The Darío Bacas goniobarimeter: building a balance based on properties of the cycloid

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Abstract
In this article we describe and build a model of a historical weighing device proposed by the Spanish engineer Darío Bacas in the second half of the 19th century. The balance was named 'goniobarimeter' by its inventor, and the weighing principle is based on a curious, and not very well known, property of the cycloid. The simplicity of the design makes it especially suitable for a hands-on educational project with pre-university physics students.

Historical introduction

Nowadays, the ready availability of modern electronic weighing apparatus has in most cases replaced the old balances in stores and laboratories. However, weighing devices based on mechanical equilibrium undoubtedly still have a historical, educational and scientific interest. In this short article we describe the operation and building of a model of one of these historical balances. Our present work could be interesting for educational purposes, as a hands-on project for pre-university physics students, as well as for science museums.

The original design of this device was due to the Spanish engineer Darío Bacas Montero (1845–1913) who named it the 'Goniobarímetro' (goniobarimeter) [1]. With this name he meant (gonio-bari-metro: angle-weight-meter) a balance that indicated on a graduated disk a lean angle proportional to the weight under measurement. This feature is common to several weighing devices; the peculiarity of the Bacas design is that it is based on the pure geometrical properties of the cycloid. In this article we summarize the working principle, and describe succinctly how to build the apparatus. Historical details on the life of the engineer Bacas can be found in [2], while a more exhaustive account of the physics behind the goniobarimeter, as well as a more detailed step-by-step building guide, can be found in [3, 4] (all in Spanish).

The physics of the goniobarimeter

Figure 1, which is an adaptation of the original Bacas illustration [1], shows a schematic representation of the goniobarimeter. The main part is a seesaw AB that is free to rotate around a fixed point O. From the point B hangs the weight P under measurement. At the other end of the seesaw beam a counterweight guiding part ATH is rigidly fixed. This part guides the counterweight Q by means of a string that gradually unfolds as the seesaw leans due to the weight P. E

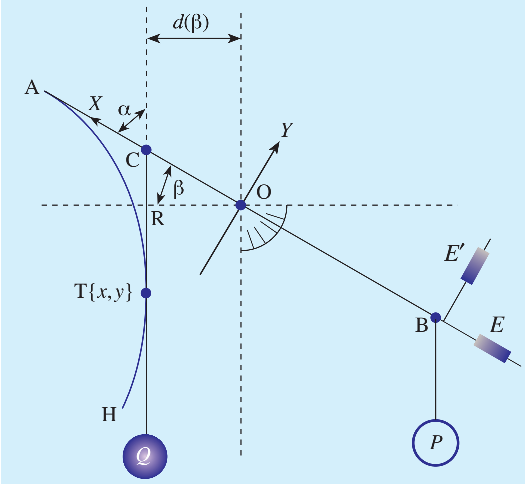

Figure 1. Schematic representation of the Bacas goniobarimeter. P is the weight under measurement, Q is the counterweight and E, E' are equilibration weights.

and E' represent two equilibration weights that are intended to locate the centre of gravity of the moving part at exactly the rotation centre O.

The shape of the director curve ATH is such that the system equilibrates at a lean angle β that is proportional to the weight P. Assuming that there exists a curve $y = y(x)$ satisfying this property, the equilibrium condition is vanishing torque with respect to O, or

$$Q\, d(\beta) = P L \cos\beta, \tag{1}$$

where L is the distance from O to B and $d(\beta)$ is the (minimum) distance from point O to the straight line tangent to the director curve $y = y(x)$ at the point $T = \{x, y\}$ from where the counterweight hangs. If the goniobarimeter works properly, the weight P is proportional to the lean angle β, then equation (1) can be rewritten as:

$$d(\beta) = k\beta \cos\beta, \tag{2}$$

where k represents all the constant parameters. The problem of the goniobarimeter is thus formulated as finding the curve $y = y(x)$ such that the distance d to the origin O of the line tangent to the curve in an arbitrary point satisfies equation (2), with $\beta = \pi/2 - \alpha$, $\tan\alpha$ being the slope of the tangent line. The distance to the origin of a straight line that goes through the point $T = \{x, y\}$ and intercepts the x-axis with angle α is given by:

$$d(\alpha) = x \sin\alpha - y \cos\alpha, \tag{3}$$

where we are implicitly assuming $x > 0, y < 0, \alpha > 0$. Combining equations (2) and (3), and taking into account that the slope of the tangent line to a curve is given by the derivative, one obtains the following nonlinear differential equation for the curve $y = y(x)$

$$y = \frac{\mathrm{d}y}{\mathrm{d}x}\left[x - k\frac{\pi}{2} + k \arctan\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)\right]. \tag{4}$$

Taking the derivative of equation (4), one finds

$$0 = \frac{\mathrm{d}^2 y}{\mathrm{d}x^2}\left[x - k\frac{\pi}{2} + k \arctan\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right) + \frac{k}{1 + (\frac{\mathrm{d}y}{\mathrm{d}x})^2}\frac{\mathrm{d}y}{\mathrm{d}x}\right]. \tag{5}$$

Then, the square bracket at the right-hand-side of equation (5) has to vanish. Combining this condition with equation (4), we finally obtain

$$\frac{\mathrm{d}x}{\mathrm{d}y} = \sqrt{\frac{y + k}{-y}}. \tag{6}$$

One identifies in equation (6) the differential equation of the cycloid. Indeed, using the parametric representation:

$$\begin{aligned} x &= \frac{k}{2}(t - \sin t) \\ y &= -\frac{k}{2}(1 + \cos t) \end{aligned} \tag{7}$$

one can readily verify by simple substitution that the cycloid, equation (7), is the solution of equation (6). Thus, as originally shown by Bacas, the goniobarimeter problem has a solution given by the cycloid generated by a circle of radius $R = k/2$ rolling over the line $y = -k$.

The goniobarimeter can hence be built. The director part ATH guiding the counterweight has to be an arc of a cycloid tangent to the vertical axis at the point $\{x = 0, y = -k\}$, and tangent to the horizontal axis at the point $\{x = k\pi/2, y = 0\}$, see figure 1 (note that the positive x-axis goes from O to A). If we choose for the distance L from O to B the value $L = k\pi/2$, then we have from equations (1) and (2):

$$\frac{P}{Q} = \frac{\beta}{\pi/2}, \tag{8}$$

meaning that the lean angle β will vary between 0 and $\pi/2$ as the weight P under measurement changes from 0 to the value Q of the counterweight.

Building a goniobarimeter

Initially, a goniobarimeter can be built in metal, wood or plastic. For the present prototype we selected wood because of its easy handling and convenience as a working material for a hands-on project with pre-university students. We used a solid wood panel of 10 mm thickness for the platform and the supporting beams, while plywood of 3 mm thickness was chosen for the moving parts. In addition, we employed nylon fishing line and lead fishing weights, a steel nail for the rotation axis, the bottom of a plastic bottle for the weighing pan, a piece of tinplate, bolts, nuts, washers and, finally, a plastic protractor for the measurement of the lean angles. Woodworking tools, like a good bowsaw and sandpaper, and a little patience and workmanship are also required.

Building the counterweight guiding part

First we plot in a piece of paper a cycloid arc with a radius $R = k/2 = 5$ cm, to use as a template. For this project we have used the printout generated by computer plotting software, however, other less sophisticated methods are also possible [5]. Next, we transfer the paper template to the plywood panel. For reasons that will be clarified below, it is convenient to extend (around 1 cm is enough) the cycloid arc a little at the two ends with straight lines: as schematically indicated in figure 2, from A to A′ with a horizontal line and from B to B′ with a vertical line. To complete the counterweight guiding part, we draw on the plywood panel a line more or less parallel to the extended cycloid arc, at a distance of about 3 cm, to give the plywood piece enough rigidity. Then, we carefully saw the part and finish it with sandpaper to the maximum accuracy possible.

The fishing line from where the counterweight is suspended has to rest over the guiding part. It is thus convenient to carve a groove on the surface in which the nylon string could fit comfortably. It is the bottom of this groove that is required to have a shape as close to a cycloid arc as possible. For this reason it is wise to saw the plywood panel not exactly over the cycloid, but approximately 1 mm above it. Finally, we need to fix the nylon string somewhere in the horizontal extension of the cycloid arc, close to the point A′. This can be easily achieved using a staple.

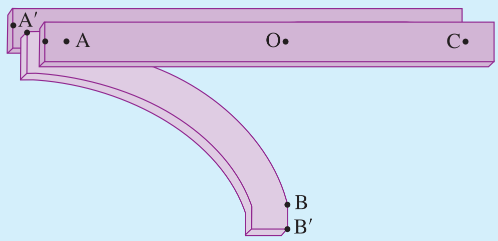

Figure 2. The moving part (seesaw beam) of the goniobarimeter. AB = cycloid arc; O = centre. The centre has to be in the vertical of point B and in the horizontal of point A.

Building the moving part

The counterweight guiding part is enclosed between two identical rectangular strips (see figure 2) cut from the plywood panel. The length of the strips has to be exactly twice the horizontal distance between the points A′ and B′ in the guiding part. For the height, something between 3 and 4 cm is advisable, to give the seesaw beam the required strength and to have enough room for the equilibration device described below.

The point A where the cycloid arc starts has to be placed at the middle of the height of the plywood strips. Then, the counterweight guiding part and the two plywood strips are solidly glued together with a strong wood adhesive. Next, holes are drilled at the point O (figure 2) to place the rotation axis. It is extremely important that the point O is placed at the intersection of the horizontal AA′ and the vertical of BB′ lines. For this reason it was convenient to extend the cycloid arc a little at both ends when building the guiding part. For the present project, for the rotation axis we use a steel nail that crosses the two plywood strips and projects about 2 cm (on each side) outside of the external surfaces. It is important for the rotation axis to be solidly glued to the seesaw

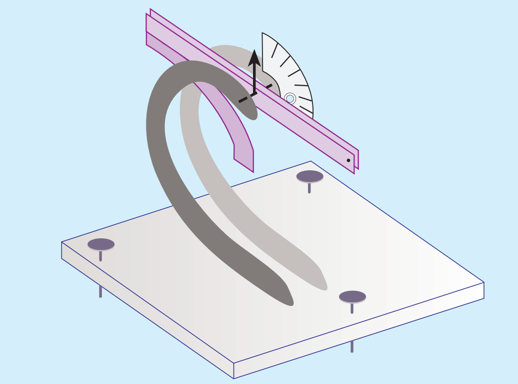

Figure 3. A schematic representation showing the supporting structure with the seesaw beam already in place.

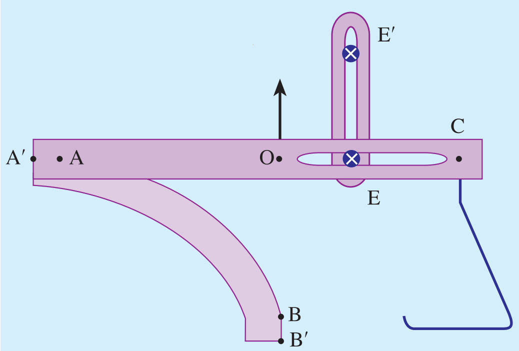

Figure 4. A schematic representation showing the seesaw beam with the equilibration device in place. The positions of E and E′ are adjusted until the centre of mass is located at point O.

beam, and to be as perpendicular as possible to the two parallel planes of the plywood strip exterior surfaces.

The point C (figure 2) has to be placed in the AO line at exactly the same distance from O as the point A. Similarly to the rotation axis at O, a second (smaller diameter) nail is placed crossing the lateral plywood strips at the point C. From this nail at point C the weighing pan will later be suspended, with the help of a hook of metallic wire and more fishing line. Finally, to give rigidity to the moving beam, plywood pieces are glued between the two external plywood strips close to the point C, so that the strips are solidly joined and as parallel as possible.

To finalize the moving part we attach the measurement needle. This needle is solidly joined to the moving seesaw beam and is as perpendicular as possible to the line AOC (figure 2). Our needle was cut from a tinplate piece and glued to one of the lateral plywood strips. If required, to avoid parallax errors, one can later bend the needle a little, bringing it closer to the measurement scale.

Building the supporting beam and the platform

Our supporting structure (figure 3) was designed with the aim of allowing the free movement of the seesaw beam. It consists of two parallel wood arches solidly joined to a flat wood platform. The platform is furnished with three levelling screws, used to adjust the verticality of the plane of rotation. To one of the supporting arches the scale for the lean angle measurement is fixed. For this project we used a simple plastic protractor as the scale.

Of course, it is extremely important that the rotation axis is as horizontal as possible. For this purpose one can use axis bearings whose position can be finely adjusted vertically, see [4] for further details. In addition, the origin of the protractor has to match the position of the rotation axis. This can be achieved with the help of a bolt, nut and washer, and an elongated hole in the protractor. Again, see [4] for further details.

Equilibration

For the goniobarimeter to work properly, the centre of gravity of the moving part has to be located at exactly the position of the rotation axis. To achieve it, we added to the moving seesaw beam a slotted plywood piece EE′ of approximately 6 cm length. The bolt E can be displaced in the line OC, and the weight E′ can be displaced along the piece EE′ perpendicularly to the line OC, as schematically shown in figure 4 [4].

Equilibration is performed with the weighing pan already hanging from the point C. First the moving seesaw beam is maintained horizontally, while the equilibration piece EE′ is moved until the seesaw remains horizontal when freed. Once this is done, the seesaw moving beam is placed and maintained vertically while the weight E′ is moved until the seesaw remains vertical when freed. Correct equilibration can be verified by

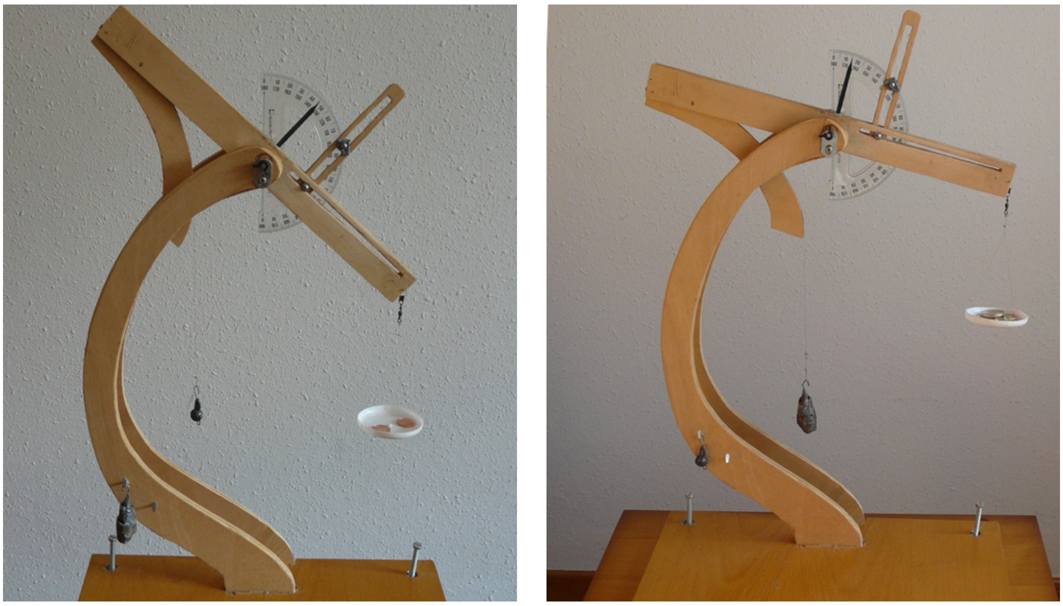

Figure 5. Left panel: the goniobarimeter with a counterweight of 9 g and loaded with two one-euro-cent coins, lean angle is about 45.5°. Right panel: the goniobarimeter with a counterweight of 90 g and loaded with two one-euro coins, lean angle is about 15.0°.

inspecting if the unloaded seesaw stands still at any angle. A good equilibration can be done with more or less difficulty depending on the axis friction, the axis horizontality, the perpendicularity of the axis to the seesaw external surfaces, the rigidity of the assembly, etc.

The working goniobarimeter

As explained by equation (8), the counterweight Q selects the range of weights that can be measured with the goniobarimeter. In our model, with a counterweight of 9 g the goniobarimeter leans 1° g^{-1} per decigram, while with a counterweight of 90 g the goniobarimeter leans 1° g^{-1}. In figure 5 we show two photographs of our model. In the left panel the counterweight is 9 g and the weighing pan is loaded with two one-euro-cent coins. In the right panel the counterweight is 90 g and the weighing pan is loaded with two one-euro coins.

We have verified the principle of the goniobarimeter, i.e., linearity of the lean angle with the load. We placed in the weighing pan of our model an increasing number of identical screws and measured the corresponding lean angle. We show graphically the results of this test in figure 6. Some small deviations are evident for lean angles close to 90°, where the goniobarimeter is less reliable, but otherwise the linearity is excellent. We conclude that our model is working as designed by Darío Bacas in the 19th century.

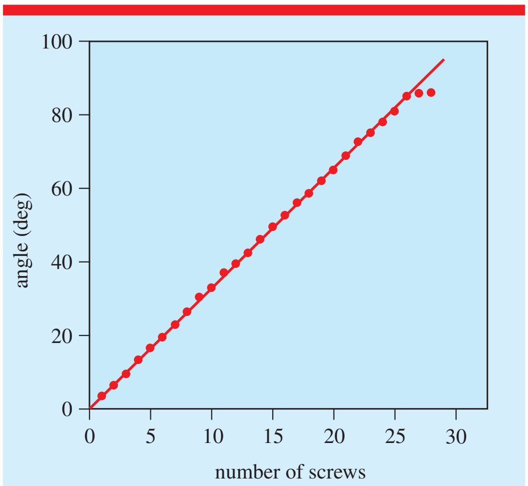

Figure 6. The lean angle measured using the goniobarimeter model of figure 5, when loaded with different numbers of identical screws. The counterweight is 90 g. The linearity is excellent.

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