Bachelor in Physics
(Academic Year 2024-25)

Differential Geometry and Tensor Calculus		**Code**	800522	**Year**	3rd	**Sem.**	2nd
Module	Transversal	**Topic**	Transversal topics		**Character**	Optional	

	Total	Theory	Exercises
ECTS Credits	6	4	2
Semester hours	45	30	15

Learning Objectives (according to the Degree's Verification Document)

- Acquire skills in different transversal subjects to be able to apply them in fourth-year courses.
- Develop the ability to apply the concepts and methods of differential geometry and tensor calculus to problems of classical and quantum physics.

Brief description of contents

Differential geometry, tensor calculus and their applications to physics.

Prerequisites

Algebra, calculus of one and several variables, and differential equations.

Coordinator	Gabriel Álvarez Galindo		**Dept.**	FT
	Office	02.317.0	**e-mail**	galvarez@ucm.es

Theory/Problems – Schedule and Teaching Staff

Group	Lecture Room	Day	Time	Professor	Period/ Dates	Hours	T/E	Dept.
B	2 7	Tu We	13:30 – 15:00 18:00 – 19:30	Gabriel Álvarez Galindo	Full term	45	T/E	FT

Office hours

Group	Professor	Schedule	E-mail	Location
B	Gabriel Álvarez Galindo	Tu : 8:00-9:00 and 10:30-12:30 (+ 3 hours online)	galvarez@fis.ucm.es	02.317.0

Syllabus

1. **Theory of curves**
   The concept of a curve. Arc length. Curvature and torsion. Formulas of Frenet.
2. **Surfaces: first fundamental form and tensor calculus**
   The concept of a surface. Curves on a surface. First fundamental form. The concept of Riemannian geometry. Covariant and contravariant vectors. Foundations of tensor calculus. Special tensors.
3. **Surfaces: second fundamental form, gaussian and mean curvature**
   Second fundamental form. Principal curvatures, mean and Gaussian curvature. Formulas of Weingarten and Gauss. Properties of the Christoffel symbols. The Riemann curvature tensor. Theorema egregium (Gauss).
4. **Geodesic curvature and geodesics**
   Geodesic curvature. Geodesics. Arcs of minimum length: introduction to the calculus of variations. Theorem of Gauss-Bonnet.
5. **Covariant differentiation and parallel transport**
   Covariant differentiation. The Ricci identity. The Bianchi identities. Parallel transport.

Bibliography

- E. Kreyszig, *Differential Geometry*, Dover (1991).
- B.A. Dubrovin, A.T. Fomenko, S.P. Novikov, *Modern Geometry–Methods and Applications (Part I. The Geometry of Surfaces, Transformation Groups, and Fields)*, Springer (1992).

Online Resources

Virtual Campus.

Methodology

The following learning activities will be developed:

• Theory lessons, in which the fundamental concepts of the subject will be explained and illustrated with examples and applications.

• Practical problem-solving sessions.

The theory lessons and problem-solving sessions will take place mainly on the blackboard, although they may be supplemented with computer projections.

The teacher will assist the students at the specified office hours in order to solve doubts or expand concepts.

A collection of problems will be made available to students in the Virtual Campus prior to their resolution in class.

Evaluation Criteria

Exams	Weight:	70%
Grade obtained in the final exam, which will consist of theoretical questions and problems of similar difficulty to those solved in class.		
Other Activities	**Weight:**	30%
Exercises handed in throughout the course or carried out during classes, or exams covering part of the syllabus.		

Final Mark

The final grade FG obtained by the student will be calculated by applying the following formula:

$$FG = \max(E, 0.7E + 0.3A),$$

where E and A are the grades obtained in the final exam and in the “other activities”, respectively, both in the interval 0–10. The grade in the extraordinary call for June-July will be obtained following the same evaluation procedure.

In order to compensate the exam grade E with the points obtained with “other activities”, E must be greater than 4.5 points.