universität wien

Deriving rankings from incomplete preference information: A comparison of different approaches

Rudolf Vetschera
University of Vienna

Universidad Complutense de Madrid
October 30, 2014

Agenda

- Motivation: Incomplete information models
- Research questions
- Models
- Computational experiments
- Conclusions

Incomplete information models

- **Decision models contain many parameters (probabilities, attribute weights etc.)**
- **Parameters are often not know precisely**
  - Lack of information
  - Elicitation too complex for decision makers
  - Different opinions in group decisions...
- **Models needed to make decisions in the presence of incompletely specified parameters**

Framework

Additive multi-attribute utility
with unknown weights:

$$u(X)=\sum_k w_k u_k(x_k)$$

Note: most concepts can also be applied to

- **other uncertain parameters**
  (e.g. partial value functions)
- **other preference models**
  (e.g. outranking models)
- **other domains**
  (e.g. group decisions)

Forms of incomplete information

- **Intervals:**
  weight is between....
  $$\underline{w}_k \leq w_k \leq \overline{w}_k$$

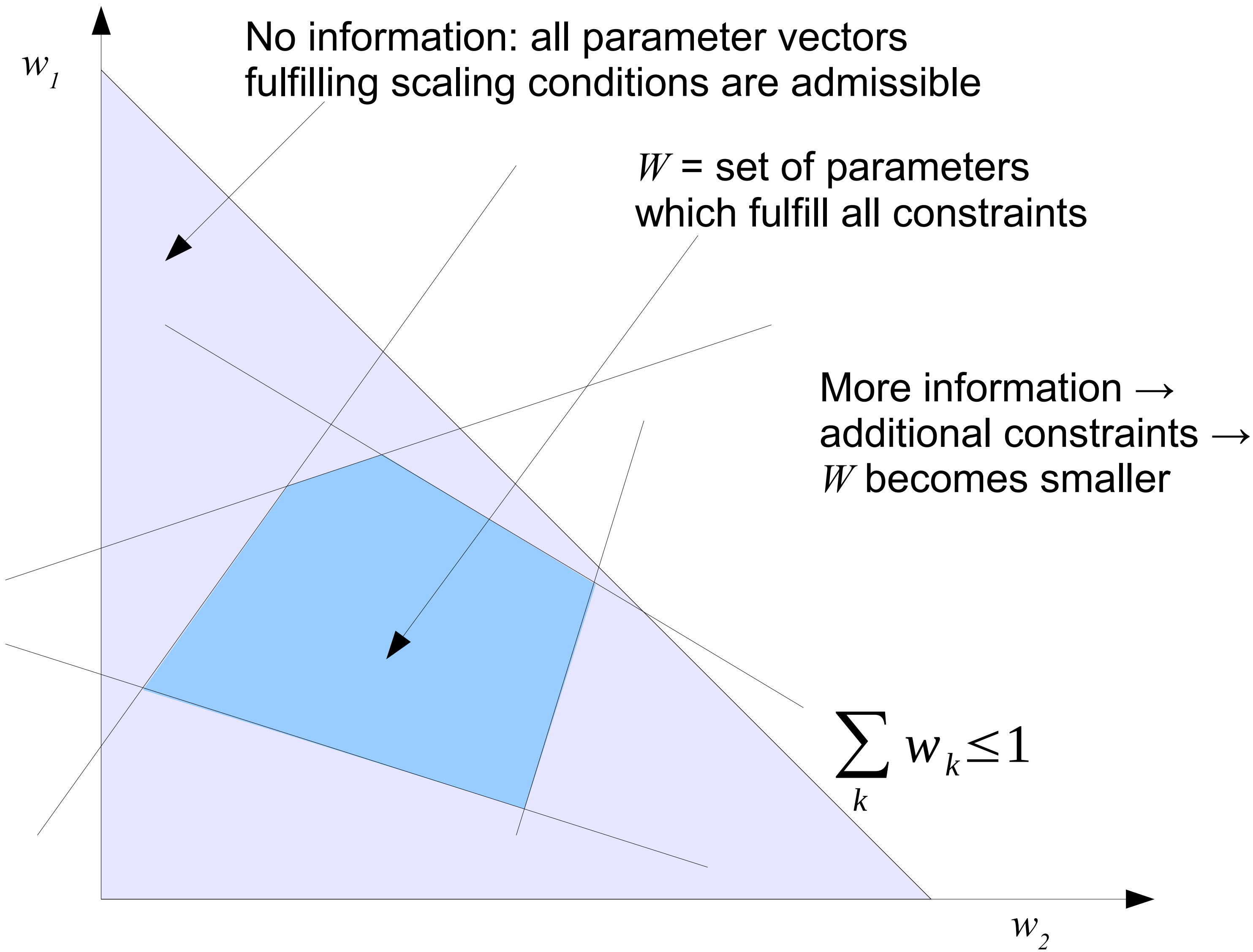
- **Rankings:**
  attribute k is more important than attribute m
  $$w_m \leq w_k$$

- **Ratios:**
  attribute k is at least twice as important as m
  $$2w_m \leq w_k$$

- **Comparison of alternatives:**
  A_i is better than A_j
  $$\sum_k w_k u_k(a_{ik}) \geq \sum_k w_k u_k(a_{jk})$$

In general: Linear constraints on w_k

Admissible parameters



Approaches

Dominance:
Establish relations that hold for all possible parameters

- Kmietowicz/Pearman 1984
- Kirkwood/Sarin 1985
- Park et al. 1996,1997, 2001
- ROR: Greco et al 2008

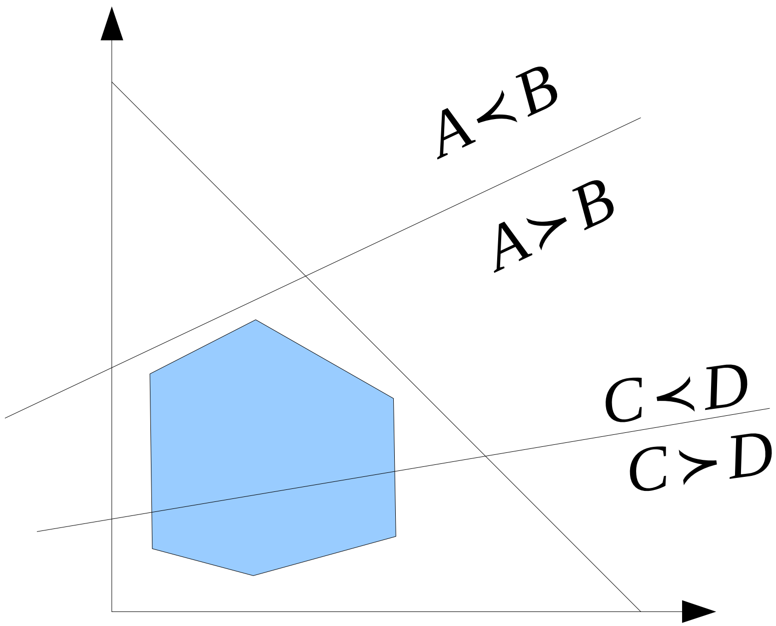
Single parameter:
Identify one "best" parameter vector

- Srinivasan/Shocker 1973
- UTA: Jacquet-Lagreze/Siskos 1982
- Representative value functions: Greco et al. 2011

Volume-based:
Relative size of regions in parameter space

- Domain criterion: Starr 1962
- Charnetski/Soland 1978
- VIP: Climaco/Dias 2000
- SMAA: Lahdelma et al 1998, 2001

Dominance based approach



A is **necessarily better** than B iff

$$\forall w \in W : u(A,w) > u(B,w)$$

C is **possibly better** than D iff

$$\exists w \in W : u(C,w) > u(D,w)$$

Necessarily better is usually **incomplete** relation on set of alternatives

universität wien

Dominance based approach: LP models

Necessarily better

$$\max z = \sum_k w_k (b_k - a_k)$$

s.t.

$$w \in W$$

Optimal $z < 0$:
A necessarily better than B

Possibly better

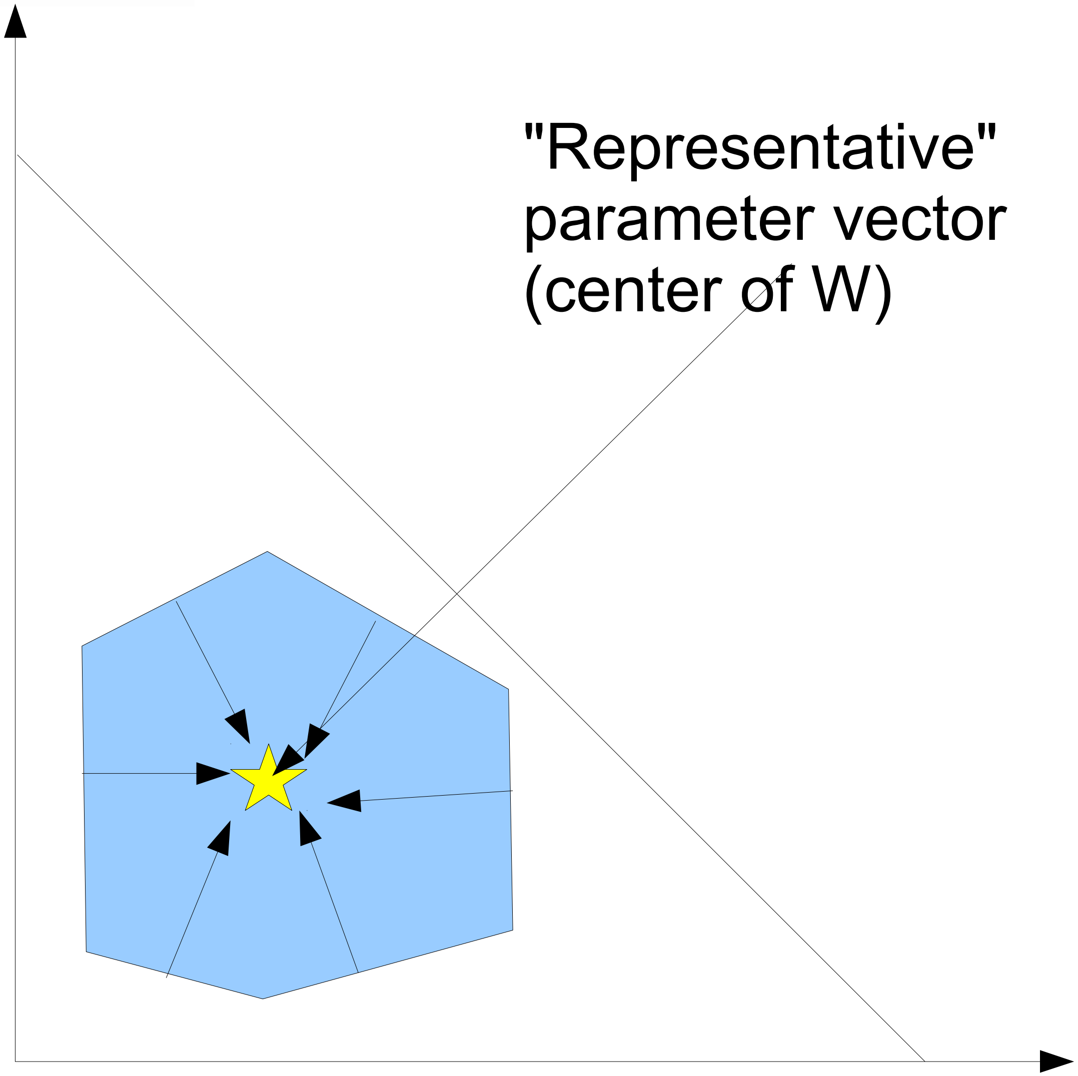
$$\max z = \sum_k w_k (a_k - b_k)$$

s.t.

$$w \in W$$

Optimal $z > 0$:
A possibly better than B

Single parameter approach



Central parameter approach: model

Example: constraints from pairwise comparison of alternatives

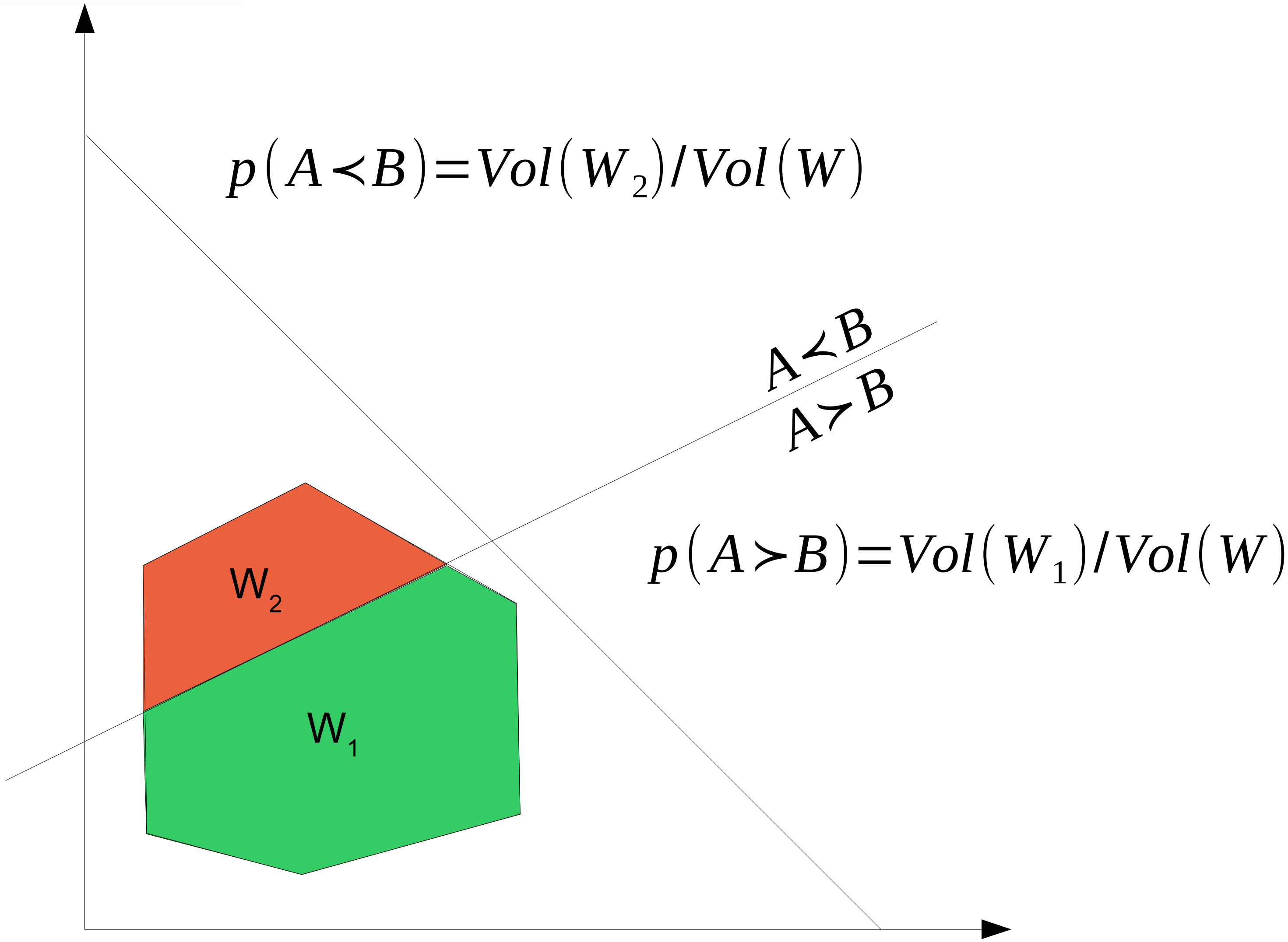
$$\max z$$

s.t.

$$\sum_k w_k (a_{ik} - a_{jk}) - z \geq 0 \quad \forall i, j : A_i \succ A_j$$

$$z \lessgtr 0$$

Volume based approach



universität wien

Results from volume-based methods (SMAA)

- **Rank acceptability index:**
  Probability r_{ik} that alternative A_i obtains rank k
- **Pairwise winning index:**
  Probability p_{ij} that alternative A_i is preferred to A_j

Approaches

Dominance:
Establish relations that hold for all possible parameters

- Kmietowicz/Pearman 1984
- Kirkwood/Sarin 1985
- Park et al. 1996,1997, 2001
- ROR: Greco et al 2008

Usually incomplete relation

Single parameter:
Identify one "best" parameter vector

- Srinivasan/Shocker 1973
- UTA: Jacquet-Lagreze/Siskos 1982
- Representative value functions: Greco et al. 2011

Complete order relation

Volume-based:
Relative size of regions in parameter space

- Domain criterion: Starr 1962
- Charnetski/Soland 1978
- VIP: Climaco/Dias 2000
- SMAA: Lahdelma et al 1998, 2001

Probabilistic information about ranks and relations

↓

Complete order

Research questions

- **How to derive a complete order relation among alternatives from the probabilistic information obtained by volume-based approaches?**
- **How are relations obtained in such way different from those obtained by other methods?**

Model for rank acceptability index

Assignment problem

$$\sum_{k=1}^{N_{alt}} x_{ik} = 1 \quad \forall i$$

each alternative is assigned to one rank

$$\sum_{i=1}^{N_{alt}} x_{ik} = 1 \quad \forall k$$

to each rank, one alternative is assigned

$$x_{ik} \in \{0,1\}$$

x_{ik}: Alternative A_i is assigned to rank k

Objective functions

Average probability of assignments

$$\max \sum_{i,k:x_{ik}=1} r_{ik} = \sum_{i=1}^{N_{alt}} \sum_{k=1}^{N_{alt}} r_{ik} x_{ik}$$

Joint probability

$$\max \prod_{i,k:x_{ik}=1} r_{ik} = \sum_{i=1}^{N_{alt}} \sum_{k=1}^{N_{alt}} \log(r_{ik}) x_{ik}$$

Minimum probability of assignment

$$\max z$$

$$z \le r_{ik} + (1 - x_{ik}) \quad \forall i, k$$

Models for pairwise winning indices

Construct complete order relation:

Complete and asymmetric

$$y_{ij} + y_{ji} = 1 \quad \forall i \neq j$$

Irreflexive

$$y_{ii} = 0 \quad \forall i$$

Transitive

$$y_{ij} \geq y_{ik} + y_{kj} - 1.5 \quad \forall k \neq i, j$$

y_{ik}: Alternative A_i is preferred to A_j

Linking models

Rank from assignment model

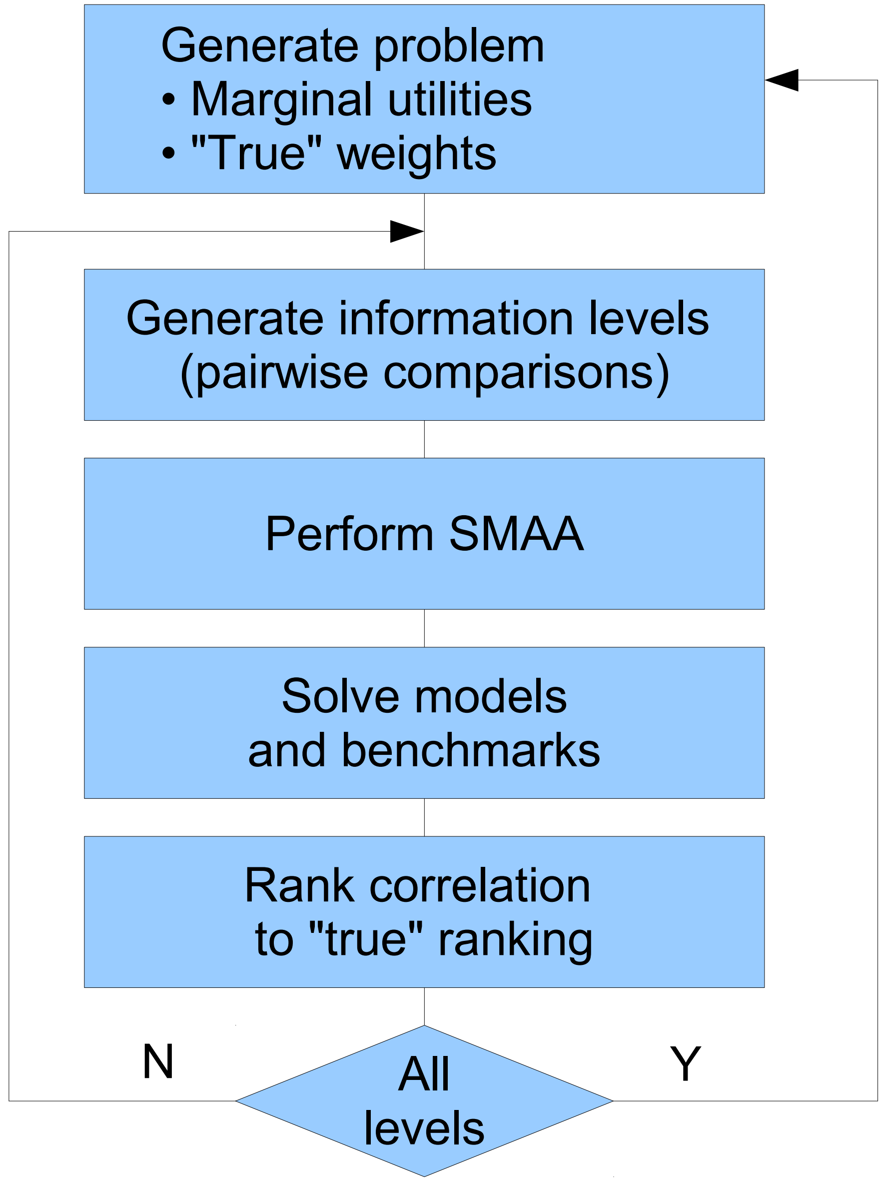
$$R_i = \sum_k k\, x_{ik}$$

Rank from relation model

$$R_i = 1 + \sum_j y_{ji}$$

=

Computational study



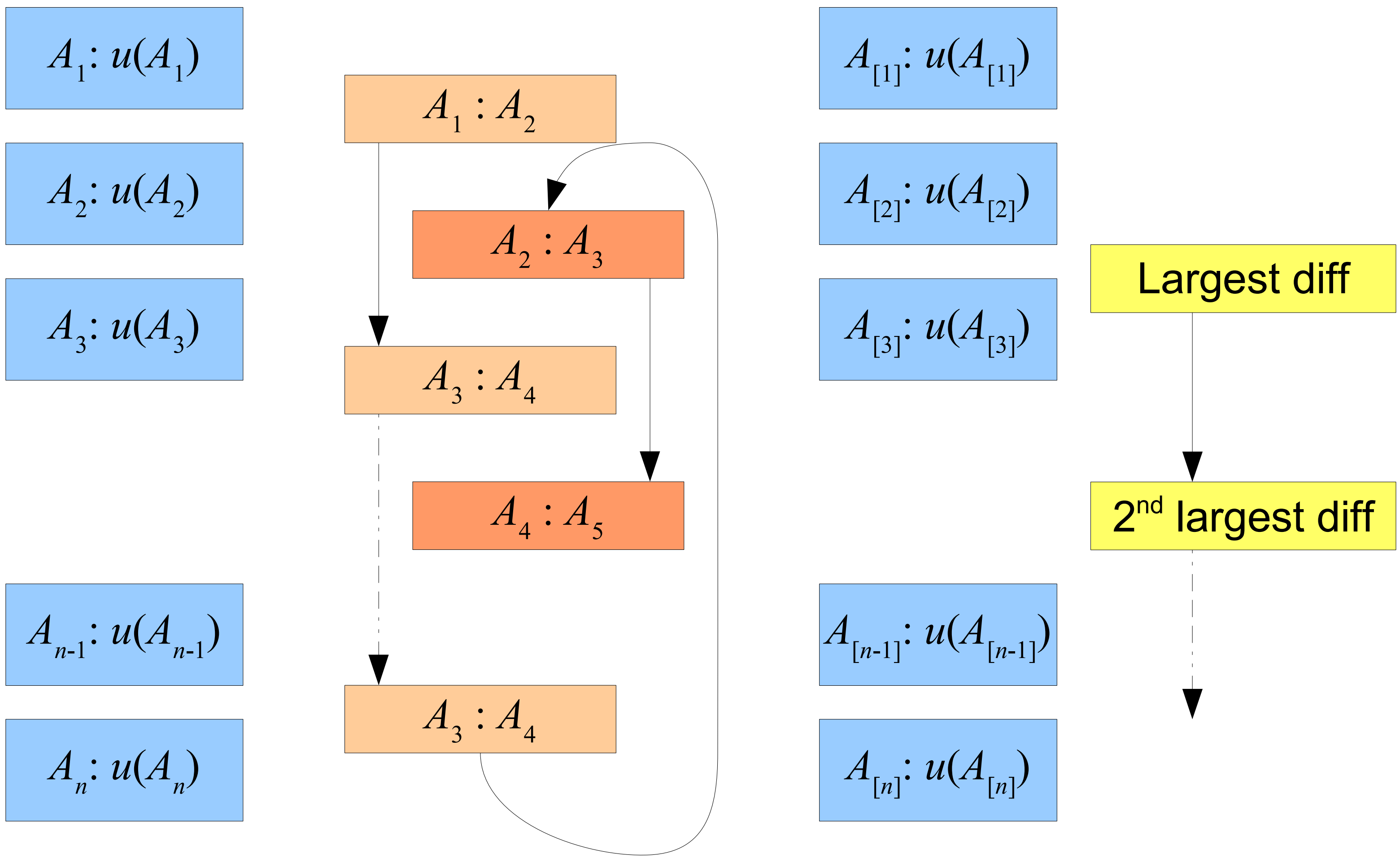
Computational study

- **Additive value function**
- **Unknown weights**
- **Incomplete information provided via pairwise comparisons of alternatives**
  - Neighboring alternatives in true ranking (sorted by utility difference)
  - Neighboring alternatives by numbers (1-2, 3-4 etc., then 2-3, 4-5)
- **Results: Rank correlation to "true" ranking of alternatives**
- **Benchmarks:**
  - Barycenter approach (average of all weights in simulation)
  - Distance based approach: LP model to find parameter vector in center of polyhedron defined by comparisons (similar to UTA)

universität wien

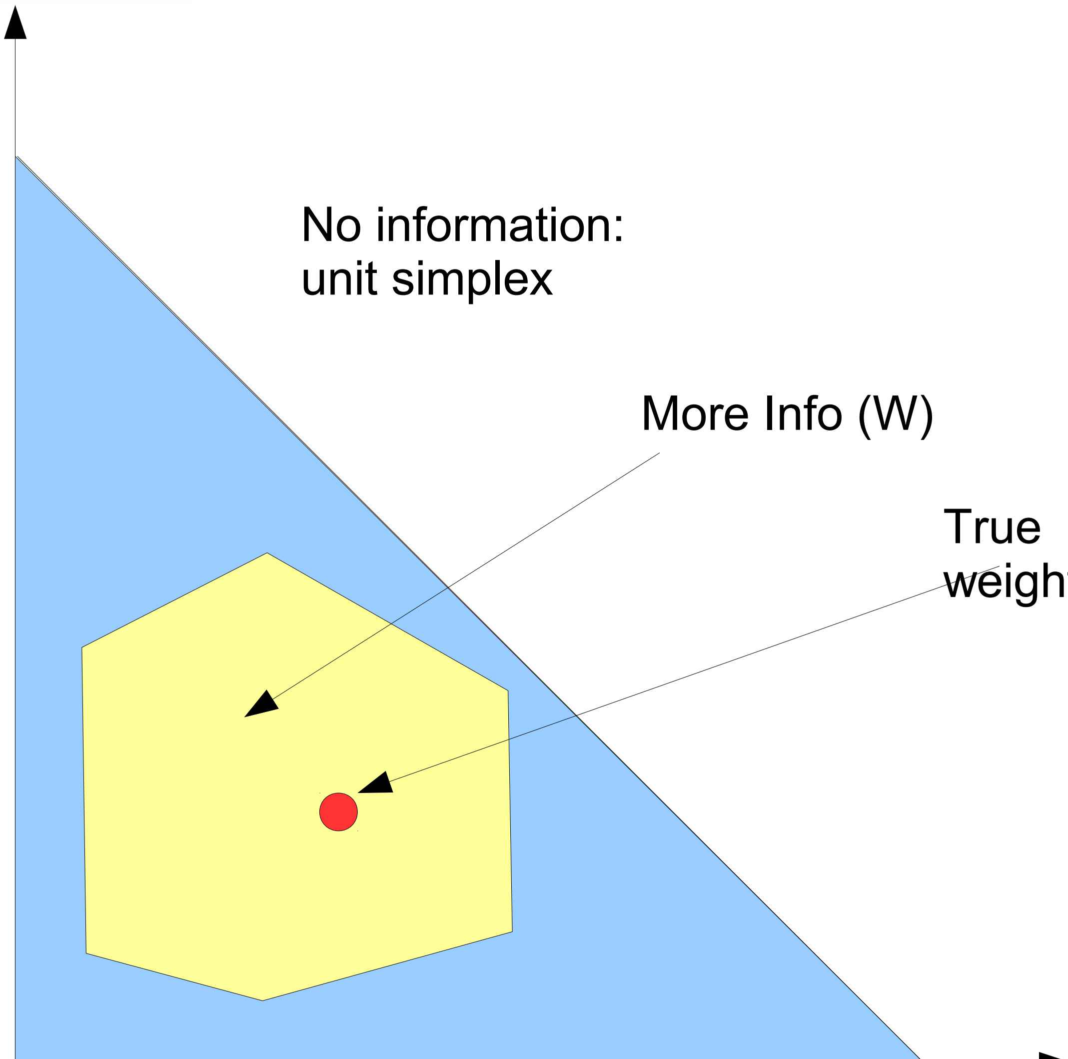
Information levels

Alternatives | Method 1: by numbers | Method 2: by preference



$u(A_{[1]}) > u(A_{[2]}) > \ldots > u(A_{[n]})$

Measuring information



Compared models

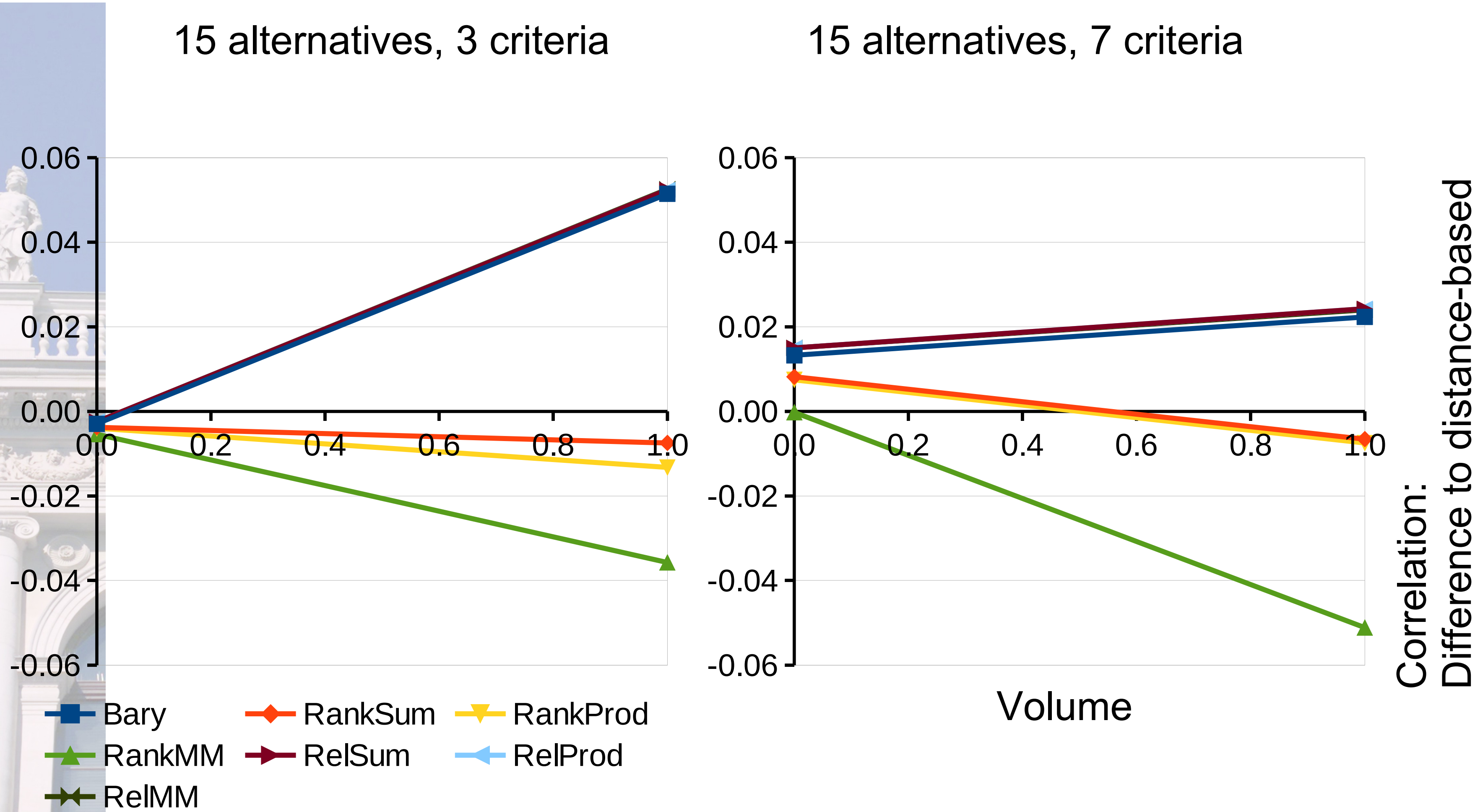
- **Distance** (Benchmark)
- **Bary**center: Average of all admissible weight vectors generated during simulation
- **Rank acceptability indices:**
  - **RankSum:** Objective sum of probabilities
  - **RankProd:** Objective joint probability
  - **RankMM:** Objective minimum probability
- **Pairwise winning indices** (relation)
  - **RelSum:** Objective sum of probabilities
  - **RelProd:** Objective joint probability
  - **RelMM:** Objective minimum probability

Analysis

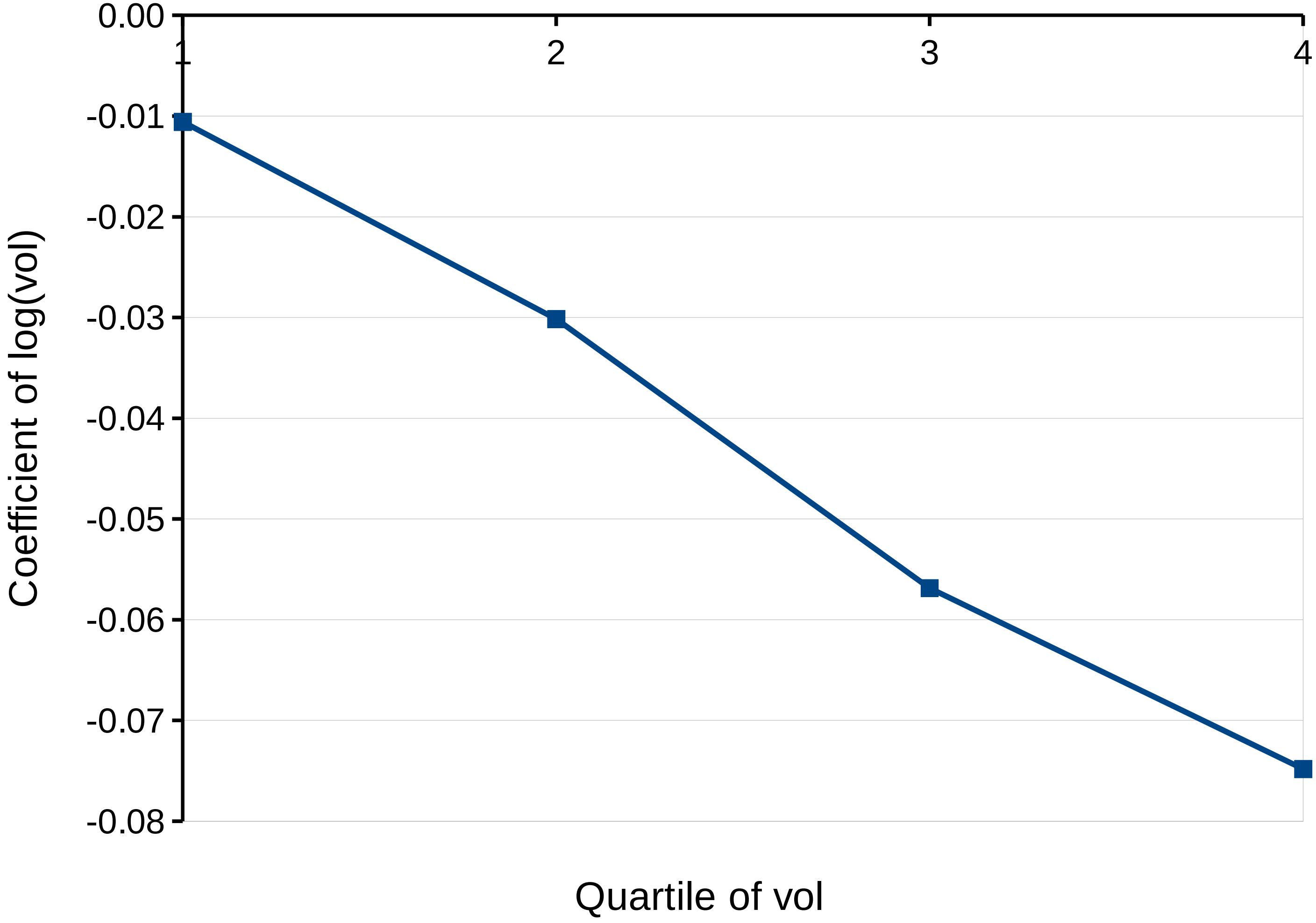
- **Nested linear regressions**
- **Dependent variable:**
  correlation between obtained ranking and “true ranking”
- **Independent variables:**
  - Problem characteristics (**M0**):
    - number of alternatives **NAlt**
    - number of criteria **NCrit**
  - Information provided (**M1**):
    Volume of admissible parameter set relative to set of all possible parameters (**Volume**)
  - Method used (**M2**)
  - Interaction effects (**M3**)

	M0	M1	M2	M3
Intercept	*** 0.8268	*** 0.9870	*** 0.9808	*** 0.9803
NAlt=9	*** 0.0032	*** -0.0351	*** -0.0351	*** -0.0351
NAlt=12	*** 0.0189	*** -0.0421	*** -0.0421	*** -0.0421
NAlt=15	*** 0.0302	*** -0.0458	*** -0.0458	*** -0.0459
NCrit=5	*** -0.0623	*** -0.0856	*** -0.0856	*** -0.0856
NCrit=7	*** -0.1005	*** -0.1300	*** -0.1300	*** -0.1300
Volume		*** -0.3530	*** -0.3530	*** -0.3510
Bary			*** 0.0118	*** 0.0089
RankSum			*** 0.0030	*** 0.0069
RankProd			*** 0.0023	*** 0.0066
RankMM			*** -0.0067	° 0.0016
RelSum			*** 0.0132	*** 0.0101
RelProd			*** 0.0132	*** 0.0101
RelMM			*** 0.0132	*** 0.0100
Vol:Bary				*** 0.0114
Vol:RankSum				*** -0.0154
Vol:RankProd				*** -0.0171
Vol:RankMM				*** -0.0331
Vol:RelSum				*** 0.0125
Vol:RelProd				*** 0.0125
Vol:RelMM				*** 0.0126
adj. R2	0.0524	0.3181	0.3195	0.3202

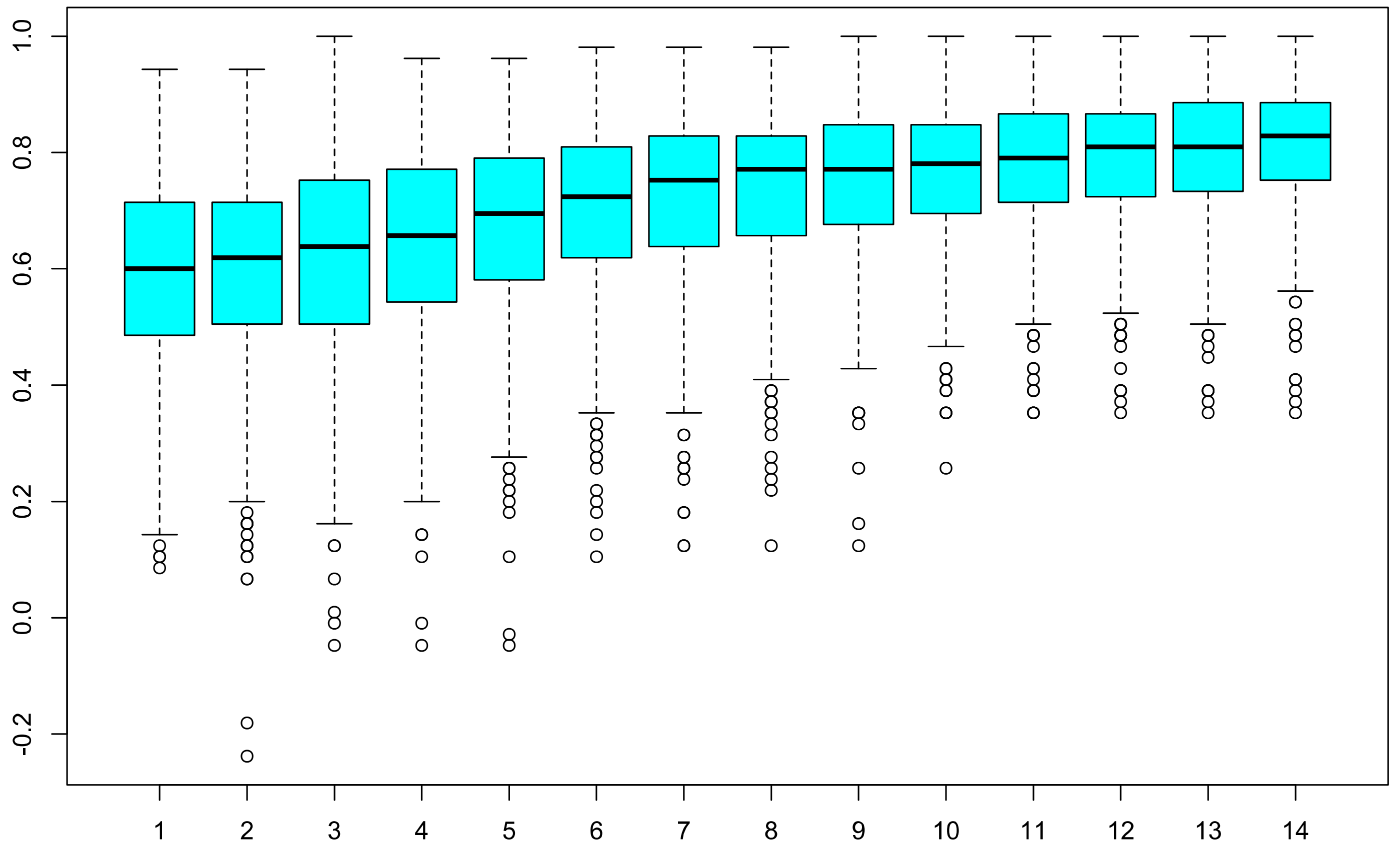
Interaction effects method and volume



"Returns to information"

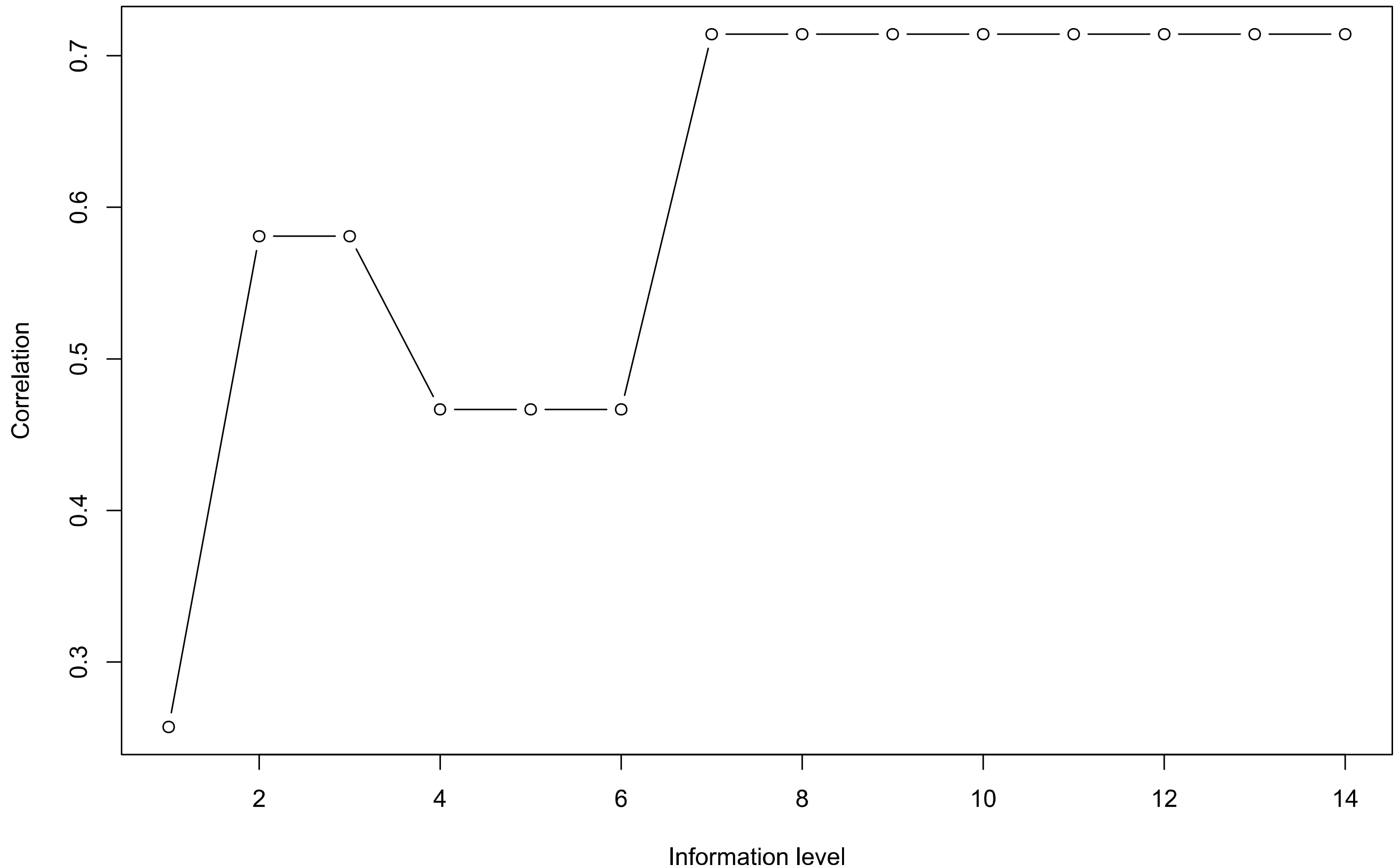


Information effect

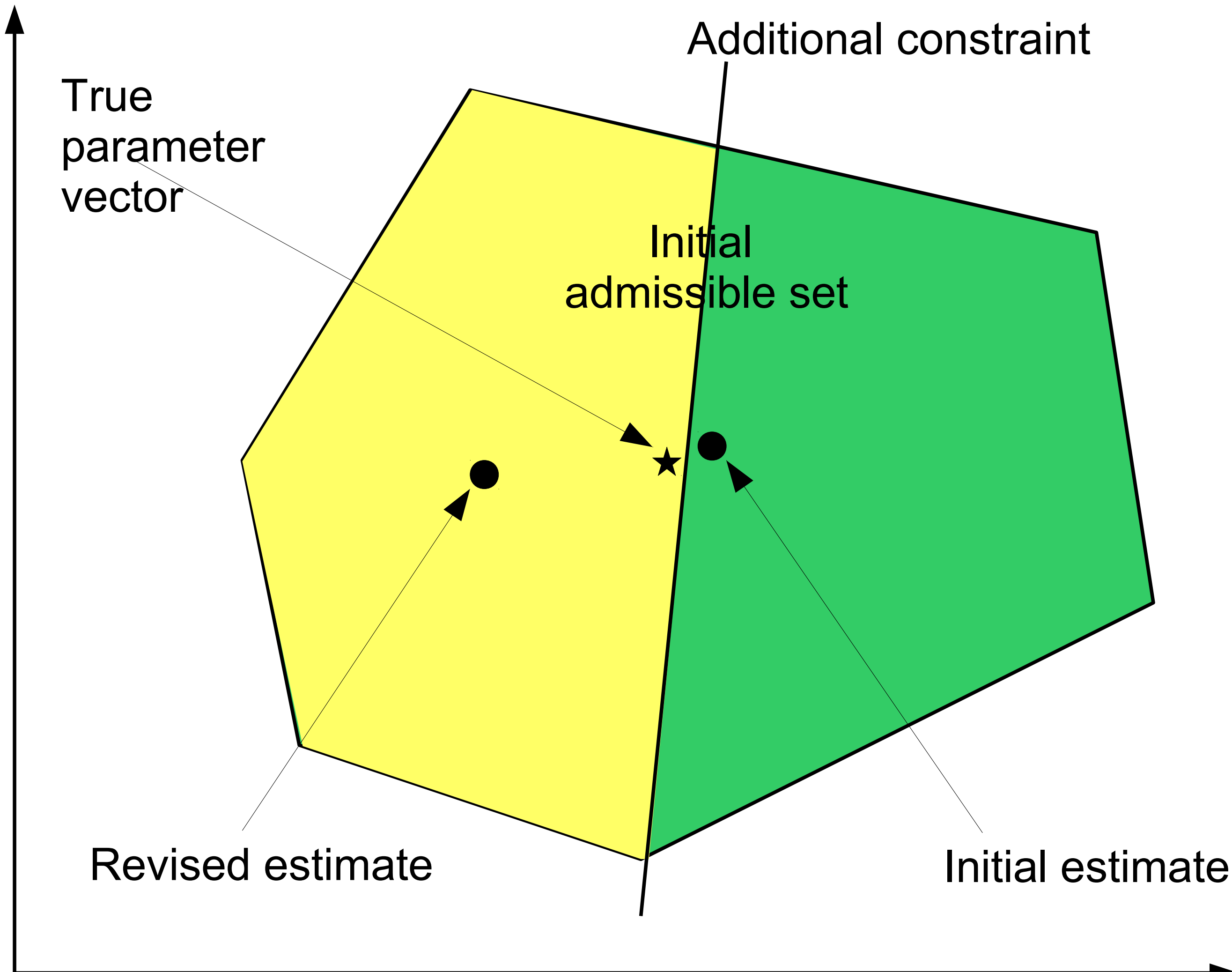


universität wien

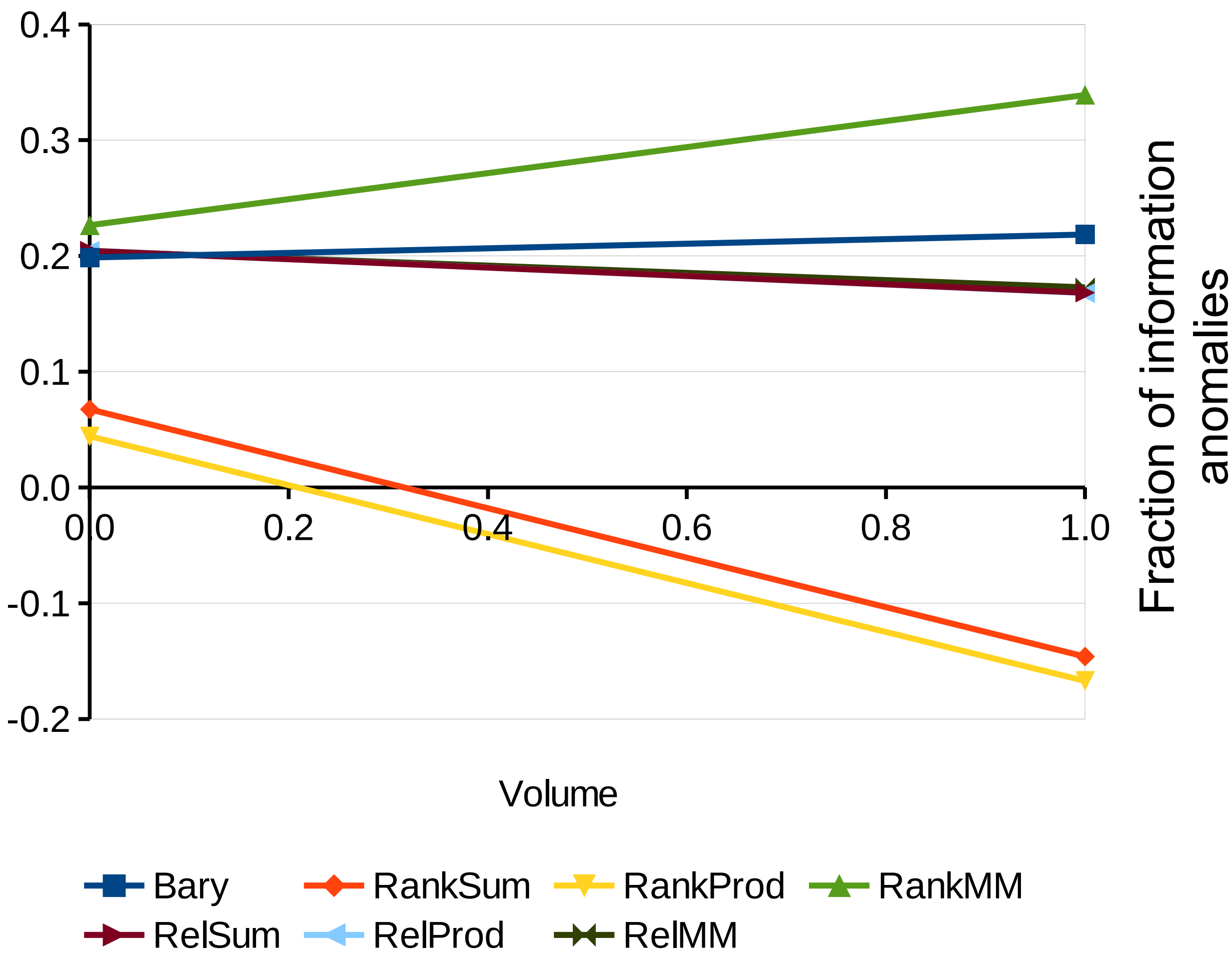
Information in one experiment



Information anomalies



Effects on information anomalies



Conclusions

- Complete ranking can be obtained from stochastic indices
- Models are independent of underlying preference model
- Differences between methods are small, but significant
- Barycenter and methods based pairwise on winning indices usually outperform benchmark (distance-based)
- Max-min probability objective function gives worse results
- Information anomalies occur quite frequently
- Models based on rank acceptability indices and average/joint probability lead to fewer information anomalies

Extensions

- **Parallel application of methods**
  - Mutual confirmation vs. differences
  - Differences could guide elicitation process
- **Uncertainty about utilities (not just weights)**
- **Other preference models**


Thank you
for your attention!