Optics Communications 281 (2008) 6040–6048

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Optical switching by controlling the double-dark resonances in a N-tripod five-level atom

M.A. Antón [a,*], F. Carreño [a], Oscar G. Calderón [a], S. Melle [a], I. Gonzalo [b]

[a] Escuela Universitaria de Óptica, Universidad Complutense de Madrid, C/Arcos de Jalón s/n, 28037 Madrid, Spain
[b] Facultad de Ciencias Fsicas, Universidad Complutense de Madrid, Ciudad Universitaria s/n, 28040 Madrid, Spain

ARTICLE INFO

Article history:
Received 29 July 2008
Accepted 3 September 2008

PACS:
42.50.Dv
03.67.−a
42.65.−k

Keywords:
Non-linear optics
Optical switching
Dark resonances

ABSTRACT

We have investigated the optical switching in a five-level atom in a novel configuration of electromagnetically induced transparency. This N-tripod type level scheme combines the attractive features of cross-phase modulation appearing in N-type atoms with the ability to slow light pulses associated with tripod atoms. The addition of a new driving field to the usual tripod configuration allows to control the double-dark resonances which appear in the four-level tripod system and thus enables to manipulate the probe absorption and dispersion properties. We have studied the temporal dynamics of two pulses, a probe pulse and a switch propagating pulse through the sample. In the presence of the switching field, a deep in the absorption at resonance due to one-photon electromagnetically induced transparency appears and the atomic system is transparent to the probe field, which propagates at a very small group velocity. By tuning the fields, one of the usual double-dark resonances appearing in tripod system can be controlled (Stark-shifted) and the medium, which is transparent in the absence of the control field, will become highly absorptive. The linear and cross-phase modulation susceptibilities have been calculated and we predict the possibility to realize two-photon switching and giant cross-phase modulation. Finally we address the question about the generation of an entangled coherent state and we show that the giant cross-phase modulation provided by this N-tripod atomic system can be used for realizing polarization quantum phase gates.

© 2008 Elsevier B.V. All rights reserved.

1. Introduction

Laser field induced quantum interferences have been proven to be crucial in controlling the optical properties of an atomic medium. Both stimulated processes such as light absorption and refraction [1,2] and also spontaneous processes were shown to be altered almost at will due to stimulated extra indistinguishable quantum mechanical pathways [3,4], and many interesting phenomena, such as electromagnetically induced transparency (EIT) [5,6], lasing without inversion (LWI) [7–10], refractive index enhancement without absorption [11–13], and giant non-linearity [14–16] have been predicted and experimentally demonstrated. Although the main features of quantum interference processes are adequately covered by the three-level approximation, there has been considerable interest in coherent effects in multilevel systems because the presence of additional levels increases multifold the possibilities of interference phenomena to exist. Since the quantum interference is usually very sensitive to the two-photon resonance, its perturbation by means of another field allows the control of the optical response. The pioneering work of Schmidt and Imamoglu [17,18] in a N-type four-level atom has opened the possibility of enhance non-linear, non-absorptive, cross-phase modulation of two fields in a medium in which giant Kerr non-linearities in the EIT regime may be obtained. The absorptive counterpart of this system was proposed by Harris and Yamamoto as a quantum switch in a N-four-level scheme that will absorb two photons but not one [19]. In these systems, adding a four atomic level to a Λ system can cause a non-linear interaction between the third field and the probe field so that photons of one light field affects the photons in another field. The experimental verification of this prediction has been carried out recently by Yan et al. and Cheng et al. [20,21] in a four-level atom realized with cold Rb. Such unusual physical effects associated with EIT are believed to be useful for the development of new techniques in quantum optics and photonics. Since light can carry quantum information in the form of single-photon polarization states [22–24], or vacuum plus single-photon superpositions, quantum light switching with single photons and the generation of entanglement of coherent states may also have important applications as a first step towards quantum

* Corresponding author. Tel.: +34 913946855; fax: +34 913946885.
E-mail addresses: antonm@fis.ucm.es (M.A. Antón), ferpo@fis.ucm.es (F. Carreño), oscargc@opt.ucm.es (O.G. Calderón), smelle@fis.ucm.es (S. Melle), igonzal@fis.ucm.es (I. Gonzalo).

0030-4018/$ - see front matter © 2008 Elsevier B.V. All rights reserved.
doi:10.1016/j.optcom.2008.09.014

information processing. Cross-phase modulation between weak signal fields is one of the most important challenges for non-linear optical switching and quantum information in order to achieve quantum logic gates of an optical quantum computer. Numerous quantum schemes have been proposed in order to generate such a kind of coherent superpositions. Cavity quantum electrodynamics seems to be a promising approach to this task [25,26]. However, the strength of the interaction of two single light quanta is too small to enhance the photon–photon interaction so the optical quantum gate operation cannot be efficiently implemented. EIT may afford a strong cross-Kerr effect at the single-photon level, and lead to a deterministic all-optical CNOT implementation. Harris and Yamamoto proved [19] that the total phase shift experienced by a field is limited by the time that the faster of the two fields spends inside the medium. The main hindrance of such schemes is the mismatch between the group velocity of the pulse subject to EIT and the control field which limits considerably the interaction time and reduces the cross-phase modulation. In order to get rid of this bottleneck, strategies to induce EIT for both fields have been developed. In particular, Lukin and Imamoglu [27] proposed an atomic system formed by two kind of atoms, one of them formed by usual N-type atoms and a second kind of the Λ-type. A variety of four-level atomic systems driven by three fields have also shown that the probe absorption can be characterized by double-dark resonances [27]. Such double-dark states can interact and produce large cross-phase modulation (XPM). For example, tripod-type atoms have proved to be robust systems for engineering arbitrary coherent superposition of atomic states [28]. Paspalakis and Knight [29] have analyzed transparency, slow light and parametric generation in a medium of tripod atoms. Petrosyan and Malakian and Petrosyan and Kurizki [30,31] have shown that this system can support large magneto-optical rotation with negligible absorption. They also found a novel regime of extremely efficient non-linear interactions of two multimode single-photon pulses in a tripod atom which allows the propagation of two orthogonally polarized weak fields with the same group velocity. In a related work, Ottaviani et al. [32] and Rebic et al. [33] have pointed out that the large Kerr XPM between the probe and trigger fields enables one to implement a phase gate with a conditional phase shift in the order of π in M-type systems. Very recently, by using an efficient preparation technique in the ^{87}RbD_1 line, large XPM via interacting double-dark resonances in an ideal four-level tripod system has been experimentally demonstrated [34], and new configuration with combined N-type and tripod-type subsystems was proposed to generate large XPM [35].

In this paper, we analyze the feasibility of controlling the double-dark resonances appearing in a five-level N-tripod level scheme with the aim of obtain optical switching and phase gate operations. Specifically, we use a new control field to couple one of the sublevels in the tripod-type atom with another excited level to involve an additional fifth level, thus we are dealing with a N-tripod configuration which exploits some of the properties of the N-type schema, in particular, the possibility of inducing giant non-linearities while modifying the two-photon resonance condition of the two Λ-subsystems of the tripod atom. Our numerical results also show the optical switching properties based on the control of two two-photon resonances in this five-level N-tripod-type atom. The switching mechanism is due to the two-photon resonance in a five-level atom in a novel configuration of EIT. The addition of a new field to the usual tripod configuration allows us to control the double-dark resonances which appear in a four-level tripod system. We will show that quantum interference lead to interesting spectral features. In particular, we will show that one weak field can be used to control another weak field, and vice versa. This new EIT configuration not only enhances the cross-phase modulation, but also allows it to transmit pulses through the atomic medium with small absorption at resonance. Compared to previous tripod or M-schema, it is shown that the flexible pumping arrangements allows for the propagation of the driving and probe pulse with nearly the same group velocity. In view of this, another motivation for this work is to analyze the possibility of a phase gate scheme using the cross-Kerr non-linearities occurring in a N-tripod five-level configuration.

The paper is organized as follows: Section 2 establishes the model, i.e., the Hamiltonian of the system and the evolution equation of the atomic operators assuming the rotating wave approximation. Section 3 is devoted to present the numerical results and to discuss the possibility of switching via the control of the double-dark resonances. In Section 4 we analyze a phase-gating scheme using the considered five-level atom. Finally, Section 5 provides the conclusions.

2. The model

The level scheme for the five-level atom as shown in a N-tripod configuration is shown in Fig. 1, which can be experimentally realized in Rb atoms [35]. The system interacts with a weak probe field of frequency ω_P ($|1\rangle \rightarrow |4\rangle$ transition), a weak trigger field, ω_T ($|3\rangle \rightarrow |4\rangle$ transition), and two strong couplings fields of frequencies ω_{2L} ($|2\rangle \rightarrow |4\rangle$ transition), and ω_{4L} ($|2\rangle \rightarrow |5\rangle$ transition), respectively. The electric field can be written as

$$\vec{E} = \frac{1}{2}\left[\vec{E}_P e^{-i\omega_P t} + \vec{E}_T e^{-i\omega_T t} + \vec{E}_2 e^{-i\omega_{2L} t} + \vec{E}_4 e^{-i\omega_{4L} t} + \text{c.c.}\right], \tag{1}$$

$\vec{E}_{(P,T,2,4)}$ being the amplitudes of the slowly varying field envelopes. The resonant frequencies between the different levels are $\omega_{ij} = \omega_i - \omega_j$, ω_j being the energy (in $\hbar$ unities) of the level $|j\rangle (j = 1 - 5)$. The corresponding atomic detuning for these transitions are $\Delta_P = \omega_{41} - \omega_P$, $\Delta_T = \omega_{43} - \omega_T$, $\Delta_2 = \omega_{42} - \omega_{2L}$, and $\Delta_4 = \omega_{52} - \omega_{4L}$.

Note that this composite system consists of two subsystems; if the atomic level $|3\rangle$ is omitted, the system is reduced to a N-type scheme. On the other hand, if the atomic level $|5\rangle$ is omitted, we have a tripod-like system. When all fields are present the system under consideration can be viewed as two adjacent Λ-N-type systems, one involving the probe and control fields Ω_P, and Ω_2, respectively, while other involving the trigger and control fields Ω_T, and Ω_2, while sharing the coupling field Ω_4.

The Hamiltonian of the system under the dipole and rotating-wave approximation can be written as

$$\begin{aligned} H = {} & \hbar[(\Delta_P - \Delta_2)\sigma_{22} + (\Delta_P - \Delta_T)\sigma_{33} + \Delta_P\sigma_{44} + (\Delta_4 + \Delta_P - \Delta_2)\sigma_{55}] \\ & - \hbar[(\Omega_P\sigma_{41} + \Omega_T\sigma_{43} + \Omega_2\sigma_{42} + \Omega_4\sigma_{52} + \text{Adj.}], \end{aligned} \tag{2}$$

where $\sigma_{mn} = |m\rangle\langle n|$ are the usual Pauli matrices and Ω_m stands for the Rabi frequencies which are defined in terms of the dipole moment μ_{ij} between the atomic levels $|i\rangle$ and $|j\rangle$ and the amplitudes

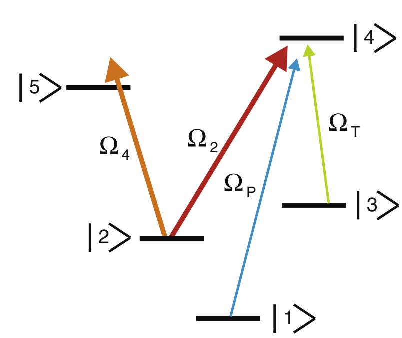

Fig. 1. Schematic diagram of a N-tripod atom driven by a probe (Ω_P) and trigger (Ω_T) fields and two control fields Ω_2 and Ω_4.

of the electric field applied to this transition E_k, as $\Omega_k = \mu_{ij} E_k / 2\hbar$, $k = 2, 4, P, T$ and $i, j = 1-5$.

In the interaction picture and the rotating-wave approximation, the equations of motion for the density matrix elements read

$$
\begin{aligned}
\frac{\partial\rho_{55}}{\partial t} &= -\gamma_{52}\rho_{55} - \gamma_{51}\rho_{55} + \mathrm{i}\Omega_4\rho_{52} - \mathrm{i}\Omega_4\rho_{25},\\
\frac{\partial\rho_{44}}{\partial t} &= -(\gamma_{41}+\gamma_{42}+\gamma_{43})\rho_{44} + \mathrm{i}\Omega_{\mathrm{P}}(\rho_{14}-\rho_{41}) + \mathrm{i}\Omega_2(\rho_{24}-\rho_{42})\\
&\quad + \mathrm{i}\Omega_{\mathrm{T}}(\rho_{34}-\rho_{43}),\\
\frac{\partial\rho_{33}}{\partial t} &= -(\gamma_{32}+\gamma_{31})\rho_{33} + \gamma_{43}\rho_{44} + \mathrm{i}\Omega_{\mathrm{T}}(\rho_{43}-\rho_{34}),\\
\frac{\partial\rho_{22}}{\partial t} &= -\gamma_{21}\rho_{22} + \gamma_{42}\rho_{44} + \gamma_{52}\rho_{55} + \gamma_{32}\rho_{33} + \mathrm{i}\Omega_2(\rho_{42}-\rho_{24})\\
&\quad + \mathrm{i}\Omega_4(\rho_{52}-\rho_{25}),\\
\frac{\partial\rho_{41}}{\partial t} &= -(\gamma_{41}+\mathrm{i}\Delta_P)\rho_{41} - \mathrm{i}\Omega_{\mathrm{P}}(\rho_{44}-\rho_{11}) + \mathrm{i}\Omega_{\mathrm{T}}\rho_{31} + \mathrm{i}\Omega_2\rho_{21},\\
\frac{\partial\rho_{42}}{\partial t} &= -(\gamma_{42}+\mathrm{i}\Delta_2)\rho_{42} - \mathrm{i}\Omega_2(\rho_{44}-\rho_{22}) + \mathrm{i}\Omega_{\mathrm{P}}\rho_{12} + \mathrm{i}\Omega_{\mathrm{T}}\rho_{32} - \mathrm{i}\Omega_4\rho_{45},\\
\frac{\partial\rho_{43}}{\partial t} &= -(\gamma_{43}+\mathrm{i}\Delta_{\mathrm{T}})\rho_{43} - \mathrm{i}\Omega_{\mathrm{T}}(\rho_{44}-\rho_{33}) + \mathrm{i}\Omega_{\mathrm{P}}\rho_{13} + \mathrm{i}\Omega_2\rho_{23},\\
\frac{\partial\rho_{32}}{\partial t} &= -[\gamma_{32}+\mathrm{i}(\Delta_2-\Delta_{\mathrm{T}})]\rho_{32} - \mathrm{i}\Omega_2\rho_{34} + \mathrm{i}\Omega_{\mathrm{T}}\rho_{42} - \mathrm{i}\Omega_4\rho_{35},\\
\frac{\partial\rho_{21}}{\partial t} &= -[\gamma_{21}+\mathrm{i}(\Delta_{\mathrm{P}}-\Delta_2)]\rho_{21} - \mathrm{i}\Omega_{\mathrm{P}}\rho_{24} + \mathrm{i}\Omega_2\rho_{41} + \mathrm{i}\Omega_4\rho_{51},\\
\frac{\partial\rho_{54}}{\partial t} &= -[\gamma_{54}+\mathrm{i}(\Delta_4-\Delta_2)]\rho_{54} + \mathrm{i}\Omega_4\rho_{24} - \mathrm{i}\Omega_{\mathrm{P}}\rho_{51} - \mathrm{i}\Omega_2\rho_{52} - \mathrm{i}\Omega_{\mathrm{T}}\rho_{53},\\
\frac{\partial\rho_{51}}{\partial t} &= -[\gamma_{51}+\mathrm{i}(\Delta_{\mathrm{P}}+\Delta_4-\Delta_2)]\rho_{51} + \mathrm{i}\Omega_4\rho_{21} - \mathrm{i}\Omega_{\mathrm{P}}\rho_{54},\\
\frac{\partial\rho_{52}}{\partial t} &= -[\gamma_{52}+\mathrm{i}\Delta_4]\rho_{52} + \mathrm{i}\Omega_4\rho_{22} - \mathrm{i}\Omega_4\rho_{55} - \mathrm{i}\Omega_2\rho_{54},\\
\frac{\partial\rho_{53}}{\partial t} &= -[\gamma_{53}+\mathrm{i}\Delta_{\mathrm{T}}]\rho_{53} + \mathrm{i}\Omega_4\rho_{23} - \mathrm{i}\Omega_{\mathrm{T}}\rho_{54},\\
\frac{\partial\rho_{31}}{\partial t} &= -[\gamma_{31}+\mathrm{i}(\Delta_{\mathrm{P}}-\Delta_{\mathrm{T}})]\rho_{31} - \mathrm{i}\Omega_{\mathrm{P}}\rho_{34} + \mathrm{i}\Omega_{\mathrm{T}}\rho_{41},
\end{aligned} \tag{3}
$$

where $\gamma_{ji} j = 1, 2, 3, 4, 5$ are the spontaneous decay rates between the level $|j\rangle$ to levels $|i\rangle$, respectively, and $\hat{\gamma}_{ij}$ are the dephasing rates $\hat{\gamma}_{ij} = (1/2)\sum_l \gamma_{il} + (1/2)\sum_l \gamma_{jl}$. We assume the closing condition $\rho_{55} + \rho_{44} + \rho_{33} + \rho_{22} + \rho_{11} = 1$.

Although it is possible to solve numerically Eq. (3), in order to obtain more transparent steady-state solutions to the Bloch equations, we will assume that the intensity of the probe and trigger fields are much weaker than the intensity of both coupling fields, i.e., $|\Omega_{\mathrm{P}}|^2, |\Omega_{\mathrm{T}}|^2 << |\Omega_2|^2, |\Omega_4|^2$ so the population remains mainly in the ground levels $|1\rangle$ and $|3\rangle$, with the populations of the other levels vanishing. If we keep two lowest-order contribution in probe and trigger fields the expressions of ρ_{41} and ρ_{43} can be written as

$$
\begin{aligned}
\rho_{41} = &\frac{\mathrm{i}\Omega_{\mathrm{P}}\left(\Gamma_{21}+\frac{|\Omega_4|^2}{\Gamma_{51}}\right)\rho_{11}^0}{\left(\Gamma_{21}+\frac{|\Omega_4|^2}{\Gamma_{51}}\right)\left(\Gamma_{41}+\frac{|\Omega_{\mathrm{T}}|^2}{\Gamma_{31}}\right)+|\Omega_2|^2}\\
&-\frac{\mathrm{i}\Omega_{\mathrm{P}}|\Omega_{\mathrm{T}}|^2\left(\Gamma_{21}+\frac{|\Omega_4|^2}{\Gamma_{51}}\right)\left(\Gamma_{32}+\frac{|\Omega_4|^2}{\Gamma_{35}}\right)\rho_{33}^0}{\Gamma_{31}\left[\left(\Gamma_{21}+\frac{|\Omega_4|^2}{\Gamma_{51}}\right)\left(\Gamma_{41}+\frac{|\Omega_3|^2}{\Gamma_{31}}\right)+|\Omega_2|^2\right]\left[\Gamma_{34}\left(\Gamma_{32}+\frac{|\Omega_4|^2}{\Gamma_{35}}\right)+|\Omega_2|^2\right]},
\end{aligned} \tag{4}
$$

$$
\begin{aligned}
\rho_{43} = &\frac{\mathrm{i}\Omega_{\mathrm{T}}\left(\Gamma_{32}^*+\frac{|\Omega_4|^2}{\Gamma_{35}^*}\right)\rho_{33}^0}{\left(\Gamma_{32}^*+\frac{|\Omega_4|^2}{\Gamma_{35}^*}\right)\left(\Gamma_{34}^*+\frac{|\Omega_{\mathrm{T}}|^2}{\Gamma_{31}^*}\right)+|\Omega_2|^2}\\
&-\frac{\mathrm{i}\Omega_{\mathrm{T}}|\Omega_{\mathrm{P}}|^2\left(\Gamma_{21}^*+\frac{|\Omega_4|^2}{\Gamma_{51}^*}\right)\left(\Gamma_{32}^*+\frac{|\Omega_4|^2}{\Gamma_{35}^*}\right)\rho_{33}^0}{\Gamma_{31}^*\left[\left(\Gamma_{32}^*+\frac{|\Omega_4|^2}{\Gamma_{35}^*}\right)\left(\Gamma_{34}^*+\frac{|\Omega_3|^2}{\Gamma_{31}^*}\right)+|\Omega_2|^2\right]\left[\Gamma_{41}^*\left(\Gamma_{21}^*+\frac{|\Omega_4|^2}{\Gamma_{51}^*}\right)+|\Omega_2|^2\right]},
\end{aligned} \tag{5}
$$

where $\Gamma_{41} = \hat{\gamma}_{41} + \mathrm{i}\Delta_{\mathrm{p}}$, $\Gamma_{31} = \hat{\gamma}_{31} + \mathrm{i}(\Delta_{\mathrm{P}} - \Delta_{\mathrm{T}})$, $\Gamma_{34} = \hat{\gamma}_{34} - \mathrm{i}\Delta_{\mathrm{T}}$, $\Gamma_{32} = \hat{\gamma}_{32} + \mathrm{i}(\Delta_2 - \Delta_{\mathrm{T}})$, $\Gamma_{51} = \hat{\gamma}_{51} + \mathrm{i}(\Delta_{\mathrm{P}} + \Delta_4 - \Delta_2)$, $\Gamma_{42} = \hat{\gamma}_{42} + \mathrm{i}\Delta_2$, $\Gamma_{35} = \hat{\gamma}_{35} + \mathrm{i}\Delta_{\mathrm{T}}$, and $\Gamma_{21} = \hat{\gamma}_{21} + \mathrm{i}(\Delta_{\mathrm{P}} - \Delta_2)$. ρ_{11}^0 and ρ_{33}^0 are the initial populations of the atomic levels $|1\rangle$ and $|3\rangle$, respectively.

The above expressions include the linear response to the probe fields (terms proportional to Ω_{P} and Ω_{T}) and the XPM between the probe and the trigger field (terms proportional to $\Omega_{\mathrm{P}}|\Omega_{\mathrm{T}}|^2$ and $\Omega_{\mathrm{T}}|\Omega_{\mathrm{P}}|^2$). Now it appears evident that the five-level system can be viewed as two adjacent subsystems, i.e., a tripod system and a N-type system. In fact, if we set $\frac{|\Omega_4|^2}{\Gamma_{51}} = 0$, the linear part of ρ_{41} reduces to the result obtained by Paspalakis et al. for a tripod system [29,33]. On the other hand if we set $\frac{|\Omega_{\mathrm{T}}|^2}{\Gamma_{32}} = 0$, it reduces, to first order in Ω_{P}, to the result obtained by Imamoglu and Smith [17].

The main difference arises from the presence of the Stark-shift-like terms such as $G_{21} = \Gamma_{21} + \frac{|\Omega_4|^2}{\Gamma_{51}}$. As we will see below these terms are very important in order to modify the cross-phase modulation between the probe and trigger field. Thus, the new incorporated transition ($|2\rangle \to |5\rangle$) driven by the field Ω_4 modifies the usual optical response of a tripod atom [33] by introducing an AC Stark-shift for the state $|2\rangle$ giving a non null cross-phase modulation term for the probe and trigger fields even at exact EIT resonance ($\Delta_{\mathrm{P}} - \Delta_2 = 0$ and $\Delta_{\mathrm{T}} - \Delta_2 = 0$).

3. Optical switching via dark resonances

We proceed to examine probe absorption profiles for the five-level atom as a function of the probe detuning Δ_{P} and for different situations of the driving fields. The set of density-matrix Eq. (3) can be easily solved to obtain the susceptibility of the atomic system. In the following analysis, Rabi frequencies, decay constants and detunings are scaled to the decay rate of the upper level γ_{41}. In (Fig. 2) we show the real and imaginary part of the probe susceptibility versus the probe laser detuning Δ_{P} under the resonance condition $\Delta_2 = \Delta_{\mathrm{T}} = 0$. The two absorption peaks in Fig. 2a near $\pm\Omega_4$ form the standard Autler-Townes doublet and corresponds to one-photon absorption from the ground state $|1\rangle$ to the dressed states generated by the field Ω_4. When the control field Ω_{T} is off, an absorption peak appears at line center $\Delta_{\mathrm{P}} = 0$ (dashed line), which corresponds to the non-linear absorption (two-photon absorption in the dressed states), manifested by constructive interference between two excitation paths. Then the probe field is absorbed for all detunings. However, when the control field is switched on ($\Omega_{\mathrm{T}} = 0.5$), a narrow feature emerges at line center where nearly perfect transparency is achieved. Hence, for proper tuning the trigger field Ω_{T} on/off, a probe field can propagate at resonance without appreciable attenuation. The narrow transmission window is accompanied with a steep, dispersive profile, and a correspondingly change from superluminal to subluminal group propagation regime. This can be appreciated from Fig. 2b where the probe dispersion is showed. Thus, the considered five-level system can be used for absorptive switching of one weak field by another weak field.

When the control fields Ω_{T} and Ω_4 are detuned from resonance, the five-level atom presents other interesting features. In this case, the field Ω_4 acts as the switching field. In Fig. 3 we plot the imaginary part of the probe susceptibility versus the probe laser detuning Δ_{P} under the resonance conditions $\Delta_2 = -\Delta_{\mathrm{T}} = 0.25\gamma_{41}$, for the Rabi frequencies $\Omega_2 = \Omega_{\mathrm{T}} = \gamma_{41}$ and two different values of the control field Ω_4. In contrast with the first case, this new configuration clearly shows the special properties of the five-level system considered which combines the properties of the N-type system scheme and those of the tripod scheme.

In the case with $\Omega_4 = 0$, the atomic system behaves as a tripod system, so we recover the EIT profile with a sharp absorption peak, and simultaneously two dark resonances at $\Delta_{\mathrm{P}} = \Delta_2$ and $\Delta_{\mathrm{P}} = \Delta_{\mathrm{T}}$. This behavior was previously found by Paspalakis and Knight in a four-level atom in a tripod configuration [29]. The inspection of Fig. 3 reveals that the spectral properties can be dramatically changed when the Rabi frequency of the control field Ω_4 is different

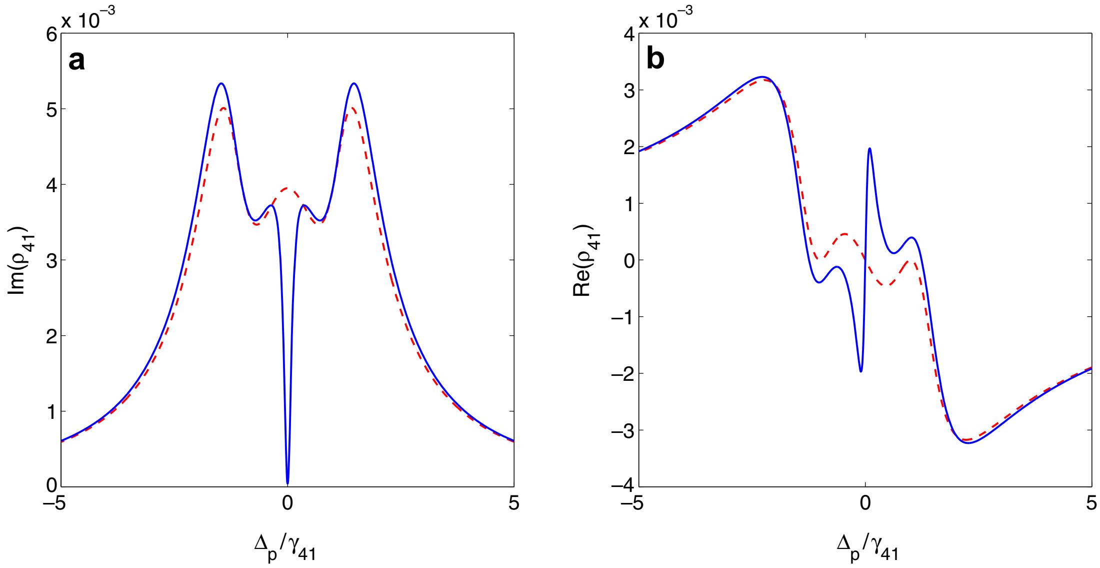

Fig. 2. (a) Absorption spectra $\mathrm{Im}(\rho_{41})$ as a function of the probe detuning for different values of the Rabi frequency of the control field Ω_T. The atomic parameters are $\Delta_2 = \Delta_T = 0$, $\Delta_4 = 4\gamma_{41}$, and $\Omega_2 = \Omega_4 = \gamma_{41}$: $\Omega_T = 0$ (dashed line), $\Omega_T = 0.5\gamma_{41}$ (solid line). (b) Dispersive spectra $\mathrm{Re}(\rho_{21})$ as a function of the probe detuning for the same parameters as in (a).

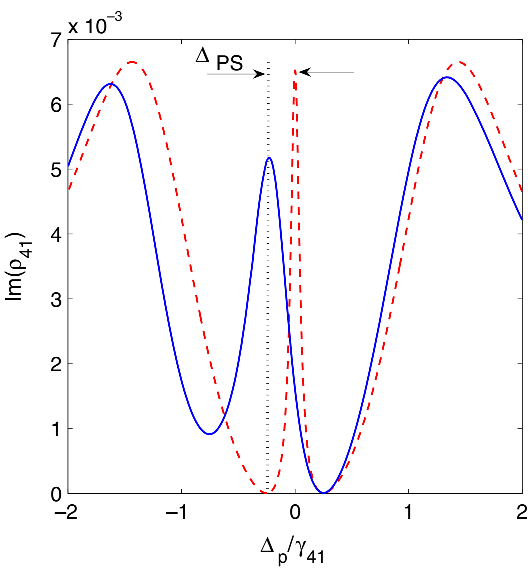

Fig. 3. Absorption spectra $\mathrm{Im}(\rho_{41})$ as a function of the probe detuning Δ_p for different values of the Rabi frequency of the control field Ω_4. $\Omega_4 = 4\gamma_{41}$ (solid line), and $\Omega_4 = 1.5\gamma_{41}$ (dashed line). Other parameters are $\Omega_2 = \Omega_T = 1\gamma_{41}$ and $\Delta_2 = -\Delta_T = 0.25\gamma_{41}$.

from zero. In the vicinity of $\Delta_P = \Delta_2$, the EIT persists because it is caused by the three-level subsystem transitions ($|1\rangle \to |4\rangle$, and $|2\rangle \to |4\rangle$). This atomic subsystem is not affected by the $|3\rangle \to |5\rangle$ transition driven by the control field Ω_4. However there is a substantial difference in the transparency window in the vicinity of $\Delta_P = \Delta_T$. The effect of switching on the control field Ω_4 is to shift the spectrum by an amount equal to Δ_{PS} (see dashed line in (Fig. 3). Thus, for a photon ω_P with a detuning $\Delta_P = \Delta_T$, the medium, which is transparent for $\Omega_4 = 0$, will become highly absorptive for $\Omega_4 \neq 0$. Since the ultranarrow peak appearing at the line center has a subnatural linewidth, a small change in the Rabi frequency of the control field is enough to lead the system from nearly transparency to high absorption, thus acting as an ultra-sensitive switch.

In order to explain the above results, we perform an analytical study of the phenomenon. In the linear regime, the linear part of the ρ_{41} given by Eq. (4) can be approximated to

$$\rho_{41} \simeq \frac{\Omega_P(\Delta_P - \Delta_3)(i\Gamma_c - \widetilde{\Delta}_0)}{\left[(\Delta_P - \Delta_T)(\hat{\gamma}_{41}\Gamma_c + |\Omega_2|^2) + \widetilde{\Delta}_0 G_T\right] + i\left[\Gamma_c G_T - \hat{\gamma}_{41}\widetilde{\Delta}_0(\Delta_P - \Delta_T)\right]}, \tag{6}$$

where

$$\Gamma_c = \hat{\gamma}_{21} + \frac{\gamma_5|\Omega_4|^2}{\Delta_4^2}, \tag{7}$$

$$\widetilde{\Delta}_0 = (\Delta_P - \Delta_T) - \frac{|\Omega_4|^2}{\Delta_4}, \tag{8}$$

$$G_T = |\Omega_T|^2 - \Delta_P(\Delta_P - \Delta_T). \tag{9}$$

The absorption curves displayed in Figs. 2 and 3 are given by $\mathrm{Im}(\rho_{41})$ which may approximate to

$$\mathrm{Im}(\rho_{41}) \simeq \frac{\Omega_P(\Delta_P - \Delta_T)^2\left[\widetilde{\Delta}_0^2 + \Gamma_c(|\Omega_2|^2 + \Gamma_c\hat{\gamma}_{41})\right]}{\left[(\Delta_P - \Delta_T)(\hat{\gamma}_{41}\Gamma_c + |\Omega_2|^2) + \widetilde{\Delta}_0 G_T\right]^2 + \left[\Gamma_c G_T - \hat{\gamma}_{41}\widetilde{\Delta}_0(\Delta_P - \Delta_T)\right]^2}. \tag{10}$$

First, we analyze the case considered in Fig. (2) where $\Delta_2 = \Delta_T = 0$. For the case $\Omega_T = 0$, Eq. (10) reduces to

$$\mathrm{Im}(\rho_{41}) \simeq \frac{\Omega_P\left[\widetilde{\Delta}_0^2 + \Gamma_c(|\Omega_2|^2 + \Gamma_c\hat{\gamma}_{41})\right]}{\left[\widetilde{\Delta}_0\Delta_P + |\Omega_2|^2\right]^2 + \left[\Gamma_c\Delta_P + \hat{\gamma}_{41}\widetilde{\Delta}_0\right]^2}. \tag{11}$$

An inspection of Eq. (11) reveals that absorption is always present at any detuning. In particular, at line center ($\Delta_p = 0$) it is in the order of

$$\mathrm{Im}(\rho_{41}) \simeq \frac{\Omega_P\Gamma_c}{|\Omega_2|^2} = \Omega_P\frac{\gamma_{51}|\Omega_4|^2}{|\Omega_2|^2}. \tag{12}$$

On the other hand, when $\Delta_2 = -\Delta_T \neq 0$, as in the case considered in Fig. 3, it is easy to see from Eq. (10) that absorption is zero if

$$\Delta_{\mathrm{P}} = \Delta_{\mathrm{T}}, \tag{13}$$

$$\left[(\Delta_{\mathrm{P}} - \Delta_2) - \frac{|\Omega_4|^2}{\Delta_4}\right]^2 + \Gamma_{\mathrm{c}}(|\Omega_2|^2 + \Gamma_{\mathrm{c}}\hat{\gamma}_{41}) = 0. \tag{14}$$

The last equation only has real solutions for Δ_{P} if $\Gamma_{\mathrm{c}} = 0$, i. e., if $\gamma_{21} = 0$ and $|\Omega_4| = 0$. In this case, two dark resonances occur at $\Delta_{\mathrm{P}} = \Delta_{\mathrm{T}}$ and $\Delta_{\mathrm{P}} = \Delta_2$. This result explain the two transparent windows appearing in Fig. (3). Moreover when $|\Omega_4| \neq 0$, null absorption persists at $\Delta_{\mathrm{P}} = \Delta_{\mathrm{T}}$ while it presents a minimum at $\Delta_{\mathrm{P}} = \Delta_2 + \frac{|\Omega_4|^2}{\Delta_4}$. Thus, the presence of the control field Ω_4 destroys one of the two dark resonances and shifts it for an amount given by $\frac{|\Omega_4|^2}{\Delta_4}$.

These interesting results could be used to develop an optical switch in which one driving field controls the absorption of another field which oscillates at a different frequency [20]. In order to show the switching characteristics, we solve the propagation equation for the probe field in the slowly varying envelope approximation

$$\frac{\partial \Omega_{\mathrm{P}}(z,t)}{\partial z} + \frac{1}{c}\frac{\partial \Omega_{\mathrm{P}}(z,t)}{\partial t} = \mathrm{i}\gamma_{41}\alpha_{\mathrm{P}}\rho_{41}, \tag{15}$$

$$\frac{\partial \Omega_{\mathrm{T}}(z,t)}{\partial z} + \frac{1}{c}\frac{\partial \Omega_{\mathrm{T}}(z,t)}{\partial t} = \mathrm{i}\gamma_{41}\alpha_{\mathrm{T}}\rho_{43}, \tag{16}$$

where $\alpha_{(\mathrm{P,T})} = \omega_{(\mathrm{P,T})}\mu^2_{4(1,3)}N/(2\hbar c\epsilon_0\hat{\gamma}_{41})$ are the absorption coefficients, with N the atomic density of the medium. In the linear regime, we easily obtain the steady-state probe field at the medium output

$$\Omega_{\mathrm{P}}^{\mathrm{out}} = \Omega_{\mathrm{P}}^{\mathrm{in}} \mathrm{e}^{-\alpha_{\mathrm{P}} L \mathrm{Im}\left(\frac{\hat{\gamma}_{41}\rho_{41}}{\Omega_{\mathrm{P}}}\right)}, \tag{17}$$

L being the medium length. The normalized absorptive loss of the transmitted amplitude function can be obtained from Eq. (17) as $T \equiv \Omega_{\mathrm{P}}^{\mathrm{out}}/\Omega_{\mathrm{P}}^{\mathrm{in}}$. Fig. 4a shows the dependence of the probe transmission spectrum on the trigger field. Close to the center of the spectrum $\Delta_{\mathrm{P}} \simeq 0$, a low transmission occurs when the trigger field is off whereas a high transmission takes place when this field is switched on. This result is in agreement with the absorption spectrum shown in Fig. 2a. If we denote T_{off} and T_{on} the probe transmission associated with the closed ($\Omega_{\mathrm{T}} = 0$) and the open switch ($\Omega_{\mathrm{T}} \neq 0$), respectively, we may define the contrast function $C \equiv (T_{\mathrm{on}} - T_{\mathrm{off}})/(T_{\mathrm{on}} + T_{\mathrm{off}})$ which measures the efficiency of the switch. In the case shown in Fig. 4a, the contrast value at $\Delta_{\mathrm{P}} = 0$ is around 0.7. The switch can be optimized by means of the Rabi frequency of the control field Ω_2. In fact, the contrast value approaches to unity as the Rabi frequency Ω_2 decreases.

These non-linear optical properties may be used to realize absorptive switching in pulsed regime. For this purpose, we numerically solve the field Eq. (15) and the density matrix Eq. (3). We consider a weak probe field at the medium entrance ($\Omega_{\mathrm{P}}^{\mathrm{in}} << \gamma_{41}$) and we use the same parameters as in Fig. 2, that is, $\Omega_2 = \Omega_4 = \gamma_{41}$, $\Delta_{\mathrm{P}} = \Delta_{\mathrm{T}} = \Delta_2 = \Delta_4 = 0$. The driving field Ω_{T} plays the role of the switching beam, changing its value at the entrance of the medium from zero (switching beam off) to $\Omega_{\mathrm{T}} = 0.5\gamma_{41}$ (switching beam on) during the whole time evolution. Fig. 5 shows the time evolution of the output probe field normalized to the input field, i.e., the transmitted amplitude function, when the control field Ω_{T} is modulated as a train of rectangular pulses. When the control beam is off, a strong absorption peak occurs at $\Delta_{\mathrm{P}} = 0$ (in agreement with dashed line in Fig. 2), then the amplitude of the output field is nearly zero. However, when the control beam is on, the system becomes transparent since a hole appears at line center (in agreement with solid line in Fig. 2).

4. Cross-phase-Kerr non-linearity and quantum gates

A major challenge for quantum information using individual photons as qubits is the implementation of logic operations, which requires efficient non-linear interactions for pairs of photons. Several EIT schemes have been analyzed in order to achieve enhanced non-linear coupling via XPM. The addition of the a new energy level to a three-level structure allows the violation of the strict two-photon resonance condition through the Stark-shift induced by the off resonant field which couples the new transition. If the disturbance of the EIT condition is small, the absorption does not increase much but the non-linearity can be very high [17].

Here we propose a phase-gating scheme using the five-level atom considered in (Fig. 1). In fact, this atom may be considered as a realization of the idea of Lukin and Imamoglu [27] but using a single spice of atoms. This atom, as discussed in Section 1, exhibits double EIT and is an alternative scheme to obtain cross-phase modulation by allowing the system to be tuned to its dark states. In addition, the inclusion of a new off-resonant field enables us to disturb the dark resonance condition, which is essential to induce large cross-phase modulation. Thus, we are interested in exploiting the optical non-linearities occurring in the N-tripod-type system for quantum gate implementations. We consider two optical weak pulses Ω_{P} and Ω_{T}, as probe and trigger fields, which couple the transitions $|1\rangle \to |4\rangle$ and $|3\rangle \to |4\rangle$, respectively. In addition, the same field E_{T} couples the transition $|2\rangle \to |5\rangle$, so $\Omega_4 = \eta\Omega_{\mathrm{T}}$, where $\eta = \mu_{52}/\mu_{43}$. This is necessary in order to have slow group velocity for both probe and trigger fields simulta-

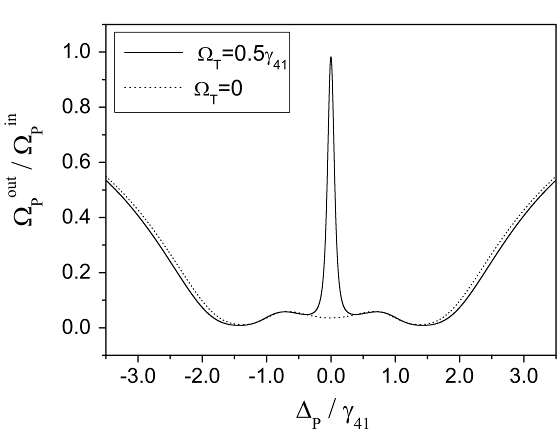

Fig. 4. Transmission spectra of the probe field $T \equiv \Omega_{\mathrm{P}}^{\mathrm{out}}/\Omega_{\mathrm{P}}^{\mathrm{in}}$ for $\alpha_P L = 3$ and different values of the driving field $\Omega_{\mathrm{T}} = 0$ (dashed line) and $\Omega_{\mathrm{T}} = 0.5\gamma_{41}$ (solid line). The rest of parameters are the same as Fig. 2a.

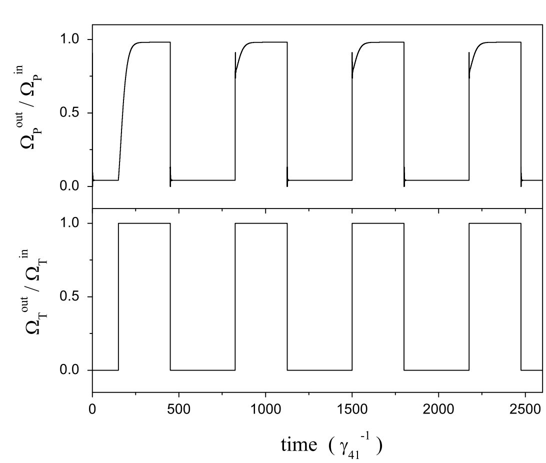

Fig. 5. Time evolution of the output probe field $\Omega_{\mathrm{P}}^{\mathrm{out}}$ normalized to the input field $\Omega_{\mathrm{P}}^{\mathrm{in}}$ and the control field $\Omega_{\mathrm{T}}^{\mathrm{out}}$ normalized to the $\Omega_{\mathrm{T}}^{\mathrm{in}}$ input field. The parameters are the same as Fig. 4.

neously. Now, we need the probe and trigger susceptibilities, which are given by

$$\chi_{\mathrm{P}}(\omega_{\mathrm{P}}) = \lim_{t\to\infty} \frac{N|\mu_{41}|^2}{\hbar\epsilon_o} \frac{\rho_{41}(\omega_{\mathrm{P}})}{\Omega_{\mathrm{P}}}, \tag{18}$$

$$\chi_{\mathrm{T}}(\omega_{\mathrm{T}}) = \lim_{t\to\infty} \frac{N|\mu_{43}|^2}{\hbar\epsilon_o} \left[\frac{\rho_{43}(\omega_{\mathrm{T}}) + \eta\rho_{52}(\omega_{\mathrm{T}})}{\Omega_{\mathrm{T}}}\right]. \tag{19}$$

If we assume that the intensity of the probe and trigger fields is much weaker than the intensity of both coupling fields, i.e., $|\Omega_{\mathrm{P}}|^2, |\Omega_{\mathrm{T}}|^2 << |\Omega_2|^2$, then populations remain mainly in the ground levels $|1\rangle$ and $|3\rangle$, while the populations of the other levels vanish. By keeping two lowest-order contributions in probe and trigger susceptibilities in Eq. (4), the susceptibilities can be expressed as

$$\chi_{\mathrm{P}} = \chi_{\mathrm{P}}^{(1)} + \chi_{P}^{(3,c)}|E_{\mathrm{T}}|^2, \tag{20}$$

$$\chi_{\mathrm{T}} = \chi_{\mathrm{T}}^{(1)} + \chi_{\mathrm{T}}^{(3,s)}|E_{\mathrm{T}}|^2 + \chi_{\mathrm{T}}^{(3,c)}|E_{\mathrm{P}}|^2, \tag{21}$$

where we have introduced the linear susceptibility, $\chi_{\mathrm{P}}^{(1)}$, the third-order self-Kerr susceptibility, $\chi_{\mathrm{T}}^{(3,s)}$, and the cross-Kerr susceptibilities, $\chi_{\mathrm{P,T}}^{(3,c)}$, which are given by

$$\begin{aligned}
\chi_{\mathrm{P}}^{(1)} &= \mathrm{i}\frac{N|\mu_{41}|^2}{\hbar\epsilon_0}\frac{\Gamma_{21}\rho_{11}^{(0)}}{\Gamma_{21}\Gamma_{41}+|\Omega_2|^2},\\
\chi_{\mathrm{T}}^{(1)} &= \mathrm{i}\frac{N|\mu_{43}|^2}{\hbar\epsilon_0}\frac{\Gamma_{23}\rho_{33}^{(0)}}{\Gamma_{23}\Gamma_{43}+|\Omega_2|^2},\\
\chi_{\mathrm{P}}^{(3,c)} &= \frac{\mathrm{i}N|\mu_{41}|^2|\mu_{43}|^2}{4\hbar^3\epsilon_0}\left[\eta^2\frac{|\Omega_2|^2\rho_{11}^{(0)}}{\Gamma_{51}(\Gamma_{21}\Gamma_{41}+|\Omega_2|^2)^2}\right.\\
&\quad\left.-\frac{\Gamma_{21}}{\Gamma_{31}(\Gamma_{21}\Gamma_{41}+|\Omega_2|^2)}\left[\frac{\Gamma_{21}\rho_{11}^{(0)}}{(\Gamma_{21}\Gamma_{41}+|\Omega_2|^2)}+\frac{\Gamma_{32}\rho_{33}^{(0)}}{(\Gamma_{34}\Gamma_{32}+|\Omega_2|^2)}\right]\right],\\
\chi_{\mathrm{T}}^{(3,c)} &= \frac{\mathrm{i}N|\mu_{41}|^2|\mu_{43}|^2}{4\hbar^3\epsilon_0}\left[\eta^2\frac{|\Omega_2|^2\rho_{11}^{(0)}}{(\Gamma_{21}\Gamma_{41}+|\Omega_2|^2)(\Gamma_{52}\Gamma_{54}+|\Omega_2|^2)}\left(\frac{1}{\Gamma_{51}}+\frac{1}{\Gamma_{24}}\right)\right.\\
&\quad\left.-\frac{\Gamma_{23}}{\Gamma_{13}(\Gamma_{43}\Gamma_{23}+|\Omega_2|^2)}\left[\frac{\Gamma_{23}\rho_{33}^{(0)}}{(\Gamma_{43}\Gamma_{23}+|\Omega_2|^2)}+\frac{\Gamma_{12}\rho_{11}^{(0)}}{(\Gamma_{14}\Gamma_{12}+|\Omega_2|^2)}\right]\right],\\
\chi_{\mathrm{T}}^{(3,s)} &= \frac{\mathrm{i}N|\mu_{43}|^4}{4\hbar^3\epsilon_0}\left[\eta^2\frac{|\Omega_2|^2\rho_{33}^{(0)}}{\Gamma_{53}(\Gamma_{43}\Gamma_{23}+|\Omega_2|^2)^2}\right].
\end{aligned} \tag{22}$$

In the derivation of the above expressions we have assumed $|\Omega_2| \ll \gamma_{21}$, and that the auxiliary transition ($|2\rangle \to |5\rangle$) is far detuned, i.e., $|\Delta_4| \ll \gamma_{51}, \gamma_{53}$.

Eq. (22) explicitly shows the effect of the cross-phase modulation induced by the interaction between the two weak fields: the polarizability at frequency ω_{P}, due to field E_{P}, depends on the intensity of field E_{T}. Since a completely analogous expression holds for the polarizability at frequency ω_{T}, the cross-phase effect becomes evident. Thus, this term yields a non-null cross-phase modulation term even at exact EIT resonance ($\Delta_{\mathrm{P}} - \Delta_2 = 0$). This situation contrasts to the results obtained in the tripod and M-type systems, where the non-linear susceptibility vanishes at two-photon resonance. Now we apply the above calculations to a real implementation of the N-tripod system following the recently proposal by Wang et al. [35] in ^{87}Rb. They consider Rubidium atoms confined in a MOT where the ultracold atoms are transferred into the state $5S_{1/2}$ with $F = 1$, $m = (-1, 0, 1)$ and with $F = 2$, and $m = (-2, -1, 0, 1, 2)$. The simplified level configuration is shown in Fig. 6. Transitions between these states and the excited levels of the state $5P_{1/2}$ with $F = 2$, $m = (-2, -1, 0, 1, 2)$ are induced by the optical fields. Specifically, the levels ($5S_{1/2}$, $F = 1$, $m = 0$), ($5S_{1/2}, F = 2$, $m = 0$), and ($5S_{1/2}$, $F = 2$, $m = 2$) correspond to the ground states $|1\rangle$, $|2\rangle$ and $|3\rangle$, respectively, while ($5P_{1/2}$, $F = 2$, $m = 1$), and ($5P_{1/2}$, $F = 2$, $m = -1$) correspond to the excited states $|4\rangle$, and $|5\rangle$, respectively. We use Steck's spectroscopic data for the decay rate the excited-states decay rate $\gamma_1 = \gamma_3 = 5.73$ MHz, close to the measured values for a sample at a temperature of 10 nK [36]. Other parameters used in the calculations are $\Omega_{\mathrm{P}} = 0.1\gamma_{41}$, and $\Omega_{\mathrm{T}} = 0.02\gamma_{41}$ which correspond to $\Omega_{\mathrm{P}} = 0.68$ MHz, and $\Omega_{\mathrm{T}} = 0.01$ MHz. A control field $\Omega_2 = 0.7\gamma_{41}$ which corresponds to $\Omega_2 = 4.76$ MHz. Note that we have chosen a small value for the trigger in comparison with the probe field in order to avoid the influence os the self-phase modulation term in Eq. (21). All these fields are at resonance with the respective transitions, i.e., $\Delta_{\mathrm{P}} = \Delta_{\mathrm{T}} = \Delta_2 \simeq 0$, while a detuning $\Delta_4 = 4\gamma_{41} \simeq 23$ MHz from the transition ($|2\rangle \to |5\rangle$) is assumed, in order to satisfy the condition $\Delta_4 \ll \gamma_{41}$. Note that the proposed model is not a complete description of the experiment as the hyperfine splitting of the $5P_{3/2}$ state is less than the Doppler broadening, so there are multiple level contributions. However, the model does give qualitative insight into the properties of the tripod atoms [34].

For these parameters, the linear absorption $\mathrm{Im}(\chi^{(1)})$, two-photon absorption $\mathrm{Im}(\chi^{(3)})$ and cross-Kerr modulation $\mathrm{Re}(\chi^{(3)})$ of the probe and trigger fields are depicted in (Fig. 7) versus their respective detunings Δ_{P}, and Δ_{T}.

It is clear that within the transparency windows, the strengths of the respective cross-phase are enhanced even at zero detuning, while the two-photon absorption remains extremely weak, which contrasts with the results obtained in N-type, tripod-type and M-type atoms. Most interestingly, a very sharp peak near zero detuning appears. This interesting result is produced by the combination of the N-type atomic subsystem with the Tripod-type atomic subsystem.

Apart from a large cross-phase modulation, a group velocity matching is another fundamental condition for achieving a large non-linear mutual phase shift [27], provided that the group velocity of the two pulses is significantly reduced. Let us see how small and equal are the probe and trigger group velocities which can be obtained in this N-tripod system. The group velocity v_g^i is given by

$$v_g^i = \frac{c}{1} + \frac{1}{2}\mathrm{Re}[\chi^{(i)}] + \frac{\omega_{\mathrm{P}}}{2}\left(\frac{\partial \mathrm{Re}[\chi^{(i)}]}{\partial\omega}\right) \quad (i = P, T). \tag{23}$$

Using the expressions of linear susceptibilities given in Eq. (22), and assuming two-photon EIT resonance condition, $(\Delta_{\mathrm{P}} - \Delta_2) \simeq 0$ and $(\Delta_{\mathrm{T}} - \Delta_2) \simeq 0$, the contribution of the non-linear susceptibilities to v_g are three orders of magnitude lower than the linear one, thus they may be neglected. Therefore, we may derive simple expressions for the group velocities

$$v_g^{\mathrm{P}} \simeq \frac{2c\hbar\epsilon_0|\Omega_2|^2}{N|\mu_{41}|^2\omega_{\mathrm{P}}\rho_{11}^0\left[1 + \beta|\Omega_{\mathrm{T}}|^2\right]}, \tag{24}$$

$$v_g^{\mathrm{T}} \simeq \frac{2c\hbar\epsilon_0|\Omega_2|^2}{N|\mu_{43}|^2\omega_{\mathrm{T}}\rho_{33}^0}, \tag{25}$$

where

$$\beta = \frac{\eta^2}{2(\gamma_{52}^2 + \Delta_4^2)^2|\Omega_{\mathrm{T}}|^2}\left[(\gamma_{52}^2 + \Delta_4^2)(2\gamma_{41}\gamma_{52} + 2\Delta_{\mathrm{P}}\Delta_4 + |\Omega_2|^2) - 2\Delta_4^2|\Omega_2|^2\right]. \tag{26}$$

Eq. (24) and (25) show that group matching at the center of the transparency window is achievable if the following condition holds

$$|\mu_{43}|^2\omega_{\mathrm{T}}\rho_{33}^0 = |\mu_{41}|^2\omega_{\mathrm{P}}\rho_{11}^0 + \beta|\Omega_{\mathrm{T}}|^2. \tag{27}$$

This condition may be achieved by properly adjusting the Rabi frequency of the control field, $|\Omega_2|$, the detuning Δ_4, and the initial populations ρ_{11}^0 and ρ_{33}^0 as in Refs. [34,35].

The probe and trigger group velocities are 4.4 m/s, and 6.6 m/s, respectively, at resonance ($\Delta_{\mathrm{P}} = \Delta_{\mathrm{T}} = 0$). Moreover, there exist a region in the vicinity of the resonance where the group velocities are very similar, ($v_g^P \approx v_g^T = 29$ m/s). In fact, for this probe and trigger detuning the cross-phase modulation reaches maximal value, as can be seen from (Fig. 7). Thus the current N-tripod system of enhancing cross-phase modulation has a very important advantage over the one using the N-type system [20], i.e., it provides large cross-phase modulation without accompanying absorption.

Now we are in position to analyze the phase gate operation, a fundamental building block for quantum information processing. In order to implement a quantum phase gate (QPG) between two optical qubits, cross-phase modulation is a very essential requirement. In a QPG, one qubit gets a phase conditioned to the other qubit state according to the transformation [37]

$$|i\rangle_1|j\rangle_2 \rightarrow e^{i\phi_{ij}}|i\rangle_1|j\rangle_2, \tag{28}$$

where $(i,j) = 0, 1$ denotes the logical qubit basis. This gate is universal when the conditional phase shift $\phi = \phi_{00} + \phi_{11} - \phi_{10} - \phi_{01}$ is non-zero, and is equivalent to a CNOT phase gate when $\phi = \pi$.

In our case, the qubit is formed by the superposition of two circularly polarized states of the field and can be written as [32]

$$|q_i\rangle = \beta_i^+|\sigma^+\rangle_i + \beta_i^-|\sigma^-\rangle_i \; i = (P, T), \tag{29}$$

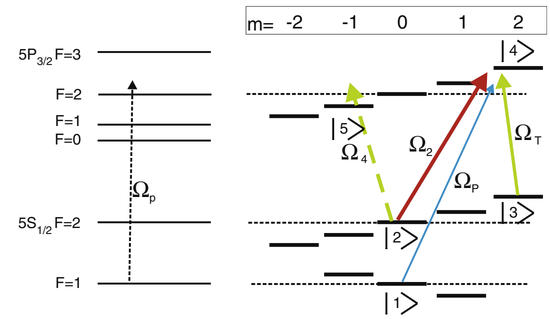

Fig. 6. Realization of the scheme of a N-Tripod atom driven by a probe (Ω_P) and trigger (Ω_T) fields and two control fields Ω_2 and Ω_4, using the D_1 line of ^{87}Rb.

where $|\sigma^\pm\rangle_i = \int d\omega\xi(\omega)a_\pm^\dagger(\omega)|0\rangle$, $\xi(\omega)$ being a Gaussian frequency distribution of incident wavepackets, centered at frequency ω_i and $\beta_i^\pm$ are constants. In order to obtain the acquired phases by the probe and trigger fields we solve the propagation equation for the slowly varying electric field amplitudes $E_i(z,t)$, $(i = P, T)$,

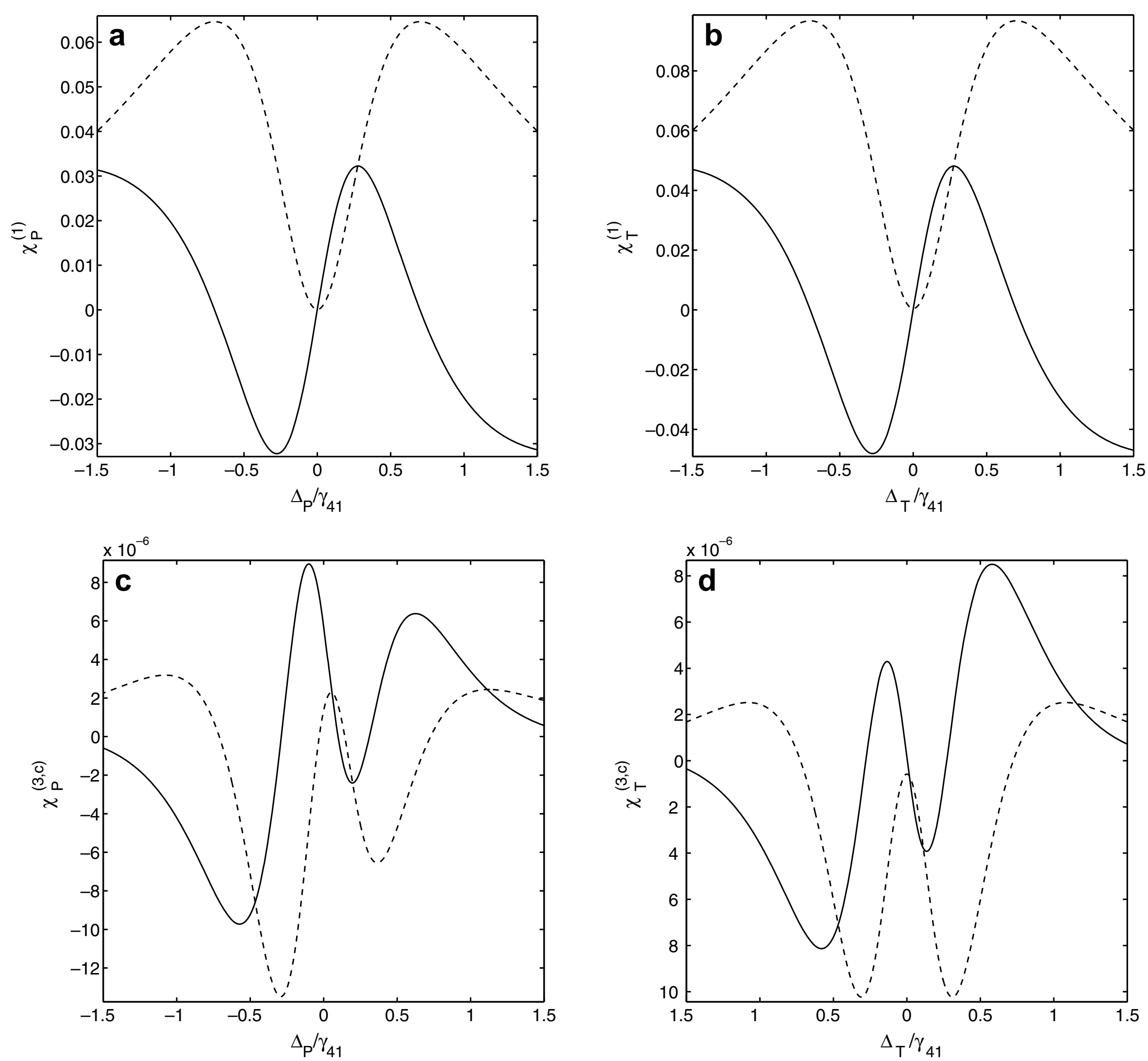

Fig. 7. (a) Linear probe susceptibility and (c) non-linear probe susceptibility as a function of the probe detuning Δ_P for $\Delta_2 = \Delta_T = 0$. (b) Linear trigger susceptibility and (d) non-linear trigger susceptibility as a function of the probe detuning Δ_T for $\Delta_2 = \Delta_P = 0$. Other parameters are $\Omega_P = 0.1\gamma_{41}$, $\Omega_2 = 0.7\gamma_{41}$, $\Omega_T = 0.02\gamma_{41}$, and $\Omega_4 = 0.1\gamma_{41}$.

$$\frac{\partial E_{\mathrm{P}}(z,t)}{\partial z}+\frac{1}{v_{\mathrm{g}}^{\mathrm{P}}}\frac{\partial E_{\mathrm{P}}(z,t)}{\partial t}=\mathrm{i}\frac{k_{\mathrm{P}}}{2}\left[\chi_{\mathrm{P}}^{(1)}+\chi_{\mathrm{P}}^{(3,c)}|E_{\mathrm{T}}(z,t)|^{2}\right]E_{\mathrm{P}}(z,t), \tag{30}$$

$$\frac{\partial E_{\mathrm{T}}(z,t)}{\partial z}+\frac{1}{v_{\mathrm{g}}^{\mathrm{T}}}\frac{\partial E_{\mathrm{T}}(z,t)}{\partial t}$$
$$=\mathrm{i}\frac{k_{\mathrm{T}}}{2}\left[\chi_{\mathrm{T}}^{(1)}+\chi_{\mathrm{T}}^{(3,c)}|E_{\mathrm{P}}(z,t)|^{2}+\chi_{\mathrm{T}}^{(3,s)}|E_{\mathrm{T}}(z,t)|^{2}\right]E_{\mathrm{T}}(z,t).$$

Under the two-photon resonance conditions and considering small values for the trigger field E_{T}, the self-phase modulation term in the non-linear susceptibility can be neglected. Thus, the solution of Eq. (30) is given by

$$E_{\mathrm{P,T}}(z,t)=E_{\mathrm{P,T}}\left(0,t-\frac{z}{v_{\mathrm{g}}^{\mathrm{P,T}}}\right)\exp\left[\mathrm{i}\frac{k_{\mathrm{P,T}}}{2}\int_{0}^{L}\mathrm{d}z'\chi_{\mathrm{P,T}}(z',t)\right], \tag{31}$$

L being the interaction length of the fields. Thus, the linear phase shift and non-linear cross-phase shift are given by

$$\phi_{\mathrm{lin}}^{\mathrm{P,T}}=\frac{k_{\mathrm{P,T}}}{2}\int_{0}^{L}\mathrm{d}z'\mathrm{Re}\left[\chi_{\mathrm{P,T}}^{(1)}\right], \tag{32}$$

$$\phi_{\mathrm{P,T}}^{(3,c)}=\frac{k_{\mathrm{P,T}}}{2}\int_{0}^{L}\mathrm{d}z'\mathrm{Re}\left[\chi_{\mathrm{P,T}}^{(3,c)}\right]|E_{\mathrm{T,P}}(z',\tau')|^{2}, \tag{33}$$

where

$$\tau'=z'\left(\frac{1}{v_{\mathrm{g}}^{\mathrm{P}}}-\frac{1}{v_{\mathrm{g}}^{\mathrm{T}}}\right). \tag{34}$$

For gaussian probe and trigger pulses of time duration τ_{P} and τ_{T}, and with peak Rabi frequencies Ω_{P0} and Ω_{T0}, respectively, the non-linear cross-phase shifts can be written as [32]

$$\phi_{\mathrm{P}}^{(3,c)}=\frac{k_{\mathrm{P}}L}{2}\frac{\sqrt{\pi}|\Omega_{\mathrm{P0}}|^{2}}{|\mu_{41}|^{2}}\frac{\mathrm{erf}[\zeta_{\mathrm{P}}]}{\zeta_{\mathrm{P}}}\mathrm{Re}\left[\chi_{\mathrm{P}}^{(3,c)}\right], \tag{35}$$

$$\phi_{\mathrm{T}}^{(3,c)}=\frac{k_{\mathrm{P}}L}{2}\frac{\sqrt{\pi}|\Omega_{\mathrm{T0}}|^{2}}{|\mu_{43}|^{2}}\frac{\mathrm{erf}[\zeta_{\mathrm{T}}]}{\zeta_{\mathrm{T}}}\mathrm{Re}\left[\chi_{\mathrm{T}}^{(3,c)}\right], \tag{36}$$

where

$$\zeta_{l,m}=\sqrt{2}L\left(1-\frac{v_{\mathrm{g}}^{l}}{v_{\mathrm{g}}^{m}}\right)\frac{1}{\tau_{m}v_{l}^{p}}\,(l,m)=(P,T). \tag{37}$$

We assume that our N-tripod system shown in (Fig. 6) is implemented only when the probe has $|\sigma^{+}\rangle$ polarization and the trigger has $|\sigma^{-}\rangle$ polarization. Thus, following Octaviani et al. [32], the QPG truth table for the N-tripod atom reads as

$$\begin{aligned}
|\sigma^{-}\rangle_{\mathrm{P}}|\sigma^{-}\rangle_{\mathrm{T}}&=\mathrm{e}^{-\mathrm{i}(\phi_{0}^{\mathrm{P}}+\phi_{\mathrm{lin}}^{\mathrm{T}})}|\sigma^{-}\rangle_{\mathrm{P}}|\sigma^{-}\rangle_{\mathrm{T}},\\
|\sigma^{-}\rangle_{\mathrm{P}}|\sigma^{+}\rangle_{\mathrm{T}}&=\mathrm{e}^{-\mathrm{i}(\phi_{0}^{\mathrm{P}}+\phi_{0}^{\mathrm{T}})}|\sigma^{-}\rangle_{\mathrm{P}}|\sigma^{+}\rangle_{\mathrm{T}},\\
|\sigma^{+}\rangle_{\mathrm{P}}|\sigma^{+}\rangle_{\mathrm{T}}&=\mathrm{e}^{-\mathrm{i}(\phi_{\mathrm{lin}}^{\mathrm{P}}+\phi_{0}^{\mathrm{T}})}|\sigma^{+}\rangle_{\mathrm{P}}|\sigma^{+}\rangle_{\mathrm{T}},\\
|\sigma^{+}\rangle_{\mathrm{P}}|\sigma^{-}\rangle_{\mathrm{T}}&=\mathrm{e}^{-\mathrm{i}(\phi_{\mathrm{lin}}^{\mathrm{P}}+\phi_{\mathrm{P}}^{(3,c)}+\phi_{\mathrm{lin}}^{\mathrm{T}}+\phi_{\mathrm{T}}^{(3,c)})}|\sigma^{-}\rangle_{\mathrm{P}}|\sigma^{-}\rangle_{\mathrm{T}},
\end{aligned} \tag{38}$$

where the conditional phase is defined through $\phi=\phi_{\mathrm{P}}^{(3,c)}+\phi_{\mathrm{T}}^{(3,c)}$.

By assuming realistic value for the electric dipole matrix elements for the system under examination $\mu=2.534\times10^{-29}$ cm [36], which are typical values for an alkali atom and atomic density in the order of $N=9\times10^{13}$ cm^{-3}, and a length of the atomic sample as long as 1 mm, we find from (Fig. 8) that a cross-phase shift of 0.03π is reached. This choice of parameters corresponds to a mean photon number around unity when the beams are focused in an area of 1 μm in diameter and for a time of flight around 1 μs. It is clear that a classical phase gate could be implemented by using more intense probe and trigger field pulses allowing a cross-phase shift of π. Although in these conditions we do not find very appreciable amount of absorption, it is worth analyzing the influence of the residual linear and non-linear losses in the degree of entanglement for two subsystems of bipartite mixed or pure states. To do

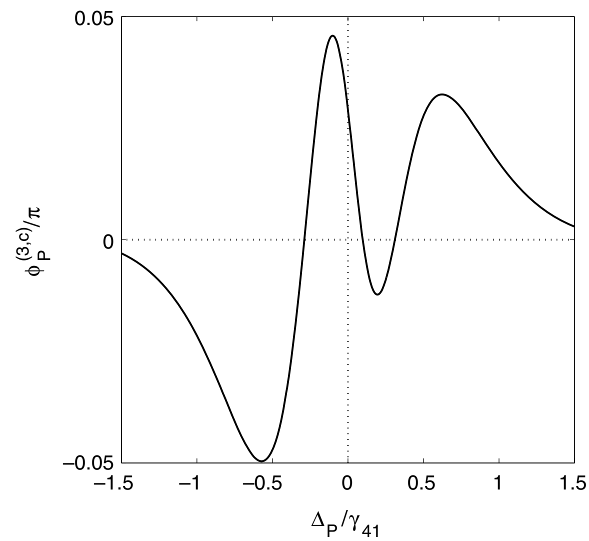

Fig. 8. Non-linear phase of the probe field induced by the trigger field as a function of the probe detuning Δ_{P}. Other parameters as in Fig. 7.

that we use the entanglement of formation proposed by Wootters [38], which, for a two-qubit system is given by

$$E_{\mathrm{F}}(C)=h\left(\frac{1+\sqrt{1-C^{2}}}{2}\right), \tag{39}$$

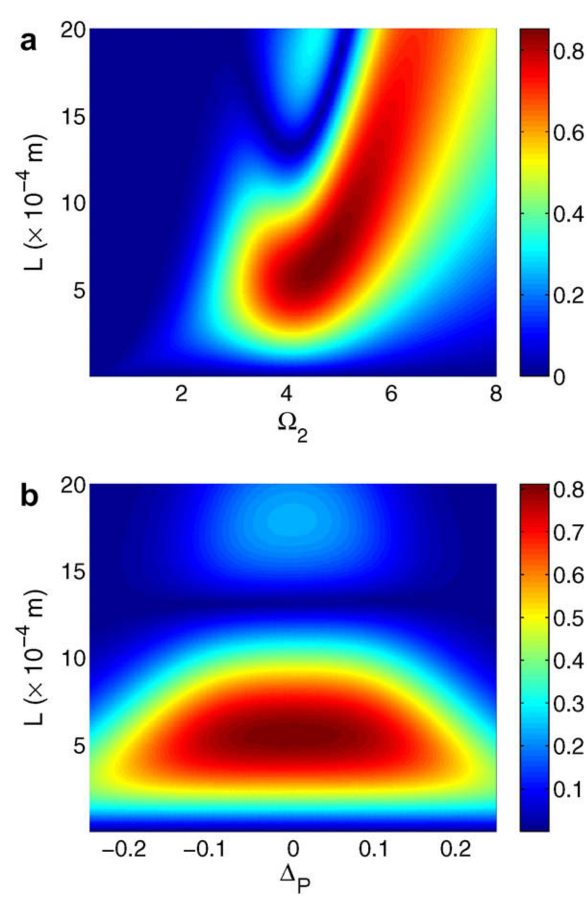

Fig. 9. Contours of the concurrence function C: (a) versus the Rabi frequency of the control field and the length of the atomic medium, and (b) versus the atomic detuning of the probe field and the length of the atomic medium. Other parameters are the same as in Fig. 7.

where $h(x) = -x\log(x) - (1-x)\log(1-x)$ is the Shannon's entropy function, and C is the so-called concurrence. The expression relating the concurrence C to the density operator for the output state defined in the computational basis $[|\sigma^-\rangle_P|\sigma^-\rangle_T, |\sigma^-\rangle_P|\sigma^+\rangle_T, |\sigma^+\rangle_P|\sigma^-\rangle_T, |\sigma^+\rangle_P|\sigma^+\rangle_T]$ is given by [38]

$$C = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}, \tag{40}$$

λ's being the square roots of the eigenvalues, in a decreasing order, of the matrix $\tilde{\rho} = \rho\sigma_y^P \otimes \sigma_y^T \rho^* \sigma_y^P \otimes \sigma_y^T$, here ρ^* denotes the complex conjugation of ρ, and σ_y is the y-component of the Pauli matrix. The case of $C = 1$ corresponds to the existence of the maximum entanglement between the two fields, and $C = 0$ means no entanglement between the fields. In (Fig. 9a and b) we plot the concurrence versus the length of the atomic medium, and the Rabi frequency of the control field and versus the atomic detuning, respectively. In both figures it can be appreciated that there exists an optimal medium length where entanglement is maximal and decreases as the length increases due to the accumulation of linear and non-linear losses. In addition, for a fixed length, the concurrence C can be controlled by changing the Rabi frequency of the driving field and the detuning of the probe field, among other parameters. The use of these figures will allow us to select the optimum parameters to produce the highest degree of entanglement between the modes of the electromagnetic fields.

5. Conclusions

In this paper we have analyzed the absorptive and dispersive properties of a five-level atom in a novel N-tripod configuration of EIT. This atomic scheme could be used for the realization of an all-optical absorptive switch. It is shown that the switching mechanism relies in the control of the double-dark resonances which appear in the system. These two dark resonances arise from the subsystem formed by levels $1|\rangle$, $|2\rangle$, $|3\rangle$, and $|4\rangle$ when the control field is turned off ($\Omega_4 = 0$). The addition of the auxiliary field which connects levels $2\rangle$ and $5\rangle$ induces a Stark-shift which breaks one of the dark resonances while maintaining the other. Simultaneously, the central peak caused by constructive interference experiences nearly the same shift, turning the system into a highly absorptive state. By numerically solving Eqs. (3) and (15) we have examined the time evolution of both the probe and trigger fields showing the dynamics of the switching process.

The linear and cross-phase modulation susceptibilities have been calculated with realistic parameters for a confined cold sample of Rb atoms showing giant cross-phase modulation even at resonance of the probe and trigger fields. We have found that the system exhibit large cross-Kerr modulation between the probe and trigger fields even at resonance with nearly null absorption. This fact may be used for realizing a phase gate with a conditional phase shift in the order of 0.03π.

Acknowledgements

This work was supported by the Project No. FIS2007-65382 (MEC) and CCG07-UCM/ESP-2179 (CAM-UCM).

References

[1] S.E. Harris, Phys. Today 50 (1997) 36.
[2] J.P. Marangos, J. Mod. Opt. 45 (1998) 471.
[3] M.O. Scully, M.S. Zubairy, Quantum Optics, Cambridge University Press, London, 1997.
[4] Z. Ficek, S. Swain, J. Mod. Opt. 49 (2002) 3.
[5] S.E. Harris, J.E. Field, A. Imamouglu, Phys. Rev. Lett. 64 (1990) 1107.
[6] K.J. Boller, A. Imamouglu, S.E. Harris, Phys. Rev. Lett. 66 (1991) 2593.
[7] O. Kocharovskaya, Y.I. Khanin, JETP Lett. 48 (1988) 580.
[8] S.E. Harris, Phys. Rev. Lett. 62 (1989) 1033.
[9] M.O. Scully, S.-Y. Zhu, A. Gavrielides, Phys. Rev. Lett. 62 (1989) 2813.
[10] J. Mompart, R. Corbalan, J. Opt. B: Quantum Semiclass. Opt. 2 R7 (2000) 175.
[11] M.O. Scully, M. Fleischhauer, Phys. Rev. Lett. 67 (1991) 1885.
[12] S.E. Harris, J.E. Feld, A. Kasapi, Phys. Rev. A 46 (1992) R29.
[13] M. Fleischhauer, C.H. Keitel, M.O. Scully, Ch. Su, B.T. Ulrich, S.-Y. Zhu, Phys. Rev. A 46 (1992) 1468.
[14] M.O. Scully, M. Fleischhauer, Phys. Rev. Lett. 69 (1992) 1360.
[15] K. Hakuta, L. Marmet, B.P. Stoicheff, Phys. Rev. Lett. 66 (1991) 596.
[16] S.E. Harris, Phys. Rev. Lett. 72 (1994) 52.
[17] H. Schmidt, A. Imamoglu, Opt. Lett. 21 (1996) 1936.
[18] S. Rebic, S.M. Tan, A.S. Parkins, D.F. Walls, J. Opt. B: Quantum Semiclass. Opt. 1 (1999) 490.
[19] S.E. Harris, Y. Yamamoto, Phys. Rev. Lett. 81 (1998) 3611; S.E. Harris, L.V. Hau, Phys. Rev. Lett. 82 (1999) 4611.
[20] M. Yan, E.G. Rickey, Y. Zhu, Phys. Rev. A 64 (2001) 041901.
[21] Y.F. Cheng, Z.-H. Tsai, Y.C. Liu, I.A. Yu, Opt. Lett. 30 (2005) 3207.
[22] N. Gising et al., Rev. Mod. Phys. 74 (2002) 145.
[23] D. Bouwmeester et al., Nature (London) 390 (1997) 575.
[24] A. Furusawa et al., Science 282 (1998) 706.
[25] Q.A. Turchette, C.J. Lange, H. Mabuchi, H.J. Kimblr, Phys. Rev. Lett. 75 (1995) 4710.
[26] A. Rauchenbeutel, G. Nogues, S. Osnaghi, P. Bertet, M. Brune, J.M. Raimond, S. Haroche, Phys. Rev. Lett. 83 (1999) 5166.
[27] M.D. Lukin, A. Imamoglu, Phys. Rev. Lett. 84 (2000) 1419.
[28] F. Vewinger, M. Heinz, R.G. Fernandez, N.V. Vitanov, K. Bergmann, Phys. Rev. Lett. 91 (2003) 213001.
[29] E. Paspalakis, P.L. Knight, J. Opt. B: Quantum Semiclass. Opt. 4 (2002) S372.
[30] D. Petrosyan, Y.P. Malakian, Phys. Rev. A 70 (2004) 023822.
[31] D. Petrosyan, G. Kurizki, Phys. Rev. A 65 (33) (2002) 833.
[32] C. Ottaviani, D. Vitali, M. Artoni, F. Catalotti, P. Tombesi, Phys. Rev. Lett. 90 (2003) 197902.
[33] S. Rebic, D. Vitali, C. Ottaviani, P. Tombesi, M. Artoni, F. Catalotti, R. Corbalán, Phys. Rev. A 70 (2004) 032317.
[34] Y. Han, J. Xiao, Y. Liu, C. Zhang, H. Wang, M. Xiao, K. Peng, Phys. Rev. A 77 (2008) 023824.
[35] Z. Wang, K.M. Marzlin, B.C. Sanders, Phys. Rev. Lett. 97 (2006) 063901.
[36] D. Steck, ^{87}Rb D Line Data, <http://steck.us/alkalidata>.
[37] M.A. Nielsen, I.L. Chuang, Quantum Computation and Quantum Information, Cambridge University Press, Cambridge, 2000.
[38] W.K. Wooters, Phys. Rev. Lett. 80 (1998) 2245.