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Programación binivel: una aplicación a mercados eléctricos liberalizados

Evaluación pública de grandes proyectos: teoría y aplicaciones

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5 de Julio de 2013

Outline

1. Motivation and Literature Review
2. Introduction to Bilevel Programming: Basic Concepts
3. Bilevel Generation Expansion Models
4. Comparison of Single-Level and Bilevel Capacity Equilibria
5. Approximation of Bilevel Generation Expansion Equilibrium Models
6. Additional Case Studies
7. Overview of Solution Techniques
8. Conclusions

1

Motivation and Literature Review

Motivation

- Sufficient and adequate **generation capacity** has to be installed to meet society’s future electricity demand.
- The **liberalization** of electricity markets has made capacity expansion planning more **challenging** especially for GENCOs.
- **Bilevel** models are **more realistic than single-level** models because they represent **sequential decision making** as opposed to simultaneous decision making.
- Thus this thesis aims at **assisting GENCOs** in taking expansion decisions more adequately via **bilevel modeling**.

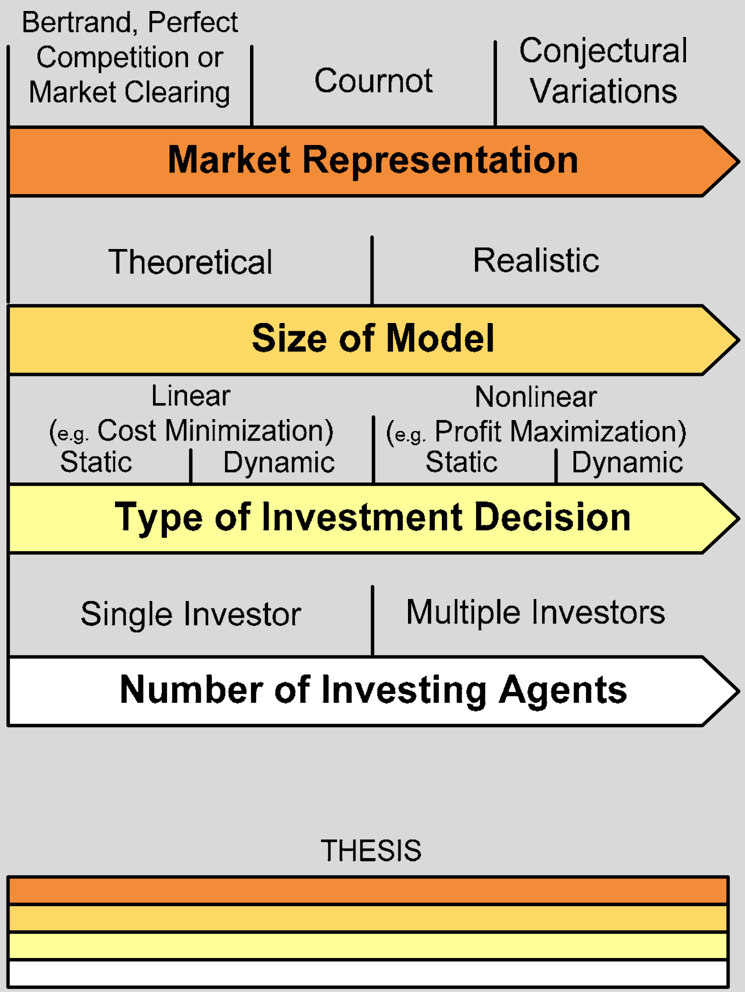
Objectives

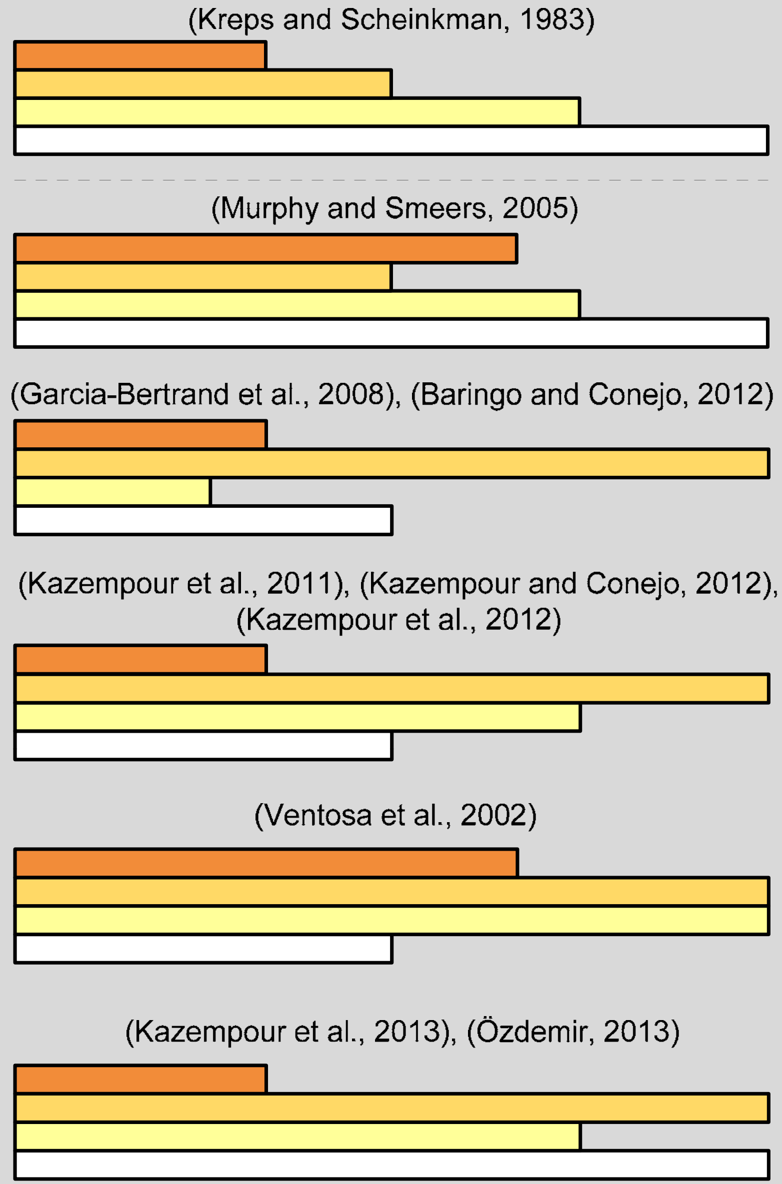
- **General Objective: Advance research in generation expansion planning for liberalized electricity markets using bilevel mathematical programming techniques.**

- **Methodological Objectives:**
  - **Compare single-level** and **bilevel** models and establish differences.
  - Propose and formulate large-scale **bilevel** generation expansion **models** that extend existing approaches in literature.
  - **Extend** and **improve** models by introducing realistic aspects.

- **Computational Objectives:**
  - Propose single-level conjectured price response expansion model.
  - Propose **single-level approximation scheme** for bilevel capacity expansion equilibria, which reduces computational time.
  - Explore and **compare** different **solution techniques** for bilevel problems.

Literature Review: Bilevel Generation Expansion Models





2

Introduction to Bilevel Programming: Basic Concepts

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Basic Concepts
Bilevel Programming Problem (BPP)

- A bilevel programming problem is a hierarchical optimization problem which is constrained by another optimization problem.

$$\min_{x,y} F(x,y)$$

$$\text{s.t. } G(x,y) \le 0, H(x,y) = 0$$

Upper level

$$\min_{y} \; f(x,y)$$

$$\text{s.t. } \mathrm{g}(x,y) \le 0, h(x,y) = 0$$

Lower level

Mathematical Program with Equilibrium Constraints (MPEC)

- An **MPEC** is an optimization model in which the essential constraints are defined by a parametric variational inequality or a **complementarity system** which typically model a certain equilibrium phenomenon.

$$
\begin{aligned}
&\min_{x,y} F(x,y) \\
&\text{s.t. } G(x,y) \geq 0 \\
&\left.\begin{aligned}
y &\geq 0 \\
f(x,y) &\geq 0 \\
y^T f(x,y) &= 0
\end{aligned}\right\}
\end{aligned}
$$

Complementarity problem

Equilibrium Problem with Equilibrium Constraints (EPEC)

- An **EPEC** can be described as finding a Nash equilibrium between i=1,…,I players that are all facing an MPEC:

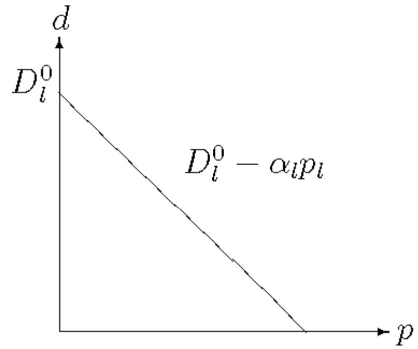
$$\left\{ MPEC_1, \ldots, MPEC_i, \ldots, MPEC_I \right\}$$

- Equilibrium in both levels – upper level and lower level.

Conjectured-Price Response

- A **conjectured-price response** θ is a type of conjectural variation which allows to express GENCO i's belief concerning its influence on price as a result of a change in its output.
- Assuming an affine relation between price and demand:

$$p(d) = \frac{D_0 - d}{\alpha} = \frac{D_0 - \sum_i q_i}{\alpha}$$



- The conjectured price response of firm *i* is defined as:

$$\theta_i = -\frac{\partial p(d)}{\partial q_i}$$

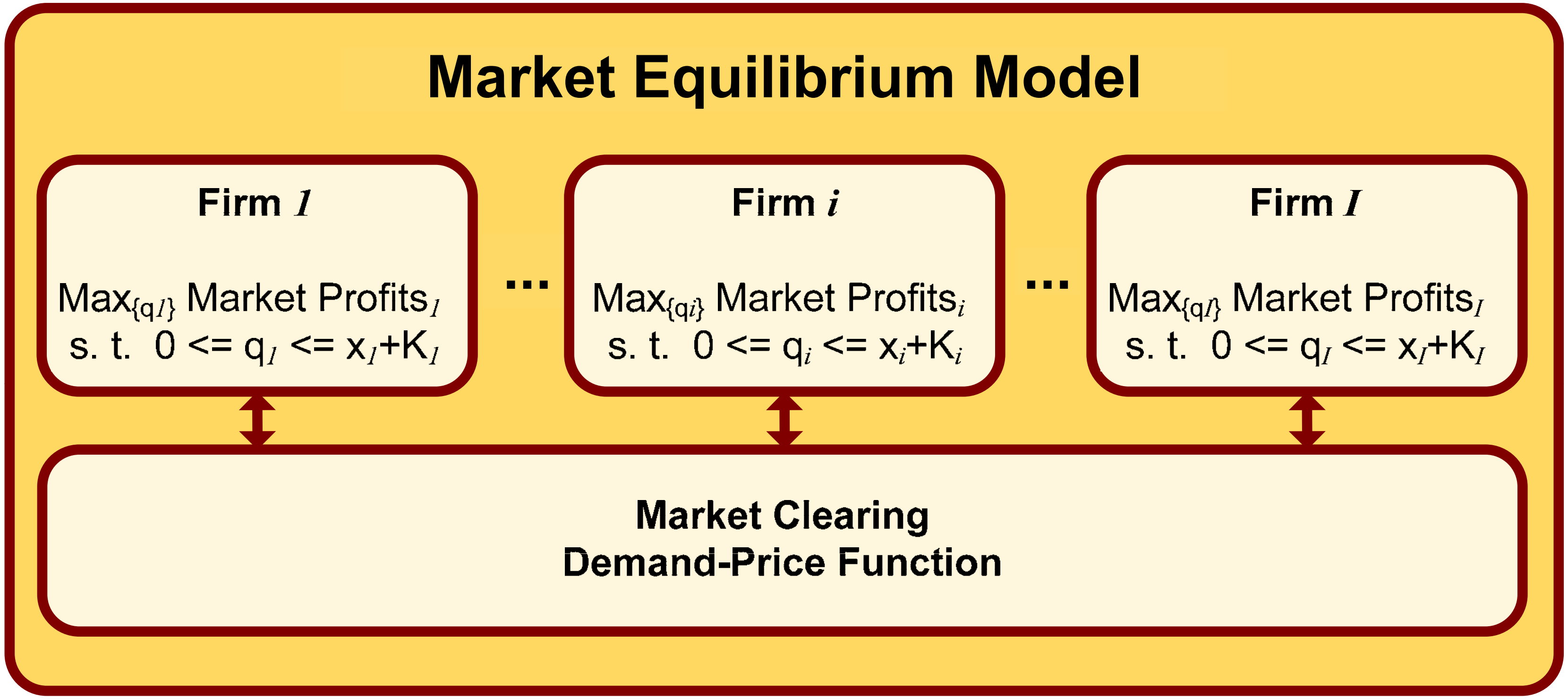
- This allows us to model different strategic market behavior (perfect competition, Cournot oligopoly, etc.)

3

Bilevel Generation Expansion Models

Model Definitions
Single-Level Market Equilibrium Model

- All GENCOs simultaneously maximize their market profits (market revenues minus production costs) subject to lower and upper bounds on production and a demand balance.



Mathematical Formulation
Single-Level Market Equilibrium

$$\forall i \begin{cases} \max_q \sum_y \frac{1}{(1+F)^y} \left\{ \sum_{jl} T_{yl}(p_{yl} - \delta_{ij}) q_{ijyl} \right\} \\ \text{s.t.} \quad q_{ijyl} \leq x_{ijy} + K_{ijy} \quad \forall jyl \quad : \lambda_{ijyl} \\ \qquad 0 \leq q_{ijyl} \quad \forall jyl \quad : \mu_{ijyl} \end{cases}$$

Market profits

Upper bound

Lower bound

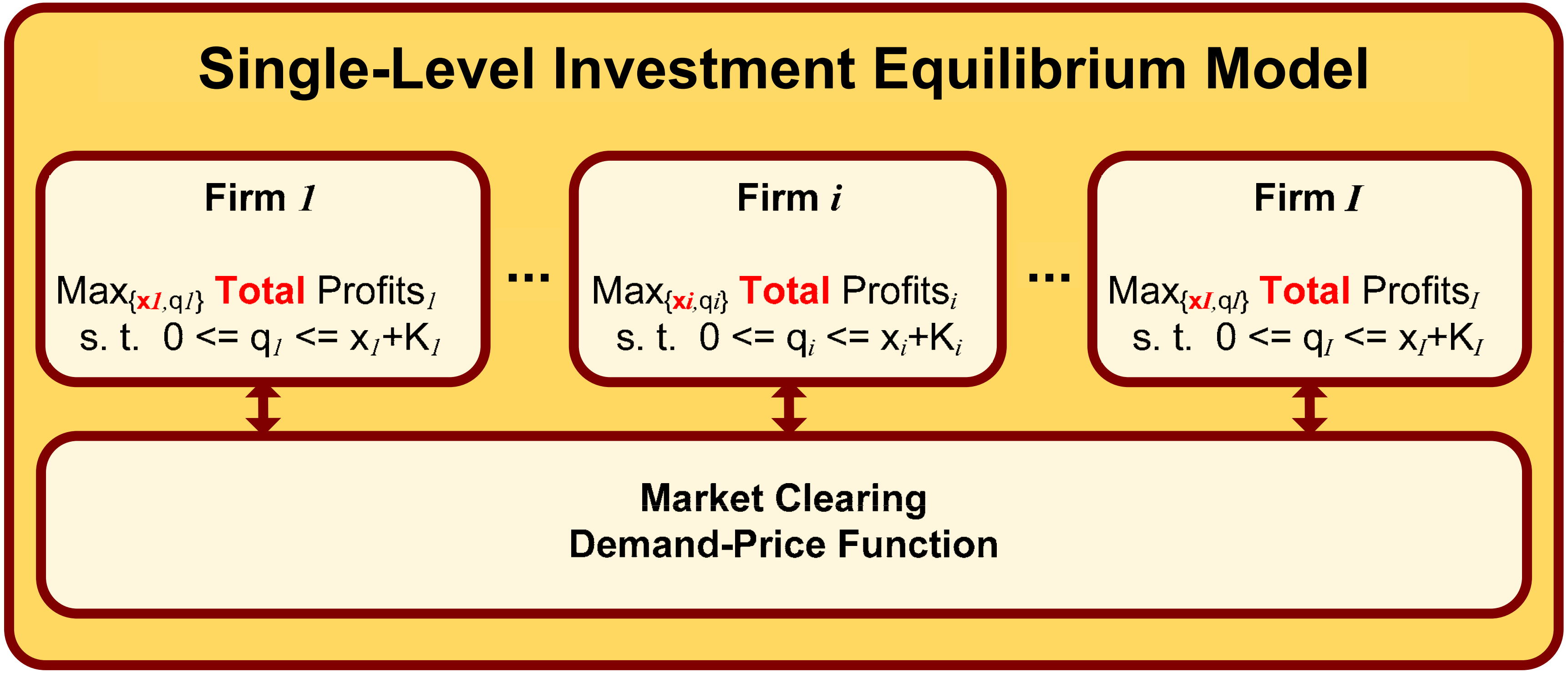
$$d_{yl} - \sum_{ij} q_{ijyl} = 0 \quad \forall yl$$

Affine relation price & demand

$$d_{yl} - D^0_{yl} + \alpha_{yl} p_{yl} = 0 \quad \forall yl$$

Model Definitions
Single-Level Investment Equilibrium Model

- All GENCOs simultaneously maximize their **total** profits (market revenues minus **investment costs** minus production costs) subject to lower and upper bounds on production and a demand balance.



Mathematical Formulation
Single-Level Investment Equilibrium

Investments x as variables

Investment costs

$$\forall i \left\{ \begin{array}{ll} \max_{x,q} \sum_y \frac{1}{(1+F)^y} \left\{ \sum_{jl} T_{yl}(p_{yl} - \delta_{ij}) q_{ijyl} - \sum_j \beta_{ijy} x_{ijy} \right\} & \\ \text{s.t.} \qquad q_{ijyl} \leq x_{ijy} + K_{ijy} \quad \forall jyl & : \lambda_{ijyl} \\ 0 \leq q_{ijyl} \quad \forall jyl & : \mu_{ijyl} \end{array} \right.$$

$$d_{yl} - \sum_{ij} q_{ijyl} = 0 \quad \forall yl$$

$$d_{yl} - D^0_{yl} + \alpha_{yl} p_{yl} = 0 \quad \forall yl$$

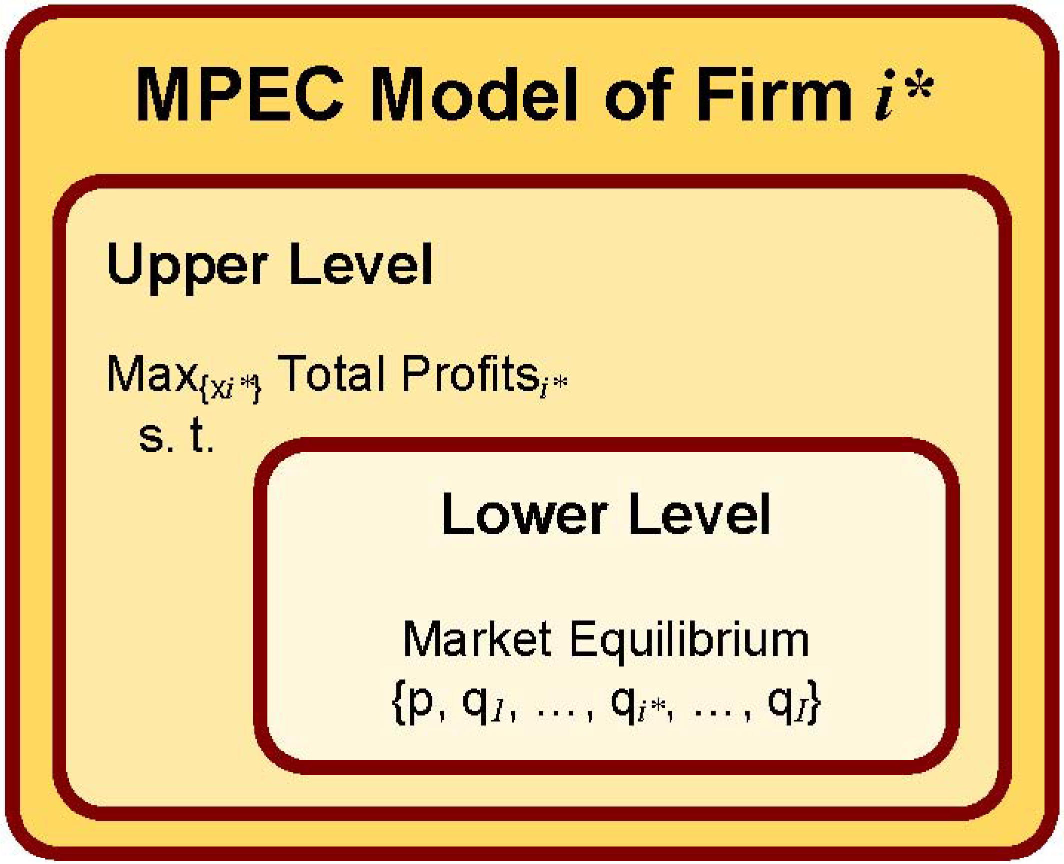
Model Definitions
BILEVEL Optimization Model (BOM)

- This model **assists one GENCO** in taking capacity decisions while considering the competitors' investments as fixed.

This model is an **MPEC**.

In the **upper level** investment decisions of firm i* are taken.

The **lower level** corresponds to the previously defined **market equilibrium**.



Mathematical Formulation
MPEC

$$\max_{x_{i^*jy}} \sum_y \frac{1}{(1+F)^y} \Big\{ \sum_{jl} T_{yl}(p_{yl} - \delta_{i^*j}) q_{i^*jyl} - \sum_j \beta_{i^*jy} x_{i^*jy} \Big\}$$

$$\text{s.t.} \qquad 0 \le x_{i^*jy} \le x_{i^*j(y+1)} \quad \forall jy$$

$$\text{s.t.}$$

Market Equilibrium Formulation

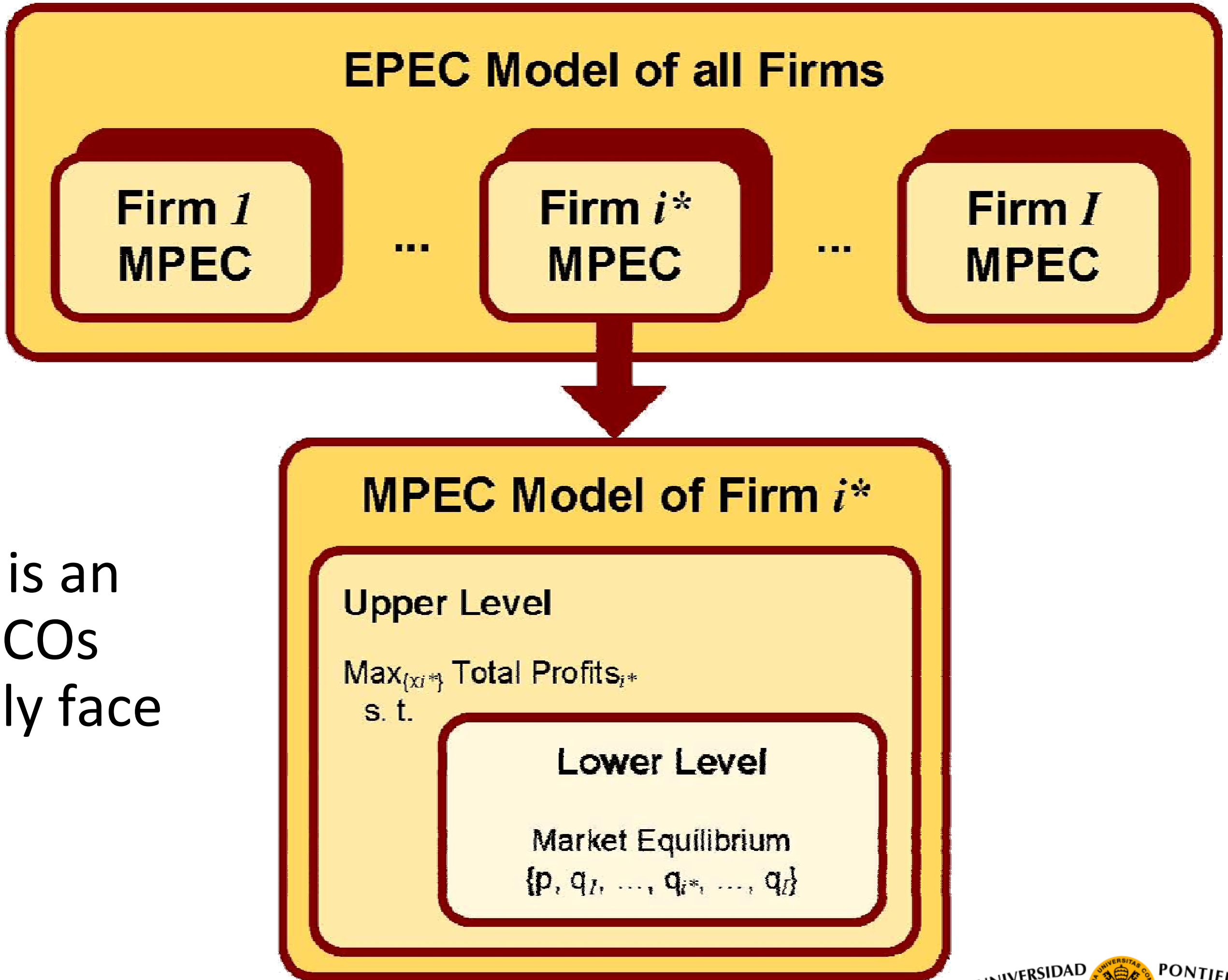
$$\forall i \begin{cases} \max_q \sum_y \frac{1}{(1+F)^y} \{ \sum_{jl} T_{yl}(p_{yl} - \delta_{ij}) q_{ijyl} \} \\ \text{s.t.} \quad q_{ijyl} \le x_{ijy} + K_{ijy} \quad \forall jyl \quad : \lambda_{ijyl} \\ \qquad 0 \le q_{ijyl} \quad \forall jyl \quad : \mu_{ijyl} \end{cases}$$

$$d_{yl} - \sum_{ij} q_{ijyl} = 0 \quad \forall yl$$

$$d_{yl} - D^0_{yl} + \alpha_{yl} p_{yl} = 0 \quad \forall yl$$

Model Definitions
BILEVEL Equilibrium Model (BEM)

- This model **assists ALL GENCOs** in taking capacity decisions.



This problem is an **EPEC**: all GENCOs simultaneously face an **MPEC**.

Mathematical Formulation
EPEC

$\forall i*$

$$\max_{x_{i*jy}} \sum_y \frac{1}{(1+F)^y} \{ \sum_{jl} T_{yl}(p_{yl} - \delta_{i*j}) q_{i*jyl} - \sum_j \beta_{i*jy} x_{i*jy} \}$$

$$\text{s.t.} \qquad 0 \leq x_{i*jy} \leq x_{i*j(y+1)} \quad \forall jy$$

s.t.

Market Equilibrium Formulation

$$\forall i \begin{cases} \max_q \sum_y \frac{1}{(1+F)^y} \{ \sum_{jl} T_{yl}(p_{yl} - \delta_{ij}) q_{ijyl} \} \\ \text{s.t.} \quad q_{ijyl} \leq x_{ijy} + K_{ijy} \quad \forall jyl \quad : \lambda_{ijyl} \\ 0 \leq q_{ijyl} \quad \forall jyl \quad : \mu_{ijyl} \end{cases}$$

$$d_{yl} - \sum_{ij} q_{ijyl} = 0 \quad \forall yl$$

$$d_{yl} - D^0_{yl} + \alpha_{yl} p_{yl} = 0 \quad \forall yl$$

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Comparison of Single-Level and Bilevel Capacity Equilibria

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Summary of Study

- We compare two generation expansion models:
  - A **single-level** model where investment and production decisions are considered to be taken **simultaneously**.
  - A **bilevel** model where first investment decisions are taken and then **sequentially** production decisions are decided in the market.

- The intensity of competition among producers in the energy market is represented using **conjectural variations**.

- For simplicity, in each of the models we consider two identical generation companies, a one-year time horizon and investment in one technology.

Theorem:
Comparison of Single-Level and Bilevel Equilibria

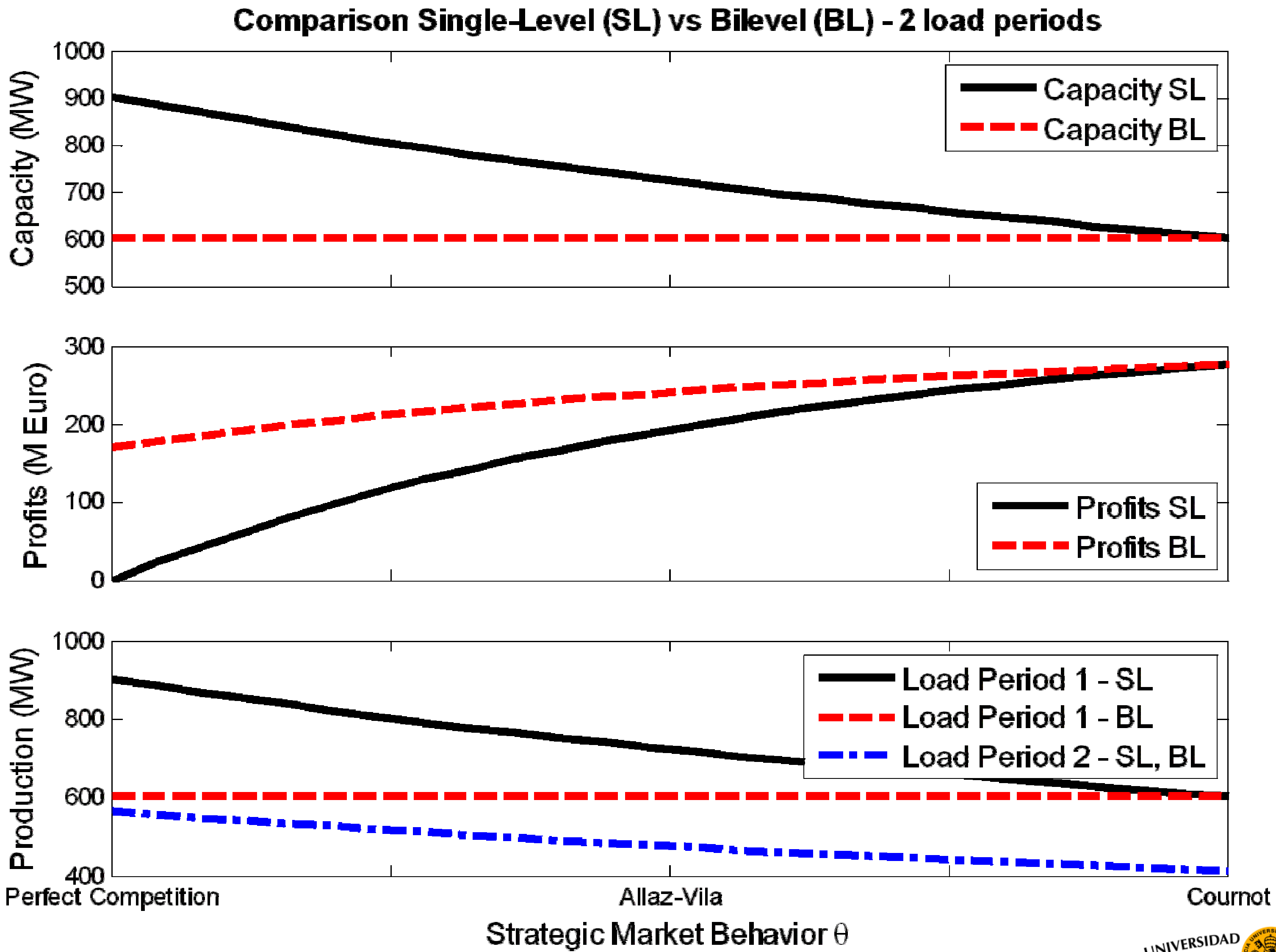
Theorem (Wogrin et al., 2012 MPB):

Let there be two identical firms with perfectly substitutable products and one load period. Let there be an affine relation between price and demand. When comparing the single and bilevel competitive equilibria for two firms, we find the following:

The single-level Cournot solution is a solution to the bilevel conjectured price response equilibrium for any choice of the conjectured price response parameter θ from perfect competition to Cournot competition.

- The result extends to multiple load periods and – under certain circumstances – to asymmetric firms.
- This is an extension of (Kreps and Scheinkman, 1983).

Comparison Single-Level and Bilevel Model
Two Load Period Example



Observation
Ranking of Bilevel Equilibria is Ambiguous

- We also prove by counter example that the **ranking of bilevel** conjectured price response **equilibria**, in terms of market efficiency (aggregate consumer surplus and market surplus) and consumer welfare, **is parameter dependent**.

- For a 20 load period example, we obtain the following results for the bilevel model:

[Billion Euro]	Perfectly competitive market	Intermediately competitive market	Cournot type market	Social welfare max solution
Market Efficiency	1.24	1.30	1.28	1.47
Consumer Surplus	0.62	0.72	0.64	1.44
Total Profits	0.62	0.58	0.64	0.03

Summary of Results

- The **bilevel** model always **yields Cournot capacities** independent of strategic spot market behavior.
- This makes them more realistic than single-level models whose capacity decisions vary with market behavior.
- Therefore bilevel models are very useful to study realistic generation expansion decisions and to evaluate the effect of alternative market designs for mitigating market power.
- Under certain circumstances (Cournot market behavior) both **single-level and bilevel results can coincide**.
- In bilevel models, **more competition can lead to less consumer surplus and less overall market efficiency**, depending on the model parameters.

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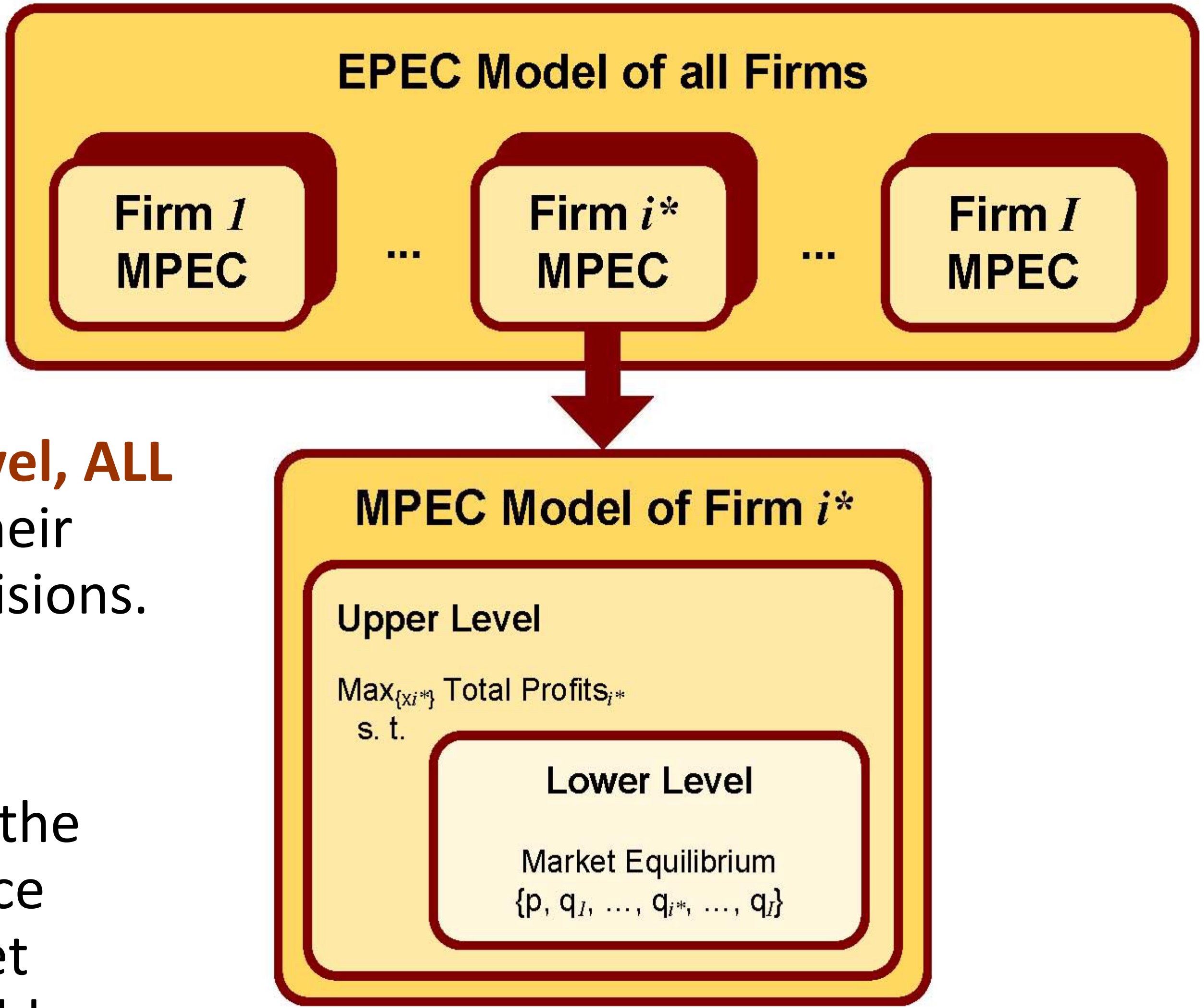
Approximation of Bilevel Generation Expansion Equilibrium Models

Graphic Representation of BILEVEL Equilibrium Model (BEM)



- In the **upper level, ALL** GENCOS take their investment decisions.

- The **lower level** corresponds to the conjectured price response market equilibrium problem.

Overview of Bilevel Equilibrium Models (BEM)

- Bilevel Equilibrium Models (BEM) **assist all GENCOs** to take generation expansion decisions.

- This type of model is usually formulated as an Equilibrium Problem with Equilibrium Constraints (**EPEC**).

- From numerical examples it becomes apparent that:
  - EPECs are very **hard to solve** and can have **multiple equilibria** (even small cases: 2 firms, 2 years, 2 load levels, 2 technologies).
  - MIP approaches to EPECs allow to choose equilibria but only allow to solve small case studies.
  - NLP approaches to EPECs allow to solve larges case studies but only yield a local solution.

Approximation of Bilevel Equilibria by Single-Level Equilibria

- We propose an **approximation scheme** of bilevel equilibria (EPECs) using only single-level equilibria (alternative version as a QCP) which allows us to reduce computational time by two orders of magnitude.
- Approximation based on results of (Wogrin et al., 2012 MPB).

Approximation Scheme:

1. Solve the **single-level equilibrium** model, assuming **Cournot** behavior in the market. This yields capacity decisions x.
2. Fix the **capacity decisions** x to values of the previous step.
3. Solve the **single-level equilibrium** model again but this time with **strategic spot market behavior** θ which yields market prices p, demand d and production decisions q.

Small Example
Application of Approximation Scheme

- **Data**: 2 GENCOs, 1 technology, 1 year, 6 load periods, θ=0.3

Actual solution of bilevel problem:

Load period	1	2	3	4	5	6
Production [MW]	13.67	13.67	11.66	13.67	13.24	9.19
Prices [Euro/MWh]	291.82	117.81	54.21	87.96	56.27	50.99

Approximation after first step:

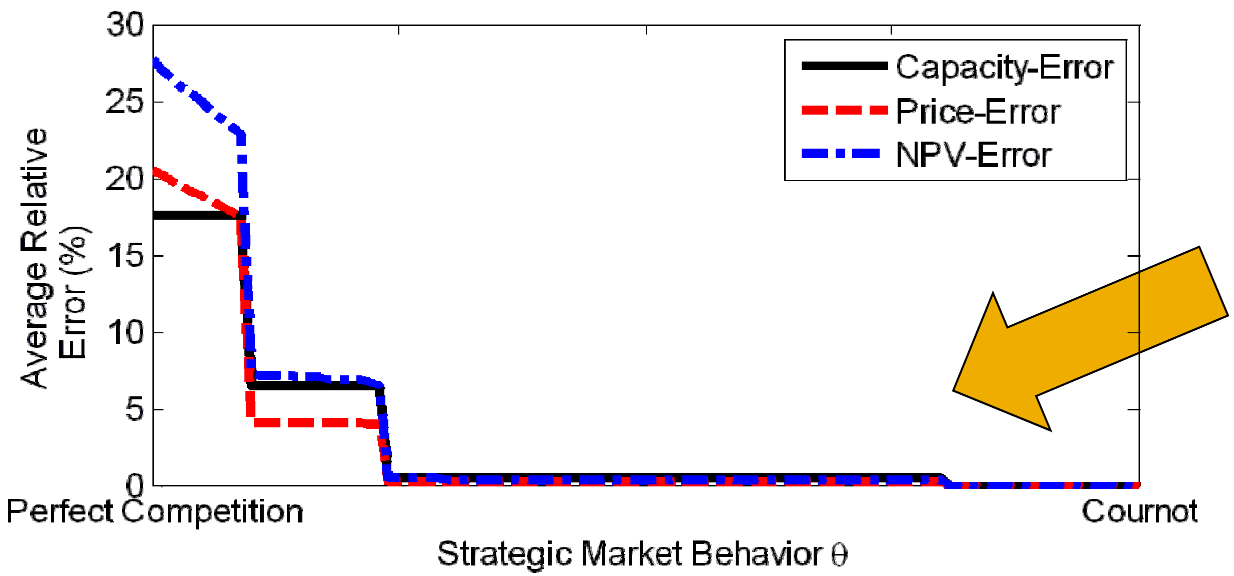
Load period	1	2	3	4	5	6
Production [MW]	13.74	13.74	8.94	12.87	10.15	7.05
Prices [Euro/MWh]	291.19	117.18	77.88	94.94	83.14	69.64

Approximation after final step:

Load period	1	2	3	4	5	6
Production [MW]	13.74	13.74	11.66	13.74	13.24	9.19
Prices [Euro/MWh]	291.19	117.18	54.21	87.32	56.27	50.99

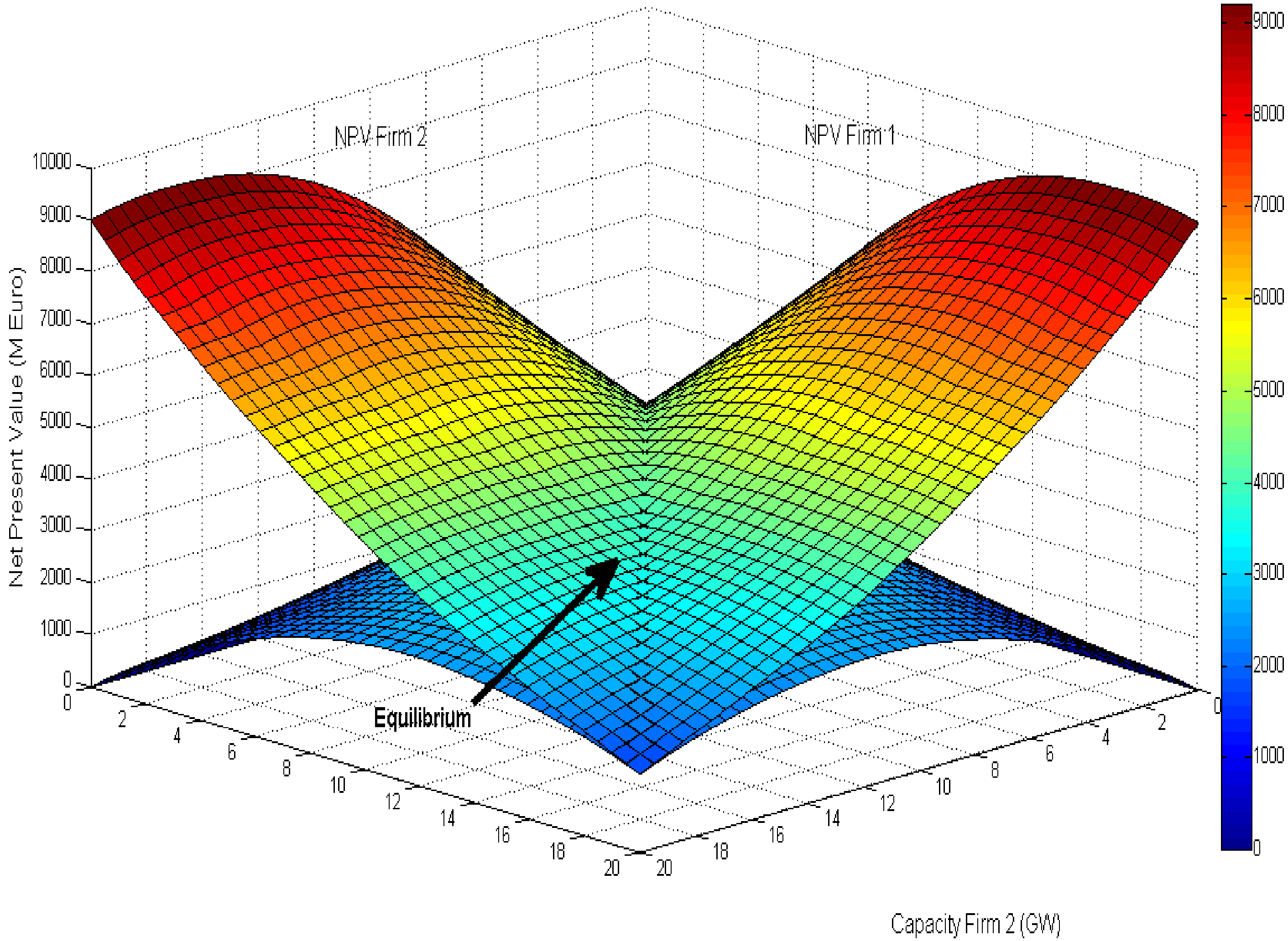
Small Example
Results of Approximation Scheme

- Approximation works **very well** for the small case study
  - Relative error in capacities 0.5%; maximum relative error in prices 0.7%; relative error in production decisions is 0% in non-binding load periods and 0.5% in binding load periods.
- Computational time **two orders of magnitude faster** than standard EPEC method (diagonalization).
- A **sensitivity analysis of θ** and its impact on results yields:

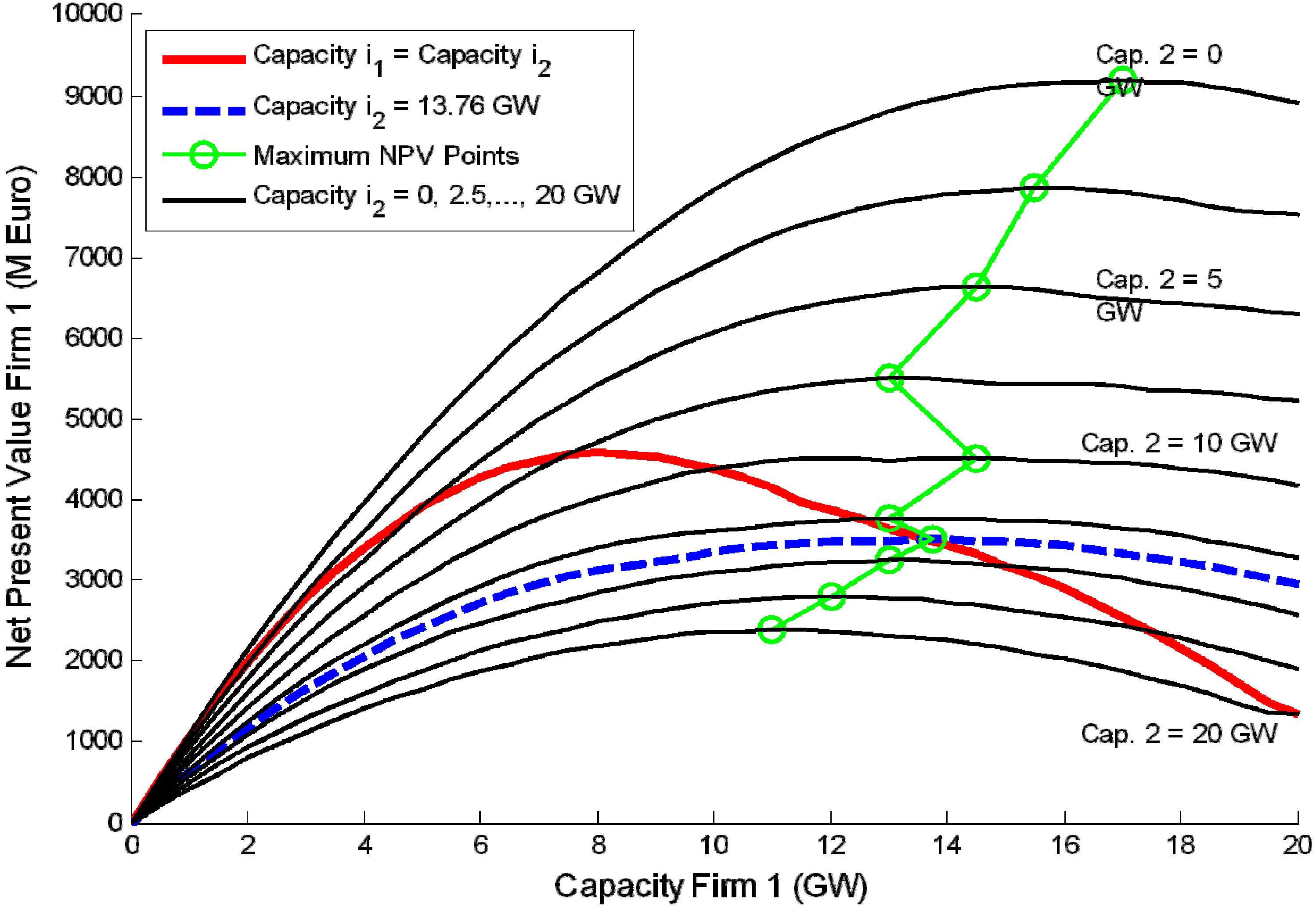


The closer strategic behavior is to Cournot, the better the approximation.

Small Example
NPV Surface of Both GENCOs



Small Example
NPV of GENCO 1 for Fixed Capacity of GENCO 2

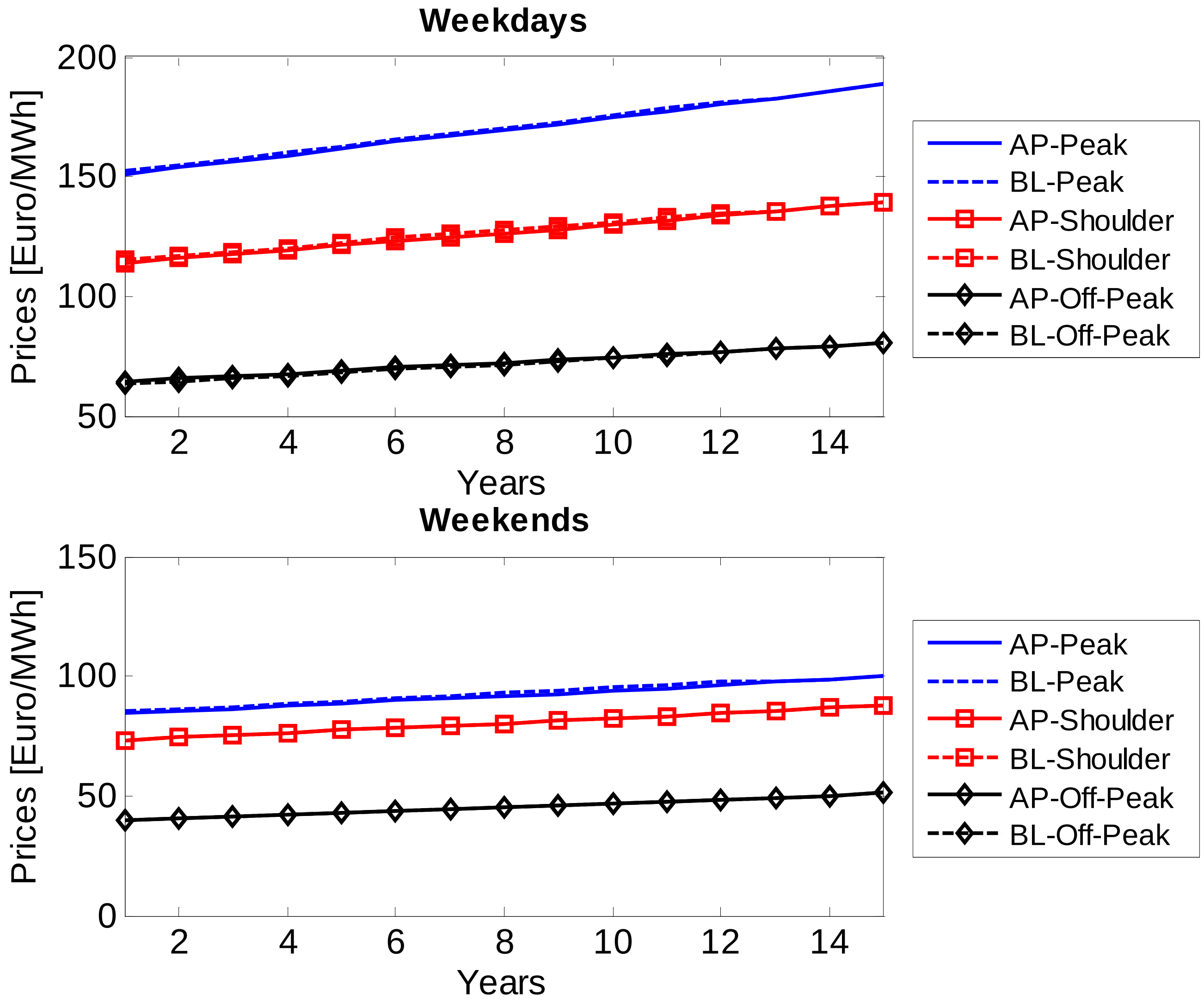


Large-Scale Example of Approximation Scheme

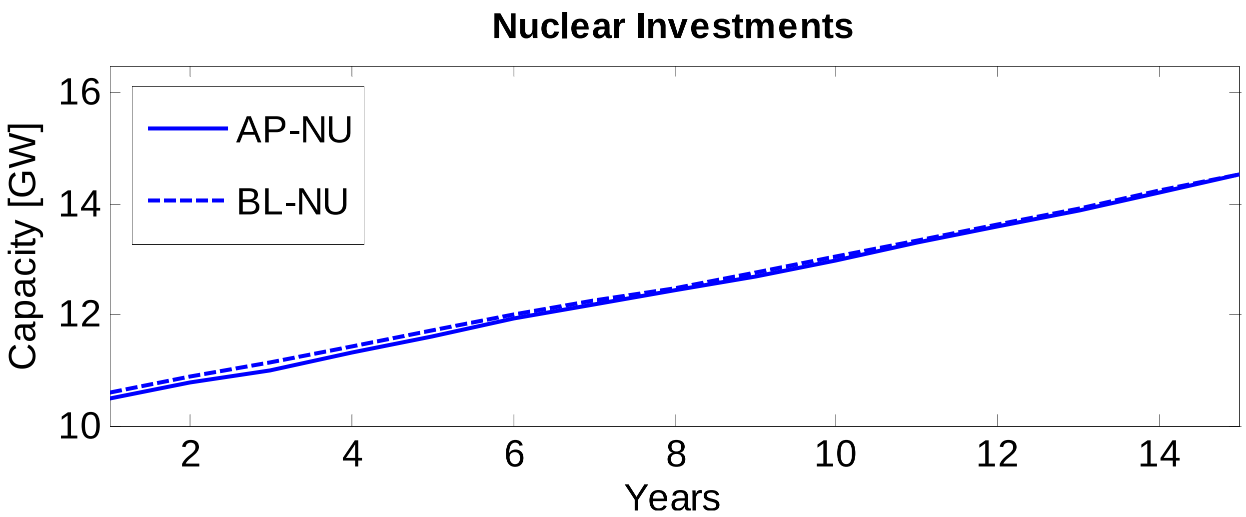
- **Data**: 2 GENCOs, 4 technologies, 6 load periods, 15 years
- The bilevel (BL) equilibrium has been solved using diagonalization (iterative method solving MPECs). Each MPEC has 2400 variables.
- The single-level (SL) model has only 930 variables.
- Approximation Scheme (AP) takes 0.6 seconds to solve.
- The bilevel (BL) model takes 68 seconds to solve.

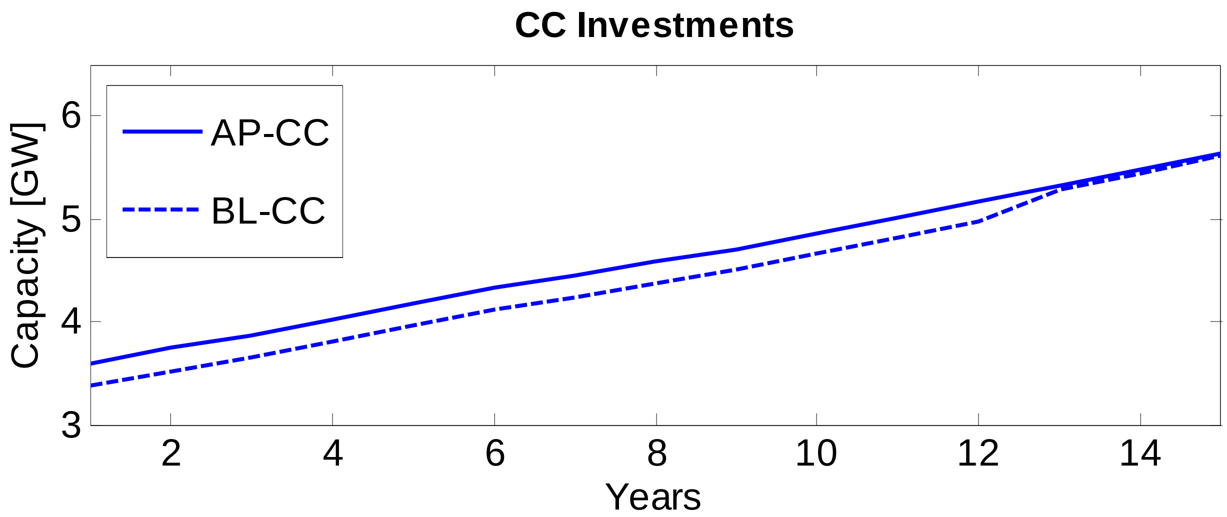
- A **reduction** in computational time of **two orders of magnitude**.

Large-Scale Example of Approximation Scheme
Results of Prices



Large-Scale Example of Approximation Scheme
Results of Capacity Investments

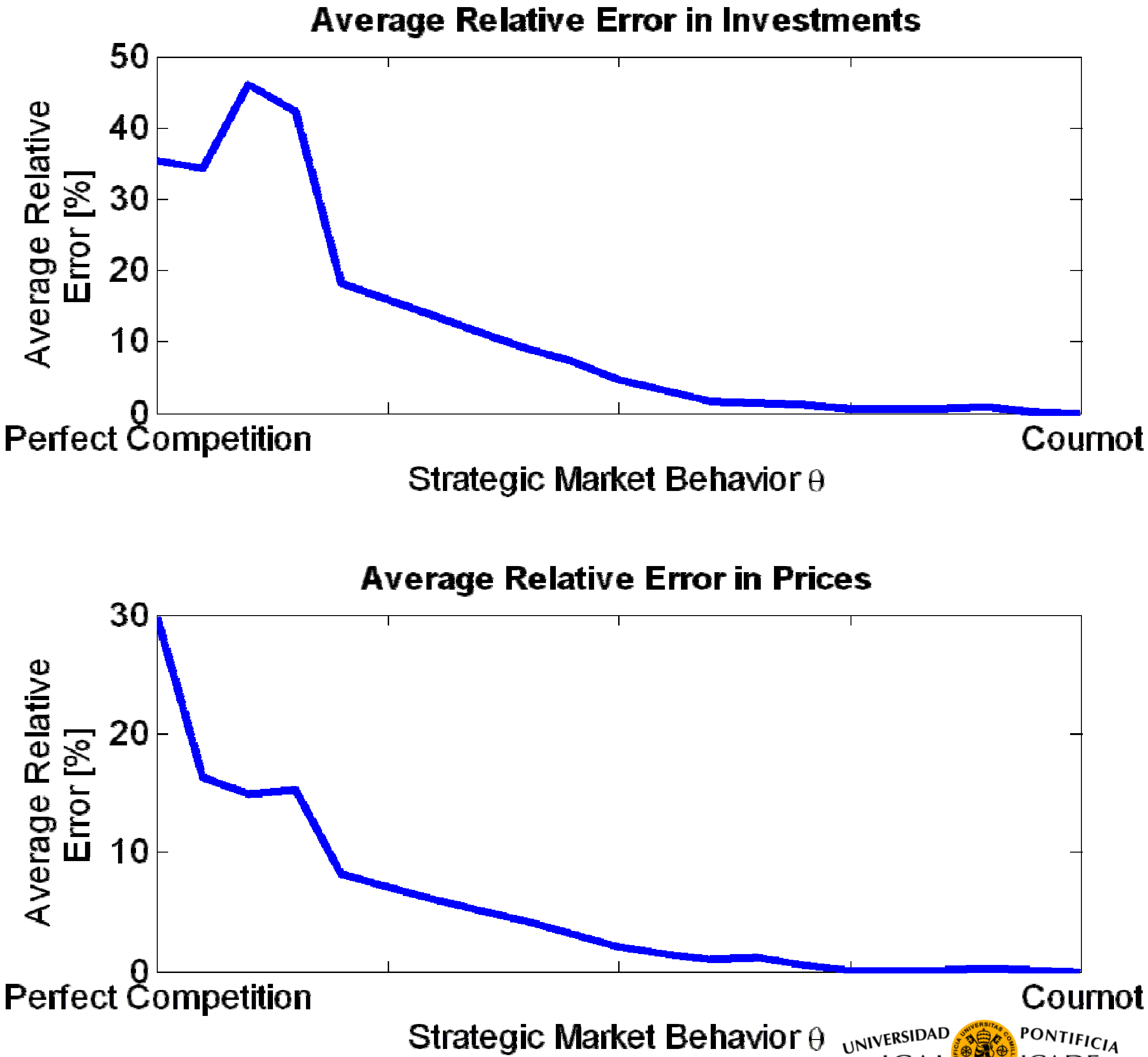




Large-Scale Example of Approximation Scheme Sensitivity Analysis - Relative Errors

- **Data**: 2 GENCOs, 4 technologies, 6 load periods, 15 years

- These **results** are **confirmed** for large-scale examples:



The AP works very well when close to Cournot.

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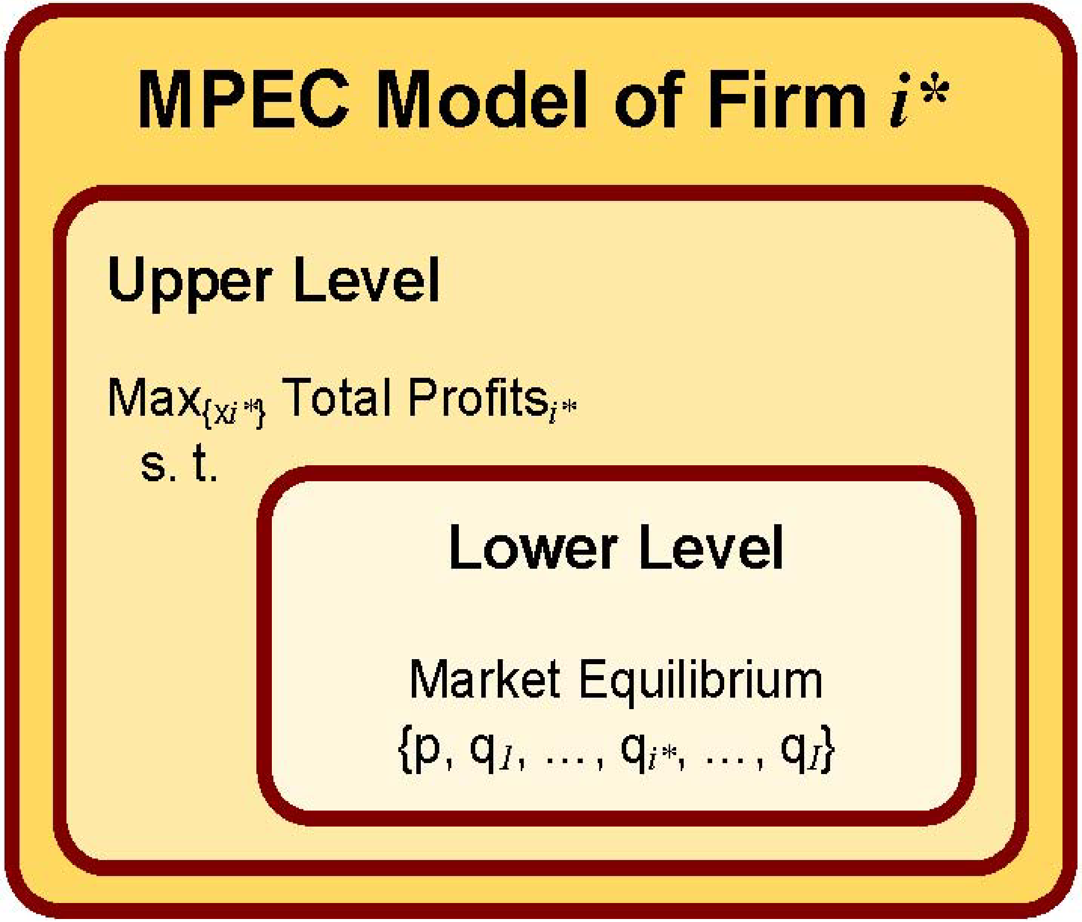
Additional Case Studies

Graphic Representation of BILEVEL Optimization Model (BOM)

- In the **upper level** investment decisions of company i* are taken while the competitors' investments are considered fixed.

- The **lower level** corresponds to the conjectured price response market equilibrium problem.



Overview of Bilevel Optimization Models (BOM)

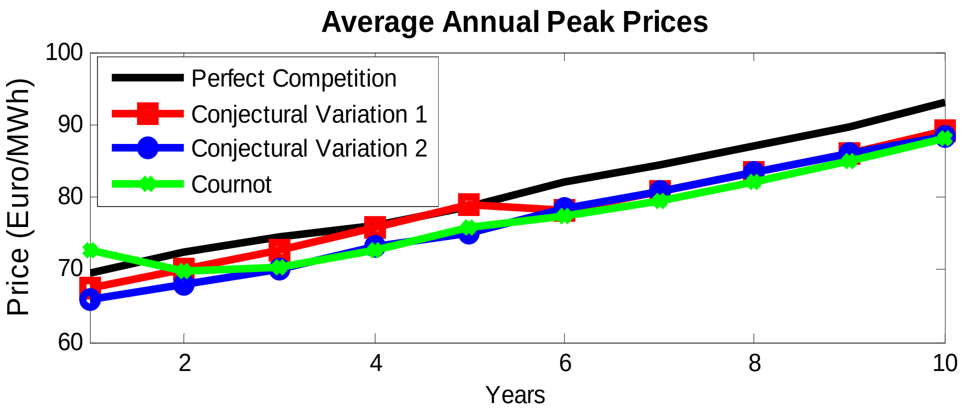
- Bilevel Optimization Models (BOM) **assist one GENCO** in particular to take generation expansion decisions.
- The **upper level** corresponds to the **investment stage** and the **lower level** represents the **market equilibrium** (production stage).
- This type of model is usually formulated as a Mathematical Program with Equilibrium Constraints (**MPEC**).
- This model is also an intermediate step on the way to EPECs.

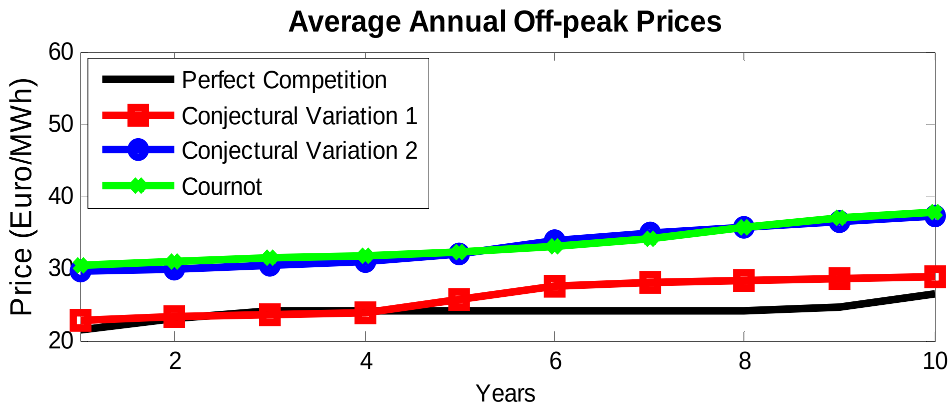
- From numerical examples it becomes apparent that the **strategic spot market behavior** has a great **impact** on **investment** decisions.

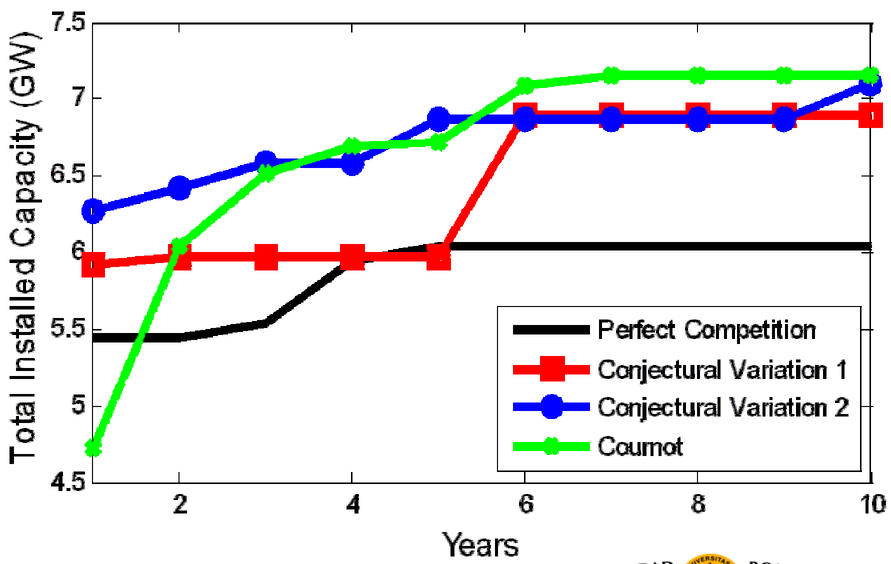
MPEC Case Study
Impact of Market Behavior

- **Data**: 3 GENCOs, 4 technologies, 6 load periods, 10 years.

Total Profits (M Euro)	
Perfect Competition	12019
Conj. Var. 1	12893
Conj. Var. 2	14854
Cournot	14977

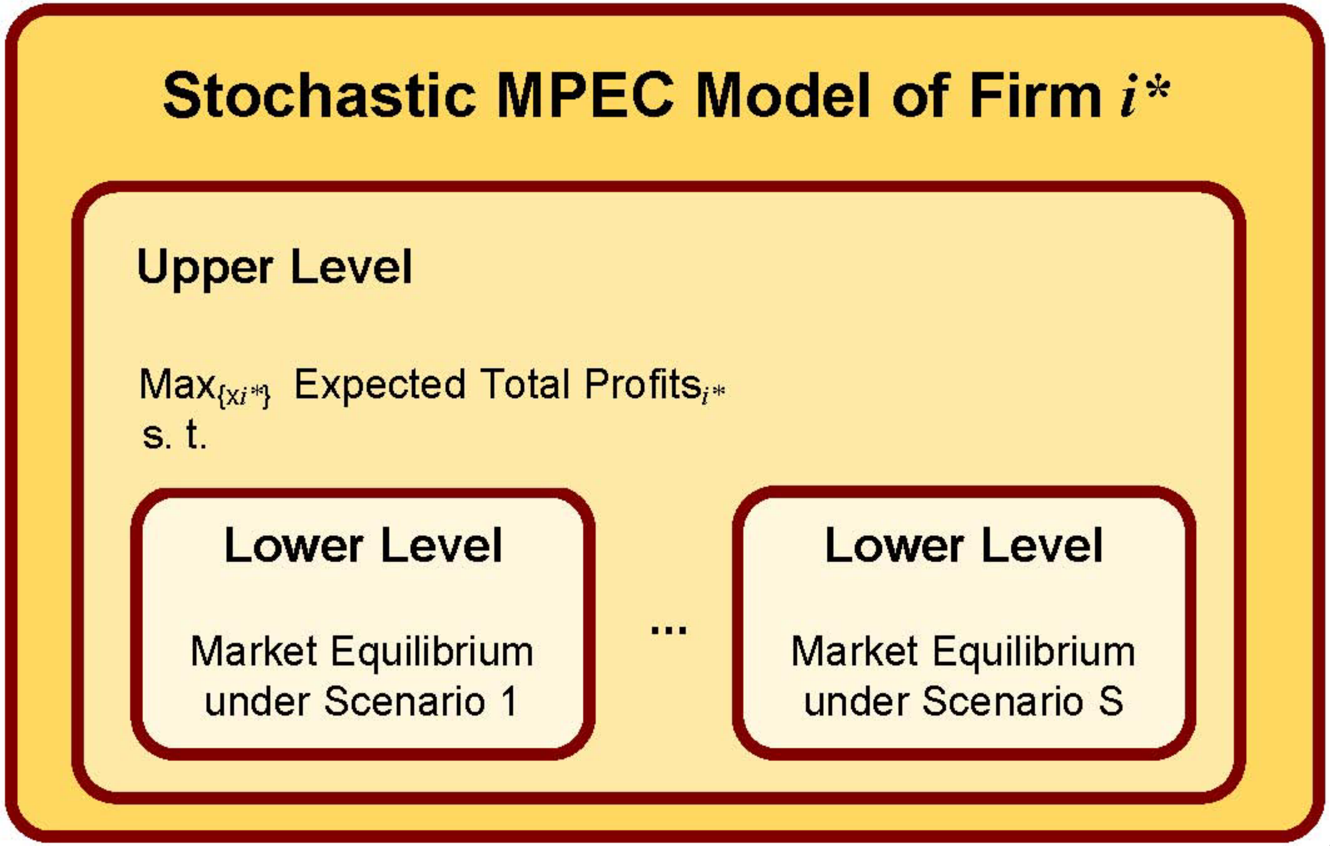






Model Extensions
Stochastic MPEC Framework (SBOM)

- This methodology allows to **cope with stochasticity** in the generation expansion framework (competitors' investment decisions, fuel prices, demand or hydro inflow).



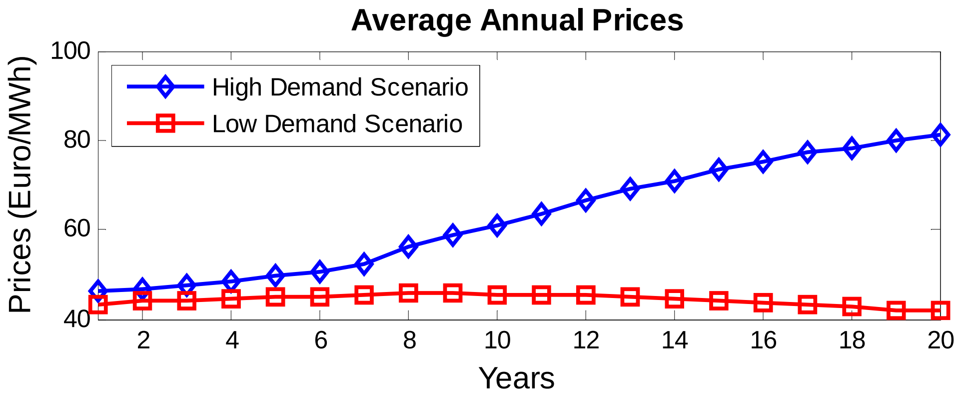
Other Model Extensions

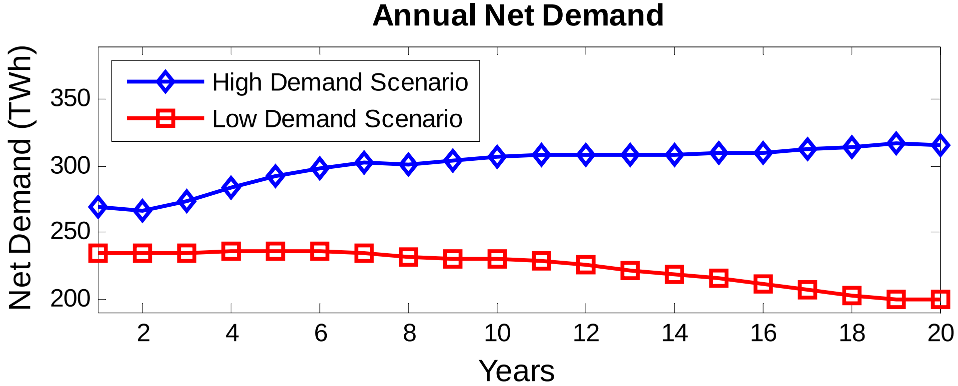
- Introduction of hydro power.
- Discrete investment decisions.
- Introduction of capacity mechanisms (capacity payments)
- Other details: contracts for differences.

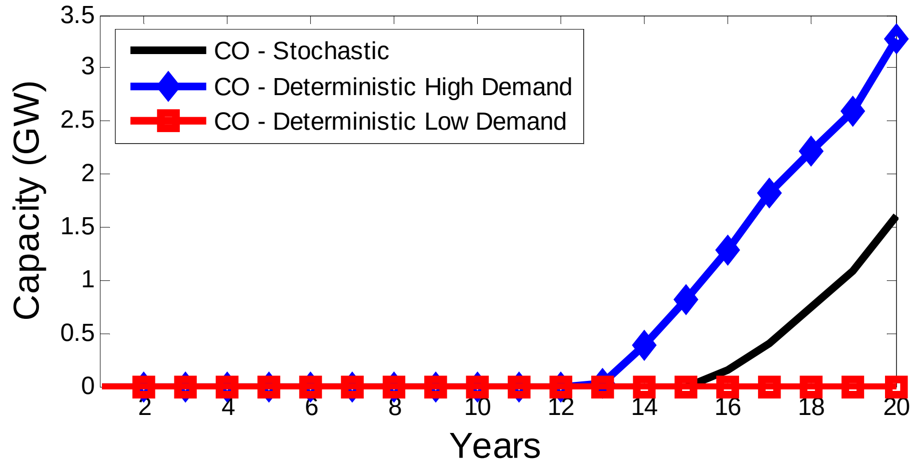
- Introducing some of these extensions into our MPEC models allows us to model **more realistic systems**, as for example the case study of a stylized Spanish system (hydro power, capacity payments, contracts for differences, demand uncertainty).
  - 6 GENCOs, 4 investment technologies, 20 years, 12 load periods per year and 2 demand scenarios were considered.

Stochastic MPEC
Spanish Case Study - Demand Uncertainty

- **Data**: 6 GENCOs, 4 technologies, 12 load periods, 20 years, 2 demand scenarios.







ROI (%)	Stochastic	High Demand	Low Demand
Year 16	0.9	24.0	-22.2
Year 17	2.4	32.7	-27.9
Year 18	2.2	37.1	-32.8
Year 19	4.5	40.3	-31.2
Year 20	3.9	41.7	-34.0

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Overview of Solution Techniques and Case Studies

Overview
Solution Techniques of Thesis

Techniques for MPECs	Techniques for EPECs
Nonlinear Programming	Nonlinear Programming (Diagonalization)
Linearization Methods (MILP)	Linearization Methods (Classification function)
Decomposition Techniques	Complementarity Problem Approach
	Approximation Scheme

Pros and Cons of Solution Techniques

Problem Type	Solution Method	References	Advantages	Disadvantages
MPEC	NLP	Centeno [27]	Realistic cases Small CPU time	Local solution
	MILP	Wogrin [116]	Global solution	Small cases High CPU time
	Decomposition	Kazempour [68]	Realistic cases Reasonable CPU time	Only works under certain circumstances[1]
EPEC	Diagonalization	Hu [62]	Realistic cases Reasonable CPU time Easy (uses MPECs)	Convergence not guaranteed Cannot classify equilibria[2]
	MCP	Gabriel [50]	Realistic cases Reasonable CPU time	Cannot classify equilibria Yields stationary points[3] Did not work in our case
	MILP	Ruiz [105] Wogrin [115]	Classifies equilibria	Small cases High CPU time Yields stationary points
	Approximation	Wogrin [117]	Realistic cases Small CPU time	Only works under certain circumstances[4]

Summary of Thesis Case Studies

Model	Problem Type	Goal of Case Study	Case Study Section	Size i	j	y	l	s	Solution Method	CPU Time
BBEM	EPEC	Theoretical Analysis	3.3.4	2	1	1	2		Analytical[1] (Diagonalization)	-
BBEM	EPEC	Theoretical Analysis	3.4.2	2	1	1	20		Analytical[1] (Diagonalization)	-
BOM	MPEC	Study Impact of θ on Investments	4.2.4	3	4	10	6		NLP	6.4 s
SBOM	MPEC	Study Stochasticity of Competitors' Investments	4.3.4	3	2	5	2	3	NLP MILP	0.8 s 11.5 h - 20 h
BEM	EPEC	Classify Multiple Equilibria	5.2.4	2	2	2	2		MILP	10 h - 24 h
BEM	EPEC	Introduce Approximation	5.4.3	2	1	1	6		Diagonalization[2] Approximation	6.5 s - 144 s 0.5 s
SBOM	MPEC	Large-Scale Example of Spanish System	7.1	6	4	20	12	2	NLP	7 min
SBOM	MPEC	Discrete Capacity Decisions	7.2	3	2	5	2	3	NLP MILP	0.8 s 20 h
BEM	EPEC	Large-Scale Validation of Approximation	7.3	2	4	15	6		Approximation Diagonalization[3]	0.6 s 68 s

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Conclusions

Conclusions (I)

- From the **comparison** of single and bilevel models
  - Independent of the strategic spot market behavior, the **bilevel** model always **yields Cournot capacities** (Theorem and Prop.).
  - These findings underline that **bilevel** models yield **more realistic** results than single-level models.
  - Thus bilevel models could be useful to evaluate the effect of alternative market designs for mitigating market power.
  - Under certain circumstances **single-level and bilevel models** indeed yield the **same result.**

Conclusions (II)

- From the **numerical MPEC analyses**

  - The **strategic spot market behavior** has a great **impact** on **investment** decisions and the optimal capacity **mix**.

  - **NLP methods** for MPEC models can only guarantee **local solutions**, however, these methods allow for large-scale problem instances to be solved.

  - **MIP methods** yield a **global solution**, however, at the cost of only being able to solve moderately-sized problems.

Conclusions (III)

- From the **numerical EPEC analyses**
  - Generation Expansion EPECs can have **multiple equilibria**, each yielding a different optimal technology mix.
  - MIP approaches allow us to **classify the equilibria** in order to explore the solution space of the EPEC.
  - **Diagonalization** allows to **solve larger problems** but only a **local solution** is obtained.
  - The proposed **approximation scheme** works very well when strategic behavior is **closer to Cournot** and is two orders of magnitude faster than standard EPEC methods.

Publications

1. S. Wogrin, B. F. Hobbs, D. Ralph, E. Centeno, and J. Barquín. **Open versus closed loop capacity equilibria in electricity markets under perfect and oligopolistic competition**. *Mathematical Programming (Series B),* 2012, accepted.
2. E. Centeno, S. Wogrin, A. López-Peña, and M. Vázquez. **Analysis of investments in generation capacity: A bilevel approach**. *Generation, Transmission Distribution, IET*, 5(8):842-849, 2011.
3. S. Wogrin, E. Centeno, and J. Barquín. **Generation capacity expansion in liberalized electricity markets: A stochastic MPEC approach**. *IEEE Transactions on Power Systems*, 24(4):2526-2532, 2011.
4. S. Wogrin, J. Barquín, and E. Centeno. **Capacity expansion equilibria in liberalized electricity markets: An EPEC approach**. *IEEE Transactions on Power Systems*, 28(2):1531 - 1539, 2012.
5. S. Wogrin, E. Centeno, and J. Barquín. **Generation capacity expansion analysis: Open loop approximation of closed loop equilibria**. *IEEE Transactions on Power Systems*, PP(99), 2013.

Future Research

- **Improvement of the formulation of bilevel models**
  - Model risk aversion, a CO_2 emissions market, pump or other storage facilities, the electricity network, reliability options, uncertainty in EPEC models etc.
- **Theoretical analysis**
  - Address the issue of existence and uniqueness of bilevel models; explore under what *a priori* conditions the active sets of single-level and bilevel generation expansion equilibria coincide.
- **Computational improvements**
  - Extend the standard Benders decomposition to tackle the non-convex MPEC problem with binary variables in the subproblem.
- **Development of new generation expansion models**
  - Investigate games with endogenous conjectural variations.
  - In a system with a high penetration of renewable energy sources, other more technical details of the market.

Thank you for your attention!

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