Cost-Loss Decision Models with Risk Aversion

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Resumen

Es generalmente aceptado que determinados fenómenos metereológicos extremos acontecidos durante las pasadas décadas están conectados con el cambio climático. Algunos de estos hechos, como sequías o heladas, afectan a la agricultura haciendo obligada la gestión del riesgo. El objetivo de este trabajo es analizar las decisiones de los agricultores respecto a la gestión del riesgo, tomando en cuenta la información climática y metereológica. El trabajo considera una situación en la que el agricultor, en su gestión de la producción agraria, tiene tecnología disponible para proteger la cosecha de los efectos del tiempo meteorológico. Este enfoque ha sido usado por Murphy et al. (1985), Katz and Murphy (1990) y otros en el caso de que los agricultores maximicen los beneficios esperados. No obstante, en esta ocasión, el modelo introduce un análisis de la actitud hacia al riesgo, de modo que se pueda evaluar cómo la decisión óptima es afectada por el coeficiente de aversión absoluta al riesgo de Arrow-Pratt; y se pueda al tiempo computar el valor económico de la información en semejante contexto.

Abstract

Extreme meteorological events have increased over the last decades and it is widely accepted that it is due to climate change. Some of the extremes, like drought or frost episodes largely affect agricultural outputs and risk management becomes crucial. The goal of this work it is to analyze farmers' decisions about risk management, taking into account climatological and meteorological information.

We consider a situation in which the farmer, as part of crop management, has available a technology to protect the harvest from weather effects. This approach has been used by Murphy et al. (1985), Katz and Murphy (1990) and others in the case that the farmer maximizes the expected returns. Nevertheless, in our model we introduce the attitude towards risk, so we can evaluate how the optimal decision is affected by the absolute risk aversion coefficient of Arrow-Pratt, and compute the economic value of the information in this context.

Key words: information value, cost-loss ratio, decision models, risk aversion.
Classification JEL : C6 and Q1.

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1. Introduction

Meteorological information affects agricultural production since it is able to change producers' decisions. Many farmers use weather forecasts to manage their activities, taking into account some meteorological variables to make better decisions when choosing the planting and harvesting time, the application of pesticides, and so on (McNew and Mapp, 1990). Cost-Loss analytical models constitute a theoretical approach to decisions under risk which are affected by weather. (See Clemen, 1996; Keeney, 1982; Winkler and Murphy, 1985; Winkler et al., 1983).

The uncertainty comes from some meteorological variable which depending on the specific case produces uncertain consequences. The meteorological forecasts help in the decision making alte-ring the conditional probability associated to these events and the economic value of these forecasts can be considered as the difference between the expected value when an imperfect forecast is available and when just basic information exists there. The basic information most commonly accepted is the climatological information. That is, to assume that the agent knows the historical relative frequencies for the meteorological events that affect his activity.

What is commonly known as the "Cost-Loss Ratio Situation" model is a particularization of prototype decision models, in which an agent must decide between two actions: (i) to protect the harvest from an adverse meteorological situation, with a cost *C*, or (II) not to protect it and expose himself to a loss *L* if the adverse event takes place $(0 < C < L < \infty)$. In the traditional cost-loss model, in its static version (Thompson, 1952, 1962; Thompson and Brier, 1955; Murphy, 1977), and also dynamic (Murphy et al., 1985; Katz and Murphy, 1990), an essential condition is assumed: the agent presents neutrality to the risk, which means that he minimizes the expected expense.

However, agents are sensitive to risk, at least where important decisions are concerned, and not taking into consideration this attitude, could lead to wrong conclusions in some cases (Wilks, 1997). Accordingly, in Section 2 we propose a model in which the risk attitude has been conside-red, and that allows us to evaluate how the optimal decision depends on the absolute risk aversion. It is assumed that individual preferences are represented by a CARA (Constant Absolute Risk Aversion) utility function. In Section 3 additional meteorological information is introduced and the economic value of this information is obtained for the risk averse agent. Section 4 concludes.

2. Climatological information

2.1. THE MODEL

In this section the role of risk aversion is analyzed by considering that the farmer simply decides between protecting and not protecting his harvest. We formulate the model in a general form.

Let K be the value of portion of loss that the farmer is able to avoid by protecting the harvest, with a cost γK, where $0 < \gamma < 1$. We assume that the avoided loss is proportional to the total value of the harvest L, so $K = \alpha L$. Thus, $\alpha = 1$ (and $K = L$), if the farmer protects all the harvest, whereas $\alpha = 0$ (and $K = 0$), if he protects nothing.

Hence, while the protect versus not to protect decision is analyzed, the chosen frame is the cost-loss traditional model, including two possible states (adverse weather $\theta = 1$ or non adverse weather $\theta = 0$) and two possible actions for the farmer (protect, $\alpha = 1$, or do not protect, $\alpha = 0$). The cost of protection is γL and the total loss if there is no harvest is *L.* The payoff matrix is in Table 1.

Table 1
Payoff Matrix

	STATE OF NATURE	
	Adverse weather	Non adverse weather
ACTION	(θ = 1)	(θ = 0)
Protect (α = 1)	$-\gamma L$	$-\gamma L$
Not protect (α = 0)	$-L$	0

In order to evaluate the risk influence over farmers decisions, and therefore to obtain the information value, we are going to analyze the decision considering the risk aversion, which provides one of the central analytical techniques in economic analysis ((Mas-Collel et al. (1995)). We assume that individual preferences can be represented by the expected utility with the utility function $U(\cdot)$, the CARA function (Constant Absolute Risk Aversion), being:

$$U(x) = -\exp\{-\rho x\}, \qquad (1)$$

where: x is monetary gains and $\rho > 0$ is the Arrow-Pratt coefficient of absolute risk aversion, which is constant for this function. This is suitable for the farmer´s decision problem since the risk aversion does not depend on the harvest value. The optimal decision in this case is obtained maximizing the expected utility, which increases with the decrease of the expected expense. (That is the reason for writing the payoffs as negative monetary costs).

2.2. SOLUTION AND THEORETICAL RESULTS

Climatological information consists of a single probability of adverse weather

$$P_\theta = \Pr\{\theta = 1\},$$

usually deriving from historical weather records. From a Bayesian perspective, the parameter P_θ can be viewed as the “prior probability” of adverse weather.

As we study the case of a risk averse farmer whose utility function is given by (1), all the elements relevant for the decision problem, including the payoff values of the farmer in accordance with the utility function, are collected in Table 2.

Table 2

	P_θ	$1 - P_\theta$
	STATE OF NATURE	
	Adverse weather	Non adverse weather

ACTION	(θ = 1)	(θ = 0)
Protect (α = 1)	$-\exp\{\rho\gamma L\}$	$-\exp\{\rho\gamma L\}$
Not protect (α = 0)	$-\exp\{\rho L\}$	-1

The optimal action to be chosen by the farmer in order to maximize the expected utility is given in the following proposition.

Proposition 1 *For the decision problem with risk, defined in Table 2, the optimal decision of the farmer considering the maximization of the expected utility criterion is*

- *Protect (α = 1), if* $A < P_\theta$. *In this case the expected utility is* $EU(1) = -\exp\{\rho\gamma L\}$.
- *Do not protect (α=0), if* $A > P_\theta$. *In this case the expected utility is* $EU(0) = -P_\theta \exp\{\rho L\} + P_\theta - 1$.
- *Indifference between both actions if* $A = P_\theta$,

where $A = \dfrac{1-\exp\{\rho\gamma L\}}{1-\exp\{\rho L\}}$.

Proof If protective action is taken (α = 1), then the expected utility of the farmer is:

$$EU(1) = P_\theta U[-\gamma L] + (1-P_\theta)U[-\gamma L] = U[-\gamma L] = -\exp\{\rho\gamma L\}.$$

If protective action is not taken (α = 0), then the expected utility of the farmer is:

$$EU(0) = P_\theta U[-L] + (1-P_\theta)U[0] = -P_\theta \exp\{\rho L\} + (1-P_\theta)\left[-\exp\{0\}\right] =$$
$$= -P_\theta \exp\{\rho L\} + P_\theta - 1.$$

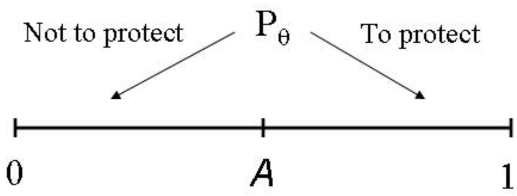
Therefore, to take protective action is strictly better if

$$EU(1) > EU(0) \Leftrightarrow -\exp\{\rho\gamma L\} > -P_\theta \exp\{\rho L\} + P_\theta - 1 \Leftrightarrow \frac{1-\exp\{\rho\gamma L\}}{1-\exp\{\rho L\}} < P_\theta,$$

and in that case, the expected utility of the farmer is $EU(1) = -\exp\{\rho\gamma L\}$.
Similarly, not to protect (α = 0) is strictly better when $EU(1) < EU(0)$, and there is indifference between the two actions when $EU(1) = EU(0)$. ■

$A = \dfrac{1-\exp\{\rho\gamma L\}}{1-\exp\{\rho L\}}$ being the probability threshold from which a farmer with constant absolute risk aversion and climatological information will protect the harvest from adverse weather, the optimal decision policy appears in Figure 1.

Figure 1
Optimal policy with climatological information and risk aversion



In the case of a risk neutral agent, it is optimal to protect the harvest whenever the cost per unit of loss to be avoided by protection is below the probability of suffering this loss (Murphy et. al., 1985), that is if $\gamma < P_\theta$. In Proposition 2 it is proved that $\gamma > \dfrac{1-\exp\{\rho\gamma L\}}{1-\exp\{\rho L\}}$ and therefore, it could happen that P_θ were in an interval, between A and γ. In that case, a risk adverse agent (with constant absolute risk aversion) will prefer to protect the harvest, although it would not be optimal for him to protect it in the event that he were risk neutral, thus minimizing the expected cost. Consequently, the risk adverse individual is more cautious and would take protection action in situations in which he would not take it if he were risk neutral.

Proposition 2 *The probability threshold values A and γ from which a farmer with constant risk absolute aversion or risk neutrality, respectively, and climatological information will protect his harvest from adverse weather verify that A < γ,*

where $A = \dfrac{1-\exp\{\rho\gamma L\}}{1-\exp\{\rho L\}}$.

Proof As

$$A = \frac{1-\exp\{\rho\gamma L\}}{1-\exp\{\rho L\}} = \frac{-\sum_{i=1}^{\infty}\frac{(\rho\gamma L)^i}{i!}}{-\sum_{i=1}^{\infty}\frac{(\rho L)^i}{i!}} = \frac{\gamma\sum_{i=1}^{\infty}\frac{\rho^i L^i \gamma^{i-1}}{i!}}{\sum_{i=1}^{\infty}\frac{(\rho L)^i}{i!}},$$ we have that

$$A < \gamma \Leftrightarrow \frac{\sum_{i=1}^{\infty}\frac{\rho^i L^i\gamma^{i-1}}{i!}}{\sum_{i=1}^{\infty}\frac{(\rho L)^i}{i!}} < 1 \Leftrightarrow \sum_{i=1}^{\infty}\frac{\rho^i L^i\gamma^{i-1}}{i!} < \sum_{i=1}^{\infty}\frac{(\rho L)^i}{i!} \Leftrightarrow \sum_{i=2}^{\infty}\frac{\rho^i L^i\gamma^{i-1}}{i!} < \sum_{i=2}^{\infty}\frac{(\rho L)^i}{i!},$$ which is satisfied, because as

$\gamma < 1$, we have that $\dfrac{\rho^i L^i \gamma^{i-1}}{i!} < \dfrac{\rho^i L^i}{i!}, \forall i = 2,3,4....$.

So, as always $\gamma < 1$, it is satisfied that $\gamma > \dfrac{1-\exp\{\rho\gamma L\}}{1-\exp\{\rho L\}}$, as we wanted to prove. ■

In Proposition 3 we prove three interesting properties of the probability threshold *A* from which a farmer with constant risk absolute aversion will protect the harvest from adverse weather.

Proposition 3 $A = \dfrac{1-\exp\{\rho\gamma L\}}{1-\exp\{\rho L\}}$ *satisfies the following properties:*

(i) $\frac{\partial A}{\partial \gamma} > 0,$

(ii) $\frac{\partial A}{\partial \rho} \le 0,$

(iii) $\lim_{\rho \to 0} A = \gamma.$

Proof (i) $\frac{\partial A}{\partial \gamma} = \frac{-\exp\{\rho\gamma L\}\rho L}{1-\exp\{\rho L\}} > 0.$

$$\text{(ii)}\ \frac{\partial A}{\partial \rho} \le 0 \Leftrightarrow \frac{\partial A}{\partial \rho} = \frac{-\left[\exp\{\rho\gamma L\}\gamma L\right]\left[1-\exp\{\rho L\}\right]+\left[1-\exp\{\rho\gamma L\}\right]\exp\{\rho L\}L}{\left[1-\exp\{\rho L\}\right]^2} \le 0 \Leftrightarrow$$

$$\Leftrightarrow -\left[\exp\{\rho\gamma L\}\gamma L\right]\left[1-\exp\{\rho L\}\right]+\left[1-\exp\{\rho\gamma L\}\right]\exp\{\rho L\}L \le 0,$$

because $\left[1-\exp\{\rho L\}\right]^2 > 0$,

so $\frac{\partial A}{\partial \rho} \le 0 \Leftrightarrow \left[1-\exp\{\rho\gamma L\}\right]\exp\{\rho L\} \le \exp\{\rho\gamma L\}\left[1-\exp\{\rho L\}\right]\gamma \Leftrightarrow$

$$\Leftrightarrow \frac{\left[1-\exp\{\rho L\}\right]\exp\{\rho\gamma L\}\gamma}{\left[1-\exp\{\rho\gamma L\}\right]\exp\{\rho L\}} \le 1.$$

Denoting $M = \frac{\left[1-\exp\{\rho L\}\right]\exp\{\rho\gamma L\}\gamma}{\left[1-\exp\{\rho\gamma L\}\right]\exp\{\rho L\}}$, we have that $\frac{\partial A}{\partial \rho} \le 0 \Leftrightarrow M \le 1$.

We can see that $M \le 1 \quad \forall \gamma \in (0,1)$, because:

- M is an increasing function of $\gamma \in (0,1)$:

  In fact:

  $M = \frac{\exp\{\rho\gamma L\}\gamma}{A\exp\{\rho L\}}, \text{ where } A = \frac{1-\exp\{\rho\gamma L\}}{1-\exp\{\rho L\}}$

  So, *A* depends on γ.

  We know that *A*>0, and that $\frac{\partial A}{\partial \gamma} = \frac{-\exp\{\rho\gamma L\}\rho L}{1-\exp\{\rho L\}} > 0$.

  So we have:

$$\frac{\partial M}{\partial \gamma} = \frac{\left[\exp\{\rho\gamma L\}\rho\gamma L + \exp\{\rho\gamma L\}\right]A\exp\{\rho L\} - \left[\frac{\partial A}{\partial \gamma}\exp\{\rho L\}\right]\exp\{\rho\gamma L\}\gamma}{\left[A\exp\{\rho L\}\right]^2} =$$

$$= \frac{\left[\exp\{\rho\gamma L\}\rho\gamma L + \exp\{\rho\gamma L\}\right] - \frac{1}{A}\frac{\partial A}{\partial \gamma}\exp\{\rho\gamma L\}\gamma}{A\exp\{\rho L\}} =$$

$$= \frac{\exp\{\rho\gamma L\}}{A\exp\{\rho L\}}\left[1+\rho\gamma L - \frac{\gamma}{A}\frac{\partial A}{\partial \gamma}\right].$$

  But as $\rho > 0,\ 0<\gamma<1 \text{ and } L > 0$ we have that

  $\exp\{\rho\gamma L\} > 0,\ \ A > 0 \text{ and } \exp\{\rho L\} > 0.$

We will see that $\left[1+\rho\gamma L-\frac{\gamma}{A}\frac{\partial A}{\partial\gamma}\right]$ is also positive:

$$\left[1+\rho\gamma L-\frac{\gamma}{A}\frac{\partial A}{\partial\gamma}\right]=1+\rho\gamma L+\gamma\frac{\left[1-\exp\{\rho L\}\right]}{\left[1-\exp\{\rho\gamma L\}\right]}\frac{\exp\{\rho\gamma L\}\rho L}{\left[1-\exp\{\rho L\}\right]}=$$

$$=1+\rho\gamma L+\frac{\rho\gamma L\exp\{\rho\gamma L\}}{1-\exp\{\rho\gamma L\}}=\frac{1-\exp\{\rho\gamma L\}+\rho\gamma L-\rho\gamma L\exp\{\rho\gamma L\}+\rho\gamma L\exp\{\rho\gamma L\}}{1-\exp\{\rho\gamma L\}}=$$

$$=\frac{-\left[\sum_{i=1}^{\infty}\frac{(\rho\gamma L)^i}{i!}\right]+\rho\gamma L}{-\left[\sum_{i=1}^{\infty}\frac{(\rho\gamma L)^i}{i!}\right]}=\frac{\left[\sum_{i=2}^{\infty}\frac{(\rho\gamma L)^i}{i!}\right]}{\left[\sum_{i=1}^{\infty}\frac{(\rho\gamma L)^i}{i!}\right]}>0.$$

- In addition, we can see that if $\gamma=1\Rightarrow M=1$.

M being an increasing function of γ, and *M* = 1 when $\gamma=1$, we have that $M\leq 1$.

So, $M\leq 1\Rightarrow\frac{\partial A}{\partial\rho}\leq 0$, as we wanted to prove.

(iii) $\lim_{\rho\to 0}A=\lim_{\rho\to 0}\frac{1-\exp\{\rho\gamma L\}}{1-\exp\{\rho L\}}=\lim_{\rho\to 0}\frac{-\gamma L\exp\{\rho\gamma L\}}{-L\exp\{\rho L\}}=\lim_{\rho\to 0}\frac{\gamma\exp\{\rho\gamma L\}}{\exp\{\rho L\}}=\gamma.$ ■

In accordance with (i) in Proposition 3, the greater the cost to protect the harvest, the smaller the caution of the farmer. In (ii) we see that the greater the risk aversion, the smaller the probability threshold from which the agent protects the harvest. If the producer is highly adverse to the risk (ρ is very high), he will maximize his expected utility by protecting the harvest from adverse weather (making sure it will not suffer the loss if the weather is adverse) although the associated probability of that adverse situation (P_θ), is small. In (iii) we see that the behaviour of a farmer whose risk aversion tends to zero is similar to that of a risk neutral agent.

Example 1 In Table 3 it can be seen how propositions 2 and 3 apply for $L=1$ and for parameters γ and ρ taking different values. The entries of the matrix correspond to the values of the thres-hold value *A.*

If for example $P_\theta=0.45$ and $\gamma=0{,}5$, the risk neutral farmer does not protect. The risk averse farmer (with CARA function) does not protect if $\rho=0{,}01$ or 0.1 but protects if $\rho=0.5,\quad 0.8,\quad 1\text{ or }5$, according to the values given in Table 3.

Table 3

	$\rho=0.01$	$\rho=0.1$	$\rho=0.5$	$\rho=0.8$	$\rho=1$	$\rho=5$
$\gamma=0.1$	0.099	0.099	0.079	0.068	0.061	0.004
$\gamma=0.3$	0.299	0.290	0.249	0.221	0.204	0.024
$\gamma=0.5$	0.499	0.488	0.438	0.401	0.378	0.076

$\gamma = 0.7$	0.699	0.689	0.646	0.613	0.590	0.218
$\gamma = 0.8$	0.799	0.792	0.758	0.731	0.713	0.364

3. Imperfect Information

As in Murphy et. al. (1985), we consider the incorporation of additional information to the model. It is introduced as an imperfect weather forecasting from a meteorological office. The goal is to obtain the optimal decision rule in this context and also to quantify the economic value of such a forecasting system, considering the information value as the benefits of changing the farmer´s behavior when he has this additional information available.

Let the random variable Z which indicates a forecast of adverse weather $(Z=1)$, or of non adverse weather $(Z=0)$ be introduced. The conditional probabilities of adverse weather are denoted by $P_1 = \Pr\{\theta = 1/Z=1\}$ and $P_0 = \Pr\{\theta = 1/Z=0\}$. In addition, as in Murphy et al. (1985) it is assumed that $\Pr\{Z=1\} = \Pr\{\theta=1\} = \mathrm{P}_\theta$, that is, the forecasting system produces adverse weather signals with the same probability that adverse weather events take place. Without loss of genera-lity, $0 \le P_0 \le P_\theta \le P_1 \le 1$, is also assumed. In these conditions it is easily obtained that $P_0 = \dfrac{(1-P_1)P_\theta}{(1-P_\theta)}$. *

For the case of a risk averse farmer whose utility function is given by (1), all the elements relevant for the decision problem with imperfect information, in the context we have just de-fined are collected in Table 4.

The quality of information is defined in terms of the following index:

$$q = Corr(\theta, Z) = \frac{(P_1 - P_\theta)}{(1-P_\theta)}.$$

The value of information is defined as

V= Value of information = *EU* (with forecas-ting) – *EU* (without forecasting),

where *EU is* the value of the expected utility corresponding to the optimal decision in both cases (with and without forecasting). Specifically, *EU*(without forecasting) is the correspond-ing value obtained in Proposition 1.

EU (with forecasting) = *EU* (Z = 1) Pr {Z = 1} + *EU* (Z = 0) Pr {Z = 0}

it is the ex-ante expected utility with forecasting.

It is interesting to obtain the value of information as a function of the quality of information *q.*

Table 4
Payoff Matrix with imperfect information

	If $Z=1$		If $Z=0$	
	P_1	$1-P_1$	P_0	$1-P_0$
	STATE OF NATURE		STATE OF NATURE	
ACTION	$\theta=1$	$\theta=0$	$\theta=1$	$\theta=0$
$\alpha=1$	$-\exp\{\rho\gamma L\}$	$-\exp\{\rho\gamma L\}$	$-\exp\{\rho\gamma L\}$	$-\exp\{\rho\gamma L\}$
$\alpha=0$	$-\exp\{\rho L\}$	-1	$-\exp\{\rho L\}$	-1

* $P_\theta = \Pr\{\theta = 1\} = \Pr\{\theta = 1/Z = 1\}\Pr\{Z = 1\} + \Pr\{\theta = 1/Z = 0\}\Pr\{Z = 0\} = P_1 P_\theta + P_0(1 - P_\theta) \Rightarrow P_0 = \frac{(1-P_1)P_\theta}{1-P_\theta}$.

In order to obtain the optimal decision rule of the farmer and also the value of the information we need to distinguish between two cases, as the expected utility of the optimal decision without forecasting enters in the calculation of the value of information.

3.1 CASE IN WHICH $0 < A \le P_\theta$.

As has been proved in Proposition 1, where $A \le P_\theta$, if we consider a situation without forecasting (that is only climatological information is used), the optimal decision of the farmer is to protect and then $EU(\text{without forecasting}) = -\exp\{\rho\gamma L\}$.

In the following proposition the optimal action to be chosen by the farmer in order to maximize the expected utility in the case of incomplete information, as well as the value of information for this case are obtained.

Proposition 4 *For the decision problem with risk and incomplete information, defined in Table 4, assuming that $0 < A \le P_\theta$, the optimal decision of the farmer considering the maximization of the expected utility criterion is:*

- *If $A < P_0$, to protect, whatever the signal is, and then the expected utility is $-\exp\{\rho\gamma L\}$.*
- *If If $A > P_0$, to protect if Z = 1 (the expected utility being $-\exp\{\rho\gamma L\}$) and not protect if Z = 0 (the expected utility being $-P_0\exp\{\rho L\} + P_0 - 1$).*
- *Indifference between both actions if A = P_0.*

The value of information is

$$V(q) = \begin{cases} 0, & \text{if } q \le q_A^* \\ (1-P_\theta)\left[\exp\{\rho\gamma L\} - 1\right] + P_\theta(1-P_\theta)\left[1 - \exp\{\rho L\}\right] - \\ \qquad - q(1-P_\theta)P_\theta\left[1-\exp\{\rho L\}\right], & \text{if } q > q_A^* \end{cases}$$

where $\quad q_A^* = 1 - \frac{A}{P_\theta}$.

Proof Since the variable Z has two possible values, we have the following possibilities:
If $Z = 1$, as $P_1 \ge P_\theta$, then $A \le P_1$, so the optimal decision is to protect and the expected utility is equal to: $-\exp\{\rho\gamma L\}$.
If $Z = 0$, as $P_0 \le P_\theta$, there are two more possibilities:

(i) $A < P_0$. In that case, the optimal decision is to protect and the expected utility is also equal to $-\exp\{\rho\gamma L\}$.

(ii) $A > P_0$, where not to protect is optimal, with an expected utility of: $-P_0\exp\{\rho L\} - (1 - P_0)$.

If $A = P_0$, the agent is indifferent between the two possible actions.

So, if $A < P_0$, the optimal decision is to protect if the information is $Z = 1$ and also to protect if the information is $Z = 0$, with the same expected utility that the farmer achieves without the forecasting system. So, in this case, meteorological information has no value because it does not affect the decision making.

If $A > P_0$, the optimal decision is different depending on the received information.

If $Z = 1$, the optimal decision is to protect and the expected utility is $-\exp\{\rho\gamma L\}$. If $Z = 0$, the optimal decision is not to protect and ex-ante expected utility is:

$$EU(\text{with forecasting}) = EU(Z=1)\Pr\{Z=1\} + EU(Z=0)\Pr\{Z=0\} =$$

$$= -P_\theta \exp\{\rho\gamma L\} + (1-P_\theta)\left[-P_0 \exp\{\rho L\} - (1-P_0)\right].$$

Accordingly, with $0 < A \le P_\theta$, and constant absolute risk aversion, meteorological information has positive economic value if and only if $A > P_0$. In this case, the economic value of the information, $V(q)$, is:

$$V(q) = \begin{cases} 0, & if \quad A \le P_0 \\ (1-P_\theta)\exp\{\rho\gamma L\} - (1-P_\theta)\left[1 - P_0\left[1-\exp\{\rho L\}\right]\right], & if \quad A > P_0 \end{cases}$$

Considering the information quality index $q = \dfrac{(P_1 - P_\theta)}{(1-P_\theta)}$, and the probabilities relation $P_0 = \dfrac{(1-P_1)P_\theta}{(1-P_\theta)}$, the information has economic value if and only if:

$$A > P_0 \Leftrightarrow A > \frac{(1-\left[q\left(1-P_\theta\right)+P_\theta\right])P_\theta}{(1-P_\theta)} \Leftrightarrow q > 1 - \frac{A}{P_\theta}.$$

Denoting: $q_A^* = 1 - \dfrac{A}{P_\theta}$, the economic value is positive if and only if $q > q_A^*$.

The economic value of the meteorological information can be expressed as a function of the quality index:

$$V(q) = \begin{cases} 0, & \text{if } q \le q_A^* \\ (1-P_\theta)\left[\exp\{\rho\gamma L\} - 1\right] + P_\theta(1-P_\theta)\left[1-\exp\{\rho L\}\right] - & \\ \qquad - q(1-P_\theta)P_\theta\left[1-\exp\{\rho L\}\right], & \text{if } q > q_A^* \end{cases}$$

■

There is a threshold, q_A^*, below which the forecast system does not improve the farmer´s expected utility. This threshold increases with the absolute risk aversion coefficient of Arrow-Pratt ρ. So, with a more risk averse agent the information quality needed to influence his decision making is higher.

In fact, $\dfrac{\partial q_A^*}{\partial \rho} = \dfrac{\partial q_A^*}{\partial A}\dfrac{\partial A}{\partial \rho} = -\dfrac{1}{P_\theta}\dfrac{\partial A}{\partial \rho}$, where $\dfrac{\partial A}{\partial \rho} \le 0$, as has been shown in Proposition 3, so necessarily: $\dfrac{\partial q_A^*}{\partial \rho} \ge 0$. The larger is the risk aversion, the higher is the quality threshold q_A^*. This result can appear as paradoxical, but it is comprehensible that a highly risk averse farmer will not change the decision of protecting his harvest (obtaining a certain result), unless the information quality is very high.

In the $q > q_A^*$ interval, it is satisfied that

$$V'(q) = -\left(1 - P_\theta\right) P_\theta \left[1 - \exp\left\{\rho L\right\}\right] > 0 ,$$

so we have that $V(q)$ is strictly increasing throughout that interval.

To see how the information value depends on the absolute risk aversion coefficient ρ, when $q > q_A^*$, we have:

$$\frac{\partial V(q)}{\partial \rho} = \gamma L(1-P_\theta)\exp\left\{\rho\gamma L\right\} - P_\theta(1-P_\theta)L\exp\left\{\rho L\right\} + qP_\theta(1-P_\theta)L\exp\left\{\rho L\right\} > 0 \Leftrightarrow$$

$$\Leftrightarrow \exp\left\{\rho L\left(\gamma - 1\right)\right\} > \frac{(1-q)P_\theta}{\gamma}.$$

That is, the value increases with ρ when $q > 1 - \frac{\gamma}{P_\theta}\frac{\exp\left\{\rho\gamma L\right\}}{\exp\left\{\rho L\right\}}$, and decreases below this level of quality. This is due to the fact that the threshold q_A^* increases with the risk aversion coefficient ρ, causing the information value changes to be zero when the risk aversion becomes higher nearby q_A^*. However, as we have seen, if the information quality is over that critical region (which happens if it exceeds the level $q = 1 - \frac{\gamma}{P_\theta}\frac{\exp\left\{\rho\gamma L\right\}}{\exp\left\{\rho L\right\}}$) is more valuable when the risk aversion is high.

Example 2 Let us consider the following values for the parameters: $\gamma = 0.3$, $L = 1$, $P = 0.4$ and ρ taking the values 0.1, 0.5 or 0.9.

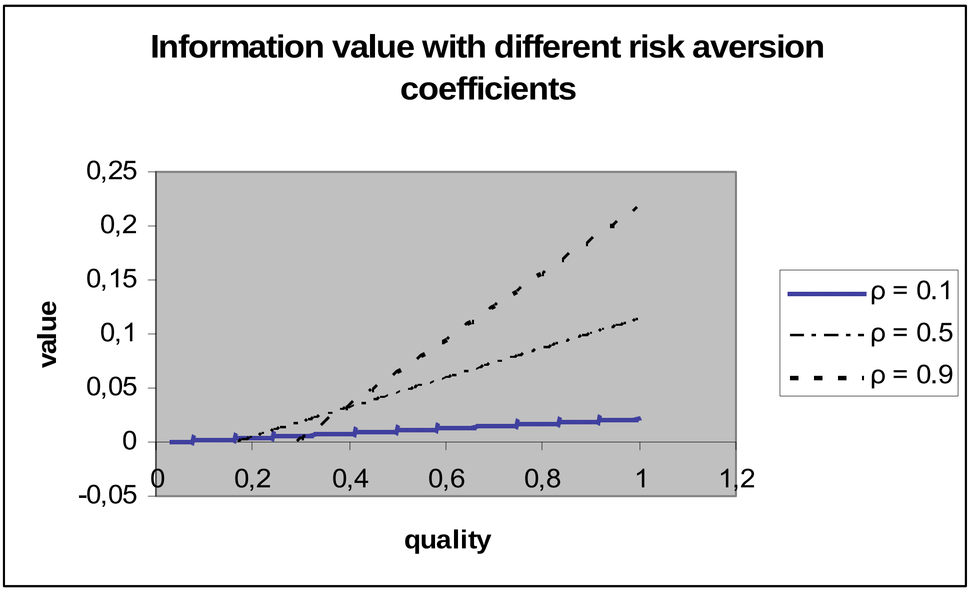
For $\rho = 0.1$ it is obtained that $A = 0.289$ and $q_A^* = 0.034$.

For $\rho = 0.5$, the corresponding values are $A = 0.249$ and $q_A^* = 0.168$.

For $\rho = 0.9$, $A = 0.212$ and $q_A^* = 0.292$ are obtained.

In Figure 2 the critical region can be observed due to changes on the quality threshold from which individual decisions with an imperfect forecast are different to those in the case of simple climatological information; and how over this region, the information value increases with the risk aversion coefficient.

Figure 2
Quality-value curve for different values of ρ, where $\gamma = 0.3$, $L = 1$, and $P_\theta = 0.4$



3.2 CASE IN WHICH $A > P_\theta$.

As has been proved in Proposition 1, when $A > P_\theta$, the optimal decision in a situation without forecasting (using just climatological information) is do not protect and then $EU\left(\text{without forecasting}\right) = -P_\theta \exp\{\rho L\} + P_\theta - 1$.

Proposition 5 *For the decision problem with risk and incomplete information, defined in Table 4, assuming that $P_\theta < A$, the optimal decision of the farmer considering the maximization of the expected utility criterion is:*

- *If $A > P_1$,* do not *protect, whatever the signal is.*
- *If $A < P_1$, to protect if Z = 1 (the expected utility being $-\exp\{\rho\gamma L\}$) and do not protect if Z = 0 (the expected utility being $-P_0 \exp\{\rho L\} + P_0 - 1$).*
- *Indifference between both actions if A = P_1.*

The value of information is

$$V(q) = \begin{cases} 0, & \text{if } q \le q_B^* \\ P_\theta\left[\exp\{\rho L\} - \exp\{\rho\gamma L\}\right] + P_\theta(1-P_\theta)\left[1-\exp\{\rho L\}\right] - & \\ \qquad - q(1-P_\theta)P_\theta\left[1-\exp\{\rho L\}\right], & \text{if } q > q_B^* \end{cases}$$

where $\quad q_B^* = \dfrac{A - P_\theta}{1 - P_\theta}$.

Proof If the signal received is $Z = 0,$ as $A > P_\theta \geq P_0,$ the optimal decision is not to protect and the corresponding expected utility is $-P_0 \exp\{\rho L\} + P_0 - 1.$

If the signal is $Z = 1,$ as $P_\theta \leq P_1,$ there are two possibilities:

(i) $A < P_1,$ in which case the optimal decision is to protect and the expected utility is $-\exp\{\rho\gamma L\}.$

(ii) $A > P_1,$ in which case the optimal decision is not to protect and the expected utility is $-P_1 \exp\{\rho L\} - (1 - P_1).$

If $A = P_1$ the agent is indifferent between the two possible actions.

Therefore, if $A > P_1,$ it is optimal not to protect, whatever the signal is, and the meteorological information has no value. If $A < P_1,$ it is optimal to protect if the signal is $Z = 1,$ (the expected utility being $-\exp\{\rho\gamma L\}$) and not to protect if Z = 0 (the expected utility being $-P_0 \exp\{\rho L\} + P_0 - 1$).

Assuming $A < P_1$, $EU(\text{with forecasting}) = EU(Z=1)\Pr\{Z=1\} + EU(Z=0)\Pr\{Z=0\} =$

$= -P_\theta \exp\{\rho\gamma L\} + (1-P_\theta)\left[-P_0 \exp\{\rho L\} - (1-P_0)\right] =$

$= -P_\theta \exp\{\rho\gamma L\} + q(1-P_\theta)P_\theta\left[\exp\{\rho L\} - 1\right] - (1-P_\theta)\left[P_\theta \exp\{\rho L\} + 1 - P_\theta\right].$

The value of the meteorological information is zero if

$$A \geq P_1 \Leftrightarrow A \geq P_\theta + q\left(1-P_\theta\right) \Leftrightarrow q\left(1-P_\theta\right) \leq A - P_\theta \Leftrightarrow q \leq \frac{A-P_\theta}{1-P_\theta}.$$

As by definition

$$V(q) = EU(\text{with forecasting}) - EU\left(\text{without forecasting}\right),$$

substituting the expressions for the expected utilities the final expression for the value of information is obtained. ■

In this case we have:

$$\frac{\partial q_B^*}{\partial \rho} = \frac{\partial q_B^*}{\partial A}\frac{\partial A}{\partial \rho} = \frac{1}{1-P_\theta}\frac{\partial A}{\partial \rho} \leq 0,$$

because $\dfrac{1}{1-P_\theta} > 0$ and $\dfrac{\partial A}{\partial \rho} \leq 0.$

Therefore, the larger is the risk aversion, the smaller is the quality threshold q_B^*. This result is reasonable, taking into account that when $A > P_\theta,$ the optimal decision with simple climatological information is not to protect. Then, the larger is the risk aversion of the farmer, the smaller are the conditions for a change to protection.

In the $q > q_B^*$ interval, it is satisfied that

$$V'(q) = -\left(1-P_\theta\right)P_\theta\left[1 - \exp\{\rho L\}\right] > 0,$$

so we have that, as in the previous case, $V(q)$ is stricly increasing throuhout that interval.

In the $q > q_B^*$ interval, we have

$$\frac{dV'(q)}{d\rho} = (1 - P_\theta)P_\theta L \exp\{\rho L\} > 0.$$

Therefore in this case, if $\rho_1 > \rho_2$, for $q > q_B^*$, the value of the information $V(q)$ corresponding to ρ_1 is always larger than the value of the information $V(q)$ corresponding to ρ_2.

Example 3 Let us consider the following values for the parameters: $\gamma = 0.3$, $L = 1$, $P = 0.2$ and ρ taking the values 0.1, 0.5 or 0.9.

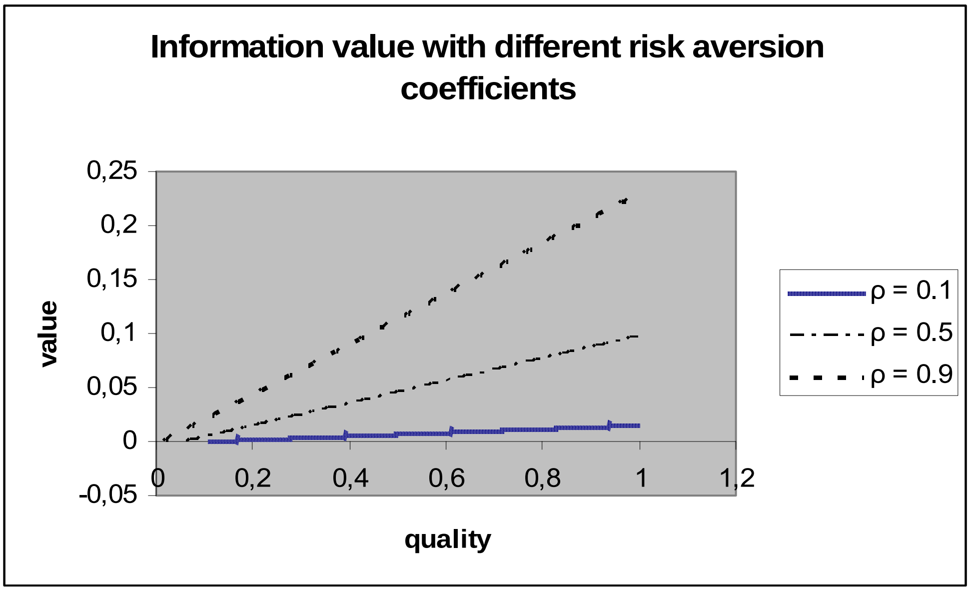
For $\rho = 0.1$ it is obtained that $A = 0.289$ and $q_B^* = 0.111$.

For $\rho = 0.5$, $A = 0.249$ and $q_B^* = 0.061$.

For $\rho = 0.9$, $A = 0.212$ and $q_B^* = 0.015$ are obtained.

In Figure 3 the quality-value curves for the different values of ρ are plotted.

Figure 3
Quality-value curve for different values of ρ, where $\gamma = 0.3$, $L = 1$, and $P_\theta = 0.2$



4. Conclusions

In this paper, the optimal decision of a farmer whose preferences are represented by a CARA utility function is obtained, in the context of cost-loss decision models under risk. The introduction of risk aversion changes the behavior of the farmer. The case of climatological information and also the case in which there is additional meteorological information are studied. A positive relation between the information value and risk aversion has been underlined, so considering neutral agents in the type of decisions analyzed underestimates the value of meteorological information.

However, the information has zero economic value below a quality threshold, which is higher in the case of risk aversion, at least in certain important cases. So, evaluating the relevance of a higher quality information system, we conclude that a forecast system whose quality is very low, does not offer an added value for the decision making with respect to the simple statistical or historical information (that is climatological information). Accordingly, an improvement in the information quality highly increases its worth in all cases, if it improves the level in which the farmers take it into account when making their decisions.

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