literature analyzes moral hazard problems in environmental economics, where the connection among agents comes from different facts. In particular, Alchian and Demsetz (1972) and Holmström (1982) analyze a team production problem, where the observable outcome is joint production. Also, Mookherjee (1984), considers a different connection among agents, assuming that the effort of a single agent influences other agent's production. Related to this, Holmström (1979) considers the existence of common-to-all agents shocks, where one agent's production might be statistically informative about other agent's effort. Xepapadeas (1991) studies a dynamic model of incentives in a moral hazard situation and proposes instruments (subsidies and fines) to be applied in situations where there exists imperfect information. The moral hazard problem in his paper arises because the individual emission levels (or abatement efforts) are not observable while the outcome of all the polluting agents combined efforts is observable. Instead, we consider a model where agents are only connected through the possibility of trading emission permits. Among others, Macho-Stadler and Pérez-Castrillo (2004) analyze an optimal audit policy in a model of environmental taxes, and Wainwright (1999) studies enforcement of environmental policies in a situation of asymmetric information. Contrarily to these works, and in order to focus on the effects of the introduction of an emission permits market, we assume that there is no monitoring on efforts and that the tax policy is perfectly enforceable.

Other literature on environmental economics has studied trading markets from a different perspective. Montgomery (1972) studies a problem of pollution control to achieve environmental goals at least cost to the polluters. In his work, the equilibrium need not be Pareto Optimal. Instead, we propose a model which implements the optimal effort on each firm. From a different point of view than informational economics, Copeland and Taylor (2005) adapt a general equilibrium trade model to analyze international emission permit trade in an open economy and the effects of free riding on the global level of emissions.

The key question of our analysis is not whether it is possible to achieve an economy-wise Pareto Optimal abatement effort through the permits market mechanism, but whether it might be reached at a lower cost for all the polluting firms if the market did not exist. We study two cases: The autarchy scenario, where there is no possibility of trading emission permits, and the market scenario. Recall that the ER specifies an assignment of permits and a system of penalties for over-polluting agents. At any of the mentioned scenarios, the ER can achieve efficiency with a continuum of policies: from low amount of permits and low penalties for over polluting agents to large amount of permits and high penalties. The

essential message of this paper is whether the choice of the ER within that continuum of policies affects crucially the welfare achieved at the market and at the autarchy situation.

Let us start considering that the ER opts for assigning a low amount of permits (low is precisely defined in the paper). Then, the ER at the autarchy scenario faces a problem that the market solves automatically. If firms are fundamentally heterogenous, the society optimally demands different abatement efforts across firms. Consequently, the ER at the autarchy scenario must discriminate among them, whereas if a market of permits is allowed to operate, it acts as a self-discriminating mechanism. The heterogeneity issue plays in favour of the market specially when the ER assigns few permits and -as mentioned- sets low penalties for over polluting firms, since there is little room left for an effective discrimination at the autarchy case. When we analyze efficiency inducing policies that are restrictive with the permits, we show that at the market case all firms might face the same penalty parameter for over-polluting agents. In other words, the market for permits does the whole discrimination task and consequently the ER saves informational costs. Moreover, we show that most of the firms would face lower penalties in case of over-polluting when a market for permits exists with respect to the autarchy situation.

The previous picture reverts if the ER assigns a *large* amount of permits. Above all, the market for permits is an instrument for the firms, not for the ER, and so each firm uses it on his own profit. Logically, this fact becomes more relevant as the potential volume of trade (the amount of permits assigned by the ER) increases. Contrarily, as the amount of permits increases, the discrimination among firms performed by the market becomes less necessary, since the ER can discriminate with her initial assignment of permits. As an illustrative case, we show that for an initial assignment of permits proxied by *historical emissions*, which is a rather natural -and widely used- discrimination form, there is no gain from the firms' perspective from introducing a market for permits in comparison to the autarchy case. The explanation of this result within an agency theory framework is standard. Roughly, if we let the ER's strategy space be large enough (i.e.: she assigns as many permits as desired), she fully captures any surplus that the firms (agents) might obtain at the market.

The paper is organized as follows. Section 2 describes the model. Section 3 analyzes the Pareto Optimal allocation. In section 4 we study Pareto optimal restoring environmental policies at the autarchy and the market scenario. Section 5 concludes. All the proofs are left to the Appendix.

#### 2 The model

We consider an economy composed of a finite number I of polluting firms that belong to a set  $\chi$ , i.e.  $\chi = \{1, 2, ... I\}$ . Generic firms will be denoted by i and j. Each firm is freely endowed with one perfectly divisible unit of resource that can be allocated either to some profitable activity or to abatement effort. The pollution level generated by firm  $i \in \chi$  is:

$$e_i = m_i + \varepsilon_i - \lambda_i g(a_i) \tag{1}$$

where  $m_i$  is the amount of resource allocated to the profitable activity, and  $a_i$  is abatement effort, provided  $m_i + a_i \le 1$ . We assume that g is twice differentiable with g' > 0,  $g'' \le 0$  and g(0) = 0,  $\varepsilon_i$  is

a firm-specific random shock, and  $\lambda_i$  is a firm specific parameter that refers to the efficiency in reducing pollution. More specifically, given any two firms i and j, we say that firm i is environmentally more efficient that firm j iff  $\lambda_i > \lambda_j$ . In order to focus on asymmetries due to these efficiency levels, we assume that  $\varepsilon_i$  is i.i.d. across firms, with density function  $\phi$ , being  $[\varepsilon_{\inf}, \varepsilon_{\sup}]$  its support, and let  $\mu = E\{\varepsilon_i\}^3$ . We also assume that  $\Pr(e_i > 0) = 1$  for every i and every pair  $(m_i, a_i)$ . This is equivalent to

$$\varepsilon_{\inf} - \lambda_i g(1) > 0$$
 (2)

We denote by  $f(m_i)$  the profit of firm i. We assume that f is twice differentiable with f' > 0, f'' < 0 and f(0) = 0.

The global level of pollution causes a damage to other individuals different than the polluting firms. Let the global level of pollution be denoted by  $\bar{e}$ , that is,  $\bar{e} = \sum_{i \in \chi} e_i$ . The social damage is  $w(\bar{e}) = \exp(\delta \bar{e})$ , being  $\delta > 0$  a fixed parameter. Thus, the society as a whole is risk averse with CARA utility function, being  $\delta$  the constant absolute risk aversion parameter.<sup>4</sup>

The Pareto Optimal allocation of the economy is a pair of vectors  $(\mathbf{m}^o, \mathbf{a}^o) = (m_1^o, ..., m_I^o, a_1^o, ...., a_I^o)$  which solves<sup>5</sup>

$$\max \left\{ \sum_{i \in \chi} f(m_i) - E\{w(\bar{e})\} \right\}$$
s.t.  $m_i + a_i \le 1 \quad \forall i \in \chi$ 

There exists an environmental regulator, ER, who decides and imposes the environmental policy. Her objective is to induce each firm to choose its corresponding part of the Pareto Optimal allocation  $(\mathbf{m}^o, \mathbf{a}^o)$ . The problem of the ER is that she cannot observe -and thus enforce directly- each firm's internal allocation. Instead, she observes and bases the environmental policy on each firm's individual emissions. The timing of the game is as follows. First, the ER sets up the environmental policy, which consists of a firmwise assignment of pollution emission permits and a system of fines for overpolluting firms. Second, firms are allowed to trade pollution permits in a competitive market. Third, firms decide their internal allocation of resource. Finally, shocks are realized, individual emissions are observed by the ER and payoffs take place.

Formally, an environmental policy is a set of I piecewise linear penalty functions, each characterized by a pair  $(q_i^*, \theta_i)$ , such that the penalty paid by firm i once  $e_i$  is observed is:

$$\tau\left(e_i; q_i^*, \theta_i\right) = \theta_i \max\left\{0, e_i - q_i^*\right\}$$

In words, the first  $q_i^*$  units of pollution of firm i are free of charge and constitute the firm's assignment of permits by the ER, whereas the firm pays a fine  $\theta_i$  for each unit exceeding  $q_i^*$ . We assume that the penalty paid by the firms does not revert to any third party. If firm i trades permits at the market, then  $q_i^*$  in the previous expression is replaced by his post-market holdings of permits, say  $q_i$ .

 $<sup>^{3}</sup>E(x)$  denotes the expectation of x.

<sup>&</sup>lt;sup>4</sup>In other words, the society is sensitive not only to the *ex ante* expected pollution level but also to eventual extremely large values of pollution. The CARA assumption implies that, in a sense, society's aversion to environmental risk is independent of society's wealth.

<sup>&</sup>lt;sup>5</sup>We omit non-negativity constraints of the decision variables throughout the text.

<sup>&</sup>lt;sup>6</sup>Since each firm's pollution is observable, we deal with a point-source pollution problem.

We assume that the emission permits market is competitive. The market clearing condition is:

$$\sum_{i \in \chi} q_i = \sum_{i \in \chi} q_i^*$$

The right hand side, that corresponds to the supply side, is decided by the ER, whereas the left hand side, the demand side, is decided by the firms. Let us explain further how demand is obtained. Analogously to society's preferences, we assume that firms are risk averse with CARA utility function, although firms' risk aversion may differ from society's.<sup>7</sup> In addition, let p denote the unit price of the permits. The net payment of firm i at the market is  $p(q_i - q_i^*)$ , which can be either positive or negative depending on firm's position (buyer or seller). In conclusion, the expected profit of firm i, once firms have decided their internal allocation of resource and transactions at the market have taken place, is the following:

$$\pi_{i} = f(m_{i}) - E\left\{\exp\left(\rho\tau\left(e_{i}; q_{i}, \theta_{i}\right)\right)\right\} - p\left(q_{i} - q_{i}^{*}\right)$$
(3)

where  $\rho > 0$  is firm's (constant) absolute risk aversion, common to all firms. Each firm selects simultaneously the tern  $(q_i, m_i, a_i)$ , that is, a position at the market and an internal allocation of resource, to maximize his expected profit  $\pi_i$ .

Since the internal allocation of resources of each firm is not observed by the ER, firms have an informational advantage that leads to a moral hazard problem, where the regulator plays as principal, each firm plays as an agent and each firm-specific penalty function set up by the regulator is an individual contract. The main interest of this paper from a purely theoretical standpoint is that the emission permits market allows the agents to trade partially their contracts (specifically, the permits) after they are assigned by the principal.<sup>8</sup>

## 3 The Pareto Optimal allocation

In this section we characterize the Pareto Optimal allocation of resources of this economy. The following result easies the subsequent analysis and states that no resources are wasted at any Pareto Optimal allocation.

**Lemma 1** At any Pareto Optimal allocation,  $m_i = 1 - a_i$  for every  $i \in \chi$ , that is, the constraint is binding at the optimum.

Given Lemma 1, selecting  $a_i$  for each firm at the Pareto Optimal allocation is equivalent to selecting  $(m_i, a_i)$  for each firm i. Moreover, since g is an increasing function and, hence, monotone, selecting  $z_i = g(a_i)$  is equivalent to selecting  $a_i$ . Let  $\gamma$  denote the inverse function of g, so that  $a_i = \gamma(g(a_i))$  for every  $a_i \in [0, 1]$ . In order to interpret  $z_i$ , notice that firm i emission reduction is  $\lambda_i z_i$ . Thus,  $z_i$  is firm's abatement effort per unit of abatement productivity and proxies firm's abatement effort. Any

<sup>&</sup>lt;sup>7</sup>In other words, we allow for a difference between the risk aversion of those who suffer the externality (pollution) and those who cause it.

 $<sup>^8</sup>$  This trade constitutes the only link between the agents in our model. In fact, without the existence of an emission permits market, the analysis reduces to I independent principal-agent models. Moreover, since uncertainty is resolved once firms have taken decisions, we do not deal with an adverse selection problem.

economywide Pareto Optimal allocation is characterized by a vector  $\mathbf{z}^o = (z_1^o, z_2^o, ..., z_I^o)$ . In what follows, we focus on *interior* Pareto Optimal allocations, that is,  $z_i^o \in (0, z_{\text{sup}})$  for every i, where  $z_{\text{sup}} = g(1)$ .

**Proposition 1** Any (interior) Pareto Optimal allocation is characterized by:

$$\frac{f'\left(1 - \gamma\left(z_{i}^{o}\right)\right)\gamma'\left(z_{i}^{o}\right)}{\gamma'\left(z_{i}^{o}\right) + \lambda_{i}} = \delta(1 + \delta E\left\{\bar{e} \mid \mathbf{z}^{o}\right\}) \tag{4}$$

for every  $i \in \chi$ .

Note that, given the assumption on society's risk aversion, the Pareto Optimal abatement effort of each firm depends on aggregate magnitudes, specifically, on the expected level of aggregate pollution conditional on socially optimal firms' abatement effort. It is straighforward to show that in a risk neutral society, the Pareto Optimal abatement effort of each firm is independent of the rest of the industry. In addition, the left hand side of (4) is the marginal rate of transformation of the profitable activity in terms of pollution abatement. As expected, the previous result imposes that such rate must be equal across firms at any Pareto Optimal allocation. The following result presents a straighforward implication of (4).

Corollary 1 At any Pareto Optimal allocation, the environmentally more efficient firms exert a larger abatement effort. Equivalently, for any two firms i and j with  $\lambda_i > \lambda_j$ , we have that  $z_i^o > z_j^o$  and, consequently,  $a_i^o > a_j^o$ .

## 4 Efficiency-inducing environmental policies

In this section we study environmental policies that induce each firm to select the Pareto Optimal allocation of resource. Our primary concern is not whether such policy exists or not whenever a market of permits is allowed to operate, but to measure how much the mere existence of the market easies this task. To this purpose, we identify an efficiency-inducing environmental policy under the market mechanism described above and compare it to its counterpart in an autarchy setting, that is, when firms are not allowed to trade permits among themselves.

A comment on the ER's excess of instruments is now in order. To this purpose, consider an economy composed of a single polluting firm. The environmental policy is a pair  $(q^*, \theta)$ , which defines a piecewise linear penalty as stated previously, and there is no room for trading after the policy is fixed. Basically, the ER might have a continuum of efficiency-inducing pairs such that the more restrictive she is with the permits (the smaller  $q^*$  is), the more permissive she is with the fine for over-polluting (the smaller  $\theta$  is). Our purpose is to be as exhaustive as possible with this fact. To this end, we first present general properties of any efficiency-inducing environmental policy. Secondly, we analyze in detail two extreme cases: one case where the ER is very restrictive with the permits and a second case where she is very permissive with the permits, in a way specified below. We claim that each of these cases has interest in itself.

<sup>&</sup>lt;sup>9</sup>In a further section we present necessary and sufficient conditions for the Pareto optimal allocation to be interior within a particular case of the above presented model. Considering economies with *fully specialized* firms does not add any new insight, whereas it makes the analysis more involved.

#### 4.1 General properties

The aim of this subsection is to illustrate that our model delivers reasonable predictions on how the efficiency-inducing environmental policies should be, either in the autarchy or in the competitive market scenario. In further subsections we deal with the comparison of such policies in both scenarios for the two extreme cases quoted above.

#### 4.1.1 Autarchy

In this setting the ER assigns emission permits to the firms and there is no possibility of trading permits thereafter. Consider a generic firm i, given an arbitrary environmental policy  $(q_i^*, \theta_i)$ , the problem that firm i solves is

$$\max_{m_i, a_i} \left\{ f(m_i) - E\left\{ \rho \exp \tau \left( e_i; q_i^*, \theta_i \right) \right\} \right\}$$
  
s.t. 
$$m_i + a_i \le 1.$$

Note that the objective function is the expected profit of the firm when  $q_i^* = q_i$ . In the following lemma we show that, as in the Pareto Optimal allocation, the constraint is binding at equilibrium.

**Lemma 2** At equilibrium,  $m_i + a_i = 1$  for every  $i \in \chi$ .

The previous lemma allows us to reduce the dimensionality of the decision variable set. Substituting  $m_i = 1 - \gamma(z_i)$  in the previous problem, we obtain the following result.

**Proposition 2** Given an arbitrary environmental policy  $(q_i^*, \theta_i)$ , the equilibrium of firm i in an autarchy setting is characterized by the following first order condition:

$$\begin{split} &\frac{f'(1-\gamma(z_{i}))\gamma'(z_{i})}{\gamma'(z_{i})+\lambda_{i}} = \rho\theta_{i}\left(\int_{t_{i}}^{\infty}\phi\left(\varepsilon\right)d\varepsilon + \rho\theta_{i}\right.\left.\int_{t_{i}}^{\infty}\left(\varepsilon-t_{i}\right)\phi\left(\varepsilon\right)d\varepsilon\right)\\ &where \ t_{i} = \lambda_{i}z_{i} + q_{i}^{*} - \left(1-\gamma\left(z_{i}\right)\right). \end{split}$$

The properties obtained from the previous expression are straighforward if we compare the previous first order condition with the result in Proposition 1. We sumarize them in the following corollary.

Corollary 2 Under the autarchy regime: (i) the abatement effort of firm i increases with  $\theta_i$  and decreases with  $q_i^*$ ; (ii) for any efficiency-inducing environmental policy  $(q_i^*, \theta_i) \, \forall i \in \chi$ , firm i has a strictly positive probability of over-polluting, and thus of paying a fine despite he undertakes the Pareto Optimal abatement effort; (iii) there exists a continuous and increasing function  $\Upsilon$  such that any environmental policy  $(q_i^*, \theta_i)$   $\forall i \in \chi$  satisfying  $\Upsilon(q_i^*) = \theta_i$  is efficiency-inducing.

#### 4.1.2 The emission permits market

Let us introduce the possibility of trading permits among firms. First, we consider each firm's decision problem. Given an environmental policy  $(q_i^*, \theta_i) \, \forall i \in \chi$ , firm i selects  $(q_i, m_i, a_i)$  in order to maximize  $\pi_i$ , as defined in (3), with eventually  $q_i^* \neq q_i$ . In the same way as for the autarchy case, we obtain that the constraint is binding at equilibrium, and therefore,  $m_i + a_i = 1$ , so we can take  $m_i = 1 - \gamma(z_i)$  and reduce the decision variables to  $z_i$  and  $q_i$ .

**Proposition 3** The equilibrium of firm i with an emission permits market is characterized by the following first order conditions:

$$\frac{f'(1-\gamma(z_i))\gamma'(z_i)}{\gamma'(z_i)+\lambda_i}=p=\rho\theta_i\left(\int_{u_i}^{\infty}\phi(\varepsilon)\,d\varepsilon+\rho\theta_i\,\int_{u_i}^{\infty}(\varepsilon-u_i)\,\phi(\varepsilon)\,d\varepsilon\right),\tag{5}$$

where  $u_i = \lambda_i z_i + q_i - (1 - \gamma(z_i)).$ 

An implication of the latter result is given in the next corollary.

Corollary 3 The existence of an emission permits market restores the Pareto Optimal allocation if and only if the market price is

$$p = \delta \left( 1 + \delta E \left\{ \bar{e} \mid \mathbf{z} \right\} \right) \tag{6}$$

Thus, the Pareto Optimal restoring market price increases linearly with the expected level of aggregate pollution conditional on firms' abatement effort,  $E\{\bar{e} \mid \mathbf{z}\}$ . This linear function shifts upwards and becomes steeper as society's risk aversion,  $\delta$ , increases. In other words, our model predicts that an emission permits market whose price does not react to changes in the aggregate level of pollution cannot restore the Pareto Optimal allocation. From the previous results, we obtain the following corollary.

Corollary 4 In an economy where firms can trade emission permits in a competitive market, the following is verified: (i)  $z_i$  increases with p, i.e., each firm increases his abatement effort when the price of the permits increases; (ii)  $q_i$  decreases with p, i.e., the demand function of permits is decreasing; (iii)  $z_i$  increases with  $\lambda_i$ , i.e., the larger  $\lambda_i$ , the larger the effort assigned to reduce pollution; (iv) the demand function of permits expands as  $\lambda_i$  decreases; (v) if all firms were equally treated by the regulator, i.e.,  $q_i^* = q_j^*$  and  $\theta_i = \theta_j$ , for every i, j = 1, ..., I,  $i \neq j$ , then those firms with higher values of  $\lambda_i$  would sell permits to those firms with lower values of  $\lambda_i$ .

#### 4.2 Restrictive permit policies

The more restrictive case we might think of is the case where no permits are available at all, i.e.  $q_i^* = 0$  for every  $i \in \chi$ . This case clearly makes the market for permits useless. A little bit less extreme case is that in which, for any possible re-assignment of permits performed at the market, all firms have probability one of over-polluting for any possible internal allocation of resources. In this subsection we deal with this latter case.

#### 4.2.1 Autarchy

For the autarchy case, the statement above is equivalent to saying that, for any  $i \in \chi$ ,  $q_i^*$  is such that firm i has probability one of over-polluting for any possible internal resource allocation. In turn, this is equivalent to:

$$\Pr(e_i > q_i^* \mid z_i) = 1 \qquad \forall z_i, \qquad \forall i \in \chi \tag{7}$$

Note that (7) holds trivially for  $q_i^* = 0$  under (2). The next result is a straightforward corollary of Proposition 2.

**Corollary 5** Under (7), the equilibrium of firm i in an autarchy setting, given an environmental policy  $(q_i^*, \theta_i) \ \forall i \in \chi$ , is characterized by:

$$\frac{f'(1-\gamma(z_i))\gamma'(z_i)}{\gamma'(z_i)+\lambda_i} = \rho\theta_i\left(1+\rho\theta_i\left(\mu-t_i\right)\right) \tag{8}$$

where  $t_i = \lambda_i z_i + q_i^* - (1 - \gamma(z_i)).$ 

In order to study the environmental policy that induces each firm to choose the Pareto Optimal abatement effort, we compare directly (4) and (8) to obtain the following result:

**Proposition 4** In an autarchy setting where the regulator assigns a number of permits  $q_i^*$  to each firm i satisfying (7), the environmental policy  $(q_i^*, \theta_i)$  induces the Pareto Optimal allocation if and only if:

$$\delta (1 + \delta E \{ \bar{e} \mid \mathbf{z} \}) = \rho \theta_i (1 + \rho \theta_i (\mu - t_i)) \qquad \forall i \in \chi$$
(9)

where  $t_i = \lambda_i z_i + q_i^* - (1 - \gamma(z_i))$ .

This condition means that the marginal disutility for the society of an increment in pollution (left hand side) must be equal to the marginal disutility for an individual firm of an increment in pollution (right hand side). From this expression, we can extract the results stated in the following corollary:

Corollary 6 In an autarchy setting where the regulator assigns a number of permits  $q_i^*$  to each firm satisfying (7): (i) the efficiency-inducing environmental policy must satisfy  $\theta_i > \frac{\delta}{\rho}$  for every  $i \in \chi$ ; (ii) the larger  $\lambda_i$ , the larger the efficiency-inducing  $\theta_i$ .

Part (i) of the previous corollary shows that the penalty parameter of the efficiency induccing environmental policy must outweight the difference in risk aversions between the society and the firms. Recall from Corollary 1 that at any Pareto Optimal allocation the society demands a larger abatement effort from the environmentally more efficient firms. Part (ii) of the latter corollary states that the ER induces those firms to exert such a larger effort by imposing larger penalty terms on them.

#### 4.2.2 The emission permits market

Recall that we are considering a situation where, for any possible re-assignment of permits performed at the market, all firms have probability one to over-pollute for any possible internal allocation of resources. Let  $Q^*$  be the total assignment of permits by the regulator. The following condition ensures that the previous statement is true:

$$\Pr(e_i > Q^* \mid z_i) = 1 \qquad \forall z_i, \qquad \forall i \in \chi$$
 (10)

In fact what we requiere that  $Q^*$  is sufficiently low to induce that the demand of permits of each firm is such that each of them over pollutes. In order to be able to compare the Pareto Optimal environmental policies in each situation, we need to obtain closed-form expressions of the demand of permits. We shall use the following specific functional forms:

$$f(m_i) = \alpha \ln(1 + m_i) \qquad \gamma(z_i) = z_i \tag{11}$$

where  $\alpha > 0$  is some exogenous parameter. The following proposition summarizes the main results.

**Proposition 5** In an economy with an emission permits market, assume that (10) and (11) hold. Then, (i) the equilibrium of firm i is defined by:

$$z_{i} = 2 - \frac{\alpha}{(1+\lambda_{i}) p}$$

$$q_{i} = \mu + \frac{1}{\rho \theta_{i}} - 1 - 2\lambda_{i} - \frac{1}{(\rho \theta_{i})^{2}} p + \frac{\alpha}{p};$$

(ii) there exists a unique equilibrium price, say  $p^e$ ; (iii) there exists a unique efficiency-inducing market price, say  $p^o$ ; (iv) a sufficient condition for  $p^o = p^e$  to hold is:

$$\delta\left(\frac{1}{\rho}\sum_{i\in\chi}\frac{1}{\theta_i}-Q^*\right) = \frac{\delta^2}{\rho^2}\sum_{i\in\chi}\frac{1}{\theta_i^2} = 1$$
(12)

(v) condition (12) implies  $\theta_i > \frac{\delta}{\rho}$  for every i; (vi) there exist efficiency-inducing vectors  $\Theta = (\theta_1, ..., \theta_I)$  others than those satisfying (12), in particular: (a) there exists  $\Theta$  satisfying  $\theta_i < \frac{\delta}{\rho}$  for at most all  $i \in \chi \setminus \{j\}$ , where j is arbitrary in  $\chi$ ; (b) there exists  $\Theta$  satisfying  $\theta_i = \theta_j$  for every  $i, j \in \chi$ .

Part (i) of the previous proposition illustrates that the previous general results on the firms' behavior hold within this particular case. Parts (ii) and (iii) show technical -and desirable- properties of our model.

Part (iv) characterizes efficiency-restoring vectors  $\Theta$  that are particularly interesting for two reasons. First, note that all we need to know about  $\Theta$  to verify if it satisfies (12) is  $\sum_{i \in \chi} \frac{1}{\theta_i}$  and  $\sum_{i \in \chi} \frac{1}{\theta_i^2}$ . These two expressions are the inverse of the harmonic mean of the entries of  $\Theta$  and of the squared entries, respectively<sup>10</sup>. Secondly, (12) does not contain  $\alpha$  neither the vector of  $\lambda$ 's. This means that our set of assumptions (10) and (11) provide a theoretical framework in which we can find socially desirable environmental policies independent of the -eventually difficult to measure- technological parameters of the economy.

Parts (v) and (vi) of the previous proposition help us to compare the market with the the autarchy scenario. More concretely, part (v) states that the policies satisfying (12) are of the same type of those in the autarchy setting (see part (i) of Corollary 6). However, part (vi) shows that, under a restrictive initial assignment of permits, the market widens the room to achieve efficiency along two dimensions: First, under the policy labelled (a) all firms but one are better off with the market (see again part (i) of Corollary 6). That is, we are able to obtain efficiency at the market scenario with lower penalty levels for almost all firms. Secondly, the policy labeled (b) is informationally very cheap for the ER, since she does not need to know who is who among the polluting firms in order to set up an efficiency-inducing policy<sup>11</sup>, which contrasts with the autarchy setting (see part (ii) of Corollary 6), where each penalty term is one-to-one linked to the  $\lambda$ 's.

<sup>&</sup>lt;sup>10</sup> To see the implication, consider a vector  $\Theta$  satisfying (12) with two entries, say  $\theta_i$  and  $\theta_j$ , satisfying  $\theta_i < \theta_j$ , that is, firm i is relatively well-treated by the ER. If we want to favour further firm i by changing from  $(\theta_i, \theta_j)$  to  $(\theta_i - \nabla \theta_i, \theta_j + \nabla \theta_j)$  with  $\nabla \theta_i > 0$  and  $\nabla \theta_j > 0$ , still within (12), it must be the case that  $\nabla \theta_i > \nabla \theta_j$ . Contrarily, if we want to favour firm j by changing to  $(\theta_i + \nabla \theta_i, \theta_j - \nabla \theta_j)$  without reverting the relative treatment, that is,  $\theta_j - \nabla \theta_j > \theta_i$ , it must be the case that  $\nabla \theta_i < \nabla \theta_j$ .

<sup>&</sup>lt;sup>11</sup>Although she needs some information on the  $\lambda$ 's, she might not have a mapping from the set of  $\lambda$ 's into the set of firms.

#### 4.3 Permissive permit policies

Before we are more concrete with the meaning of permissive, let us introduce an additional assumption. In the remainder of this section, we assume that there are only two types of firms, say 0 and 1. Formally, there are two real positive numbers,  $\lambda_0$  and  $\lambda_1$ , with  $\lambda_0 < \lambda_1$ , such that

$$\lambda_i \in \{\lambda_0, \lambda_1\} \qquad \forall i \in \chi \tag{13}$$

Thus,  $\lambda_0$  and  $\lambda_1$  define the inefficient and efficient types, respectively. Since any two firms of the same type are identical, in what follows the subscript refers to type, whereas we keep no track of any individual firm hereafter. Let  $I_0$  and  $I_1$  be the number of firms of each corresponding type, so that  $I_0 + I_1 = I$ . In addition, we maintain the functional forms specified in (11). Of course, the Pareto Optimal allocation in the two-type economy characterized by (11) and (13) resembles the general properties of the Pareto Optimal allocation presented for the more general case. The next result summarizes this point.

**Proposition 6** Consider the two-type economy characterized by (11) and (13) and let  $\lambda_0 < \lambda_1$ . (i) A necessary and sufficient condition for the Pareto Optimal allocation to be interior is  $\lambda_1 < 1 + 2\lambda_0$ ; (ii) the subset of the parameter space for which there exist an interior Pareto Optimal allocation has non-empty interior. Moreover, at any interior solution: (iii)  $z_0^o < z_1^o$ ; (iv) both  $z_0^o$  and  $z_1^o$  increase when I (keeping  $\frac{I_0}{I}$  constant),  $\delta$ ,  $\mu$ ,  $\frac{I_0}{I}$  (keeping I constant),  $\lambda_1$  increase and/or  $\alpha$  decreases; (v)  $z_0^o$  increases as  $\lambda_0$  increases and  $z_1^o$  increases as  $\lambda_0$  decreases.

Condition (i) has a straightforward interpretation. In order to avoid a specialization where the inefficient firms optimally exert no environmental effort at all, the relative inefficency of those firms should not be too large. Assertion (ii) is rather technical, whereas (iii) just replicates Corollary 1. Most of the comparative statics results in parts (iv) and (v) are fairly intuitive. Perhaps the most striking fact is that an increase in the abatement efficiency of the efficient firms increases the Pareto Optimal environmental effort that the inefficient firms must exert. This result amounts to the idea that it is not socially optimal that the difference in the abatement efforts exerted by the two types of firms,  $z_1^o - z_0^o$ , be too large, when firms are identical from the profitable activity point of view. An increase in  $\lambda_1$  (keeping  $\lambda_0$  constant) increases  $z_1^o$ , which increases the difference in the abatement efforts, and the only way to reduce it is to increase  $z_0^o$  as well. Conversely, the previous result states that  $z_1^o$  decreases with  $\lambda_0$ : the difference in the abatement efforts becomes smaller as the difference in efficiency,  $\lambda_1 - \lambda_0$ , is reduced.

Next we turn our attention to the problem of characterizing efficiency-inducing environmental policies in which the amount of permits assigned by the regulator is relatively large. Specifically, we consider a case where each firm expects to pay no fine if he keeps exactly the permits assigned by the ER and undertakes the socially optimal effort. Formally, for type i firms, this means that:<sup>12</sup>

$$q_i^* = E\left\{e_i \mid z_i^o\right\} \tag{14}$$

Fixing (14) and the two-type economy just presented, an environmental policy either at the autarchy or at the market case is defined by a pair  $(\theta_0, \theta_1)$ .

<sup>&</sup>lt;sup>12</sup>Using this rule, we get close to the European Union system that bases the assignment of permits on historic emissions.

#### 4.3.1 Autarchy

The main findings concerning the autarchy setting are presented in the next proposition.

**Proposition 7** Consider the two-type economy characterized by (11) and (13) and an autarchy setting defined by (14). (i) The efficiency-inducing environmental policy satisfies  $\theta_0 = \theta_1 = \theta$ ; (ii) the efficiency-inducing environmental policy  $\theta$  increases with  $\Pr(\varepsilon - \mu > 0)$ ,  $z_0^o$  and  $z_1^o$ . Assume, in addition, that  $\varepsilon$  is uniformly distributed with support  $[\varepsilon_{\inf}, \varepsilon_{\sup}]$  and let  $\sigma = \varepsilon_{\inf} - \varepsilon_{\sup}$ . Then: (iii) there exists an efficiency-inducing environmental policy  $\theta$  if and only if  $\sigma > 0$ , and it is defined by

$$\rho\theta = \frac{2}{\sigma} \left( \sqrt{1 + \frac{2\alpha\sigma}{\tilde{z}}} - 1 \right) \tag{15}$$

where  $\tilde{z} = (2 - z_i^o)(1 + \lambda_i)$ , for every  $i \in \{0, 1\}$ .

The more remarkable statement of the previous result is part (i): the environmental policy that restores Pareto Optimality is not type specific. This is a widely observed fact in real life policy making: different polluters are assigned a different quantity of emission permits, typically based on historic emissions, but the unitary pollution penalty is common to all polluters. This simple two-type economy model rationalizes this fact and constitutes a theoretical scenario where, conditioned on a discrimination of permits assignment, it is Pareto Optimal not to discriminate on the penalty parameter. Part (ii) is straightforward.

In part (iii) of the latter proposition we have added a distributional assumption on  $\varepsilon$  that helps us to gain some intuitition. It emphasizes an important characteristic of our model: for an efficiency-inducing environmental policy to exist, the probability distribution function of  $\varepsilon$  must be non-degenerated, i.e.:  $\sigma > 0$ . Contrarily, if it was degenerated, the abatement effort would be observable and the moral hazard problem would dissapear. Finally, notice that the definition of  $\tilde{z}$  conveys that it is socially optimal that the inefficient type firms pollute more and consequently receive a larger amount of permits at the autarchy case. In addition, since  $E\{\bar{e} \mid \mathbf{z}^o\}$  increases (linearly) with  $\tilde{z}$ , part (iii) shows that, as expected, the efficiency-inducing penalty parameter decreases with  $E\{\bar{e} \mid \mathbf{z}^o\}$ .

#### 4.3.2 The emission permits market

Clearly, the counterpart of (14) when a market for permits exists within our two-type economy is

$$Q^* = I_0 E \{ e_0 \mid z_0^o \} + I_1 E \{ e_1 \mid z_1^o \}$$
(16)

The following result shows the basic properties of the efficiency-inducing environmental policy at the market scenario. On the analogy of the autarchy setting, we restrict ourselves to policies where  $\theta_0 = \theta_1$ . Moreover, we focus on the analysis of strictly positive prices of the permits.

**Proposition 8** Consider the two-type economy characterized by (11), (13) and a market setting in which the supply of permits is given by (16). Let us restrict to the case  $\theta_0 = \theta_1$ . Then: (i) the individual demand function of permits is continuous and strictly decreasing for any p > 0. Moreover, for any such price we have that  $\Pr(e_i > q_i \mid z_i) > 0$ ; (ii) the value of  $\theta$  that restores efficiency coincides with the one at the autarchy case, presented in proposition 7.

Part (i) of the previous proposition is again an extension of the more general analysis carried out in Corollary 4. Part (ii) shows that, when the ER assigns a large number of permits to be distributed in the market, then the environmental policy that she chooses in order to implement the Pareto Optimal allocation is the same that the one she would have chosen at the autarchy scenario. This implies that, in this setting, the market is not capable of improving the agents. This contrasts with the result we obtained at the restrictive permits policy case, where there was a chance for the market to improve most of the firms. From these results we are able to extract the following insight: when an ER who faces a moral hazard problem increases the amount of permits to be distributed among the agents, the introduction of a market for trading permits reduces the capability of the ER to set a more favorable environmental policy to induce efficiency.

## 5 Conclusion

We have proposed a moral hazard model to analyze the implications of introducing a market for permits when an environmental regulator designs the environmental policy in order to achieve the Pareto optimal abatement effort on firms.

We have studied the efficiency-inducing environmental policy at an autarchy situation (with no possibility of trading permits) and at the market scenario. To this purpose, we have distinguished between a situation where the regulator assigns a *low* amount of permits (restrictive policies) and a situation where the amount of permits assigned is *larger*, in particular, the assignment of permits is based on historical emissions (permissive policies).

When the policy is restrictive, we observe two important implications: first, at the autarchy scenario, the environmental policy set up by the regulator to obtain efficiency must discriminate among agents, that is, those firms with a higher efficiency on the abatement technology face larger penalty terms in case of overpolluting. However, when a market for permits is allowed, the market may help the regulator to save informational costs and there exists efficiency-inducing environmental policies where the penalty term is the same for all agents. Secondly, the introduction of a market for permits is able, in some cases, to improve the welfare of the agents, that is, there exists a reason to defend the existence of a market for permits since we can obtain the same efficiency that at the autarchy case with low penalty levels.

When we study *permissive* policies, the result is completely the opposite: there is no room for the market to improve the welfare of the agents. In this case, the efficiency-inducing environmental policy is the same at the autarchy and at the market situation. All the discrimination among firms is given by the assignment of permits made by the regulator at the environmental policy and the market does not add any additional help.

Finally, it is worth mentioning that this model opens different lines of research within the economics of information literature. In this paper, we have dealt with a moral hazard problem. We believe that studying the impact that a market for permits can have when there exist adverse selection problems will provide interesting insights that did not arise in our setup.

## 6 Appendix

**Proof. Lemma 1.** We use a second order Taylor's expansion for the exponential function:  $\exp \{\delta x\} \simeq 1 + \delta x + \frac{1}{2}\delta^2 x^2$ . The objective function of the problem that characterizes the Pareto Optimal allocation of the economy is

 $J := \sum_{i \in \chi} f(m_i) - \delta E \left\{ \sum_{i \in \chi} e_i \right\} - \frac{1}{2} \delta^2 E \left\{ \left( \sum_{i \in \chi} e_i \right)^2 \right\}$ 

Use (1), the i.i.d. assumption of the shocks and the definition  $z_i := g(a_i)$  to write (ommitting the summation sign index on i):

 $E\left\{\left(\sum e_i\right)^2\right\} = \left(\sum m_i\right)^2 + \left(\sum \lambda_i z_i\right)^2 + E\left\{\left(\sum \varepsilon_i\right)^2\right\} + 2\left(\sum m_i\right)I\mu - 2\left(\sum m_i\right)\left(\sum \lambda_i z_i\right) - 2\left(\sum \lambda_i z_i\right)I\mu$  where  $\mu = E\left\{\varepsilon_i\right\}$ . In addition, it is:  $E\left\{\sum e_i\right\} = \sum m_i + I\mu - \sum \lambda_i z_i$ . Assume that  $m_i + a_i < 1$  holds for some i, so that we can increase  $a_i$ , or equivalently  $z_i$ , keeping  $m_i$  constant. By doing so, we have:  $\frac{\partial J}{\partial z_i} = \delta \lambda_i \left(1 + \delta E\left\{\sum e_i\right\}\right) > 0$ , where the inequality follows from (2).

**Proof. Proposition 1.** Take  $m_i = 1 - \gamma(z_i)$  and  $g(a_i) = z_i$  in J, differentiate J with respect to  $z_i$  and set the derivative equal to zero to obtain:  $-f'\left(1 - \gamma\left(z_i^o\right)\right)\gamma'\left(z_i^o\right) + \delta\left(1 + \delta E\left\{\bar{e} \mid \mathbf{z}^o\right\}\right)\left(\gamma'\left(z_i^o\right) + \lambda_i\right) = 0$ , where  $E\left\{\bar{e} \mid \mathbf{z}^o\right\} = I - \sum_{i \in \chi} \gamma\left(z_i^o\right) - I\mu - \sum_{i \in \chi} \lambda_i z_i^o$ . The expression in the proposition is straight forward from the previous expression.

**Proof.** Corollary 1. It follows trivially from the fact that the right hand side of (4) is constant across i's whereas its left hand side increases monotonically with  $z_i^o$  and decreases monotonically with  $\lambda_i$ .

**Proof.** Lemma 2. Assume that for some pair  $(m_i, a_i)$  satisfying  $m_i + a_i < 1$ , it is the case that  $\Pr(e_i - q_i^* > 0) = 0$  for firm i. The firm increases his expected profit by increasing both  $m_i$  and  $a_i$  in such a way that  $\Pr(e_i - q_i^* > 0)$  remains zero and  $f(m_i)$  increases. Alternatively, assume that for some pair  $(m_i, a_i)$  satisfying  $m_i + a_i < 1$ , it is the case that  $\Pr(e_i - q_i^* > 0) > 0$  for firm i. Then, the firm increases his expected profit increasing a, which reduces the latter probability.

**Proof. Proposition 2.** Using  $z_i$  as the unique decision variable and taking the second order Taylor expansion for the exponential function, the first order condition of firm i's problem is

$$-f'\left(1-\gamma\left(z_{i}\right)\right)\gamma'\left(z_{i}\right)-\rho\theta_{i}\frac{\partial}{\partial z_{i}}h_{1}-\frac{1}{2}\rho^{2}\theta_{i}^{2}\frac{\partial}{\partial z_{i}}h_{2}=0$$
 where  $h_{r}=E\left\{\max\left\{0,1-\gamma\left(z_{i}\right)+\varepsilon_{i}-\left(\lambda_{i}z_{i}+q_{i}^{*}\right)\right\}^{r}\right\}$ . It is 
$$h_{r}=\int_{t_{i}}^{\infty}\left(1-\gamma\left(z_{i}\right)+\varepsilon_{i}-\left(\lambda_{i}z_{i}+q_{i}^{*}\right)\right)^{r}\phi\left(\varepsilon\right)d\varepsilon.$$
 Using Leibniz's rule, we have  $\frac{\partial}{\partial z_{i}}h_{1}=-\left(\gamma'\left(z_{i}\right)+\lambda_{i}\right)\int_{t_{i}}^{\infty}\phi\left(\varepsilon\right)d\varepsilon$ , and 
$$\frac{\partial}{\partial z_{i}}h_{2}=-2\int_{t_{i}}^{\infty}\left(1-\gamma\left(z_{i}\right)+\varepsilon_{i}-\left(\lambda_{i}z_{i}+q_{i}^{*}\right)\right)\left(\gamma'\left(z_{i}\right)+\lambda_{i}\right)\phi\left(\varepsilon\right)d\varepsilon=$$
 
$$=-2\left(1-\gamma\left(z_{i}\right)-\left(\lambda_{i}z_{i}+q_{i}^{*}\right)\right)\left(\gamma'\left(z_{i}\right)+\lambda_{i}\right)\int_{t_{i}}^{\infty}\phi\left(\varepsilon\right)d\varepsilon-2\left(\gamma'\left(z_{i}\right)+\lambda\right)\int_{t_{i}}^{\infty}\varepsilon\phi\left(\varepsilon\right)d\varepsilon=$$
 
$$=-2\left(\gamma'\left(z_{i}\right)+\lambda\right)\int_{t_{i}}^{\infty}\left(\varepsilon-t_{i}\right)\phi\left(\varepsilon\right)d\varepsilon$$

Substituting in the previous first order condition and re-arranging terms, we obtain the expression of the proposition. ■

**Proof.** Corollary 2. (i) Differentiating the first order condition obtained in Proposition 2 with respect to  $z_i$  and  $\theta_i$ , and collecting terms we obtain:

$$\left[\frac{\lambda_{i}f'\left(1-\gamma\left(z_{i}\right)\right)\gamma''\left(z_{i}\right)}{\left(\gamma'\left(z_{i}\right)+\lambda_{i}\right)^{2}}-\frac{f''\left(1-\gamma\left(z_{i}\right)\right)\left(\gamma'\left(z_{i}\right)\right)^{2}}{\gamma'\left(z_{i}\right)+\lambda_{i}}+\rho\theta_{i}\left(\gamma'\left(z_{i}\right)+\lambda_{i}\right)\left(\phi(t_{i})+\rho\theta_{i}\int_{t_{i}}^{\infty}\phi(\varepsilon)d\varepsilon\right)\right]dz_{i}=0$$

$$= \left[ \rho \int_{t_i}^{\infty} \phi(\varepsilon) d\varepsilon + 2\rho^2 \theta_i \int_{t_i}^{\infty} (\varepsilon - t_i) \phi(\varepsilon) d\varepsilon \right] d\theta_i$$

Since the terms multiplying  $dz_i$  and  $d\theta_i$  are both positive (provided f'' < 0,  $\gamma'' \ge 0$ ,  $\gamma' > 0$  and f' > 0), then we must have  $\frac{dz_i}{d\theta_i} > 0$ , and, therefore, we can conclude that the abatement effort of firm i increases with  $\theta_i$  (since g and  $\gamma$  are both increasing functions). Differentiating the first order condition in Proposition 2 with respect to  $z_i$  and  $q_i^*$  and collecting terms, we obtain:

$$\left[\frac{\lambda_{i}f'\left(1-\gamma\left(z_{i}\right)\right)\gamma''\left(z_{i}\right)}{\left(\gamma'\left(z_{i}\right)+\lambda_{i}\right)^{2}}-\frac{f''\left(1-\gamma\left(z_{i}\right)\right)\left(\gamma'\left(z_{i}\right)\right)^{2}}{\gamma'\left(z_{i}\right)+\lambda_{i}}+\rho\theta_{i}\left(\gamma'\left(z_{i}\right)+\lambda_{i}\right)\left(\phi(t_{i})+\rho\theta_{i}\int_{t_{i}}^{\infty}\phi(\varepsilon)d\varepsilon\right)\right]dz_{i} = \\
=-\rho\theta_{i}\left(\phi(t_{i})+\rho\theta_{i}\int_{t_{i}}^{\infty}\phi(\varepsilon)d\varepsilon\right)dq_{i}^{*}$$

Since the terms multiplying  $dz_i$  and  $dq_i^*$  have different signs, then,  $\frac{dz_i}{dq_i^*} < 0$ . (ii) From Proposition 1 and 2, given an efficiency-inducing policy  $(q_i^*, \theta_i) \forall i \in \chi$ , we know that firm i exerts an abatement effort at equilibrium such that:

$$\delta(1 + \delta(E\{\bar{e} \mid \mathbf{z}^o\})) = \rho\theta_i \left( \int_{t_i}^{\infty} \phi(\varepsilon) d\varepsilon + \rho\theta_i \int_{t_i}^{\infty} (\varepsilon - t_i) \phi(\varepsilon) d\varepsilon \right)$$
(17)

Take  $\Pr(t_i < \varepsilon_i) = 0$ , i.e.  $\Pr(\varepsilon_i < t_i) = 1$ . In this case, the right hand side of (17) becomes zero, while its left hand side is strictly positive, so we must have that  $\Pr(t_i < \varepsilon_i) > 0$ , even when each firm exerts the Parero Optimal equilibrium effort. (iii) Any efficiency-inducing environmental policy  $(q_i^*, \theta_i) \forall i \in \chi$  satisfies (17) with  $t_i = \lambda_i z_i^o + q_i^* - (1 - \gamma(z_i^o))$ . Then, given  $q_i^*$  there exists a unique value of  $\theta_i$ ,  $\Upsilon(q_i^*)$ , that satisfies the first equality. Differentiability (and thus continuity) of  $\Upsilon$  follows trivially. Differentiating (17) with respect to  $q_i^*$  and  $\theta_i$  and collecting terms leads to:

$$0 = \rho \left( \int_{t_i}^{\infty} \phi\left(\varepsilon\right) d\varepsilon + 2\rho \theta_i \int_{t_i}^{\infty} \left(\varepsilon - t_i\right) \phi\left(\varepsilon\right) d\varepsilon \right) d\theta_i + \rho \theta_i \left( -\phi\left(t_i\right) - \rho \theta_i \int_{t_i}^{\infty} \phi\left(\varepsilon\right) d\varepsilon \right) dq_i^*$$

Since the terms multiplying  $dq_i^*$  and  $d\theta_i$  in the above expression have different signs, then  $\Upsilon$  is increasing.

**Proof. Proposition 3.** The first order condition with respect to  $z_i$  mimics the corresponding one in proposition 2 just replacing  $q_i^*$  by  $q_i$  and  $t_i$  by  $u_i$ . Similarly, the first order condition with respect to  $q_i$ , can be written as

$$-p - \rho \theta_i \frac{\partial}{\partial q_i} \hat{h}_1 - \frac{1}{2} \rho^2 \theta_i^2 \frac{\partial}{\partial q_i} \hat{h}_2 = 0$$

where we have denoted  $\hat{h}_r = E\{\max\{0, 1 - \gamma(z_i) + \varepsilon_i - (\lambda_i z_i + q_i)\}^r\}$ . It is

$$\widehat{h}_{r} = \int_{u_{i}}^{\infty} (1 - \gamma(z_{i}) + \varepsilon_{i} - (\lambda_{i} z_{i} + q_{i}))^{r} \phi(\varepsilon) d\varepsilon$$

Using Leibniz's rule, we have  $\frac{\partial}{\partial q_i} \hat{h}_1 = -\int_{u_i}^{\infty} \phi(\varepsilon) d\varepsilon$ , and

$$\frac{\partial}{\partial q_{i}}\widehat{h}_{2} = -2\left(\left(1 - \gamma\left(z_{i}\right) - \left(\lambda_{i}z_{i} + q_{i}\right)\right)\int_{u_{i}}^{\infty}\phi\left(\varepsilon\right)d\varepsilon + \int_{u_{i}}^{\infty}\varepsilon\phi\left(\varepsilon\right)d\varepsilon\right) = -2\int_{u_{i}}^{\infty}\left(\varepsilon - u_{i}\right)\phi\left(\varepsilon\right)d\varepsilon$$

Substituting in the first order condition and re-arranging terms, we obtain the expression in the proposition. ■

**Proof. Corollary 3.** Necessity (*only if* part) follows trivially from the comparison of (4) to the first equality in (5) and the fact that the left side of (4) is monotone in  $z_i^o$ . To prove sufficiency (*if* part), consider an allocation  $\mathbf{z} = (z_1, ..., z_I)$  and a market price p such that (5) and (6) are met. Then  $\mathbf{z}$  satisfies (4).

**Proof. Corollary 4.** (i) Differentiating the first equality in (5) with respect to  $z_i$  and p and collecting terms we obtain:  $\left(-f''\left(1-\gamma\left(z_i\right)\right)\gamma'\left(z_i\right)^2+\left(f'\left(1-\gamma\left(z_i\right)\right)-p\right)\gamma''\left(z_i\right)\right)dz_i=\left(\gamma'\left(z_i\right)+\lambda_i\right)dp$ . Use (5) to observe that  $f'\left(1-\gamma\left(z_i\right)\right)-p=\frac{\lambda p}{\gamma'(z_i)}$ , then, we have that:  $\left(-f''\left(1-\gamma\left(z_i\right)\right)\gamma'\left(z_i\right)^2+\frac{\lambda p}{\gamma'(z_i)}\gamma''\left(z_i\right)\right)dz_i=\left(\gamma'\left(z_i\right)+\lambda_i\right)dp$ . The expressions multiplying  $dz_i$  and dp are both positive provided f''<0 and  $\gamma''\geq0$ . Therefore  $\frac{dz_i}{dp}>0$  holds. (ii) Consider equation (5). When p increases,  $z_i$  also increases, and, hence,  $u_i$  also increases. Moreover, as  $u_i$  increases, the integrals in (5) with  $u_i$  as their lower bound cannot increase. In summary, if  $q_i$  remains constant as p increases, the rightest hand side of (5) decreases. On the other hand, the rightest hand side of (5) decreases with  $q_i$ . Therefore, p and  $q_i$  must move in the opposite direction to satisfy (5). (iii) It follows trivially from the fact that the leftest term in (5) decreases monotonically with  $\lambda_i$  whereas it increases monotonically with  $z_i$ . (iv) From (iii) we have that, given p, an increase of  $\lambda_i$  conveys an increase in  $z_i$  as well, and thus an increase in  $u_i$  in the rightest term of (5). Consequently, for the second equality of (5) to hold, we must decrease  $q_i$ . (v) It follows trivially from (iv).

**Proof. Corollary 5.** Condition (7) is equivalent to  $t_i < \varepsilon_{\inf}$ . Thus, under (7) the integrals in the left hand side of proposition 2 become:  $\int_{t_i}^{\infty} \phi(\varepsilon) d\varepsilon = 1$  and  $\int_{t_i}^{\infty} \varepsilon \phi(\varepsilon) d\varepsilon = \mu$ . The result follows trivially.

**Proof. Proposition 4.** It follows trivially from the direct comparison of (4) to (8) and monotonicity in  $z_i^o$  of the left hand side of (4).

**Proof. Corollary 6.** (i) For any arbitrary  $\mathbf{z} = (z_1, ..., z_I)$ , it is  $E\{\bar{e} \mid \mathbf{z}\} > E\{e_i \mid z_i\} = m_i + \mu - \lambda_i z_i > m_i + \mu - \lambda_i z_i - q_i^* = \mu - t_i$  for all  $i \in \chi$ , where we have used (2) for the first inequality. Therefore, for (9) to hold, it must be the case that  $\delta < \rho \theta_i$  holds for all  $i \in \chi$ . (ii) The larger  $\lambda_i$ , the larger the distance between  $E\{\bar{e} \mid \mathbf{z}\}$  and  $\mu - t_i$ , and therefore, the larger the value of  $\theta_i$ .

**Proof. Proposition 5.** (i) Condition (10) implies  $u_i < \varepsilon_{\inf}$ , where  $u_i$  is defined in proposition 3. Thus, under (10) the integrals in the rightest term of (5) become:  $\int_{u_i}^{\infty} \phi(\varepsilon) d\varepsilon = 1$  and  $\int_{u_i}^{\infty} \varepsilon \phi(\varepsilon) d\varepsilon = \mu$ . Consequently, under (10) and (11), (5) becomes:

$$\frac{\alpha}{(2-z_i)(1+\lambda_i)} = p = \rho \theta_i \left(1 + \rho \theta_i \left(\mu - u_i\right)\right)$$

The latter two equalities together with the definition of  $u_i$  constitute a system of three equations on three unknowns:  $u_i$ ,  $q_i$  and  $z_i$ . The unique solution is:  $u_i = \mu + \frac{1}{\rho\theta_i} \left(1 - \frac{p}{\rho\theta_i}\right)$ ;  $q_i = \mu + \frac{1}{\rho\theta_i} - 1 - 2\lambda_i - \frac{1}{(\rho\theta_i)^2}p + \frac{\alpha}{p}$  and  $z_i = 2 - \frac{\alpha}{(1+\lambda_i)p}$ , respectively. (ii) Since  $u_i < \varepsilon_{\inf}$  holds under (10), the latter solution in  $q_i$  constitutes the individual demand function of permits. The equilibrium price equals aggregate demand to total supply. Then, the equilibrium price is a solution in p to

$$I(\mu - 1) + \frac{1}{\rho} \sum_{i \in \chi} \frac{1}{\theta_i} - 2 \sum_{i \in \chi} \lambda_i - \frac{1}{\rho^2} p \sum_{i \in \chi} \frac{1}{\theta_i^2} + I \frac{\alpha}{p} = Q^*$$

Multiplying by p and collecting terms, we obtain

$$I\alpha + \left(I(\mu - 1) + \frac{1}{\rho} \sum_{i \in \chi} \frac{1}{\theta_i} - 2\sum_{i \in \chi} \lambda_i - Q^*\right) p - \left(\frac{1}{\rho^2} \sum_{i \in \chi} \frac{1}{\theta_i^2}\right) p^2 = 0.$$
 (18)

The left hand side of this equation is a quadratic concave function which is strictly positive at p = 0. Thus, there exists a unique positive value of p which solves it. (iii)  $p^o$  is characterized in corollary 3. Using (11) and the solution to  $z_i$  given in part (i), after some algebra, the equation in corollary 3 becomes

$$-\delta^2 I\alpha - \delta (1 + \delta M) p + p^2 = 0, \tag{19}$$

where  $M=I\left(\mu-1\right)-2\sum_{i\in\chi}\lambda_i$ . The left hand side of the previous equality is a quadratic convex function strictly negative at p=0. This implies that there exists a unique positive value of p which solves it. (iv) By direct comparison of (18) and (19), we obtain trivially that (12) is a sufficient condition. (v) From the second equality in (12), it follows trivially that it must be  $\frac{\delta^2}{\rho^2\theta_i^2} < 1$  for all  $i \in \chi$ , or, equivalently, the statement in the proposition. (vi) Both (18) and (19) have the same positive solution in p if and only if:

$$\delta (1 + \delta M) + \sqrt{\delta^2 (1 + \delta M)^2 + 4\delta^2 I \alpha} = \delta \frac{k_0 + \delta M}{k_1} + \sqrt{\delta^2 \left(\frac{k_0 + \delta M}{k_1}\right)^2 + 4\frac{1}{k_1} \delta^2 I \alpha},$$
 (20)

where  $k_0 = \delta\left(\frac{1}{\rho}\sum_{i\in\chi}\frac{1}{\theta_i} - Q^*\right)$  and  $k_1 = \frac{\delta^2}{\rho^2}\sum_{i\in\chi}\frac{1}{(\theta_i)^2}$ . Equation (20) allows for:  $\frac{k_0+\delta M}{k_1} > 1 + \delta M$  together with  $\frac{1}{k_1} < 1$ . Assume first that M > 0 holds. Then the previous inequalities together imply that  $k_0 > k_1 > 1$  must hold, or, equivalently,  $\sum_{i\in\chi}\frac{\delta}{\rho\theta_i} - \delta Q^* > \sum_{i\in\chi}\left(\frac{\delta}{\rho\theta_i}\right)^2 > 1$ . This latter chain of inequalities can hold if  $\frac{\delta}{\rho\theta_i} > 1$  holds for all  $i \in \chi \setminus \{j\}$ , being j arbitrary in  $\chi$ , which is part (a) of (vi). If M < 0 holds, we can still have  $k_0 > k_1$ , and the subsequent argument follows. To prove part (b) of (vi) it suffices to let  $r = \theta_i$  for all  $i \in \chi$  and to show that (20) has a positive solution in r. The left hand side of (20) is strictly positive (regardless the sign of M). The right hand side of (20) is a continous function of r that tends to zero as r tends to  $\infty$  whereas it tends to  $\infty$  as r tends to zero.

**Proof. Proposition 6.** (i), (ii) and (iii) It is convenient to introduce the following notation:  $\overline{m} = \sum_{i \in Y} (1 - \gamma(z_i^o))$  and  $\overline{s} = \sum_{i \in Y} \lambda_i z_i^o$ . Thus  $E\{\bar{e} \mid \mathbf{z}^o\} = \overline{m} + I\mu - \overline{s}$ , and (4) becomes:

$$\frac{f'\left(1-\gamma\left(z_{i}^{o}\right)\right)\gamma'\left(z_{i}^{o}\right)}{\gamma'\left(z_{i}^{o}\right)+\lambda_{i}}=\delta(1+\delta\left(\overline{m}+I\mu-\overline{s}\right))\tag{21}$$

for all  $i \in \{0, 1\}$ , where we must recall that the subscript i refers to type and not to a specific firm. Using (11) and the fact that the left hand side of (21) must be equal across types, we have:

$$z_0^o = 2 - (2 - z_1^o) \frac{1 + \lambda_1}{1 + \lambda_0} \tag{22}$$

Note that (22) together with  $\lambda_0 < \lambda_1$  implies part (iii) of the proposition. Using (22) we can write

$$\bar{m} = I_0 \left( \frac{1+\lambda_1}{1+\lambda_0} - 1 \right) + \left( I_0 \frac{1+\lambda_1}{1+\lambda_0} + I_1 \right) (1-z_1^o)$$

and

$$\bar{s} = 2I_0\lambda_0 \left(1 - \frac{1+\lambda_1}{1+\lambda_0}\right) + \left(I_0\lambda_0 \frac{1+\lambda_1}{1+\lambda_0} + I_1\lambda_1\right)z_1^o$$

After some algebra, these two previous expressions lead to

$$\bar{m} - \bar{s} = I + 2I_0(\lambda_1 - \lambda_0) - I(1 + \lambda_1)z_1^o$$

Using this expression, equation (21) for type 1 firms can be writen as follows:

$$\frac{\alpha}{2 - z_1^o} \frac{1}{1 + \lambda_1} = \delta \left( 1 + \delta I \left( 1 + 2 \frac{I_0}{I} (\lambda_1 - \lambda_0) + \mu \right) \right) - \delta^2 I (1 + \lambda_1) z_1^o$$
 (23)

The left hand side of (23) increases with  $z_1$  whereas the right hand side decreases with  $z_1$ . Thus, if there exists a solution in  $z_1$  to (23), it is unique. In addition, such solution exists and lies in the interval (0, 1) if and only if: (a) the left hand side of (23) is smaller than the right hand side at  $z_1 = 0$ , and thus the solution is larger than zero, and (b) the left hand side of (23) is larger than the right hand side at  $z_1 = 1$ , and thus the solution is smaller than 1. Condition (b) is:

$$\frac{\alpha}{1+\lambda_1} > \delta\left(1+\delta I\left(1+2\frac{I_0}{I}(\lambda_1-\lambda_0)+\mu\right)\right) - \delta^2 I(1+\lambda_1)$$

or, equivalently:

$$\frac{\alpha}{1+\lambda_1} > \delta \left( 1 + \delta I \left( 2\frac{I_0}{I} (\lambda_1 - \lambda_0) + \mu - \lambda_1 \right) \right) \tag{24}$$

Notice that, since  $z_0 < z_1$  holds, (24) ensures both  $z_1 < 1$  and  $z_0 < 1$ . Note that condition (b) ensures  $z_1 > 0$  but not  $z_0 > 0$ . Thus, we need to replace (a) with a stronger condition that ensures  $z_0 > 0$ . Using (22), we can write  $z_0 > 0$  as  $z_1 > \hat{z}$ , where  $\hat{z} = 2\left(1 - \frac{1+\lambda_0}{1+\lambda_1}\right)$ . Since we have that  $\hat{z} > 0$ ,  $z_1 > \hat{z}$  is stronger than condition (a). Therefore, replacing 0 with  $\hat{z}$  in condition (a) leads to the following condition: (a') the left hand side of (23) is smaller than the right hand side at  $\hat{z}$ , and thus the solution is larger than  $\hat{z}$ . Condition (a') ensures that  $z_0 > 0$  and consequently  $z_1 > 0$  holds. We can write (a') as

$$\frac{\alpha}{2-\hat{z}}\frac{1}{1+\lambda_1}<\delta\left(1+\delta I\left(1+2\frac{I_0}{I}(\lambda_1-\lambda_0)+\mu\right)\right)-\delta^2 I(1+\lambda_1)\hat{z}.$$

Using  $(2 - \hat{z})(1 + \lambda_1) = 2(1 + \lambda_0)$  within the left hand side of the latter inequality and using  $(1 + \lambda_1)\hat{z} = 2(\lambda_1 - \lambda_0)$  within the right hand side, we can write it as follows:

$$\frac{\alpha}{2(1+\lambda_0)} < \delta \left( 1 + \delta I \left( 1 - 2\frac{I_1}{I} (\lambda_1 - \lambda_0) + \mu \right) \right) \tag{25}$$

Conditions (24) and (25) characterize the subset of parameter values for which the Pareto Optimal interior solutions exist and are unique. Let us denote that subset by  $\Omega$ . Next, we analyze the non-emptyness of  $\Omega$ . First, since the interior Pareto Optimal solution  $z_1$ , defined by the solution to (23) in  $z_1$ , must lie in the interval  $(\hat{z}, 1)$ , it must be the case that  $\hat{z} < 1$ . The latter inequality is equivalent to

$$\lambda_1 < 1 + 2\lambda_0 \tag{26}$$

This condition is thus necessary for  $\Omega$  to be non-empty. Now we show that it is also sufficient. The previous assumptions ensure that the right hand side of (24) is strictly positive. Thus, there exists a unique value of  $\alpha$ , say  $\alpha^*$ , such that (24) holds with equality. Write (25) as follows, replacing  $\alpha$  by  $\alpha^*$ :

$$\frac{1}{2} \frac{1+\lambda_1}{1+\lambda_0} \delta(1+\delta IT_1) < \delta(1+\delta IT_2).$$

where we have denoted  $T_1 = 2\frac{I_0}{I}(\lambda_1 - \lambda_0) + \mu - \lambda_1$  and  $T_2 = 1 - 2\frac{I_1}{I}(\lambda_1 - \lambda_0) + \mu$ . Condition (26) ensures that  $\frac{1}{2}\frac{1+\lambda_1}{1+\lambda_0} < 1$ . Therefore, for the previous inequality to hold, it suffices that  $\delta(1+\delta IT_1) < \delta(1+\delta IT_2)$  or, equivalently,  $T_1 < T_2$ , but this latter inequality is equivalent to (26). The non-emptyness of  $\Omega$  follows now trivially using a continuity argument on  $\alpha$ . Furthermore, if, for every  $\alpha$  satisfying both (24) and (25), we use a continuity argument on the rest of the parameters, we obtain that  $\Omega$  has a non-zero measure along any dimension in  $(\alpha, I_0, I_1, \lambda_0, \lambda_1, \delta, \mu)$ , i.e.  $\Omega$  has a non-empty interior. (iv) and (v) Write (23) as

$$\frac{\alpha}{2 - z_1^o} \frac{1}{1 + \lambda_1} = k + \delta^2 I(1 + \lambda_1)(2 - z_1^o)$$

where:

$$k = \delta \left( 1 + \delta I \left( 1 + 2 \frac{I_0}{I} (\lambda_1 - \lambda_0) + \mu \right) \right) + 2\delta^2 I (1 + \lambda_1)$$

Thus, defining  $\tilde{z} = (1 + \lambda_1)(2 - z_1^o)$  and taking into account the definition of k, equation (23) can be written as  $\delta^2 I \tilde{z}^2 + k \tilde{z} - \alpha = 0$ , which is a quadratic equation in  $\tilde{z}$ . More specifically, the left hand side of this equation is convex and stricly negative at  $\tilde{z} = 0$ , thus it has exactly one positive root. Once  $\tilde{z}$  is obtained, we have that:  $z_1^o = 2 - \frac{1}{1+\lambda_1}\tilde{z}$  and  $z_0^o = 2 - \frac{1}{1+\lambda_0}\tilde{z}$ , where we have used (22) for the latter equality. The signs of the comparative statics analysis are straightforward: when a parameter changes, first look at the changes in  $\tilde{z}$  and then at the changes in  $z_i^o$ .

**Proof. Proposition 7.** (i) Consider the expressions in proposition 2 taking  $z_i = z_i^o$ . It is  $t_i = q_i^* + (1 + \lambda_i)z_i^o - 1 = \mu$ , where we have used (14). Using in addition that  $z_i^o = 2 - \frac{1}{1 + \lambda_i}\tilde{z}$  holds for every  $i \in \{0, 1\}$ , where  $\tilde{z}$  is defined in the proof of proposition 6, the expression in proposition 2 becomes

$$\frac{\alpha}{\tilde{z}} = \rho \theta_i \left( \int_{\mu}^{\infty} \phi(\varepsilon) d\varepsilon + \rho \theta_i \int_{\mu}^{\infty} (\varepsilon - \mu) \phi(\varepsilon) d\varepsilon \right)$$
(27)

Clearly, the solution in  $\theta_i$  to this latter equation is type-independent. (ii) It follows trivially just noting that  $\Pr(\varepsilon - \mu > 0) = \int_{\mu}^{\infty} \phi(\varepsilon) d\varepsilon$  and that both  $z_0^o$  and  $z_1^o$  decrease with  $\tilde{z}$ . (iii) Notice that  $\sigma > 0$  is necessary and sufficient for the integrals on the right hand side of the above equation to be strictly positive, which in turn is a necessary and sufficient condition for the above equation to have a (positive) solution in  $\theta$ . Using the uniform distribution, we have:  $\int_{\mu}^{\infty} \phi(\varepsilon) d\varepsilon = \frac{1}{2}$  and  $\int_{\mu}^{\infty} (\varepsilon - \mu) \phi(\varepsilon) d\varepsilon = \frac{\sigma}{8}$ , which substituted in the above equation leads to the expression in the proposition.

**Proof. Proposition 8.** (i) Using (11), equation (5) becomes

$$\frac{\alpha}{(2-z_i)(1+\lambda_i)} = p = \pi(u_i) \tag{28}$$

where  $\pi(u_i) = \rho\theta \left( \int_{u_i}^{\infty} \phi\left(\varepsilon\right) d\varepsilon + \rho\theta \int_{u_i}^{\infty} \left(\varepsilon - u_i\right) \phi\left(\varepsilon\right) d\varepsilon \right)$  and  $u_i$  is defined in proposition 3. Under (11), we have  $u_i = q_i - 1 + (1 + \lambda_i)z_i$ . It is  $\pi(u_i) \geq 0$  for all  $u_i$ , and the strict inequality holds iff  $u_i < \varepsilon_{\sup}$ . Thus, for any p > 0, from the second equality in (28),  $u_i < \varepsilon_{\sup}$  must hold. Also,  $\pi'(u_i) = -\rho\theta \left(\phi\left(u_i\right) + \rho\theta \int_{u_i}^{\infty} \phi\left(\varepsilon\right) d\varepsilon\right)$ , thus it is  $\pi'(u_i) \leq 0$  for all  $u_i$ , and the strict inequality holds iff  $u_i < \varepsilon_{\sup}$ . Thus, under the latter inequality, the inverse of  $\pi$ , say  $\pi^{-1}$ , is well defined, and the second equality in (28) is  $u_i = \pi^{-1}(p)$ . Using this latter equality together with the first equality in (28) and the definition of  $u_i$ , we have:

$$q_i = \pi^{-1}(p) - (1 + 2\lambda_i) + \frac{\alpha}{p}$$

The latter expression is the individual demand of permits for any p > 0, and it is continuous and strictly decreasing in p. Notice that  $u_i < \varepsilon_{\sup}$  is equivalent to  $\Pr(e_i > q_i \mid z_i) > 0$ . (ii) Consider the first equality in (28) with  $z_i = z_i^o$  and use the definition of  $\tilde{z}$  given in proposition 7. The unique price that restores efficiency is  $p = \alpha/\tilde{z}$ . Of course, for it to be an equilibrium price, the market must clear at that price. The aggregate demand at that price is

$$D = I\pi^{-1} \left(\frac{\alpha}{\tilde{z}}\right) - \left(I + 2\left(I_0\lambda_0 + I_1\lambda_1\right)\right) + I\tilde{z}$$

whereas the agregate suply (at any price) is given by (16). Note that

$$E\{e_i \mid z_i^o\} = 1 + \mu - (1 + \lambda_i)z_i^o = \mu + \tilde{z} - (1 + 2\lambda_i)$$

where, for the latter equality, we have used the definition of  $\tilde{z}$ . Using the latter expression into (16) to obtain the aggregate supply, the market clearing condition, after cancelling out common terms on both sides, becomes  $\pi^{-1}(\alpha/\tilde{z}) = \mu$  or, equivalently:

$$\frac{\alpha}{\tilde{z}} = \pi(\mu)$$

but the latter equation is exactly (27).

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