COOPERATIVE BEHAVIOR IN THE PRISONER DILEMMA: THE ROLE OF RULES IN INSTITUTIONS

José Casas Pardo
Juan D. Montoro-Pons

IUDEM
Documento de Trabajo 2000-3
Año 2000

ABSTRACT

A basic problem that may arise in situations of collective decision-making is that of individual rationality vs. collective welfare which can be depicted by the prisoner dilemma. The problem has been widely analyzed in the game theory literature yielding several insights to the problem of social cooperation mostly based in the differences posed by short term vs. long term relationships between the players. The following paper deals with a dynamic setting of the prisoner dilemma, in which a coordination mechanism, to be understood as the set of norms of the institution, play a key role in achieving cooperation between the participants. Moreover cooperation is maintained for any number of players.

Keywords: Cooperation, legal systems, collective institutions, strategic behavior, controlled game.

JEL: C72, C73, D74.

RESUMEN

Un problema que puede surgir en situaciones de decisión colectiva es la contraposición racionalidad individual vs. racionalidad colectiva, que puede ser representado a través del dilema del prisionero. El problema ha sido ampliamente analizado en la literatura de teoría de juegos, dando lugar a diversas aproximaciones al problema de la cooperación social, principalmente basadas en las diferencias que se plantean entre las relaciones a corto y a largo plazo. El presente trabajo considera el dilema del prisionero en un contexto dinámico, en el cual un mecanismo de coordinación, que podemos entender como el conjunto de normas de las instituciones, juegan un papel clave en la determinación de la cooperación entre los participantes. Adicionalmente, se mostrará cómo la cooperación se mantiene para cualquier número de jugadores.

Palabras clave: cooperacion, sistemas legales, instituciones colectivas, comportamiento estrategico, juego controlado.

JEL: C72, C73, D74.

El texto completo de este trabajo puede descargarse (en formato PDF) de la página web del IUDEM: http://www. ucm.es/info/iudem.

Cooperative behavior in the prisoner dilemma: the role of rules in institutions

Jose Casas-Pardo, Juan D. Montoro-Pons

Departamento de Economía Aplicada,Universitat de València, 46022 Valencia, Spain, e-mail: Jose.Casas@uv.es, and e-mail: Juan.D.Montoro@uv.es

Abstract A basic problem that may arise in situations of collective decision-making is that of individual rationality vs. collective welfare which can be depicted by the prisoner dilemma. The problem has been widely analyzed in the game theory literature yielding several insights to the prblem of social cooperation mostly based in the differences posed by short term vs. long term relationships between the players. The following paper deals with a dynamic setting of the prisoner dilemma, in which a coordination mechanism, to be understood as the set of norms of the institution, play a key role in achieving cooperation between the participants. Moreover cooperation is maintained for any number of players.

Resumen Un problema que puede surgir en situaciones de decisión colectiva es la contraposición racionalidad individual vs. racionalidad colectiva, que puede ser representado a través del dilema del prisionero. El problema ha sido ampliamente analizado en la literatura de teoría de juegos, dando lugar a diversas aproximaciones al problema de la cooperación social, principalmente basadas en las diferencias que se plantean entre las relaciones a corto y a largo plazo. El presente trabajo considera el dilema del prisionero en un contexto dinámico, en el cual un mecanismo de coordinación, que podemos entender como el conjunto de normas de las instituciones, juegan un papel clave en la determinación de la cooperación entre los participantes. Adicionalmente, se mostrará como la cooperación se mantiene para cualquier número de jugadores.

1 Introduction

One of the main issues that may arise when analyzing collective institutions (CI) is the one regarding the sub-optimality of most collective processes due to the lack of coordination of individual actions. In many social environments the interaction of fully rational individuals may yield suboptimal outcomes, being game theory and specifically the prisoner dilemma (PD) the main tool used to gain insights in the problem. From this point of view the problems of coordination and the design of mechanisms that lead to pareto improvements are in the core of CI analysis.

As it is well known, markets (under perfect competition) yield pareto efficient outcomes from the decisions taken by a myriad of independent individuals. That means that when individuals pursue their own goals by taking rational selfish decisions, normally expressed as the constrained maximization of some objective function, the best collective solution is attained. Shall CI perform as competitive markets the usual conclusions would apply; but the complex exchange arising from collective decision making is better depicted as the outcome of both conflict and cooperation. When these two characteristics arise, welfare improvements are usually attainable provided individuals coordinate their strategies; in this case there is a need for inducing some behavior on individuals. Consider the PD: while both players will be better off by cooperating it is true that a rational player, given the behavior of the other player, will choose to defect. A Nash (non-cooperative) solution is attained and the aggregated output is below the maximum attainable. In fact this simple framework has been widely used in assessing many policy affairs (see Axelrod (1984) for a non restricted application) and in describing the (poor) performance of CI.

The purpose of this paper is to extend the game theoretic analysis of CI in a dynamic framework. We will show how the introduction of institutional arrangements allow for efficient outcomes, even for a increasing number of individuals involved in the process.

2 Controlled games and collective institutions: a redistribution model

One of the most fruitful approaches in the analysis of institutions and the behavior of individuals within institutions (either private or collective, but specially in the latter case) is the game theoretical one. Game theory deals with strategic situations in which players must implement an action in order to achieve certain well defined goals. If this process of strategic choice is modeled in terms of conflict and cooperation many of these situations may be solved in terms of the degree of cooperation individuals show: as Binmore (1992) points out, the question is whether it is rational to cooperate with other rational players. Whenever cooperation is not an equilibrium, it may be attained by means of the introduction of institutional arrangements that allow to coordinate the outcomes of the players, as it will be later shown.

New economic institutionalists (Nelson 1995) have emphasized a rule-oriented definition of institutions in which these are the way a game is played. If multiple equilibria arise in a game, specific institutions define its outcome. If collective processes involve repeated PD-type games individuals face a potential coordination failure, that is, the optimal, or pareto-efficient, situation may not be achieved due to a lack of institutional mechanisms within the game that shift the players into a more cooperative solution. To reach superior collective situations arrangements need to be introduced in the game. The attainment of efficiency may be done in different ways: when considering a (one-shot) static game different behavioral assumptions (for instance in the form of preplay communication, Sorin 1997) may help in enlarging the strategy sets and in achieving outcomes pareto preferred; if the game is considered to be played for more than one period individual strategic decisions may lead to pareto improvements. In fact this is the main result of the *Folk Theorem* for repeated games which states that asymptotically any payoff over the threat point is sustainable as an equilibrium point; an example would be the *trigger strategy* for repeated games in which efficient solutions become feasible for suitable discount factors (see for example Friedman 1991, Eichberger 1993, or Aumann 1986). As our interest remains in the interaction between individuals when engaged in CI, as most of them imply a long term relationship between its members, it seems plausible to choose a dynamic framework.

Dynamic games allow for a wider range of equilibria than the constituent one-shot game. As the *Folk theorem* states, in a infinitely repeated game every individually rational payoff is attainable if players are patient enough: when committed in a long term relationship individuals may be willing to cooperate inasmuch there are benefits arising from this cooperation. Turning again into the PD, intuition points to the fact that if both individuals are engaged in a repeated relationship, they both could benefit from the outcomes that might arise from cooperation; in this case it is not rational to maximize the present payoffs, as the game will evolve in time, but to maximize the flow of (discounted) payoffs (in this respect the degree of impatience showed by the players will be reflected in their degree of cooperation: the less impatient are the players, i.e.the closer to one is the discount rate, the more likely is the cooperative behavior). When dealing with games with a known ending period T, the backwards induction procedure leads us to the same situation than in one-shot non-cooperative games; in order to attain a higher efficiency in this type of games institutional arrangements (see for instance Hirshleifer and Rasmusen (1989), where they consider the expulsion of the defector in the next round of the game, a threat that under some assumptions may become credible) or more restrictive assumptions are needed, as informational deficiencies (incomplete information becomes important in achieving cooperation in Kreps et al. (1982); Kreps and Wilson (1982) show how choosing in an uncertain environment any outcome is possible in a game; similar results are obtained by analyzing problems of reputation, as Milgrom and Roberts (1982) do). This problem does not arise in infinitely repeated games. But saying that the playing horizon is infinite is not to suppose that the game is to be played forever; in an uncertain environment (when the ending period is not known) the discount factor of the payoffs may be interpreted as the probability at which the game is to end in some period.

Finally there is an additional fact that must be considered when dealing with repeated games: cooperation may be affected by the number of players.

As Myerson (1991) points out

> ...the feasibility theorem can be interpreted as a statement about the power of social norms in small groups...when we move from small groups into larger social structures the assumption that everyone observe everyone else may cease to hold and general feasibility can fail...

The problem posed is evidently an informational one, and spreading the information available over the set of players will help to recover the original situation (see for example Milgrom et al. (1990) where the addition of a central mediator that keeps trace of the non-cooperative individuals sustains cooperation).

Summarizing, in a game theoretical context it is considered that the repetition of a game may allow for a cooperative behavior in the players, which will be directly related to the degree of impatience the individuals show, the information set available to the individuals and inversely related to the size of the social group, all the things being equal. The aim of this paper is to implement a setting that allow the achievement of cooperation by means of a coordination mechanism that will act inducing the commitment of the players with a previously specified strategy.

2.1 Collective institutions as controlled games: basic definitions

In what follows we will develop a simple game in which the presence of a coordinator may lead to an efficient outcome. Our aim in this section is to implement the dynamical game under the existence of a coordinator (or a *controlled game*) for analyzing the performance of CI. We begin by defining the most basic concepts: define a *coordination mechanism* (or *the coordinator*) as the institutional arrangements of a particular CI that induce some behavior in the agents. Individuals have the possibility to choose; this is reflected in their strategic spaces. Nevertheless, this possibility is normally coerced within certain institutions. The rules of a CI act, in fact, as a coercive mechanism that seeks to induce (or rule out) certain behaviors. What norms, rules, or more generally constitutions do is to modify (reducing) the strategic space of the participants in the game. This coercive mechanism is but a way of controlling the outcomes of the game. To some extent the coordinator can be seen as the set of rules that coerces individual actions, or put in another way, the legal constraints faced in the game.

Let $\Gamma = (\mathcal{N}, \mathcal{S}, \mathcal{P})$ be a one period non-cooperative game with complete information in strategic form, being $\mathcal{N}$ the set of n players, $\mathcal{S} = \times_{i=1}^{n} \mathcal{S}_i$ the n-dimensional strategic space (with $\mathcal{S}_i$ the strategic space of player i), and $\mathcal{P}$ the $1 \times n$-dimensional payoffs array (i.e. the values that the n players get from playing). Now assume that the game Γ depends (in some predefined way) on a parametric set Θ. At any given instant of time a CI can be viewed as a pair $\langle \Gamma, \Theta \rangle$. However the parametric set Θ varies in time according to the decisions of a coordinator; let F be the mechanism that define the state transitions of Θ.

We define a *collective institution* as a tuple $\text{CI} = \langle \Gamma, \Theta, F \rangle$, where Γ is a one period non-cooperative game with perfect information, and Θ and F represent a coordination mechanism. Given the previous definition a CI is a game under the effect of a coordination mechanism, which will be called as well a *controlled game.* Even though the game is static by definition, due to the fact that the coordinator is a dynamic mechanism we will find the game to be played during a (possibly) infinite number of periods and this will define a dynamical system that will evolve in time.

According to the previous definition a CI includes the following elements:

- a game that defines its components (i.e., the players), their possible actions (strategic space), and the output of their interaction (the payoff vector);
- a coordination mechanism defined by a parametric space and its evolution over time that affects the outcomes of the game. It can be seen as a set of rules that induces some behavior in the players.

The parametric set is defined as:

$$\Theta = \{\theta_i \in \mathbf{R} : \theta_i \geq 0 \wedge \sum_{i=1}^{n} \theta_i = 1,\, i = 1 \ldots n\} \tag{1}$$

and it affects in a known way the strategies of the players. The value of θ_i is determined by the coordinator by means of a decision rule taking into consideration the past actions of the players; these in turn implies that the actual behavior of any player will affect her future payoffs. The one-shot *controlled game* can be seen as the modified game under the action of the coordinator: $\Gamma(\Theta) = (\mathcal{N}, \mathcal{S}(\Theta), \mathcal{P}(\Theta))$, where $\mathcal{N}$ is (as usual) the set of n players, $\mathcal{S}(\Theta) = \times_{i=1}^{n} \mathcal{S}_i(\theta_i)$ is the strategic n-dimensional space and $\mathcal{P}(\Theta)$ is the n-dimensional payoff vector. In this setting the coordinator defines, given the strategies of the different players, a decision rule for the parametric set:

$$\dot{\Theta} = F(\Theta, \mathcal{S}(\Theta)) \tag{2}$$

where

$$\dot{\Theta} = \frac{d\Theta}{dt}$$

Given the game $\Gamma(\Theta)$, the rational player will try to maximize the payoff stream from any moment until the end of the game. Let J_i be the stream of payoffs for the i-th player; then the individual maximization problem is given by:

$$J_i = \int_{t_0}^{\infty} e^{-\rho(t-t_0)} p_i(\theta_i, s_1(\theta_1), s_2(\theta_2), \ldots, s_n(\theta_n))dt \tag{3}$$

in the discounted infinite horizon game, where $s_i(\theta_i) \in \mathcal{S}_i(\theta_i)$. That is to say, any player's objective is to maximize the stream of future payoffs given by (3).

Define $\tilde{s}_i$ the Nash (or non-cooperative) equilibrium of the game; when no coordination mechanism exists, this may be the strategic option in the one shot-game: it maximizes the payoff of any player given the strategies of the rest of the players. Introducing the coordinator may yield a (probably) different strategy if the parameter θ_i modifies the current behavior of the player. Suppose that the parameter affects in such a way that a cooperative behavior is induced; define the cooperative strategy of player i as $\hat{s}_i$. Finally define the degree of cooperation of player i by means of a parameter ν_i, such that $0 \leq \nu_i \leq 1$. Now assume that the action taken by any player under the coordinated game is to choose between the cooperative and non-cooperative strategy, so its final strategy σ_i will be a linear combination of both:

$$\sigma_i(\nu_i) = \nu_i \hat{s}_i + (1 - \nu_i)\tilde{s}_i \tag{4}$$

Let $\pi_i(\theta_i, ..., \sigma_i ...)$ be the payoffs of player i when her strategy follows the process in (4); in this case, and once the function $F(\cdot)$ of equation (2) is specified, the value function of the controlled game becomes:

$$J_i = \int_{t_0}^{\infty} e^{-\rho(t-t_0)} \pi_i(\theta_i, \sigma_1, \sigma_2, \ldots, \sigma_n)dt \tag{5}$$

Any rational player objective is to maximize J_i, which involves solving an optimal control problem, being Θ the state variables and ν_i the control variables. In the next section a redistribution model (as in Weeren et al. (1996)) will be derived and it will be shown how cooperative solutions might be achieved in this setting.

2.2 Coordinating cooperative outcomes in a simple two players PD game

The main point of the model will be developed in a two person setting as it will help us to clarify the role of the coordinator and how its behavior influences the agents decisions. As it is usual, any player can assure for herself the corresponding Nash equilibrium payoffs; this means that by acting non-cooperatively individuals are best replying to any other participant in the exchange. This by no means will necessarily imply maximizing the collective output, as already mentioned, as in a PD setting the individual maximization is suboptimal; in order to do so a coordination mechanism should exist. What we propose is to define this mechanism as a redistributional one, in which the agents receive (in addition to their Nash values) part of the benefits of their cooperation. In fact the coordination mechanism is a type of legal system: Kuhn and Moresi (1995) define a legal system in terms of the monetary transfers between players when committed strategies are not followed. In a more general sense, a legal system may be viewed as one that redistributes the utility in a social setting when rules are violated. This is in fact the perspective we use in the paper.

Let the game in the previous section be defined as a two person game plus a coordinator. In this case the (controlled) game will be defined as follows:

- Let θ_1 and θ_2 be (a transformation of) the parameters that control the game. The parametric set is defined as per (1).

– From (2) the decision rule of the coordinator in a two player setting is

$$\dot{\vartheta}_i = f_i(\vartheta i, s_1(\Theta), s_2(\Theta))$$

. Define

$$\theta_i = \frac{\vartheta_i}{\vartheta_i + \vartheta_j} \tag{6}$$

$\forall\ i \neq j$ and $i, j = 1, 2$; the following dynamic process will be used:

$$\dot{\vartheta}_i = \beta\vartheta_i(1 - \vartheta_i)(\nu_j - \nu_i) \tag{7}$$

with $\beta < 0$ the meaning of which will be discussed later. Equation (7) ensures that the ϑ's lie within the interval $[0, 1]$, while (6) ensures the restrictions of the parametric set (1).

– The strategies of the players evolve according to (4); the payoffs of any agent at any stage of the game will be $p_i(\theta, \sigma_1, \sigma_2)$. In this case the payoffs of any player will be the minimum non-cooperative payoffs plus the amount exceeding those. Let the extra payoff (those above the Nash point) of the game be defined as

$$\begin{aligned}\pi^*(\nu_1, \nu_2) &= p_1(\sigma_1(\nu_1), \sigma_2(\nu_2)) + p_2(\sigma_1(\nu_1), \sigma_2(\nu_2)) \\ &\quad -p_1(\tilde{s}_1, \tilde{s}_2) - p_2(\tilde{s}_1, \tilde{s}_2)\end{aligned} \tag{8}$$

This implies that whenever the strategy of any player is different from the non-cooperative one there will be extra benefits (this may be positive or negative) in the institution. Specifically, when acting cooperatively the players generate a surplus for the group. The redistribution of this extra-outcomes will follow the rule:

$$\pi_1(\theta_1, \nu_1, \nu_2) = \theta_1\pi^*(\nu_1, \nu_2) \tag{9}$$

$$\pi_2(\theta_2, \nu_1, \nu_2) = \theta_2\pi^*(\nu_1, \nu_2) \tag{10}$$

So there is a proportional redistribution with respect to the control parameter θ.

In order to fully understand the implications of the above setting we will begin by examining the behavior of the coordinator. Given the initial conditions of the game (the values $\theta(t_0)$, $\nu_1(t_0)$ and $\nu_2(t_0)$), and $\beta < 0$[1], the evolution of the strategies of the players induces changes in θ; whenever one player is more cooperative than the other, (7), (9) and (10) ensure that the redistribution mechanism will benefit her. The controlled game punishes non-cooperative behavior, or put in other words, the coordinator seeks for pareto efficient outcomes by means of redistributing the extra benefits of the institution depending on the willingness to cooperate that the players show: this is achieved by setting $\beta < 0$. In this setting each player faces a problem of maximization of the flow of extra benefits during the game, that is:

$$\text{Maximize}\, J_i = \int_{t_0}^{\infty} e^{-\rho(t-t_0)}\pi_i(\theta, \nu_1, \nu_2)dt \tag{11}$$

s.t.

$$\begin{gathered}\dot{\vartheta}_1 = f_1(\vartheta_1, \nu_1, \nu_2) \\ \dot{\vartheta}_2 = f_2(\vartheta_2, \nu_1, \nu_2) \\ 0 \leq \nu_i \leq 1 \\ \vartheta_i(t_0) = \vartheta_i^0 \quad \nu_i(t_0) = \nu_i^0 \quad i = 1, 2\end{gathered}$$

The above gives rise to a differential game, and in order to derive the optimal strategy for each player an optimal control problem must be solved.

[1] We will suppose for simplicity that it is announced at the beginning of the game and it will be fixed in the rest of the setting.

2.3 Solving the model

Suppose a game in which the payoff matrix of the interaction between column and row is given by:

$$\begin{pmatrix} b, b & d, a \\ a, d & c, c \end{pmatrix} \tag{12}$$

where the first element of each pair is row's payoff and the second is column's one; the first column/row is cooperate (C) and the second column/row is defect (D).

Definition 1 (Strictly utilitarian PD, Kuhn and Moresi 1995) *Assume the bimatrix game (12), and let the following restrictions hold among the payoffs:*

(i) $a > b > c > d$
(ii) $b + b > d + a > c + c$

Then the only equilibrium of the game is given by (D,D), and the maximum aggregated payoffs are reached at the stratety pair (C,C).

Definition 1 depicts the main and paradoxical outcome of the PD: as $b > c$ the equilibrium payoffs are below the cooperative strategy and rationality leads to a suboptimal situation. More important, Kuhn and Moresi's taxonomy helps to clarify when cooperation can be assured by means of utility transfers. Assume mixed strategies for this game, that is probability distributions over the set of pure strategies. This in turn leads us to the concept of expected payoff of the game[2]; these are:

$$\begin{aligned} p_1(\sigma_1(\nu_1), \sigma_2(\nu_2)) &= p_1^1(\hat{s}_1)\nu_1\nu_2 + p_1^2(\hat{s}_1)\nu_1(1-\nu_2) + \\ &\quad p_1^1(\tilde{s}_1)(1-\nu_1)\nu_2 + p_1^2(\tilde{s}_1)(1-\nu_1)(1-\nu_2) \qquad (13) \\ p_2(\sigma_1(\nu_1), \sigma_2(\nu_2)) &= p_2^1(\hat{s}_2)\nu_1\nu_2 + p_2^2(\hat{s}_2)(1-\nu_1)\nu_2 + \\ &\quad p_2^1(\tilde{s}_2)\nu_1(1-\nu_2) + p_2^2(\tilde{s}_2)(1-\nu_1)(1-\nu_2) \qquad (14) \end{aligned}$$

Keeping in mind that

$$\begin{aligned} p_1^1(\hat{s}_1) &= p_2^1(\hat{s}_2) = b \\ p_1^2(\tilde{s}_1) &= p_2^2(\tilde{s}_2) = c \\ p_1^2(\hat{s}_1) &= p_2^2(\hat{s}_2) = d \\ p_1^1(\tilde{s}_1) &= p_2^1(\tilde{s}_2) = a \end{aligned}$$

The summation of payoffs is

$$p_1(\cdot) + p_2(\cdot) = (a+d)(\nu_1 + \nu_2 - 2\nu_1\nu_2) + 2b\nu_1\nu_2 + 2c(1-\nu_1)(1-\nu_2)$$

As the redistribution mechanism that controls the game effects the output level over the Nash Equilibrium, we shall deduct this amount from the above expression in order to get π^*; this yields:

$$\pi^* = \alpha(\nu_1 + \nu_2 - 2\nu_1\nu_2) + \lambda\nu_1\nu_2 + \gamma(1-\nu_1)(1-\nu_2) - \gamma \tag{15}$$

where $\alpha = a + d$, $\lambda = 2b$, and $\gamma = 2c$. So the problem now is reduced to a dynamic maximization one, in which (15) is the instantaneous payoffs of the game, and each

[2] Although the concept of mixed strategies relies on probabilistic basis this may not be the case in our work; suppose a PD in which there is a continuum of strategies between non-cooperation and cooperation. Suppose a mapping $f : \mathcal{S}_i \mapsto [0,1]$, where $f(\mathrm{D}) = 0$ and $f(\mathrm{C}) = 1$; in this case we may identify the cooperative behavior of any individual in the interval [0,1], that is exactly what the parameter ν_i does.

player's sharing in these extra-payoffs is given by (9) and (10). This in turn implies maximizing the hamiltonian of the optimal control model, given by:

$$H_i = e^{-\rho(t-t_0)}\pi_i(\theta_i,\nu_1,\nu_2) + q_{ii}f_i(\vartheta_i,\nu_1,\nu_2) + q_{ij}f_j(\vartheta_j,\nu_1,\nu_2) \tag{16}$$

where ϑ are the state variables, ν_i are the controls, and q_{ii} and q_{ij} the costate variables, subject to the evolution of the state variable and the boundary conditions. The *Pontryagin maximum principle* (see for example Takayama 1994; Tu 1992; or Intrilligator 1971) defines, for ν_i^* being a solution of the control problem with the corresponding state variable ϑ, the following necessary optimality conditions:

1. q_{ii}^*, q_{ij}^* and θ^* are the solution of the *hamiltonian system*:

$$\dot{\vartheta}_i = \frac{\partial H_i}{\partial q_{ii}} \tag{17}$$

$$\dot{q}_{ii} = -\frac{\partial H_i}{\partial \vartheta_i} \tag{18}$$

$$\dot{q}_{ij} = -\frac{\partial H_i}{\partial \vartheta_j} \tag{19}$$

2. ν_i^* maximizes the hamiltonian H_i; which, for the first player, means that:

$$\begin{aligned} e^{-\rho(t-t_0)}\pi_1(\theta_1^*,\nu_1^*,\nu_2^*) + q_{11}f_1(\vartheta_1^*,\nu_1^*,\nu_2^*) + q_{12}f_2(\vartheta_2^*,\nu_1^*,\nu_2^*) \geq \\ e^{-\rho(t-t_0)}\pi_1(\theta_2^*,\nu_1,\nu_2^*) + q_{11}f_1(\vartheta_1^*,\nu_1,\nu_2^*) + q_{12}f_2(\vartheta_2^*,\nu_1,\nu_2^*) \end{aligned} \tag{20}$$

   (and the same substituting the corresponding subindex for player 2). This means that, at any instant of time, player 1 selects ν_1^* so as it maximizes the redistributed benefits towards her.
3. Transversality conditions are satisfied:

$$\lim_{t\to\infty} q_{i,j}\theta = 0 \qquad \forall i,j \tag{21}$$

From expression (20), and as our problem has a boundary solution, we derive a switching rule for ν_1 that maximizes the hamiltonian at any instant of time (given the values for ν_2^*, and the rest of the parameters in the model). Define:

$$A_i = \vartheta_i\left(\nu_j^*(\lambda+\gamma-2\alpha) + (\alpha-\gamma)\right) + e^{\rho(t-t_0)}\beta\left(q_{ii}\vartheta_i(\vartheta_i-1) + q_{ij}\vartheta_j(1-\vartheta_j)\right)$$

with $i \neq j$. Then:

$$\nu_i^* = \begin{cases} 1 \text{ if} & A_i > 0 \\ 0 \text{ if} & A_i < 0 \end{cases} \tag{22}$$

where $0 \leq d_1 \leq 1$. In order to obtain the optimal rule for player one it is necessary to determine the sign of the condition in the switching rule. The first step will be to give an intuitive contents to the costates; q_{ii} represents the marginal valuation of player i of an increase in the state variable ϑ_i (that is the marginal value of being more cooperative than player j) and need be positive as increases in the state variable yield a redistribution towards this player. If $\beta < 0$ and $(\vartheta_i - 1) \leq 0$, then all the product involving q_{ii} is greater than or equal to zero. On the other hand, q_{ij} represents the marginal value of an increase of the state variable ϑ_j that will necessarily be negative (a more cooperative behavior in the other player, all the things equal, will imply a negative redistribution); now $(1-\vartheta_j) \geq 0$ and $\beta < 0$, so this expression is again positive.

Proposition 1 *Given a strictly utilitarian PD (definition 1) Γ, and a legal system (Θ, F) which redistributes utility towards the more cooperative player; then the optimal policy of any player in $\langle\Gamma,\Theta,F\rangle$ is $\nu_i = 1$, $\forall i$.*

Full cooperation is the only equilibrium path of the model, and is achieved regardless of the values for ρ (the impatience of players or the probability of continuation of the game in the next stage), β (the degree of punishment) and θ_0 the initial redistribution of benefits (provided it is within the interval $]0,1[$). Cooperation is attained and it is a structural feature of the model robust to changes in parameters. Symmetry is not needed for supporting this conclusion and there is a wide range of payoffs structure that will support cooperation. The case for both negative conditions require such a specific payoff structure that we will be no longer talking about a PD; in this case, as cooperation will not necessarily be a desired outcome of the game (and perhaps it will turn out to be a non desirable one) there will be no interest in promoting cooperation. It seems evident that the problem we are facing rules out this possibility by the structure of the game and we will not consider it.

If we focus on the optimal policy, the phase space of the dynamical system, applying (17) and (18), yields:

$$\dot{\vartheta}_i = \beta\vartheta_i(1-\vartheta_i)(\nu_j-\nu_i) \tag{23}$$

$$\dot{q}_{ii} = -e^{-\rho(t-t_0)}\pi^*\left(\frac{1}{\vartheta_i+\vartheta_j}-\frac{\vartheta_i}{(\vartheta_i+\vartheta_j)^2}\right)+ q_{ii}\beta(2\vartheta_i-1)(\nu_j-\nu_i) \tag{24}$$

$$\dot{q}_{ij} = e^{-\rho(t-t_0)}\pi^*\frac{\vartheta_i}{(\vartheta_i+\vartheta_j)^2}+q_{ij}\beta(2\vartheta_j-1)(\nu_j-\nu_i) \tag{25}$$

The solutions for the dynamic processes are characterized by steady ϑ_i parameters (as the solutions imply $\dot{\vartheta}_i = 0$), and an exponential decay in q_{ii} and an exponential increase in q_{ij} function[3], which implies that eq.(21) holds.

It has been proved that cooperation may be sustained in a PD setting when institutional arrangements that induce the players to cooperation are introduced. This is different that saying that cooperation may be attained: most of dynamical examples of PD condition the strategic choice of players by means of more or less sophisticated strategic policies. The possibility of cooperation is mentioned in the literature as an outcome, but why cooperation arises in a world of self interested individuals is related to the set of rules under which individuals interact: the coercive nature of rules is the origin of many cooperative behavior. And this is the point at which we were focusing. Of course rules may not work as the perfect mechanism described in this paper; the less capacity of punishing individual defections, the less effective they will become in the induction of behavior. This is not to deny a kind of inductive behavior even when perfect assessment of any individual reaction is impossible in real world. In fact the probability of being punished remains in the model. And of course individuals shall be tempted to value any departure from the stated behavior by a rational cost-benefit analysis.

2.4 Generalization to the n-person PD

Keeping the structure of the PD in a n-person setting implies considering n^2 pure strategy combinations; in this case, the defection of all players is a Nash equilibrium and the greater the number of individuals that cooperate, the higher the total collective returns. The main problem in a n-person PD is that lack of cooperation is more likely to arise due, amongst other facts, to:

- As n increases the marginal effect of an individual's defection decay; this will cause an individuals defection being less perceived by the rest of the players.

[3] This can be easily shown by integrating the rhs of 24 and 25.

– In addition to this, there shall be difficulties to focus the punishment on the individual that failed to cooperate.

As has been shown, punishing an individual's defection is a way of improving the performance of the CI; in most settings, the possibility of threats to defectors by means of behavioral strategies are not credible: consider the extreme case of the *trigger strategy* (see for example Friedman) in which all individuals cooperate until one fails to do so; from this moment on, and as a punishment, all decide not to cooperate, imposing higher social costs than the former defection. The *Tit-for-Tat* strategy could be seen as an strategy of this type in a less stringent manner. As Axelrod (1997) points out there are ways to overcome this problem: social norms are a powerful means of sustaining cooperation. In fact any kind of norms may play this role.

In what follows we will show how a generalization of the previously developed model is successful (provided a PD dilemma payoffs structure) by inducing a cooperative behavior in the individuals. There is a Nash equilibrium that is suboptimal and the degree of optimality of the solution is given by the degree of cooperation of the individuals (that is given by the number and the extent, given infinite mixed strategies, to which individuals cooperate). The full cooperative solution is the only pareto optimum and, implies all individuals cooperating. Again, the degree of cooperation shown by the individuals is given by a set of n variables ν_1, $\nu_2 \dots \nu_n$, ranging in the closed $[0, 1]$. In order to attain cooperation, there is a redistribution mechanism that controls the game, by which all payoffs above the Nash equilibrium are redistributed according to the parametric set (1); each player's sharing of extra-benefits $\pi^*(\nu_1, \dots \nu_n)$, is given by the following expression:

$$\pi_i(\theta_i, \nu_1, \dots \nu_n) = \theta_i \pi^*(\nu_1, \dots \nu_n) \tag{26}$$

The controls evolve according to the following system of differential equations:

$$\begin{aligned} \dot{\vartheta}_1 &= \beta\vartheta_1(1 - \vartheta_1)(\zeta_1 - \nu_1) \\ \dot{\vartheta}_2 &= \beta\vartheta_2(1 - \vartheta_2)(\zeta_2 - \nu_2) \\ &\dots\dots\dots\dots\dots\dots \\ \dot{\vartheta}_n &= \beta\vartheta_n(1 - \vartheta_n)(\zeta_n - \nu_n) \end{aligned} \tag{27}$$

where

$$\theta_i = \frac{\vartheta_i}{\sum_{i=1}^{n} \vartheta_i}$$

and

$$\zeta_i = \frac{1}{n-1} \sum_{j \neq i} \nu_i$$

So the evolution of the ϑ's is bounded within the closed $[0, 1]$ and assures the desired behavior for the control variables. Meanwhile the deviation of the cooperative behavior of any individual is considered in relation with the average behavior of the group (excluding herself). As a generalization of the two players case the optimal decision of the individual will be determined by the payoffs structure of the game; in the case of the utilitarian PD cooperation arises as the individual maximizing rational behavior. So theoretically, full cooperation may be attained in a generalized PD, when institutional arrangements are included in the framework.

3 Conclusions

While one of the most important questions of CI involve the coordination of individual actions in order to achieve efficient outcomes, the design of rules that help achieving this goal is a matter of interest. In this sense, the PD is a paradigm of lack of coordination and its analysis helps in giving insights about how can coordination be achieved in social groups where strategic decision involve conflict and mutual benefits. For a long time the *Folk theorem* has become a major source of understanding the behavior of individuals in long term relationships, but it has given little theoretical insights about what the outcomes will be, unless specific patterns of behavior were assumed. In addition to it, the possibly cooperative behavior in dynamic settings was strictly considered in a two person setting, as the incentives for cooperation tend to decay when the number of players increases. The present work has overcome the limitations of the *Folk theorem* that in its strict meaning is just a possibility theorem that states which outcomes are feasible in a repeated game. First we have shown that by means of institutional designs, rules play a substantial role in this process by means of punishing any deviation from the optimal. Moreover, in our setting threats are credible as they do not imply that cooperators will be better off by not punishing, as could be the case of some strategies that rely on non credible threats. Second, the model is robust for any number of players, of importance when analyzing CI that involve complex exchange. Two remarkable aspects of the model are that it does not rely on symmetry assumptions and on initial conditions, as long as they rule out the possibility of one individual being excluded of the redistribution mechanism (i.e., setting $\vartheta_i(t_0) = 0$).

References

1. Aumann, R. J.: 1986, *Repeated Games, in George R. Feiwel(ed.), Issues in Contemporary Microeconomics and Welfare.* Albany: State University of New York Press.
2. Axelrod, R.: 1984, *The evolution of cooperation.* Basic Books, inc, Publishers.
3. Axelrod, R.: 1997, *The complexity of cooperation.* Princeton University Press.
4. Binmore, K.: 1992, *Fun and Games.* Lexington, Massachusetts: D.C. Heath and Company.
5. Eichberger, J.: 1993, *Game Theory for Economists.* San Diego: Academic Press Inc.
6. Friedman, J. W.: 1991, *Game Theory with Applications to Economics.* Oxford University Press.
7. Friedman, J. W. (ed.): 1994, *Problems of Coordination in Economic Activity.* Kluwer Academic Publishers.
8. Greenberg, J.: 1990, *The Theory of Social Situations. An Alternative Game-Theoretic Approach.* Cambridge: Cambridge University Press.
9. Hirshleifer, D. and E. Rasmusen: 1989, 'Cooperation in a Repeated Prisoners' Dilemma with Ostracism'. *Journal of Economic Behavior and Organization* **12**, 87–106.
10. Intriligator, M. D.: 1971, *Mathematical Optimization and Economic Theory.* Prentice Hall.
11. Kreps, D., P. Milgrom, J. Roberts, and R. Wilson: 1982, 'Rational Cooperation in the Finitely Repeated Prisonner's Dilemma'. *Journal of Economic Theory* **27**, 245–252.
12. Kreps, D. and R. Wilson: 1982, 'Reputation and Imperfect Information'. *Journal of Economic Theory* **27**, 253–279.
13. Kuhn, S. T. and S. Moresi: 1995, 'Pure and Utilitarian Prisoner's Dilemmas'. *Economics and Philosophy* **11**, 333–343.
14. Mas-Colell, A.: 1997, *Bargaining Games, in Sergiu Hart and Andreu Mas-Colell (eds.), Cooperation: Game-Theoretic Approaches.* Berlin: Springer-Verlag.
15. Mehlmann, A.: 1988, *Applied Differential Games.* New York: Plenum Press.
16. Milgrom, P., D. C. North, and B. R. Weingast: 1990, 'The role of institutions in the revival of trade: the Medieval Law Merchant'. *Economics and Politics* **2**, 1–23.
17. Milgrom, P. and J. Roberts: 1982, 'Predation, Reputation and Entry Deterrence'. *Journal of Economic Theory* **27**, 280–312.
18. Myerson, R. B.: 1991, *Game Theory. Analysis of Conflict.* Cambridge, Massachusetts: Harvard University Press.
19. Nelson, R. R.: 1995, 'Recent evolutionary theorizing about economic change'. *Journal of Economic Literature* **33**, 48–90.
20. Olson, M.: 1965, *The Logic of Collective Action.* Cambridge, Massachusetts: Harvard University Press.
21. Samuelson, P. A.: 1983, *Foundations of Economic Analysis, enlarged edition.* Cambridge, Massachusetts: Harvard University Press.
22. Schick, F.: 1992, 'Cooperation and Contracts'. *Economics and Philosophy* **8**, 209–229.
23. Sorin, S.: 1997a, *Communication, correlation and cooperation, in Sergiu Hart and Andreu Mas-Colell (eds.), Cooperation: Game-Theoretic Approaches.* Springer-Verlag.
24. Sorin, S.: 1997b, *Cooperation through repetition: complete information, in Sergiu Hart and Andreu Mas-Colell (eds.), Cooperation: Game-Theoretic Approaches.* Berlin: Springer-Verlag.
25. Stewart, H.: 1997, 'The Law of Damages and the Prisoner's Dilemma: A Comment on Pure and Utilitarian Prisoners' Dilemmas". *Economics and Philosophy* **13**, 231–240.
26. Suber, E.: 1992, 'Stable cooperation in iterated prisoners' dilemmas'. *Economics and Philosophy* **8**, 127–139.
27. Takayama, A.: 1994, *Analytical Methods in Economics.* Harvester Wheatsheaf.
28. Thompson, E. and R. Faith: 1981, 'A Pure Theory of Strategic Behavior and Social Institutions'. *American Economic Review* **71**, 366–380.
29. Tu, P. N.: 1992, *Dynamical Systems. An Introduction with Applications in Economics and Biology.* Springer-Verlag.
30. Weeren, A. J., J. M. Schumacher, and J. C. Engwerda: 1996, 'Strategic Behavior and Noncooperative Hierarchical Control'. *Journal of Economic Dynamics and Control* **23**. 641–669.

IUDEM
DOCUMENTOS DE TRABAJO PUBLICADOS

DT 2OOO-1	LA MODERNA MACROECONOMÍA CLÁSICA Y SUS CONSECUENCIAS PARA LA FORMULACIÓN DE LA POLÍTICA ECONÓMICA, Ramón Febrero Devesa
DT 2OOO-2	LA LIBERACIÓN DEL LIBRO. UNA CRÍTICA DEL SISTEMA DE PRECIO FIJO, Pedro Schwartz
DT 2OOO-3	COOPERATIVE BEHAVIOR IN THE PRISONER DILEMMA: THE ROLE OF RULES IN INSTITUTIONS, José Casas Pardo y Juan D. Montoro-Pons
DT 2OOO-4	SEX DISCRIMINATION. IS THE LABOUR MARKET A MARKET FOR LEMONS ?, Rocío Albert y Francisco Cabrillo
DT 2OOO-5	ORÍGENES DEL SOCIALISMO LIBERAL. EL CASO DE JUAN B. JUSTO, Carlos Rodríguez Braun
DT 2OOO-6	LA REGULACIÓN DE LOS SERVICIOS PROFESIONALES: UNA GUÍA PARA LAS DECISIONES PÚBLICAS EN TIEMPOS DE CAMBIO, Benito Arruñada

IUDEM

DOCUMENTOS DE TRABAJO PUBLICADOS

DT 2OOO-1	LA MODERNA MACROECONOMÍA CLÁSICA Y SUS CONSECUENCIAS PARA LA FORMULACIÓN DE LA POLÍTICA ECONÓMICA, Ramón Febrero Devesa
DT 2OOO-2	LA LIBERACIÓN DEL LIBRO. UNA CRÍTICA DEL SISTEMA DE PRECIO FIJO, Pedro Schwartz
DT 2OOO-3	COOPERATIVE BEHAVIOR IN THE PRISONER DI-LEMMA: THE ROLE OF RULES IN INSTITUTIONS, José Casas Pardo y Juan D. Montoro-Pons
DT 2OOO-4	SEX DISCRIMINATION. IS THE LABOUR MA RKET A MARKET FOR LEMONS ?, Rocío Albert y Francisco Cabrillo
DT 2OOO-5	ORÍGENES DEL SOCIALISMO LIBERAL. EL CASO DE JUAN B. JUSTO, Carlos Rodríguez Braun
DT 2OOO-6	LA REGULACIÓN DE LOS SERVICIOS PROFESIO-NALES: UNA GUÍA PARA LAS DECISIONES PÚBLI-CAS EN TIEMPOS DE CAMBIO, Benito Arruñada