Capital Cash Flows: A Simple Approach to Valuing Risky Cash Flows

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This paper presents the Capital Cash Flow (CCF) method for valuing risky cash flows. I show that the CCF method is equivalent to discounting Free Cash Flows (FCF) by the weighted average cost of capital. Because the interest tax shields are included in the cash flows, the CCF approach is easier to apply whenever debt is forecasted in levels instead of as a percent of total enterprise value. The CCF method retains its simplicity when the forecasted debt levels and the implicit debt-to-value ratios change throughout forecast period. The paper also compares the CCF method to the Adjusted Present Value (APV) method and provides consistent leverage adjustment formulas for both methods.

The most common technique for valuing risky cash flows is the Free Cash Flow (*FCF*) method. In that method, interest tax shields are excluded from the *FCF*s and the tax deductibility of interest is treated as a decrease in the cost of capital using the after-tax weighted average cost of capital (*WACC*). Because the *WACC* is affected by changes in capital structure, the *FCF* method poses several implementation problems in highly leveraged transactions, restructurings, project financings, and other instances in which capital structure changes over time. In these situations, the capital structure has to be estimated and those estimates have to be used to compute the appropriate *WACC* in each period. Under these circumstances, the *FCF* method can be used to correctly value the cash flows, but it is not straightforward.

This paper presents an alternative method for valuing risky cash flows. I call this method the Capital Cash Flow (*CCF*) method, because the cash flows include all of the cash available to capital providers, including the interest tax shields. In a capital structure with only ordinary debt and common equity, *CCF*s equal the flows available to equity—*NI* plus depreciation less capital expenditure and the increase in working capital—plus the interest paid to debtholders. The interest tax shields decrease taxable income, decrease taxes and, thereby, increase after-tax cash flows. In other words, *CCF*s equal *FCF*s plus the interest tax shields. Because the interest tax shields are included in the cash flows, the appropriate discount rate is before-tax and corresponds to the riskiness of the assets.

Although the *FCF* and *CCF* methods treat interest tax shields differently, the two methods are algebraically equivalent. In other words, the *CCF* method is a different way of valuing cash flows using the same assumptions and approach as the *FCF* method. The advantage of the *CCF* method is its simplicity. Whenever debt is forecasted in levels, instead of as a percent of total enterprise value, the *CCF* method is much easier to use, because the interest tax shields are easy to calculate and easy to include in the cash flows. The *CCF* method retains its simplicity when the forecasted debt levels and the implicit debt-to-value ratios change throughout the forecast period. Also, the expected asset return depends on the riskiness of the asset and, therefore,

I would like to thank Malcolm Baker, Ben Esty, Stuart Gilson, Paul Gompers, Bob Holthausen, Chris Noe, Paul Maleh, Scott Mayfield, Lisa Meulbroek, Stewart Myers, Denise Tambanis, Peter Tufano, the Editors, Lemma Senbet and Alexander Triantis (the Editors), the referees, and seminar participants at Duke, Georgetown, and Harvard for comments on earlier drafts and helpful discussions.

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Financial Management • Summer 2002 • pages 85 - 103

does not change when capital structure changes. As a result, the discount rate for the *CCF*s does not have to be re-estimated every period. In contrast, when using the *FCF* method, the after-tax *WACC* has to be re-estimated every period. Because the *WACC* depends on value-weights, the value of the firm has to be estimated simultaneously. The *CCF* method avoids this complexity so that it is especially useful in valuing highly levered firms whose debt is usually forecasted in levels and whose capital structure changes substantially over time.

The *CCF* method is closely related to my work on valuing riskless cash flows (Ruback, 1986) and to Stewart Myers' work on the Adjusted Present Value (*APV*) method (Myers, 1974). In my paper on riskless cash flows, I showed that the interest tax shields associated with riskless cash flows can either be equivalently treated as increasing cash flows by the interest tax shield, or as decreasing the discount rate to the after-tax riskless rate. The analysis in this paper presents similar results for risky cash flows; namely, risky cash flows can be equivalently valued by using the *CCF* method with the interest tax shields in the cash flows or by using the *FCF* method with the interest shields in the discount rate.

The *APV* method is generally calculated as the sum of *FCF*s discounted by the cost of assets plus interest tax shields discounted at the cost of debt. It results in a higher value than the *CCF* method, because it assigns a higher value to interest tax shields. The interest tax shields that are discounted by the cost of debt in the *APV* method are discounted by the cost of assets explicitly in the *CCF* method and implicitly in the *FCF* method. Stewart Myers suggests the term "Compressed APV" to describe the *CCF* method, because the *APV* method is equivalent to *CCF* when the interest tax shields are discounted at the cost of assets. However, most descriptions of *APV* suggest discounting the interest tax shields at the cost of debt (Taggart, 1991 and Luehrman, 1997).

The *APV* method treats the interest tax shields as being less risky than the assets, because the level of debt is implicitly assumed to be a fixed dollar amount. The intuition is that interest tax shields are realized roughly when interest is paid so that the risk of the shields matches the risk of the payment. This matching of the risk of the tax shields and the interest payment only occurs when the level of debt is fixed. Otherwise, the risk of the shields depends on both the risk of the payment and systematic changes in the amount of debt. Because the risk of a levered firm is a weighted average of the risk of an unlevered firm and the risk of the interest tax shields, the presence of less risky interest tax shields reduces the risk of the levered firm. As a result, a tax adjustment has to be made when unlevering an equity beta to calculate an asset beta.

The *CCF* method, like the *FCF* method, assumes that debt is proportional to value. The higher the value of the firm, then the more debt the firm uses in its financial structure. The more debt used, then the higher the interest tax shields. The risk of the interest tax shields, therefore, depends on the risk of the debt as well as the changes in the level of the debt. When debt is a fixed proportion of value, the interest tax shields will have the same risk as the firm, even when the debt is riskless. Because the interest tax shields have the same risk as the firm, leverage does not alter the asset beta of the firm. As a result, no tax adjustment has to be made when calculating asset betas.

The primary contributions of this paper are to introduce the *CCF* method of valuation, to demonstrate its equivalency to the *FCF* method, and to show its relation to the *APV* method. The *CCF* method has been used in teaching materials to value cash flow forecasts, in Kaplan and Ruback (1995) to value highly levered transactions, and in Gilson, Hotchkiss, and Ruback (2000) to value firms emerging from Chapter 11 reorganizations.[1] Also, finance textbooks contain some of the ideas about the relation between the discount rate for interest tax shields, unlevering

[1]Teaching materials include Ruback (1989, 1995a, 1995b) and Holthausen and Zmijewski (1996).

formulas, and financial policy. This paper provides the basis for the applications of *CCF*s and highlights the linkages between the three methods of cash flow valuation.

Although the focus of this paper is on cash flow methods that yield estimates of total enterprise value, the results have implications for the Equity Cash Flow (*ECF*) and Residual Income (*RI*) methods. In the *ECF* method, the *FCF*s to equity are discounted at the cost of equity capital. That method is equivalent to the *FCF* method and has the same drawbacks.[2] The cost of equity capital, like the *WACC*, changes as leverage changes; it requires a simultaneous estimation of the equity-to-value ratios and the value whenever the debt is forecasted in levels. In those situations, the *CCF* method is a simpler approach. Similarly, as Lundholm and O'Keefe (2001) stress, the *RI* approach is equivalent to the *FCF* method as long as consistent assumptions are used—including the assumption that the discount rate consistently incorporates the assumed debt policy. Thus, the *RI* approach does not mitigate the valuation issues addressed in this paper.

Section I describes the mechanics of the *CCF* method, including the calculation of the cash flows and the discount rate. Section II shows that the *CCF* method is equivalent to the *FCF* method through an example and, then, with a more general proof. Section III relates the *CCF* and the *APV* methods and shows that the difference between the two methods depends on the implicit assumption about the financial policy of the firm. I also show that the assumption about financial policy has implications regarding the impact of taxes on risk and, thereby, determines the approach used to transform equity betas into asset betas. Section IV concludes.

I. Mechanics of Capital Cash Flow Valuation

The present value of *CCF*s is calculated by discounting them by the expected asset return, K_A. This section details the calculation of the *CCF*s in subsection A and explains the calculations of K_A in subsection B. An example is presented in subsection C.

A. Calculating Capital Cash Flows

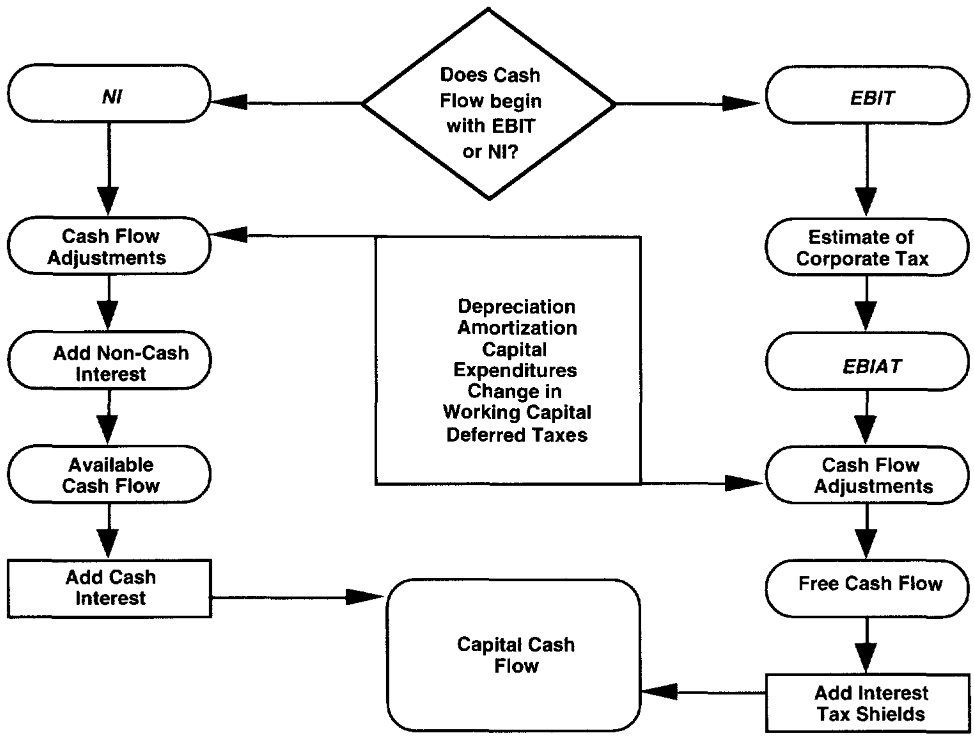
*CCF*s include all of the cash flows that are paid or could be paid to any capital provider. By including cash flows to all security holders, *CCF*s measure all of the after-tax cash generated by the assets. Since *CCF*s measure the after-tax cash flows from the enterprise, the present value of these cash flows equals the value of the enterprise. Figure 1 summarizes the calculation of *CCF*s. The calculations depend on whether the cash flow forecasts begin with net income (*NI*) or earnings before interest and taxes (*EBIT*).

1. The *NI* Path

NI includes any tax benefit from debt financing because interest is deducted before computing taxes. The interest tax shields, therefore, increase *NI*. Available cash flow is *NI* plus cash flow adjustments and non-cash interest. Cash flow adjustments include those adjustments required to transform the accounting data into cash flow data. Typical adjustments include adding depreciation and amortization, because these are non-cash subtractions from *NI*. Non-cash interest occurs when the interest is paid in kind by issuing additional debt, instead of paying the interest in cash. These non-cash interest payments are deducted from *NI* like cash interest but are not a cash outflow and, therefore, must be

[2]See Ruback (1995b) and Esty (1999) for a discussion of the relation between the *FCF* and *ECF* methods.

Figure I. Calculating Capital Cash Flows



added to *NI* to calculate cash flow available.

Capital expenditures are subtracted from *NI* because these cash outflows do not appear on the income statement and, thus, are not deducted from *NI*. Subtracting the increases in working capital transforms the recognized accounting revenues and costs into cash revenues and costs. The label "available cash flow" often appears in projections and measures the funds available for debt repayments or other corporate uses. *CCF* is computed by adding cash interest to available cash flow so that cash flows represent the after-tax cash available to all cash providers.

2. The *EBIT* Path

When cash flow forecasts present *EBIT*, instead of *NI*, corporate taxes have to be estimated to calculate earnings before interest and after taxes (*EBIAT*). Typically the taxes are estimated by multiplying *EBIT* by a historical marginal tax rate. *EBIAT* is then adjusted using the cash flow adjustments that transform the accounting data into cash flow data. *EBIAT* plus cash flow adjustments equals *FCF*, which is used to compute value using the after-tax *WACC* (*WACC*). *FCF*s equal *CCF*s less the interest tax shields. Interest tax shields, therefore, have to be added to the *FCF*s to arrive at the *CCF*s. The interest tax shields on both cash and non-cash debt are added because both types of interest tax shields reduce taxes and, thereby, increase after-tax cash flow.

The *EBIT* path should yield the same *CCF*s as the *NI* path. In practice, however, the *NI* path is usually easier and more accurate than the *EBIT* path. The primary advantage of the *NI* path is that it uses the corporate forecast of taxes, which should include any special circumstances of the firm. Taxes are rarely equal to the marginal tax rate times taxable income. The *EBIT* path involves estimating taxes, usually by assuming a constant average tax rate. This ignores the special circumstances of the firm and adds a likely source of error.

B. Calculating the Expected Asset Return

The appropriate discount rate to value *CCF*s is a before-tax rate because the tax benefits of debt financing are included in the *CCF*s. The pre-tax rate should correspond to the riskiness of the *CCF*s. One such discount rate is the pre-tax *WACC*:

$$\text{Pre-tax WACC} = \frac{D}{V} K_D + \frac{E}{V} K_E \tag{1}$$

where D/V is the debt-to-value ratio; E/V is the equity-to-value ratio, and K_D and K_E are the respective expected debt and equity returns. Using the pre-tax *WACC* as a discount rate is correct, but there is a much simpler approach. If expected returns in equation (1) are determined by the Capital Asset Pricing Model (*CAPM*):

$$K_D = R_F + \beta_D R_P \tag{2}$$

$$K_E = R_F + \beta_E R_P \tag{3}$$

where R_F is the risk-free rate, R_P is the risk premium, and β_D and β_E are the debt and equity betas, respectively. Substituting equations (2) and (3) into equation (1) yields:

$$\text{Pre-tax WACC} = \frac{D}{V}(R_F + \beta_D R_P) + \frac{E}{V}(R_F + \beta_E R_P) \tag{4}$$

Simplifying:

$$\text{Pre-tax WACC} = R_F + \left(\frac{D}{V}\beta_D + \frac{E}{V}\beta_E\right) R_P \tag{5}$$

The beta of the assets, β_U, is a weighted average of the debt and equity beta:

$$\beta_U = \frac{D}{V}\beta_D + \frac{E}{V}\beta_E \tag{6}$$

Substituting equation (6) into equation (5) provides a simple formula for the pre-tax *WACC* which is also labeled as the Expected Asset Return, K_A:

$$K_A = \text{Pre-tax WACC} = R_F + \beta_U R_P \tag{7}$$

Note that the pre-tax expected asset return depends only on the market-wide parameters for the risk-free rate, R_F, the risk premium, R_P, and on the unlevered asset beta, β_U. The debt-to-value and equity-to-value ratios are not in equation (7). K_A, therefore, does not depend on capital structure and does not have to be recomputed as capital structure changes. This means that the debt-to-value and equity-to-value ratios do not have to be estimated to use the *CCF* valuation method. This eliminates much of the complexity encountered when applying the *FCF* method.

The discount rate for the *CCF*s is simple to calculate regardless of the capital structure. It takes two steps. First, estimate the asset beta, β_U. The asset beta is usually estimated using equation (6) by multiplying the equity beta, β_E, by the equity-to-value ratio and adding an estimate of the debt beta multiplied by the debt-to-value ratio. Second, use β_U, together with the risk-free rate, R_F, and the risk premium, R_P to compute the expected asset return, K_A. For example, if the equity beta is 1.25, the debt beta is 0.25, and the equity-to-value ratio is 0.75, then the asset beta is 0.75 X 1.25 + 0.25 X 0.25 = 1.0. With an asset beta of 1.0, an assumed risk free rate of 10%, and an assumed risk premium of 8%, the expected asset return is 10% + 1.0 X 8% = 18%.

C. Numerical Example

Table I contains a numerical example that demonstrates the *CCF* method. The example assumes an initial investment of $100,000 to be depreciated equally over three years. Panel A details the assumptions. The asset beta is assumed to equal 1.0 in all three years.[3] The forecasted expected pre-tax operating profits are $50,000 in year one, $60,000 in year two, and $70,000 in year three. The risk-free rate is assumed to be 10%, the risk premium is assumed to be 8%, and the tax rate is assumed to be 33%. The debt is assumed to be risky, with a debt beta of 0.35 in the first year, 0.30 in the second year, and 0.25 in the third year. The project is financed with debt so that the initial debt is $100,000 at the beginning of year one, $50,000 at the beginning of year two, and $20,000 at the beginning of year three.

The *CCF* is calculated by following the *NI* path. The cash flow available is equal to *NI* plus noncash adjustments. *CCF* is calculated by adding the expected interest to the cash flow available.

The value of the *CCF*s is calculated using the expected asset return. The easiest way to calculate the asset return is to use the asset beta in the *CAPM*. Using a risk-free rate of 10%, an asset beta of 1.0, and a risk premium of 8% yields an expected asset return of 18%. The asset return does not depend on leverage because it is a pre-tax cost of capital. It remains constant even though the leverage changes through time. As Panel B of Table I shows, discounting the *CCF*s at the expected asset return results in a value of $136,996.

II. The Relation Between Capital Cash Flow and Free Cash Flow Valuation

The *FCF* and *CCF* methods are equivalent. I demonstrate this equivalency in subsection A by extending the numerical example of Table I and showing that the *FCF* valuation is the same as the *CCF* valuation. Subsection B presents a more general proof of the equivalence of the *CCF* and *FCF* methods. Because the two methods are equivalent, the choice between them is governed by the ease of use. Subsection C presents some suggestions on choosing between the methods.

A. Numerical Example

Panel C of Table I presents a *FCF* valuation of the same cash flows valued using *CCF*s in Panel B. The *FCF*s are calculated from *EBIT*, which is reduced by the hypothetical taxes on *EBIT* to determine *EBIAT*. Adding the non-cash adjustments to *EBIAT* results in *FCF*s.

The *FCF*s are valued using the after-tax *WACC*. The *WACC* has two components: the after-tax cost of debt and the levered cost of equity. The after-tax cost of debt depends on the assumed riskiness of the debt with the cost of debt calculated as its *CAPM* expected return using equation (2). The levered cost of equity is calculated by levering the asset beta to

[3]The equivalency of the *CCF* and *FCF* methods does not depend on a constant asset beta. The asset beta could change each year. In practice, however, the asset beta is related to the assets of the company and, thus, is assumed to be constant unless the asset composition changes.

Table I. An Example of Capital Cash Flow and Free Cash Flow

	Year 1	Year 2	Year 3
Panel A. Assumptions			
Market Parameters:			
Riskless Debt Rate = 10%			
Risk Premium = 8%			
Tax Rate = 33%			
Asset Beta	1.00	1.00	1.00
Debt Beta	0.35	0.30	0.25
Expected Cash Flows:			
Operating Profit	50,000	60,000	70,000
Less: Depreciation	33,333	33,333	33,333
EBIT	16,667	26,667	36,667
Less: Expected Interest[a]	12,800	6,200	2,400
Pre-Tax Income	3,867	20,467	34,267
Less: Taxes	1,276	6,754	11,308
Net Income	2,591	13,713	22,959
Non-Cash Adjustments[b]	43,333	43,333	43,333
Cash Flow Available	45,924	57,046	66,292
Beginning Debt	100,000	50,000	20,000
Panel B. Capital Cash Flow Valuation			
Cash Flow Available	45,924	57,046	66,292
Plus: Expected Interest[a]	12,800	6,200	2,400
Capital Cash Flow	58,724	63,246	68,692
Cost of Assets[c]	18.0%	18.0%	18.0%
Discount Factor	0.8475	0.7182	0.6086
Present Value of CCFs	49,766	45,422	41,808
Total Enterprise Value	136,996		

[a]Expected Interest is calculated using the Expected Cost of Debt from the CAPM (riskfree rate plus the debt beta times the risk premium).
[b]Noncash adjustments include depreciation plus $10,000 of other adjustments.
[c]Expected asset return is calculated using the assumed asset beta in the CAPM with the assumed riskless debt rate and risk premium.

determine the levered equity beta. Because the fraction of debt is not the same each year, the *WACC* and its components need to be recomputed each year.

The formula for levering the asset or unlevered beta is:

$$\beta_E = \left(\beta_U - \frac{D}{V}\beta_D\right) / \frac{E}{V} \tag{8}$$

which requires information on the value of the firm to compute the percentage of debt and equity in the capital structure.[4] Generally, an iterative or dynamic programming approach is used to solve for a consistent estimate of enterprise value.[5] However, because the value is already computed in Panel B, that value can be used to compute the debt and equity proportions. Based on the implied equity-to-value ratio of 27.0% in the first year, the asset beta of 1.0, and the debt beta of 0.35, the implied equity beta is 2.76. Using the *CAPM* and the

[4]This formula is derived in Section III.B of this paper.
[5]See Esty (1999) an explanation of the iterative technique and a project finance application of that approach.

Table I. An Example of Capital Cash Flow and Free Cash Flow (*Continued*)

Panel C. Free Cash Flow Valuation

		Year 1	Year 2	Year 3
EBIT		16,667	26,667	36,667
Less: Tax on EBIT		5,500	8,800	12,100
EBIAT		11,167	17,867	24,567
Non-Cash Adjustments[b]		43,333	43,333	43,333
Free Cash Flows		54,500	61,200	67,900
Capitalization:				
Total Enterprise Value[d]		136,996	102,932	58,214
Debt		100,000	50,000	20,000
WACC Calculations:				
Debt				
	Percent	73.0%	48.6%	34.4%
	After-Tax Cost[e]	8.6%	8.3%	8.0%
	Contribution[f]	6.3%	4.0%	2.8%
Equity				
	Percent	27.0%	51.4%	65.6%
	Equity Beta[g]	2.76	1.66	1.39
	Cost[h]	32.1%	23.3%	21.1%
	Contribution[i]	8.7%	12.0%	13.9%
WACC		14.9%	16.0%	16.6%
Discount Factor		0.8702	0.7501	0.6431
Present Value of FCFs		47,426	45,905	43,665
Total Enterprise Value		136,996		

[b]Noncash adjustments include depreciation plus $10,000 of other adjustments.
[d]Total Enterprise Value is the present value of the remaining cash flows.
[e]After-tax cost of debt is the Expected Cost of Debt times (1-tax rate).
[f]Debt contribution is the After-tax Expected Cost of Debt times the percent debt.
[g]Equity is determined by levering the asset beta ((asset beta - debt beta contribution)/percent equity).
[h]Cost of equity is calculated using the CAPM as the riskfree rate plus the equity beta times the risk premium.
[i]Equity contribution is the cost of equity times the percent equity.

assumed market parameters, the expected cost of equity is 32.1% in the first year. Weighting the expected after-tax cost of debt and the expected cost of equity by their proportions in the capital structure results in a *WACC* of 14.9% for the first year.

The capital structure changes in each period, because the ratio of the value of the remaining cash flows, and the amount of debt outstanding does not remain constant throughout the life of the project. Repeating the process of valuing the enterprise, determining the debt and equity proportions, unlevering the asset beta, and estimating the equity cost of capital according to the *CAPM*, results in a *WACC* of 16.0% for the second year and 16.6% for the third year. These after-tax *WACC*s rise as the percentage of debt in the capital structure, and the corresponding amount of the interest tax shields, fall.

Total Enterprise Value (*TEV*) is calculated by discounting the *FCF*s by the after-tax *WACC*s. Since the after-tax *WACC*s change, the discount rate for each period is the compounded rate that uses the preceding after-tax *WACC*s. The resulting value of the *FCF*s is $136,996, exactly the same value as obtained in the *CCF* calculations in Panel B of Table I.

B. Proof of Equivalency

This section shows that the *CCF* method is equivalent to the *FCF* method. To keep the analysis simple, assume the asset being valued generates a constant pre-tax operating cash flow. This cash flow is before tax, but after cash adjustments such as depreciation, capital expenditures, and changes in working capital. The after-tax operating cash flow equals earnings before interest and after-tax plus the cash flow adjustments and equals *FCF*, which measures the cash flow of the firm if it were all equity financed.

The value, V_{FCF}, is calculated using the *FCF* method by discounting the *FCF*s by the after-tax *WACC*:

$$V_{FCF} = \frac{FCF}{WACC} \tag{9}$$

where V is the value of the project being valued. *WACC*, the after-tax *WACC*, is defined as:

$$WACC = \frac{D}{V} K_D (1-\tau) + \frac{E}{V} K_E \tag{10}$$

with D and E equal to the market value of debt and equity, respectively; τ is the tax rate; $K_D(1-\tau)$ is the after-tax expected cost of debt; and K_E is the expected cost of equity.

The *CCF* is the expected cash flow to all capital providers with its projected financing policy, including any benefits of interest tax shields from its financial structure. Since *FCF* measures the cash flow assuming a hypothetical all equity capital structure, then *CCF* is equal to *FCF* plus interest tax shields:

$$CCF = FCF + Interest\,Tax\,Shield = FCF + \tau K_D D \tag{11}$$

where $K_D D$ is the interest tax shield calculated as the tax rate, τ, times the interest rate on the debt, K_D, times the amount of debt outstanding, D.

Value is calculated using the *CCF* method, V_{CCF}, by discounting the *CCF*s by the expected return on assets. The expected asset return is measured using the Capital Asset Pricing Model (*CAPM*) and the asset beta (β_U) of the project being valued:

$$V_{CCF} = \frac{FCF + \tau K_D D}{R_F + \beta_U R_P} \tag{12}$$

where R_F is the risk-free rate and R_P is the risk premium.

The goal is to show that the value obtained using *FCF*s and *WACC* is the same as the value obtained using *CCF*s and K_A. In other words, the goal is to show that equation (9) is identical to equation (12). By combining equations (9) and (10):

$$V_{FCF} = \frac{FCF}{K_D \frac{D}{V}(1-\tau) + \frac{E}{V} K_E} \tag{13}$$

In equation (13), K_E and K_D are measured using the *CAPM* according to equations (2) and (3). By substituting the equality between the pre-tax *WACC* and the cost of assets from equation (7):

$$V_{FCF} = \frac{FCF}{(R_F + \beta_U R_P) - \tau K_D \frac{D}{V}} = \frac{FCF}{K_A - \tau K_D \frac{D}{V}} \tag{14}$$

Multiplying both sides by the denominator on the right-hand-side of equation (14) yields:

$$V_{FCF}(K_A) - \tau K_D D = FCF \quad (15)$$

Rearranging terms by adding $\tau K_D D$ to both sides and dividing by the cost of assets shows that:

$$V_{FCF} = \frac{FCF + \tau K_D D}{K_A} = V_{CCF} \quad (16)$$

which is identical to equation (12). Thus, this proof shows that the *FCF* approach in equation (9) and the *CCF* approach of equation (12) will, when correctly applied, result in identical present values for risky cash flows.[6]

C. Choosing Between Capital Cash Flows and Free Cash Flow Methods

The proof in subsection II.B shows that the *CCF* method and the *FCF* method are equivalent because they make the same assumptions about cash flows, capital structure, and taxes. When applied correctly using the same information and assumptions, the two methods provide identical answers. The choice between the two methods, therefore, is governed by ease of use. The ease of use, of course, is determined by the complexity of applying the method and the likelihood of error.

The form of the cash flow projections generally dictates the choice of method. In the simplest valuation exercise, when the cash flows do not include the interest tax shields and the financing strategy is specified as broad ratios, the *FCF* method is easier than the *CCF* method. To apply the *FCF* method, the discount rate can be calculated in a straightforward manner using prevailing capital market data and information on the target capital structure. Because that target structure does not (by assumption) change over time, a single *WACC* can be used to value the cash flows. This type of valuation often occurs in the early stages of a project valuation before the detailed financial plan is developed. When the goal is to get a simplified 'back-of-the-envelope' value, the *FCF* method is usually the best approach.

When the cash flow projections include detailed information about the financing plan, the *CCF* method is generally the more direct valuation approach. Because such plans typically include the forecasted interest payments and *NI*, the *CCF*s are simply computed by adding the interest payments to the *NI* and making the appropriate non-cash adjustments. These cash flows are valued by discounting them at the expected cost of assets. This process is simple and straightforward even if the capital structure changes through time. In contrast, applying the *FCF* method is more complex and more prone to error because, as illustrated in subsection II.A and Panel C of Table I, firm and the equity values have to be inferred to apply the *FCF* method. Also, the *CCF* method can easily incorporate complex tax situations. Therefore, in most transactions, restructurings, leverage buyouts, and bankruptcies, the *CCF* method will be the easier to apply.

[6]Taggart (1991) analyzes the impact of personal taxes on the *FCF* approach and shows that the corporate tax rate is the only tax rate that explicitly enters the valuation equation. The algebraic equivalence of the *CCF* and *FCF* implies that the corporate tax rate is also the only tax rate in the *CCF* valuation equation. Nevertheless, personal taxes may affect the expected debt and equity returns and, thereby, affect the *FCF* and *CCF* value.

III. Capital Cash Flows and Adjusted Present Value

Both *CCF* and *APV* methods can be expressed as:

$$Value = Free\ Cash\ Flows\ Discounted\ at\ K_A + Interest\ Tax\ Shields\ Discounted\ at\ K_{ITS}$$

where K_{ITS} is the discount rate for interest tax shields. For both methods, the discount rate for the *FCF*s is the cost of assets, K_A, which is generally computed using the *CAPM* with the beta of an unlevered firm.The methods differ in K_{ITS}, the discount rate for interest tax shields: the *APV* method generally uses the debt rate and the *CCF* method uses the cost of assets, K_A. *APV* assigns a higher value to the interest tax shields so that values calculated with *APV* will be higher than *CCF* valuations.[7]

To gauge how much higher *APV* valuations are relative to *CCF* valuations, Table II calculates the difference in values assuming perpetual cash flows and interest tax shields. I define the value of the interest tax shields in the *CCF* valuation as a proportion, γ, of the all equity value. The ratio of V_{APV} to V_{CCF} becomes:

$$\frac{V_{APV}}{V_{CCF}} = \frac{1+\gamma\left(\dfrac{K_A}{K_D}\right)}{1+\gamma} \qquad (17)$$

Table II presents the percentage differences between the *APV* and *CCF* valuations. For example, if K_D =10% and K_A =15%, then the ratio of the expected asset return to the expected debt return is 1.5, locating it in the middle column of Table II. If the tax rate is 36% and the debt is 42% of the all equity value, then the value of the interest tax shield is about 15% of the all equity value, locating it in the middle row of Table II. In this example, therefore, the *APV* approach would provide a discounted cash flow value that is 7% higher than the *CCF* value.

In the *CAPM* framework, the discount rate for the interest tax shields should depend on the beta of the interest tax shields:

$$K_{ITS} = Risk\ free\ rate + \beta_{ITS} * Risk\ Premium \qquad (18)$$

When debt is assumed fixed, subsection A shows that the beta of the interest tax shields equals the beta of the debt. This implies that the appropriate discount rate for the interest tax shields is the debt rate, which is the rate used in the *APV* method. It also implies that the interest tax shields reduce risk so that a tax effect should appear when unlevering equity betas. When debt is assumed proportional to value, subsection B shows that the beta of the interest tax shields is equal to the unlevered or asset beta. This implies that the appropriate discount rate is the cost of assets, which is the rate used in the *CCF* method. It also implies that taxes have no effect on the transformation of equity betas into asset betas. Debt could also be a linear function of firm value with both fixed and proportional components. Subsection C shows that the beta of the interest tax shields with a linear debt policy is a value-weighted average of the interest tax shield betas for the fixed and proportional debt policies described

[7]Inselbag and Kaufold (1997) present examples of *FCF* and *APV* valuations that result in identical values for debt policies with both fixed debt and proportional debt. This occurs because they infer the equity costs that result in equivalence in their *FCF* valuations instead of obtaining discount rates from the *CAPM*.

Table II. Percentage Differences Between *APV* Values and *CCF* Values (V_{APV}/V_{CCF})

	Ratio of Expected Asset Return to Debt Rate (K_A/K_D)[a]		
Tax Shield/ All Equity Value[b]	**1.25**	**1.50**	**1.75**
10%	2%	5%	7%
15%	3%	7%	10%
20%	4%	8%	13%

[a]Calculations assume perpetual cash flows and interest tax shields.
[b]All Equity Value is the FCFs discounted at the cost of assets.

in subsections A and B, respectively.

A. Fixed Debt Policy

When debt is perpetual and fixed as a dollar amount, D, which does not change as the value of the firm changes, then the value of the interest tax shields is:

$$V_{ITS,t} = \frac{\tau \overline{K}_D D}{K_{D,t}} \tag{19}$$

where $\overline{K}_D$ is the fixed yield on the debt, $K_{D,t}$ is the cost of debt in period t from equation (2), and τ is the tax rate. The value of the debt can change through time if $\overline{K}_D$ is fixed and the cost of debt changes. Assuming $\overline{K}_D$ is the fixed yield,

$$V_{D,t} = \frac{D\overline{K}_D}{K_{D,t}} \tag{20}$$

By substituting equation (20) into equation (19), the value of the interest tax shield at time t can, therefore, be expressed as:

$$V_{ITS,t} = \tau V_{D,t} \tag{21}$$

The beta of the interest tax shields, β_{ITS}, equals:

$$\beta_{ITS} = \frac{Cov(V_{ITS,t}, R_M)}{V_{ITS,t-1} Var(R_M)} \tag{22}$$

By substituting equation (21) into equation (22) and simplifying,

$$\beta_{ITS} = \frac{Cov(V_{D,t}, R_M)}{V_{D,t-1} Var(R_M)} = \beta_D \tag{23}$$

The beta of the interest tax shields is, therefore, equal to the beta of the debt when the debt is assumed to be a fixed dollar amount.[8,9] If the debt is assumed to be riskless, then the interest tax shields will also be riskless. If the debt is risky, then the interest tax shields will have the same amount of systematic risk as the debt. This result shows that the practice of discounting interest tax shields by the expected return on the debt is appropriate when the debt is assumed to be a fixed dollar amount.

The assumption of fixed debt and the result that the beta of interest tax shields equals the debt beta implies that leverage reduces the systematic risk of the levered assets. The value of a levered firm, V_L, exceeds the value of an unlevered or all equity firm, V_U, by value of the interest tax shields from the debt of the levered firm, V_{ITS}:

$$V_L = V_U + V_{ITS} \tag{24}$$

Equation (24) holds in each time period and abstracts from differences between levered and unlevered firms other than taxes. Also, the analysis assumes strictly proportional taxes. I assume that interest is deductible and that interest tax shields are realized when interest is paid.

The beta of the levered firm, β_L, is a value-weighted average of the unlevered beta, β_U, and the beta of the interest tax shields, β_{ITS}:

$$\beta_L = \frac{V_U}{V_L}\beta_U + \frac{V_{ITS}}{V_L}\beta_{ITS} \tag{25}$$

When the beta of the interest tax shields equals the debt beta, equation (25) simplifies to:

$$\beta_L = \beta_U - \tau\frac{D}{V_L}(\beta_U - \beta_D) \tag{26}$$

The beta of a levered firm, β_L, can also be expressed as a value weighted average of the debt and equity of the levered firm:

$$\beta_L = \frac{E}{V_L}\beta_E + \frac{D}{V_L}\beta_D \tag{27}$$

Where E is the equity of the levered firm, β_E is the equity beta and β_D is the debt beta. By setting equation (26) equal to equation (27):

$$\frac{E}{V_L}\beta_E + \frac{D}{V_L}\beta_D = \beta_L = \beta_U - \tau\frac{D}{V_L}(\beta_U - \beta_D) \tag{28}$$

which can be simplified as:

$$\beta_E = \left(\beta_U - \frac{D}{V_L}(\beta_D + \tau(\beta_U - \beta_D))\right) / \frac{E}{V_L} \tag{29}$$

Thus, the equity beta is equal to the asset beta less the proportion of debt borne by the debt holder and the reduction due to the tax effect and scaled by leverage. The equity beta is reduced by the tax effect, because the government absorbs some of the risk of the cash

[8]When debt is assumed to be fixed in value instead of a fixed dollar amount, then the beta of the interest tax shields is zero regardless of the debt beta.

[9]If the debt is not principal, then the beta of the interest tax shields would equal the beta of the debt when the interest payment and principle payments have the same beta.

flows. With fixed debt, the interest tax shields portion of the cash flows are insulated from fluctuations in the market value of the firm.

When the debt is riskless, the beta of the debt is zero. Therefore, equation (29) simplifies to:

$$\beta_E = \frac{E + D(1-\tau)}{E}\beta_U \tag{30}$$

Equation (30) is the standard unlevering formula that correctly includes tax effects when the debt is assumed to be fixed and assumes a zero debt beta. In the next subsection, I show that when debt is assumed to be proportional to firm value, taxes do not appear in the unlevering formula.

B. Proportional Debt Policy

When the value of debt is assumed to be proportional to total enterprise value, the firm varies the amount of debt outstanding in each period so that:

$$V_D = \delta V_U \tag{31}$$

where δ is the proportionality coefficient and V_U is the value of the unlevered firm. The value of the interest tax shields is the tax rate times the value of the debt so that:

$$V_{ITS} = \tau V_D = \tau\delta V_U \tag{32}$$

By substituting equation (32) into the definition of the beta of the interest tax shields from equation (22):

$$\begin{aligned}\beta_{ITS} &= \frac{Cov(V_{ITS,t}, R_M)}{V_{ITS,t-1}Var(R_M)} \\ &= \frac{Cov(\tau\delta V_{U,t}, R_M)}{\tau\delta V_{U,t-1}Var(R_M)} \\ &= \frac{Cov(V_{U,t}, R_{M,t})}{V_{U,t-1}Var(R_M)} \\ &= \beta_U\end{aligned} \tag{33}$$

The equality between the beta of the interest tax shields and the beta of the unlevered firm implies that the rate used to discount the interest tax shields is equal to K_A, the unlevered or asset cost of capital.[10]

The equality between the betas for the interest tax shields and the assets also implies that there is no levering/unlevering tax effect. From equation (25), the beta of a levered firm is a weighted average of the beta of the unlevered firm and the beta of the interest tax shields. Since the asset beta equals the interest tax shield beta, the beta of the levered firm equals the beta of the

[10]Harris and Pringle (1985) also show that the interest tax shields should be discounted by the pre-tax weighted average cost of capital when debt is assumed to be proportional to value.

unlevered firm. To calculate the beta of levered equity, equation (29) can be restated as:

$$\beta_E = \left(\beta_U - \frac{D}{V}\beta_D\right) / \frac{E}{V} \tag{34}$$

This result means that tax terms should not be include when applying the *CCF* or *FCF* methods.[11]

C. Choosing Between Capital Cash Flows and Adjusted Present Value Methods

Subsection B shows that the difference between the *CCF* and the *APV* methods is the implicit assumption about the determinants of leverage. *CCF* (and equivalently *FCF*) assumes that debt is proportional to value; *APV* assumes that debt is fixed and independent of value. Debt cannot literally be strictly proportional to value at all levels of firm value. For example, when a firm is in financial distress, the option component of risky debt increases, thereby, distorting the proportionality. Nevertheless, Graham and Harvey (2001) report that about 80% of firms have some form of target debt-to-value ratio, and that the range around the target is tighter for larger firms. That suggests that the *CCF* approach is more appropriate than the *APV* approach when valuing corporations.

There are circumstances when the fixed debt assumption is more accurate. These cases typically involve some tax or regulatory restriction on debt, such as industrial revenue bonds that are fixed in dollar amounts. Luehrman (1997) presents an example of *APV* valuation in which debt is assumed to be a constant fraction of book value. To the extent that book value does not respond to market forces, a fraction of book value is a fixed dollar amount.

In practice, valuations are often performed on forecasts that make assumptions about debt policy. When that policy is characterized as a target debt-to-value ratio, the proportional policy seems more accurate. In project finance or leveraged buyout situations, however, the forecasts typically are characterized as a changing dollar amount of debt in each year. These amounts can, of course, be characterized as a changing percentage of value or as a changing dollar amount through time. It is not obvious from the forecasts themselves whether the assumption of proportional debt or fixed debt is the better description of debt policy. The answer in these circumstances depends on the likely dynamic behavior. If debt policy adheres to the forecasts regardless of the evolution of value through time, the fixed assumption is probably better. Alternatively, if debt is likely to increase as the firm expands and value increases, then the proportional assumption is probably better.

Debt policy can, of course, be more complex than either an exclusively fixed debt or proportional debt policy; whatever the debt policy, valuation depends on that policy. For example, debt policy can include a fixed component and a component that is proportional to value:

$$V_D = V_F + \delta V_U \tag{35}$$

Such a linear debt policy could occur in a project finance application where a fixed amount of debt is subsidized or guaranteed by a government agency and the remaining debt is roughly proportional to the value of the project.

[11] Kaplan and Ruback (1995) incorrectly uses tax adjustments to unlever observed equity betas to obtain asset betas when applying the *CCF* method. Correcting this error does not meaningfully change the results of Kaplan and Ruback (1995).

The valuation of cash flows from a project with a linear debt policy such as equation (35) will combine features of the fixed debt and proportional debt policies. The beta of the interest tax shields for the linear debt policy, for example, is a value-weighted average of the beta with a fixed debt policy (equation 23) and the beta with a proportional debt policy (equation 33) with the value-weights equal to the relative values of the fixed and proportional debt components:

$$\begin{aligned}\beta_{ITS} &= \frac{Cov(\tau V_{D,t} R_M)}{\tau V_{D,t-1} Var(R_M)} \\ &= \frac{Cov(\tau V_{Ft} + \delta V_{Ut}, R_M)}{\tau V_{D,t-1} Var(R_M)} \\ &= \frac{V_{F,t-1}}{V_{D,t-1}}\beta_D + \frac{\delta V_{U,t-1}}{V_{D,t-1}}\beta_A \end{aligned} \tag{36}$$

The value of a project with a linear capital structure could be valued by valuing the interest tax shields using the pre-tax expected return implied by the beta of the interest tax shields from equation (36) and adding that value to the value of the *FCF*s:

$$V_{APV} = \frac{FCF}{K_A} + \frac{\tau K_D D}{K_{ITS}} \tag{37}$$

where D is the amount of debt including the fixed and proportional component, and K_{ITS} is calculated using the *CAPM* with β_{ITS} from equation (37).

The value of the project can also be valued more simply by adding the value of the fixed interest tax shields to the *CCF* value:

$$V_{CCF} = \frac{FCF + \tau K_D D_P}{K_A} + \frac{\tau K_D D_F}{K_D} \tag{38}$$

where D_P and D_F are the amount of proportional debt and fixed debt, respectively. The first term on the right-hand-side of equation (38) is the formula for the *CCF* value when the debt is proportional to value (equation 16). The discount rate for the interest tax shields from the proportional debt is the expected asset return for the same reasons it is the correct rate for the proportional interest tax shields discussed in subsection B. In short, interest tax shields are proportional to the value of the debt so that when debt is proportional to value, the interest tax shields will have the same risk as the value of the firm.

The second term on the right-hand side of equation (38) is the value of the interest tax shields associated with the fixed portion of the linear debt policy. The discount rate for the interest tax shields from the fixed portion is the expected debt rate for the same reason that it is the correct rate for the fixed interest tax shields in discussed in subsection A. When the amount of debt is fixed, the interest tax shields are also fixed, and the value of the interest tax shields will vary as the value of the debt varies. The fixed interest tax shields, therefore, will have the same risk as the fixed debt.

Equation (38) is consistent with the generally accepted approach of identifying project cash flows with different risk characteristics and valuing those components at an expected return that reflects their risk. In Gilson, Hotchkiss, and Ruback (2000), for example, the value of firms emerging from bankruptcy are valued as the *CCF* value of their continuing operations plus the value of their fixed net operating losses discounted at a debt rate. The different risk characteristics of the interest tax shields in equation (38) arise because of the combination of fixed and proportional debt in the linear debt policy. Similarly, Miles and Ezzell (1980, 1983) model the debt policy as mixed through time with fixed debt in the first period and proportional debt in subsequent periods.

The best approach to estimating the value of interest tax shields is to model the debt policy, and then appropriately value resulting interest tax shields using the corresponding discount rate. As an example, Arzac (1996) recognizes that excess available cash flow is typically used to repay senior debt after a leveraged buyout and suggests a "recursive APV approach" to value the transactions. In most corporate circumstances, however, the valuation, at least at its initial stage, will not have the information to model the debt policy in detail and with precision. The practical alternatives may be to simply choose between *APV* approach with its assumed fixed debt policy and the *CCF* (or equivalent *FCF*) approach with its assumed proportional debt policy.

Beyond the Graham and Harvey (2001) evidence that most corporations have target debt ratio, theories of debt policy generally suggest that debt changes as value changes. For example, in the static tradeoff between tax benefits and bankruptcy costs, doubling the operations of a firm would double its value, which, in turn, doubles the tax benefits of debt financing and bankruptcy costs so that the amount of debt would also double. Thus, for most applications, the proportional debt assumption appears to be a more accurate description of corporate behavior. That means that the *CCF* or the equivalent *FCF* method of valuation will generally be preferred to *APV* and that asset beta calculations should not include tax adjustments.

IV. Conclusions

This paper presents the Capital Cash Flow (*CCF*) method of valuing risky cash flows. The *CCF* method is simple and intuitive. The after-tax *CCF*s are just the before-tax cash flows to both debt and equity, reduced by taxes including interest tax shields. The discount rate is the same expected return on assets that is used in the before-tax valuation. Because the benefit of tax deductible interest is included in the cash flows, the discount rate does not change when leverage changes.

The *CCF* method is algebraically equivalent to the popular method of discounting *FCF*s by the after-tax *WACC*. But in many instances, the *CCF* method is substantially easier to apply and, as a result, is less prone to error. The ease of use occurs because the *CCF* method puts the interest tax shields in the cash flows and discounts by a before-tax cost of assets. The cash flow calculations can generally rely on the projected taxes, and the cost of assets does not generally change through time even when the amount of debt changes. In contrast, when applying the *FCF* method, taxes need to be inferred, and the cost of capital changes as the amount of debt changes.

The *CCF* method is closely related to the *APV* method. *APV* is generally calculated as the sum of operating cash flows discounted by the cost of assets plus interest tax shields discounted at the cost of debt. The interest tax shields that are discounted by the cost of debt in the *APV* method are discounted by the cost of assets in the *CCF* method. The *APV* method results in a higher value than the *CCF* method, because it treats the interest tax shields as being less risky than the firm as a whole, because the level of debt is implicitly assumed to be a fixed dollar amount. As a result, a tax adjustment is made when unlevering an equity beta to calculate an asset beta. In contrast, the *CCF* method, like the *FCF* method, makes the more economically plausible assumption that debt is proportional to value. The risk of the interest tax shields, therefore, matches the risk of the assets.

Beyond introducing the *CCF* method, demonstrating its conceptual equivalence to the *FCF* method, and showing its relation to the *APV* method, this paper makes the more general point that the financial policy affects the choice of valuation technique. A proportional debt policy, for example, implies that interest tax shields are valued at the cost of assets and that taxes do not affect the measure of risk that goes into calculating the discount rate. In contrast, when the amount of debt is fixed, interest tax shields are valued at the expected return of debt and taxes do affect the measure of risk. Furthermore, the debt policy need not be exclusively proportional or fixed, and, as an example, I provide a *CCF* valuation for a linear debt policy that has both fixed and proportional components. Whatever the debt policy, valuation depends on that policy and the challenge is to value the cash flows using an approach that consistently incorporates the assumption about debt policy.■

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