On the Efficiency of Partial Information in Elections*

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September 23, 2011

Abstract

We study the relation between the electorate's information about candidates' policy platforms during an election, and the subsequent provision of inefficient local public goods (pork) by the elected government. More information does not lead to better outcomes. We show that the efficient outcome in which no candidate proposes to provide any inefficient good is sustained in equilibrium only if voters are not well informed. If the electorate is well informed, electoral competition leads candidates to provide inefficient pork in all equilibria. We show that this result is robust even if candidates care about efficiency.

Keywords: Elections, information, inefficiency, pork, campaigns.

*We thank Dan Bernhardt, Sandy Gordon, Dunia Lopez-Pintado, Becky Morton and participants at workshops in Baiona, Barcelona, Malaga and at U.Autonoma Barcelona; at SAET 2011 in Faro; and at a seminar at U. Carlos III in Madrid for comments and suggestions. J. Eguia thanks the Ford Motor Company Center for Global Citizenship and the Center for Mathematical Studies in Economics and Management Science at the Kellogg School of Management for their funding and hospitality during the 2010-11 academic year.

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1 Introduction

During electoral campaigns, candidates running for office make policy proposals to woo voters. Voters pay only limited attention to electoral campaigns and as a result they do not become fully informed about the policies proposed by the candidates. We study the relation between the information acquired by the voters, and the policies that the candidates announce during the campaign and execute once in office. In particular, we explain the effect of voters' information on the provision of socially inefficient particularistic public goods.

A particularistic or local public good provides a benefit only to the members of a single district or group. If the costs of provision are spread across society at large by general taxation, voters in each district want their own particularistic public good to be provided, while they prefer the public good in any other district not to be provided. Because voters enjoy the benefits of the public good provided to their own district fully, while they only pay a fraction of the cost of any given public good, they care more about the provision of their own good than about the non-provision of the public good in any one other given district. This leads politicians to promote inefficient policies that result in the over provision of particularistic public goods.

We find that a more informed electorate does not obtain a more efficient policy outcome. In fact, if inefficient local public goods (pork) provide a benefit that is at least two thirds of their cost, the efficient policy –no provision of pork- can be sustained in equilibrium if and only if the electorate is *not* well informed. If the electorate is better informed, all equilibria are inefficient, even if candidates also care about social efficiency along with their desire to gain office.

Examples of policies that distribute targeted goods of dubious efficiency abound: we highlight farm subsidies, military procurement, and some infrastructure investment in the United States and Europe.

Farm subsidies distribute between $12 billion and $25 billion every year in the United States, and around €55 billion per year in the European Union (almost one half of the European Union budget),[1] distorting the market and creating an aggregate welfare loss.

Parochial interests also trump efficiency in contracts for military equipment. The US Air Force saw a $40 billion contract to replace its refuelling tankers repeatedly delayed from 2002 to 2011 amid controversies over waste and fraud; Congress spent $65 billion from 1991 to 2009 on the F-22 jet fighter, designed to counter a Soviet threat and found unsuitable by the Air Force for any combat mission in the post-Soviet world; controversy surrounds the plans to develop two parallel versions of the engine for the F-35 fighter at an additional cost of $3 billion. President Obama declared purchases of the F-22 aircraft an "inexcusable waste" and the second engine for the F-35 an "unnecessary and extravagant expense." These projects gain political support based on the funding and jobs they bring to specific districts, irrespective of their merit as a cost-efficient mean to satisfy military needs. Procurement decisions degenerate into contests for pork that serve as local jobs programs. Companies disperse production among multiple districts to maximize political support: Boeing promised to "create up to 50,000 jobs in 40 states" in its bid for the refueling tanker; building the

[1] Source: US Department of Agriculture, http://www.ers.usda.gov/briefing/farmincome/govtpaybyfarmtype.htm and the European Union portal at http://ec.europa.eu/agriculture/publi/capexplained/cap_en.pdf

F-22 that the Air Force has never used provided 25,000 jobs in 44 states, etc.[2] In Europe, production of the European Typhoon jet fighter was assigned to countries in proportion to their procurement orders and not based on any measure of efficiency.

Younossi, Stem Lourell and Lussier [63] show that the dispersion of production among many districts that makes a project politically viable, is precisely the cause of large cost overruns and delays. An audit on the European Typhoon conducted by the United Kingdom's Ministry of Defense, which alone has spent over £20 billion on the project, agrees, identifying "the inefficient commercial and managerial arrangements on the project as the root cause of much of the cost escalation and schedule slippage on the project."[3]

With regard to investment in public infrastructure, an egregious example in the United States was the practice (halted in 2011) of approving earmarks, which allocate funds to local projects avoiding the scrutiny and debate of regular appropriations. The total cost of these projects ascended to $16 billion in 2010.[4] Aside from earmarks, the cost effectiveness of other projects such as high speed rail lines, both existing ones in Europe (Ginés and Inglada [28]) and future ones in the United States[5] appears to be rather questionable.[6]

[2] For the F-22 story, see "Obama wins crucial Senate vote on F-22", New York Times, July 21st, 2009; for the F-35, "House votes to end alternate jet engine program", New York Times, February 16th, 2011. For the Boeing claim on the tanker, "Boeing Submits Final NewGen Tanker Proposal" at www.boeing.mediaroom.com.

[3] See the Report by the Comptroller and Auditor General, HC 755, session 2010–2011, 2 march 2011. These "inefficient commercial arrangements" were such that the left wings were built in Italy, the right ones in Spain, the front of the plane in the United Kingdom and the back in Germany.

[4] Taxpayers for Common Sense (www.taxpayer.org) defines earmarks as "legislative provisions that set aside funds within an account for a specific program, project, activity, institution, or location. These measures normally circumvent merit-based or competitive allocation processes."

[5] See Edward Glaeser, "Running the numbers on High-Speed Trains", in the Economix blog at the New York Times, August 4, 2009.

[6] For instance, in June 2011, Spanish state-controlled rail operator RENFE suspended service along its newest high speed line, after noting that its average daily passenger volume was nine passengers. See "Spain cuts high speed 'ghost train' " from *The Telegraph*, 28 June 2011.

We investigate whether this inefficient allocation of funds occurs because voters are not sufficiently informed about issues that do not concern them directly: Farm subsidies are important to farmers, and aircraft procurement to aviation industry workers. Most voters are not farmers or aviation industry workers, and they do not follow political campaigns closely enough to know each candidate's funding plans for farming or aircraft purchases, or other projects that are not directly relevant to the voter. To the extent that voters remain unaware of policy details on issues that do not concern them directly, legislators have no incentives to pursue efficiency gains by eliminating programs that generate targeted benefits and diffused costs, even if the aggregate costs surpass the localized benefits.

This intuition, while true, is incomplete. A society with a perfectly informed electorate can also suffer from inefficient policies that favor organized lobbies and special interests (Grossman and Helpman [31]). Does more information at least alleviate the problem? Does a more informed electorate obtain less inefficient policy outcomes?

To answers these questions, we find the equilibrium policy proposals in an election with two parties in a Westminster democracy with multiple districts. Candidates compete by proposing to implement local public good projects that are inefficient for society, but beneficial to the district in which they are developed. The party that wins in most districts implements its policy proposal. The model also fits presidential elections, where the two agents competing for votes are two individual politicians; we refer to the two agents contesting the elections as candidates to accommodate both cases.

Since voters care more about policies that directly affect their districts, if their attention span is limited, they naturally become better informed about these proposals than about projects in other districts that only affect them indirectly through general taxation. We solve the game in which voters observe the policy proposals for their own district with certainty, but observe the candidates' spending plans in other districts only with some probability. Uninformed voters form beliefs about a candidate's proposal based on what they observe in their district.

We first find that if pork is very inefficient, an equilibrium with no pork exists for any level of information possessed by the electorate. We say pork is "very" inefficient if it is so inefficient that no majority of districts is better off providing pork only to itself and distributing costs equally among all districts in society. With three districts, it is pork that provides less than 67cts of benefit per unit of cost; with n districts, less than $\frac{n+1}{2n}$ units of benefit per unit of cost.

Our main results concern pork that is inefficient, but not very inefficient. The efficient policy outcome cannot always be sustained in equilibrium, and information affects incentives perversely:

If the probability that voters are perfectly informed is low, the efficient outcome with no pork is sustained in an equilibrium in which voters believe that a candidate who deviates to offer them their local public good must be a big spender who made similar inefficient offers in other districts. If the probability that voters are perfectly informed is high, candidates can effectively campaign by targeting pork to a minimal-winning majority of districts, destroying the efficient equilibrium. With an informed electorate, the competition among districts to be in the winning coalition and between candidates to attract votes induce an inefficient overprovision of public goods: all equilibria are inefficient.

The unsettling conclusion is that some degree of voters' ignorance is necessary to sustain the efficient policy outcome in equilibrium.

Seminal contributions to the literature on distributive politics by Weingast [61], Shepsle and Weingast [56], Weingast, Shepsle and Johnsen [62] and Niou and Ordeshook [47] analyze the provision of local inefficient public goods as the result of a legislative bargaining game, and predict a "pork for everybody" outcome. Legislators commit to a norm of universalism by which every district gets its own inefficient project, rather than letting a minimal-winning majority distribute public goods only to the districts in this majority. An objection to this seminal theory is that it cannot explain why legislators do not embrace instead a Pareto-superior universalist norm by which no inefficient local public goods are ever provided. In fact, if legislators do not commit to any norm, Ferejohn, Fiorina and McKelvey [26] and Baron [7] show that only a minimal winning majority of districts benefit from the provision of inefficient projects. More recent developments study how pork provision is affected by the number of districts in the polity (Baqir [6], Primo and Snyder [50], Chen and Malhotra [16]) or the distribution of legislators' preferences for local or collective public goods (Volden and Wiseman [60]).[7]

A stream of economic theories explain targeted redistribution as the equilibrium outcome of a game in which candidates compete in elections (Cox and McCubbins [18], Lindbeck and Weibull [36]; Myerson [45]; Dixit and Londregan [20] and [21]; Lizzeri and Persico [37]; Chari, Jones and Marimon [15]; the survey by Persson and Tabellini [49]; and more recent work by Milesi-Ferreti, Peroti and Rostagno [43]; Roberson [52]; Fernandez and Levy [27]; Smith

[7] In a departure from most of the literature, Drazen and Izetzki [22] argue that pork is welfare-improving when the agenda-setter uses it as a costly signal to inform other legislators.

and Bueno de Mesquita [58] and Huber and Ting [32]). These theories assume that citizens are fully informed about the policy proposals made by the candidates. The assumption is unrealistic. The empirical literature on voter behavior has conclusively established that in practice voters have a sketchy idea of these policy proposals (Campbell, Converse, Miller and Stokes [14]; Bartels [9] and Alvarez [2]).

Our contribution narrows the gap between the assumptions in the theoretical literature, and the accepted stylized facts of the empirical literature, by recognizing that voters have only partial information about candidates' policy proposals.

Ours is not the first theory of elections with voters that are not fully informed. At the opposite extreme, Grosser and Palfrey [30] assume that citizens do not know anything about the candidates. McKelvey and Ordeshook [39], [40] and [41] assume that voters infer candidates' policy positions indirectly from polls and endorsements. Snyder and Ting [59] argue that party labels serve as cues to provide some information to voters who do not directly know the ideological preferences of individual candidates. Baron [8] assumes that some voters are fully informed, while others are fully uninformed. Glaeser, Ponzetto and Shapiro [29] assume that each voter becomes either fully informed or fully uninformed about the policy proposal of each of the two candidates, so that a given voter may become perfectly informed about one candidate's proposal but remain entirely unaware of the other candidate's proposal. All these models deal with ideological preferences in the real line.[8] Also

[8] Other models consider electorates that are not perfectly informed about the state of the world (Feddersen and Pesendorfer [24] and [25]) or about candidates' competence (Krishna and Morgan [35]), or allow candidates to deviate from their campaign proposals once in office (Banks [5] and Callander and Wilkie [12]). These models are more distantly related because they do not deal with distributive policies, and assume that voters are perfectly informed about candidates' proposals.

working with a one dimension policy space with uncertainty about candidates' preferences, Dhami [19] studies general redistribution from rich to poor agents, and Bernhard, Dubey and Hughson [11] introduce exogenous, fixed transfers from districts with a junior representative to districts with a senior representative.

To our knowledge, ours and an independent working paper by Aidt and Shvets [1] are the first electoral theories on the provision of local public goods to imperfectly informed voters. Aidt and Shvets [1] do not study policy proposals; they are concerned about the principal-agent problem of selecting high type politicians and motivating them to exert effort. Their main finding is that under term limits, politicians exert less effort in their last term in office.

Our paper is a unique study on the effect of voters' information about policy proposals over the provision of inefficient local public goods and the efficiency of policy outcomes. We believe that in applications to large elections, voters posses some but not all the information about candidates' policy proposals, hence instead of studying fully informed or fully uninformed voters, it is more appropriate to build theories of elections with imperfectly informed voters. This is the approach we pursue.

2 The Model

We consider a society partitioned into three subsets, with one representative voter $i \in \{a, b, c\}$ in each subset. We refer to these subsets as districts, but they could also be population groups of similar size divided by ethnicity, age, profession or class.

Two candidates A and B compete for election. Let $J \in \{A, B\}$ denote an arbitrary candidate and let $-J$ denote the other candidate, so that $\{J, -J\} \equiv \{A, B\}$. We assume that candidates and voters are fully strategic, rational agents who evaluate lotteries according to standard expected utilities.

The policy space consists on whether or not to provide a public good in each district. A strategy for each candidate consists on proposing a policy in the policy space. Let $S^J = S^{-J} = \{0,1\}^3$ be the strategy set of each candidate. Let $s^J = (s_a^J, s_b^J, s_c^J) \in \{0,1\}^3$ be a strategy by candidate J, where $s_i^J = 1$ indicates that J proposes to provide the public good in district i and $s_i^J = 0$ indicates that J proposes not to provide it. Let s_{-i}^J denote the proposals for the other two districts, not including i.

The timing is simple:

One – Candidates announce their policy proposals simultaneously.

Two – Nature determines how much information voters get about these proposals, as detailed below.

Three – An election is held, where voters simultaneously choose to vote for A, for B or to abstain.

Four – The candidate who receives most votes, or a randomly selected candidate in case of a tie, wins the election and implements her policy proposal.

Each public good brings a benefit β to the voter in the district where it is provided and no benefit to the other districts. The cost of each public goods is identical across districts and it is normalized to one. All the costs of local public good provision are equally borne by all voters, regardless of which districts actually receive their local public good.

We assume that local public goods are inefficient, so no district would like to provide its own public good if it had to bear its full cost, but we assume that projects are sufficiently efficient so that, ceteris paribus, each voter prefers to have the public good provided in her district given that the district bears only $\frac{1}{3}$ of the cost of provision. This means that $\beta \in \left(\frac{1}{3}, 1\right)$.[9]

Voter $i's$ preference order over policies depends on β. If $\beta \in \left(\frac{2}{3}, 1\right)$, the preference order of voter a over the eight possible policy outcomes is

$$\{1,0,0\} \succ_a \{1,1,0\} \sim_a \{1,0,1\} \succ_a \{0,0,0\} \succ_a \{1,1,1\} \succ_a \{0,0,1\} \sim_a \{0,1,0\} \succ_a \{0,1,1\}. \tag{1}$$

If $\beta \in \left(\frac{1}{3}, \frac{2}{3}\right)$,

$$\{1,0,0\} \succ_a \{0,0,0\} \succ_a \{1,1,0\} \sim_a \{1,0,1\} \succ_a \{0,0,1\} \sim_a \{0,1,0\} \succ_a \{1,1,1\} \succ_a \{0,1,1\}. \tag{2}$$

Voters' preferences over candidates depend exclusively on the expected utility of the candidates' proposals. Each voter i uses all the information available to her about the policy proposal of candidate J to calculate the (subjective) expected utility of candidate J to voter i. If candidate J proposes to provide the public good in district i, then voter i's expected utility of $J's$ proposal is equal to $\beta - \frac{1}{3}k$ where $k \in \{0,1,2,3\}$ is the total number of public goods proposed by candidate J. If candidate J does not propose to provide the public good in district i, then voter i's expected utility is equal to $-\frac{1}{3}k$. The expectation is over k, which may be unknown to the voter.[10]

[9] If $\beta < 1/3$, the policy decision is simple, since all voters agree that it is best not to implement any project.

[10] When voter i does not observe the policy proposal s^J, the subjective expected payoff depends on the

We assume that voters vote for the candidate with the highest expected payoff, that is, voters are sequentially rational (Kreps and Wilson [34]). We assume that voters do not use weakly dominated strategies. This rules out equilibria in which all voters vote for the same candidate even though some voters prefer the losing candidate's proposal. If the subjective expected payoffs of both candidates to voter i coincide given voter $i's$ beliefs, voter i is indifferent about the candidates. We assume that in this case she abstains, unless abstention has been eliminated as a weakly dominated strategy.

We consider two alternative assumptions on candidates' motivations: Candidates who care exclusively about winning office, and candidates who care about winning office and about the policies they enact in office. All candidates seek to be elected. For the purely office motivated, this is their only concern: They obtain utility one if they win and zero otherwise. Candidates who also care about the policies they enact are efficiency concerned; these candidates want to win, but they prefer to win proposing and implementing more efficient policies. Let k the number of projects in the proposal of candidate J.

Candidate $J's$ preferences are represented by the utility function:

$$\begin{aligned} U_J(k) &= \frac{1}{1+\alpha_J(1-\beta)k} \text{ if } J \text{ wins the election, where } \alpha_J \in \{0,1\}. \\ U_J(k) &= 0 \text{ otherwise.} \end{aligned}$$

If candidate J is purely office motivated, $\alpha_J = 0$; if candidate J is efficiency concerned, $\alpha_J = 1$. The term $(1-\beta)k$ measures the inefficiency of a policy that implements k projects,

beliefs of voter i about s^J, and if the beliefs are not correct, the subjective expected payoff may not coincide with the objective expected payoff (note however that this can only occur outside of the equilibrium path).

and the utility that an efficiency concerned candidate experiences if she wins proposing such policy decreases in this term.

We solve the model with two purely office motivated candidates, with two efficiency concerned candidates, and with one office motivated and one efficiency concerned candidate. All previous electoral theories on distributive politics or local public good provision assume that candidates have homogenous motivations: either all candidates care only about winning (Person and Tabellini [49]), or they all care about both winning and policy outcomes (McKelvey and Riezman [42]). Roemer [53], Aragones and Palfrey [3] and Callander [13] introduce heterogenous motivations to the classical one dimensional spatial model.[11] To the best of our knowledge, our theory is the first to allow for heterogenous candidate motivations in an electoral theory of distributive politics or pork provision.

The solution concept we use is Pure Perfect Bayesian Equilibrium in which no agent uses weakly dominated strategies. If (and only if) there exists no equilibrium in pure strategies, we look at mixed strategy equilibria. When we study mixed strategy equilibria, we let σ_k be the probability of playing strategy s_k in a mixed strategy σ, and we let $p^J \in \{0,1\}^3$ be the policy actually proposed (action taken) by J. Clearly, if J chooses a pure strategy, $p^J = s^J$. Let $p = (p^A, p^B)$.

We are particularly interested in voters' behavior when they have only have partial information on candidates' policy proposals. We analyze how voters modify their behavior in response to changes in their information about candidates' proposals. We assume that each voter is well informed about candidates' proposals for the project in the voter's district, but

[11] See as well Saporiti, Drouvelis and Vriend [54].

it is harder for a voter to keep track of information about projects in other districts.

We parameterize how informed is the electorate by a single parameter ε, which captures the probability that all voters are perfectly informed. Nature determines whether candidates' policy proposals become common knowledge or not. Policy proposals become common knowledge with probability $\varepsilon \in [0,1]$. With probability $1-\varepsilon$, each voter $i \in \{a,b,c\}$ only observes what candidates commit to do in her district, and she is completely unaware of what candidates promise in the other districts. Notice that the extreme case $\varepsilon = 1$ corresponds to the standard model with perfect information, and $\varepsilon = 0$ corresponds to an electorate in which voters only have local information about the proposals for their district, and never learn about the proposals to provide public goods in other districts.[12]

If Nature makes proposals public knowledge, voters observe p^A and p^B and compare the two proposals and vote for the one they prefer, according to the preference order (1) or (2).

If proposals do not become public knowledge, each voter i remains unaware about what each candidate proposes in districts other than i : Voter i only observes either $p_i^J = 0$ or $p_i^J = 1$ for each candidate J, that is, voter i observes $(p_i^A, p_i^B) \in \{0,1\} \times \{0,1\}$, so that voter i has four information sets in which to make a decision, and three possible actions (vote A, vote B or abstain) in each of these sets.

Under this informational structure, in which voters have imperfect information, an equilibrium must describe strategies for voters and candidates, and beliefs for voters. Beliefs along the equilibrium path must be correct. Beliefs off the equilibrium path must assign all the probabilities to undominated strategies if any such strategy is consistent with the

[12] As we discuss below on a section on extensions, all the results are robust if we let each district become informed with independent probability $\varepsilon_i = \varepsilon$.

information possessed by the agent.[13]

The strategy pair (s^A, s^B) determines the information (s_i^A, s_i^B) received by each voter i. This information, together with beliefs, determine the subjective expected utility for i if A or B wins, which in turn determines agent $i's$ vote and therefore, aggregating over all three agents, it determines the electoral outcome and the payoffs to A and B.

The set of equilibria depend on the efficiency of the public goods measured by β, and on how informed is the electorate, measured by ε.

3 Results

We say that public goods are very inefficient if they are worth less than two thirds of their total cost. If local public goods are very inefficient, any minimal winning coalition of districts prefers a policy outcome in which none of the districts in the coalition receive their local public good, rather than an outcome in which all the districts in the coalition receive it, keeping fixed the policy for districts outside the coalition. Our first result concerns these goods: If voters are poorly informed there are multiple equilibria and the efficient equilibrium is one of them. If voters are sufficiently well informed, the outcome is efficient: no local public goods are provided in the unique equilibrium. The result is robust to either assumption on the motivation of candidates, whether they exclusively seek to win office, or whether they also have care about policy outcomes. The following proposition summarizes this result.

[13] If no undominated strategy is consistent with the information possessed by the agent, then we let the agent hold any beliefs over the entire strategy set.

Proposition 1 *For any* $(\alpha_A, \alpha_B) \in \{0,1\}^2$*, any* $\varepsilon \geq 0$ *and any* $\beta \in \left(\frac{1}{3}, \frac{2}{3}\right)$*, an equilibrium in which both candidates propose to implement the efficient policy exists, and if* $\varepsilon > \frac{1}{2}$*, it is the unique equilibrium.*

We relegate to an appendix all the proofs, together with more extensive results including a full characterization of the set of equilibria.

If voters are poorly informed ($\varepsilon < \frac{1}{2}$), they are unlikely to know the true proposals at the time of voting, and their votes more often than not depend on their beliefs: different equilibria can be sustained for certain out of equilibrium beliefs, and the set of Pure Perfect Bayesian Equilibria is large, regardless of the efficiency of the public good.

In contrast, if the probability that citizens are informed is high ($\varepsilon > \frac{1}{2}$), out of equilibrium beliefs are more often irrelevant since with high probability voters are able to fully observe a deviation. If $\beta < \frac{2}{3}$, the efficient policy $(0,0,0)$ is the second best policy for each voter i, second only to the policy that provides pork only to district i; if $\beta < \frac{2}{3}$ and voters are fully informed, $(0,0,0)$ defeats any other policy in pairwise comparisons by simple majority rule. If both candidates propose the efficient policy $(0,0,0)$, each candidate wins the election with probability $\frac{1}{2}$. Any deviation makes the deviating candidate lose when information is fully revealed; if $\varepsilon > \frac{1}{2}$, it follows than any deviation makes the deviating candidate strictly worse off. Hence the strategy profile such that both candidates propose the efficient policy $(0,0,0)$ is an equilibrium. There is no other equilibrium: If both candidates propose inefficient policies, each could deviate to the efficient policy and win when information is fully revealed, which occurs with probability $\varepsilon > \frac{1}{2}$; if one candidate proposes the efficient policy and the other candidate an inefficient one, the candidate with the efficient proposal wins with probability one, and the losing candidate is strictly better deviating to propose the efficient policy.

If at least one candidate is efficiency concerned, the efficient equilibrium such that both candidates promise the efficient policy $(0,0,0)$ is the unique equilibrium for any $\varepsilon > \bar{\varepsilon}$, for some threshold $\bar{\varepsilon}$ strictly below one half: An efficiency concerned candidate cares her about her policy and therefore she will deviate from any other strategy profile if deviating causes only a slight decrease in her probability of victory.

If local public goods are very inefficient, it is normatively desirable that voters become better informed, because a well informed electorate leads an efficient equilibrium outcome that maximizes aggregate social welfare.

In contrast, if the public goods are inefficient but their benefit/cost ratio is closer to one, the favorable result on the effect of information on efficiency does not hold. Our more striking finding is that if local public goods are inefficient but not too inefficient, an increase in voters' information about candidates' policies is detrimental because it destroys the possibility of reaching an efficient outcome. The efficient outcome can only be obtained if citizens are *not* well informed.

Theorem 2 below states our main result.

Theorem 2 *For any* $(\alpha_A, \alpha_B) \in \{0,1\}^2$ *and any* $\beta \in \left(\frac{2}{3}, 1\right)$,

a) if $\varepsilon \leq \varepsilon(\alpha_A, \alpha_B, \beta)$ *an equilibrium in which both candidates propose to implement the efficient policy exists, and*

b) if $\varepsilon > \varepsilon(\alpha_A, \alpha_B, \beta)$, *all equilibria are inefficient,*

where $\varepsilon(0,0,\beta) = \varepsilon(0,1,\beta) = \varepsilon(1,0,\beta) = \frac{1}{2}$ *and* $\varepsilon(1,1,\beta) = \frac{3-2\beta}{2} > \frac{1}{2}$.

If citizens are poorly informed, there exist many equilibria, which we characterize in the appendix. Very different equilibria can be supported by the same pessimistic beliefs about any observed deviation. Equilibria in which both candidates propose zero projects and in which both candidates propose to provide an inefficient local public good to every district exist and are sequential (Kreps and Wilson [34]), trembling hand perfect (Selten [55]) and rationalizable (Pearce [48] and Battigali [10]). These equilibria are sustained for any β by the following voters' beliefs off the equilibrium path: a voter who observes a deviation in her district believes that the deviating candidate has proposed to provide the public goods in the other two districts. Given these beliefs, no voter votes for a deviating candidate unless information is fully revealed. It follows that if the probability that full information is revealed is low, deviating is never profitable. With a poorly informed electorate, comparative statics on β show that if pork becomes more inefficient –if β decreases-, the efficient equilibrium can be sustained by a larger set of off-equilibrium beliefs: the efficient equilibrium holds if each voter who observes a deviation believes that the deviator offers pork to at least $3\beta - 1$ other districts.

In contrast, with a more informed electorate, the efficient equilibrium breaks down. For ε sufficiently large, the unique equilibrium which emerges is an equilibrium in mixed strategy where in expectation at least 9/7 projects are implemented, as we show in the appendix. The result is more intuitive than it might initially appear to be. Suppose that both candidates are office motivated and they both play pure strategies. At least one of the two candidates wins the election with probability at most $\frac{1}{2}$. However, if citizens are fully informed, simple majority generates a Condorcet cycle: for any pure strategy, there exists another pure strategy that defeats it.[14] Each candidate could deviate to the strategy that defeats her opponent's strategy, and in this manner win whenever Nature reveals the policy proposals, which occurs with probability ε; hence if $\varepsilon > \frac{1}{2}$, the initial (arbitrary) strategy pair cannot be supported in equilibrium.

The intuition holds for efficiency concerned candidates too. In this case the threshold above which no pure strategy equilibrium exists is larger than $\frac{1}{2}$ since candidates may prefer to win with probability $\frac{1}{2}$ proposing the efficient policy, rather than to win with slightly larger probability proposing to implement the inefficient policy that provides the local public good to two districts. However if ε is sufficiently large, the incentive to deviate and win the election when information is fully revealed becomes strong enough to destroy the efficient equilibrium.

It follows that if public goods are inefficient but not too inefficient, some degree of voters' ignorance is necessary to sustain an efficient outcome. The intuition behind this distressing result is straightforward. Candidates, even those concerned about efficiency, care about winning the election: we assume that they prefer to win with a bad policy than to lose with a good one. If public goods are not too inefficient, districts compete in order to belong to a minimal winning majority. If citizens are well informed, the efficient proposal cannot be part of an equilibrium because it is defeated by any proposal that provides the public good to two districts. If full information is revealed with sufficiently high probability the unique equilibrium is in mixed strategy in which the public good in each district is provided with positive probability.

[14] Proposing zero projects is defeated by proposing two, which is defeated by proposing one of those two, which is defeated by proposing all three, which is defeated by proposing zero.

The efficient equilibrium in which both candidates propose the policy $(0, 0, 0)$ can only exist if candidates are not tempted to deviate by offering pork to two districts. Lack of information makes this deviation unfruitful if voters who do not observe it, do not believe it: voters who do not observe the true proposals and believe that a candidate who offers them pork offers pork to everybody will not support a candidate who deviates to offer them pork. Voters who are skeptical about candidates who promise to favor them and not others with pork, together with the inability for candidates to credibly announce that they favor a particular subset of districts sustain the efficient equilibrium. Campaign promises to carve out a minimal winning coalition are not credible if the electorate is poorly informed, but they are credible and they destroy the efficient equilibrium, if voters are well informed.

The finding that more information does not lead to more efficient outcomes if public goods are not too inefficient is reinforced if candidates are efficiency concerned. In this case, we find a stronger result: too much information makes voters unambiguously and strictly worse off.

Proposition 3 *Suppose that both candidates are efficiency concerned and public goods are not very inefficient* $\beta \in (\frac{2}{3}, 1)$.

If $\varepsilon \leq \frac{2-\beta}{6-4\beta}$, *there exist multiple pure equilibria, including the efficient one;*

If $\varepsilon \in \left(\frac{2-\beta}{6-4\beta}, \frac{3-2\beta}{2}\right]$, *there is a unique pure strategy equilibrium in which both candidates propose the efficient policy; and*

If $\varepsilon > \frac{3-2\beta}{2}$ *there is no pure strategy equilibrium, there exists a mixed strategy equilibrium, an in expectation at least 9/7 projects are implemented.*

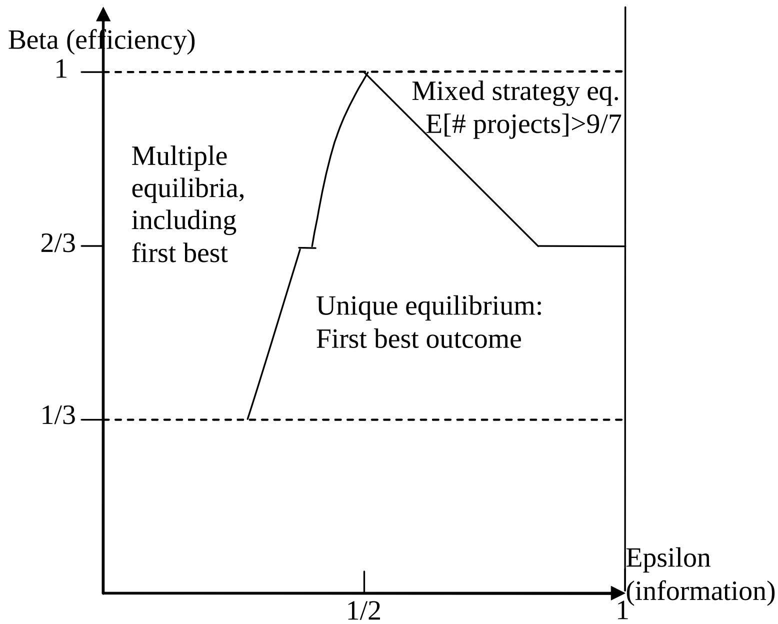

Figure 1: Equilibria and policy outcomes with efficiency-concerned candidates

An increase from low to intermediate information is at least weakly beneficial because it makes the efficient equilibrium unique; however, a further increase of information is strictly detrimental, as it leads to inefficient outcomes that reduce the ex-ante welfare of each voter. We illustrate these results in figure 1. As shown, if public goods are not too inefficient, some voters' ignorance is necessary to sustain the first best in equilibrium.

A similar result holds if only one candidate is efficiency concerned, or if candidate's types are private information: if ε is in an intermediate interval below $\frac{1}{2}$, the unique equilibrium is efficient, but if citizens are more informed, the equilibrium is inefficient, in mixed strategies. More information strictly reduces aggregate welfare.

4 Extensions and Robustness

We consider several extensions and generalizations to the model, to check that the intuition underpinning our results is robust.

Our main generalization is to consider a society with an arbitrary number of districts. While the case with three districts suffices to convey the intuition of the results in the clearest manner, we show that results are robust to a society with an arbitrary odd number n of districts. The relevant efficient cutoff is no longer 66.7cts of benefit per unit of cost, but rather, $\frac{n+1}{2n}$ units of benefit per unit of cost. In a society with a large number of districts, and projects that deliver at least 51cts of benefit, we predict that the efficient outcome with no pork is only observed if voters are poorly informed.

The most relevant extensions and variations to the model that we have analyzed are the following:

a) A model in which each district becomes informed with independent probability ε_i.

b) A model in which candidates' motivations are private, unobserved by the other candidate or by voters.

c) A citizen-candidate model in which each candidate is biased toward her own district so that she favors allocating pork to that district.

We show that our results are robust to these variations: if $\beta \in (2/3, 1)$, the efficient equilibrium can only be sustained if citizens are poorly informed, while if $\beta \in (1/3, 2/3)$, it can always be sustained; furthermore, if candidates are efficiency concerned, there is an interval for $\varepsilon \leq \frac{1}{2}$ such that the efficient equilibrium is unique. The set of parameter values for which each equilibrium exists vary with each variation of the model, but the qualitative results are robust.

A precise formulation and proof of each of the claims in this section is available from the authors, along with the following additional results:

1. A complete characterization of the pure strategy equilibria for two purely office motivated candidates, one purely office motivated and one efficiency concerned, or two efficiency concerned candidates, for any $\varepsilon \in [0,1]$.

2. A complete characterization of the symmetric mixed strategy equilibria for the range of parameter values for which no pure strategy equilibrium exists, for the cases with two purely office motivated or two efficiency concerned candidates.

3. A proof that all the equilibria are extensive form rationalizable (Pearce [48] and Battigali [10]), and a characterization of the set of trembling hand perfect (Selten [55]) and proper (Myerson [44]) equilibria for the case with two purely office motivated candidates.

5 Discussion

We have developed a theory on the provision of inefficient particularistic goods to an imperfectly informed electorate. We argue that citizens are better informed about government expenditures in their own district (which they favor) than about government expenditure in other districts (which they oppose). We analyze how this informational bias affects the provision of socially inefficient local public goods.

If particularistic goods are very inefficient, returning less than 66cts of benefit per unit of cost, the efficient policy with no provision of pork can be supported in equilibrium, whether or not voters are fully informed.

If particularistic goods are inefficient but provide more than 67cts of benefit per unit of cost, the equilibrium outcome depends on the information held by the electorate:

a) If voters are poorly informed, many different strategies and levels of provision of inefficient goods, including the efficient zero provision, can be sustained in equilibrium.

b) If voters are well informed, there is no pure strategy equilibrium because majority preferences exhibit a Condorcet cycle: a majority prefers to provide goods to no district rather than to all districts, and to provide to two districts over no district; a different majority prefers to provide the good to only one of those two districts instead of to both of them; and yet a different majority prefers to provide goods to all three districts instead of to just that one district, completing the cycle. Equilibria are in mixed strategies, and are ex-ante inefficient, since in expectation at least 43% of districts receive a unit of pork.

These results hold whether candidates care only about winning the election, or also about the efficiency of the policies they propose and implement once in office. The negative effect of information on social welfare is reinforced if candidates care about efficiency. In this case, there is an intermediate range of information for which the unique pure strategy equilibrium is the efficient one. A higher level of information destroys this equilibrium, as each candidate is then always able to best respond to any pure strategy by the other candidate by crafting a proposal that is more beneficial to a simple majority of districts.

We therefore find that a more informed electorate can make every voter ex-ante worse off.

In a survey on the role of the media, Stromberg and Prat [51] argue that an electorate that ignores what is the state of the world may become worse off if it gains information about candidates' actions but not about outcomes, because it makes candidates' pander by choosing actions that match the prior of the voter about the right action to take (Maskin and Tirole [38], Ashworth and Shotts [4]). In their framework, the electorate would always be better off learning about outcomes. We identify a novel channel by which information hurts the electorate: without any uncertainty about the state of the world, an informed electorate leads candidates to defeat the efficient policy by proposing inefficient policies designed to benefit a simple majority of districts, which ex-ante makes every voter worse off.

Our results have normative implications with regard to voter education: making the electorate fully informed does not suffice and in fact harms the prospects of obtaining efficient policies from the political process. The classic solution to restore efficiency –to fund local public goods with locally raised revenue– appears more promising. In the words of Adam Smith [57]:

> When high roads, bridges, canals, etc. are in this manner made and supported by the commerce which is carried on by means of them, they can be made only where that commerce requires them, as consequently where it is proper to make them... A magnificent high road cannot be made through a desert country where there is little or no commerce. (p. 683).

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6 Appendix

In this appendix, we first introduce additional notation, which we need to prove all our formal results.

Thereafter, we characterize the set of pure strategy equilibria for any β and any ε; and for any parameter values such that there exists no pure strategy equilibrium, we analyze the set of mixed strategy equilibria. We separate the results according to candidates' motivations, solving first the model with two purely office motivated candidates, then with two efficiency concerned candidates, and finally the asymmetric motivations case with one purely office motivated and one efficiency concerned candidate (in this case we only find the pure strategy equilibria). Proposition 1 and Theorem 2 aggregate the results in this appendix for $\beta \in \left(\frac{1}{3}, \frac{2}{3}\right)$ in the case of the proposition and $\beta \in \left(\frac{1}{3}, \frac{2}{3}\right)$ in the case of the theorem. Their proofs follow as immediate corollaries of the more detailed results in the appendix.

Due to space constraints, we provide the proofs for the two purely office motivated candidates in this section, and the proofs for the other two cases (efficiency concerned candidates, and the hybrid case), in a continuation of the appendix online.

6.1 Preliminaries

For each candidate $J \in \{A, B\}$, the set of pure strategies S^J consists of the following eight 3-dimensional vector.

$s_1 = (0,0,0)$, $s_2 = (1,0,0)$, $s_3 = (0,1,0)$, $s_4 = (0,0,1)$, $s_5 = (1,1,0)$, $s_6 = (1,0,1)$, $s_7 = (0,1,1)$, $s_8 = (1,1,1)$.

It is useful to classify strategy pairs in classes of strategic equivalence, as follows:

Let $S_1 = \{(s_1, s_1)\}$; $S_2 = \{(s_1, s_2), (s_1, s_3), (s_1, s_4), (s_2, s_1), (s_3, s_1), (s_4, s_1)\}$;

$S_3 = \{(s_1, s_5), (s_1, s_6), (s_1, s_7), (s_5, s_1), (s_6, s_1), (s_7, s_1)\}$; $S_4 = \{(s_1, s_8), (s_8, s_1)\}$;

$S_5 = \{(s_2, s_2), (s_3, s_3), (s_4, s_4)\}$; $S_6 = \{(s_2, s_3), (s_2, s_4), (s_3, s_4), (s_3, s_2), (s_4, s_2), (s_4, s_3)\}$;

$S_7 = \{(s_2, s_5), (s_2, s_6), (s_3, s_5), (s_3, s_7), (s_4, s_6), (s_4, s_7), (s_5, s_2), (s_6, s_2), (s_5, s_3), (s_7, s_3), (s_6, s_4), (s_7, s_4)\}$;

$S_8 = \{(s_2, s_7), (s_3, s_6), (s_4, s_5), (s_7, s_2), (s_6, s_3), (s_5, s_4)\}$;

$S_9 = \{(s_2, s_8), (s_3, s_8), (s_4, s_8), (s_8, s_2), (s_8, s_3), (s_8, s_4)\}$;

$S_{10} = \{(s_5, s_5), (s_6, s_6), (s_7, s_7)\}$; $S_{11} = \{(s_5, s_6), (s_5, s_7), (s_6, s_7), (s_6, s_5), (s_7, s_5), (s_7, s_6)\}$;

$S_{12} = \{(s_5, s_8), (s_6, s_8), (s_7, s_8), (s_8, s_5), (s_8, s_6), (s_8, s_7)\}$; $S_{13} = \{(s_8, s_8)\}$.

Note that $S = S^A \times S^B = \prod_{k=1}^{13} S_k$ is the set of all possible candidates' pure strategy pairs. Within each class, it is without loss of generality to establish whether any one of the elements can or cannot be supported in equilibrium.

To simplify notation, and given that voters' strategies are straightforward when full information is revealed, in all the analysis below we implicitly assume that if Nature fully reveals the policy proposals, voters vote according to their preferences and abstain when indifferent. This allows us to focus our analysis of voters on the branches of the game in which Nature does not reveal the full information so that voters face uncertainty.[15] For each voter $i \in \{a, b, c\}$, let $s^i : \{0, 1\} \times \{0, 1\} \longrightarrow \{A, B, \emptyset\}$ be a behavioral strategy for voter i, which is a function that maps each information set of the voter when Nature does not reveal the policy proposals fully, into an action by the voter. A complete strategy for the voter

[15]We stress that this simplifies notation, but does not change our behavioral assumption that voters are fully strategic and rational. In the branches of the game with full information the voters' decision problem can be solved by simple domination arguments, and we directly anticipate and impose the outcome that follows from the unique undominated solution.

specifies s^i, and the actions to be taken when information is fully revealed. We also express s^i as a vector $s^i = (s^i_1, ..., s^i_4)$, where s^i_k is the action chosen under the k-th information set according to the following order: $\{(0,0),(0,1),(1,0),(1,1)\}$.

The belief of voter i, denoted δ_i, is a vector $\delta_i = (\delta_i^A, \delta_i^B)$, where $\delta_i^J = (\delta_i^J(0), \delta_i^J(p_i^J))$ for each $J \in \{A, B\}$, and $\delta_i^J(p_i^J)$ is the probability distributions over the set of strategies played by candidate J that voter i holds as a belief after observing p_i^J. For $s_i^J \in \{0,1\}$, let $\omega_k^{i,J}(s_i^J)$ be the sum of weights assigned by $\delta_i^J(s_i^J)$ to the set of strategies where J proposes to carry out k projects in districts other than i.

It is also useful to define $v : S \longrightarrow \{A, B, \emptyset\}^3$ as the list of votes by voters $\{a, b, c\}$ as a function of candidates' strategies, given some beliefs. We stress that this function is only defined given beliefs, and we will only use it after specifying beliefs, or providing a strategy pair that the voters believe is being played.

Because the number of players is finite, and each player has a finite number of possible strategies, the game is finite, and hence an equilibrium (possibly in mixed strategies) exists (see for instance Myerson [46] pg 177).

The set of equilibria depend on the efficiency of the projects measured by β, and on how informed is the electorate, measured by ε. We focus in the following propositions on the case in which $\varepsilon \in (0,1]$. All equilibria that exist for $\varepsilon \in (0, \frac{1}{2})$, also exist if $\varepsilon = 0$, but if $\varepsilon = 0$ we can sustain additional equilibria that do not seem very plausible, in which one candidate wins all votes with probability one. For instance, if $\varepsilon = 0$ there exists an equilibrium in which candidate A proposes $s^A = (0,0,0)$ and candidate B proposes $s^B = (1,1,1)$ and all voters vote for candidate A. This equilibrium is supported by voters' beliefs such that each voter i

who observes $p_i^B = 0$ assigns probability one that $p_k^B = 1$ for both $k \neq i$. These equilibria are fragile because they fail for any $\varepsilon > 0$, because in this case by mimicking A and proposing $\tilde{s}^B = (0, 0, 0)$ candidate B can tie the election with positive probability. This argument can be used to prove the following preliminary claim that holds both for office motivated and efficiency concerned candidates

Lemma 4 *If* $\varepsilon > 0$, *in any pure strategy equilibrium both candidates propose to carry out the same number of projects and the election is tied.*

Proof. of Claim 4. Consider any strategy profile such that candidate J wins with probability less than $\frac{1}{2}$. Since in equilibrium voters hold correct beliefs, the probability that J wins conditional on full information being revealed, or not revealed, is less than $\frac{1}{2}$ in each case. In pure strategies, the probability of victory is in the set $\{0, \frac{1}{2}, 1\}$ so if it is less than $\frac{1}{2}$, it is zero. Deviating to $s^J = s^{-J}$, candidate J ties the election if full information is revealed, so the probability of winning is at least $\frac{\varepsilon}{2}$. ■

6.2 Office Motivated Candidates

6.2.1 Pure strategy equilibria

Proposition 5 *Assume* $\varepsilon \in \left(0, \frac{1}{2}\right)$. *For any* $\beta \in \left(\frac{2}{3}, 1\right)$, *an equilibrium in which candidates use the strategy pair* (s^A, s^B) *exists if and only if* $(s^A, s^B) \in S_k$ *for some* $k \in \{1, 11, 13\}$. *For any* $\beta \in \left(\frac{1}{3}, \frac{2}{3}\right)$, *an equilibrium in which candidates use the strategy pair* (s^A, s^B) *exists if and only if* $(s^A, s^B) \in S_k$ *for some* $k \in \{1, 5, 6, 11, 13\}$.

Assume $\varepsilon \in (\frac{1}{2}, 1]$. *For any* $\beta \in \left(\frac{2}{3}, 1\right)$ *there is no equilibrium in pure strategies. For any*

$\beta \in \left(\frac{1}{3}, \frac{2}{3}\right)$, *there is a unique equilibrium such that candidates propose* $k = 0$.

Proof. We prove the high benefit case first.

S_1 :Voter strategy $s^i = (\emptyset, A, B, \emptyset)$ for each voter i and beliefs such that $\omega_0^{i,J}(0) = 1$ and $\omega_2^{i,J}(1) = 1$ for any voter i and any candidate J make the election tied and if candidate J deviates to any $s^J \neq s_1$, then J loses the election. It is also straightforward to check that the voting strategy is a best response given the strategy of the candidates and the beliefs of the voters are correct along the equilibrium path, so these strategies and beliefs are an equilibrium. If $\varepsilon \leq \frac{1}{2}$, no gain when full information is revealed can compensate for this loss. Suppose now that $\varepsilon > \frac{1}{2}$. If candidate A deviates and proposes $s^A = s_5$ she wins the election with probability ε and therefore the deviation is profitable.

S_2 : Assume without loss of generality that $(s^A, s^B) = (s_1, s_2)$. Given (s_1, s_2), $v(s_1, s_2) = (B, A, A)$. By Lemma 4, this cannot occur in equilibrium.

S_3 : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_5)$. Given (s_1, s_5), $v(s_1, s_5) = (B, B, A)$. Ruled out by Lemma 4.

S_4 : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_8)$. Every voter votes for A. Ruled out by Lemma 4.

S_5 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_2)$. Given (s_2, s_2), every voter i abstains. If candidate J deviates to $s^J = s_8$ wins the election both in case the information is revealed and in case it is not.

S_6 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_3)$. Given (s_2, s_3), $v(s_2, s_3) = (A, B, \emptyset)$. If A deviates to $s^A = s_6$ voters a and c vote for A both in case full information is revealed and in case it is not revealed. Hence by deviating, A wins with for sure.

S_7 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_5)$. Given (s_2, s_5), $v(s_2, s_5) = (A, B, A)$. Ruled out by Lemma 4.

S_8 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_7)$. Given (s_2, s_7), $v(s_2, s_7) = (A, B, B)$. Ruled out by Lemma 4.

S_9 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_8)$. Given (s_2, s_8), $v(s_2, s_8) = (A, B, B)$. Ruled out by Lemma 4.

S_{10} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_5)$. Given (s_5, s_5), all voters abstain. Suppose first that $\varepsilon < \frac{1}{2}$. If candidate A deviates to $s^A = s_8$ and full information is not revealed, only voter c observes the deviation and $v(s_8, s_5) = (\emptyset, \emptyset, A)$. Hence by deviating candidate A wins with probability at least $1 - \varepsilon > \frac{1}{2}$. If $\varepsilon = \frac{1}{2}$ then voter strategy $s^i = (\emptyset, A, B, \emptyset)$ for each voter i and beliefs such that $\omega_2^{i,J}(0) = 1$ for $i = a, b$ and $J = A, B$ make the election tied and if candidate J deviates to any $s^J \neq s_5$, then J wins the election with probability at most $\frac{1}{2}$ and with complementary she looses the election. Hence no deviation is profitable. If $\varepsilon > \frac{1}{2}$ then candidate A can win the election with probability ε deviating to $s^A = s_2$.

S_{11} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_6)$. Given $(s^A, s^B) = (s_5, s_6)$, beliefs such that $\omega_2^{i,J}(1 - s_i^J) = 1$ for each $i \in \{a, b, c\}$ and $J \in \{A, B\}$ support an equilibrium in which $v(s_5, s_6) = (\emptyset, A, B)$ and each candidate wins with equal probability. Suppose $\varepsilon \leq \frac{1}{2}$. It suffices to check that A has no incentives to deviate. If A deviates to $s^A \in \{s_1, s_2, s_3, s_4, s_6, s_7\}$ and full information is not revealed, A loses the election. If A deviates to $s^A = s_8$ and full information is not revealed, the election is tied, but if full information is revealed, A loses the election. In any case, after a deviation A wins the election with probability less than $\frac{1}{2}$. If $\varepsilon > \frac{1}{2}$ candidate A can win the election with probability ε deviating to $s^A = s_2$.

S_{12} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_8)$. Given (s_5, s_8), $v(s_5, s_6) = (A, A, B)$. Ruled out by Lemma 4.

S_{13} :Voter strategy $s^i = (\emptyset, B, A, \emptyset)$ for each voter i and beliefs such that $\omega_2^{i,J}(0) = \omega_2^{i,J}(1) = 1$ for any voter i and any candidate J make the election tied. Suppose $\varepsilon \leq \frac{1}{2}$; if candidate J deviates to any strategy $s^J \neq s_8$ and full information is not revealed, J loses the election. Suppose $\varepsilon > \frac{1}{2}$; candidate J wins the election probability ε deviating to $s^J = s_1$.

Next we prove the low benefit case. To sustain equilibria, assume that off-equilibrium path beliefs in cases S_1, S_5, S_6, S_{11} and S_{13} are such that given the equilibrium proposal s_i^J, $\omega_2^{i,J}(1 - s_i^J) = 1$ for each $i \in \{a, b, c\}$ and $J \in \{A, B\}$. That is, a voter who observe a deviation believes that the deviating candidate proposes to carry out the projects in the other two districts.

S_1 : Suppose $\varepsilon \leq \frac{1}{2}$. Voter strategy $s^i = (\emptyset, A, B, \emptyset)$ for each voter i and beliefs such that $\omega_0^{i,J}(0) = 1$ and $\omega_2^{i,J}(1) = 1$ for any voter i and any candidate J make the election tied and if candidate J deviates to any $s^J \neq s_1$, then J loses the election. It is also straightforward to check that the voting strategy is a best response given the strategy of the candidates and the beliefs of the voters, and that the beliefs are correct along the equilibrium path, so these strategies and beliefs are an equilibrium. If full information is revealed there is not a different proposal that can defeat s_1. Hence there are no profitable deviations for all $\varepsilon \in [0, 1]$.

S_2 : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_2)$. Given (s_1, s_2), $v(s_1, s_2) = (B, A, A)$. Ruled out by Lemma 4.

S_3 : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_5)$. Given (s_1, s_5), $v(s_1, s_5) = (A, A, A)$. Ruled out by Lemma 4.

S_4 : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_8)$. Given (s_1, s_5), $v(s_1, s_5) = (A, A, A)$. Ruled out by Lemma 4.

S_5 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_2)$. Given (s_2, s_2), every voter abstains. If J deviates and full information is not revealed, any voter who observes the deviation votes for $-J$ and J loses the election. If $\varepsilon > \frac{1}{2}$ then if candidate J deviates and proposes $s^J = s_1$ then she wins with probability ε and therefore the deviation is profitable.

S_6 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_3)$. Given (s_2, s_3), $v(s_2, s_3) = (A, B, \emptyset)$. If J deviates and full information is not revealed, any voter who observes the deviation votes for $-J$ and J loses the election. If $\varepsilon > \frac{1}{2}$ then candidate A wins with probability ε deviating to $s^A = s_1$

S_7 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_5)$. Given (s_2, s_5), $v(s_2, s_5) = (A, B, A)$. Ruled out by Lemma 4.

S_8 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_7)$. Given (s_2, s_7), $v(s_2, s_7) = (A, B, B)$. Ruled out by Lemma 4.

S_9 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_8)$. All three voters vote for A. Ruled out by Lemma 4.

S_{10} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_5)$. Given (s_5, s_5), all voters abstain. Suppose $\varepsilon < \frac{1}{2}$. If candidate A deviates to $s^A = s_8$ and full information is not revealed, only voter c observes the deviation and $v(s_8, s_5) = (\emptyset, \emptyset, A)$. Hence by deviating candidate A wins with probability at least $1 - \varepsilon > \frac{1}{2}$. Suppose $\varepsilon = \frac{1}{2}$, for any deviation played by candidate J, she wins with probability $\frac{1}{2}$ and looses with the same probability. Hence there is not a profitable deviation. If $\varepsilon > \frac{1}{2}$, candidate J wins when full information is revealed deviating to $s^J = s_1$.

S_{11} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_6)$. Given (s_5, s_6), $v(s_5, s_6) = (\emptyset, A, B)$. Suppose $\varepsilon \leq \frac{1}{2}$. It suffices to check that A has no incentives to deviate. If A deviates to $s^A \in \{s_1, s_2, s_3, s_4, s_6, s_7\}$ and full information is not revealed, A loses the election. If A deviates to $s^A = s_8$ and full information is not revealed, the election is tied, but if full information is revealed, A loses the election. In any case, after a deviation A wins the election with probability less than $\frac{1}{2}$. Suppose $\varepsilon > \frac{1}{2}$ if candidate J deviates to $s^J = s_1$ she wins the election with probability ε.

S_{12} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_8)$. Given (s_5, s_8), $v(s_5, s_6) = (A, A, B)$. Ruled out by Lemma 4.

S_{13} : All voters abstain and the election is tied. Suppose $\varepsilon \leq \frac{1}{2}$. If candidate J deviates and full information is not revealed, any voter who observes the deviation votes for $-J$ and J loses the election. Suppose $\varepsilon > \frac{1}{2}$; if candidate J deviates to $s^J = s_1$ she wins the election with probability ε. ■

6.2.2 Mixed strategy equilibria

We look at the set of mixed equilibria in case there are no equilibria in pure strategies. By the previous proposition this happens when $\varepsilon > \frac{1}{2}$ and $\beta \in (\frac{2}{3}, 1)$.We know from (Eguia and Nicolo 2011) that if $\varepsilon = 1$, the equilibrium strategy pair is $\sigma^A = \sigma^B = (1/7, ..., 1/7, 0)$. Let $s_L = (0, 1/3, 1/3, 1/3, 0, 0, 0, 0)$ and $s_H = (0, 0, 0, 0, 1/3, 1/3, 1/3, 0)$ be the special mixed strategies that consist, respectively, on proposing exactly one project and randomizing which one, and proposing exactly two projects and randomizing which two. The first proposition almost characterizes the set of mixed equilibria for $\varepsilon \in (\frac{1}{2}, \frac{3}{4})$

Lemma 6 *If* $\beta \in (\frac{2}{3}, 1)$ *and* $\varepsilon \in (\frac{1}{2}, \frac{3}{4})$, *candidate strategies* (s_H, s_H) *are supported in equilibrium. Conversely, in any symmetric equilibrium,* $\sigma_5^J + \sigma_6^J + \sigma_7^J = 1$ *for* $J \in \{A, B\}$.

Proof. First we prove that (s_H, s_H) can be supported in equilibrium. Suppose (s_H, s_H) is played, for any candidate J and voter i, if i does not observe the full proposals, beliefs are such that $\omega_1^{i,J}(1) = \omega_2^{i,J}(0) = 1$ and hence $s^i(1,0) = A$, $s^i(0,1) = B$ and $s^i(p_i^A, p_i^B) = \varnothing$ if $p_i^A = p_i^B$. Therefore, given that (s_H, s_H) is played as expected by voters, either all voters abstain if $p^A = p^B$ or one voter votes for A, one for B and one abstains if $p^A \neq p^B$; in either case the election is tied and the probability that each candidate is elected is $\frac{1}{2}$, so the expected payoff for each candidate is $\frac{1}{2}$. Suppose A deviates to s_1, A loses $0-2$ if Nature does not reveal p, and A loses $1-2$ if Nature reveals p. Suppose A deviates to $s_k \in \{s_2, s_3, s_4\}$. If Nature reveals p, with probability $\frac{2}{3}$ A wins and with probability $\frac{1}{3}$ A loses; while if p is not revealed, A loses for sure. Hence, A wins with probability $\frac{2}{3}\varepsilon$, the expected utility deviating is $\frac{2}{3}\varepsilon$ and the deviation is profitable if and only if $\frac{2}{3}\varepsilon > \frac{1}{2}$, that is, $\varepsilon > \frac{3}{4}$. Suppose A deviates to $s_k \in \{s_5, s_6, s_7\}$. Then A achieves the same expected electoral outcomes and utilities as not deviating. Suppose A deviates to s_8. If Nature reveals p, A loses $1-2$. If Nature does not reveal p, then A wins $2-1$. But Nature reveals p with probability ε, hence the expected utility for A is $1-\varepsilon$ which is less than $\frac{1}{2}$ for any $\varepsilon > \frac{1}{2}$. Hence, there is no profitable deviation.

Second, we prove that in any symmetric equilibrium both candidates propose two projects.

Suppose (σ^J, σ^J) is part of a symmetric equilibrium. Since the equilibrium is symmetric, for any $i \in \{a, b, c\}$, if i does not observe p, then $s^i(p_i^A, p_i^B) = \varnothing$ if $p_i^A = p_i^B$ and furthermore, for $k \in \{0, 1\}$, $s^i(k, 1-k) = A$ if and only if $s^i(1-k, k) = B$. Suppose $s^i(1,0) \neq A$, so $s^i(0,1) \neq B$. Then s_8 is not a best response and it is not played in equilibrium. But if s_8 is not played, the expected payoff for i if A wins is strictly higher than if B wins given $(p_i^A, p_i^B) = (1, 0)$, so by assumption $s^i(1,0) = A$, a contradiction. Thus, it must be $s^i(1,0) = A$ and $s^i(0,1) = B$. Then, given that $\varepsilon \in (\frac{1}{2}, \frac{3}{4})$, for any strategy σ^{-J}, a best response by J must propose two projects. Thus no strategy that proposes any other number of projects can be used in a symmetric equilibrium. ■

Proposition 7 *If $\beta \in (\frac{2}{3}, 1)$ and $\varepsilon \in [\frac{3}{4}, \frac{11+\sqrt{61}}{20}]$, there is a unique symmetric mixed strategy equilibrium, in which $\sigma^J = (0, \frac{2\varepsilon-1}{10\varepsilon-3}, \frac{2\varepsilon-1}{10\varepsilon-3}, \frac{2\varepsilon-1}{10\varepsilon-3}, \frac{1}{10\varepsilon-3}, \frac{1}{10\varepsilon-3}, \frac{1}{10\varepsilon-3}, \frac{4\varepsilon-3}{10\varepsilon-3})$. The expected number of projects is weakly larger than $\frac{5}{3}$ and converges to $\frac{5}{3}$ as $\varepsilon \to \frac{3}{4}$. If $\beta \in (\frac{2}{3}, 1)$ and $\varepsilon \in [\frac{11+\sqrt{61}}{20}, 1)$, there is a unique symmetric mixed strategy equilibrium, in which $\sigma^J = (\frac{4\varepsilon-3}{10\varepsilon-3}, \frac{1}{10\varepsilon-3}, \frac{1}{10\varepsilon-3}, \frac{1}{10\varepsilon-3}, \frac{2\varepsilon-1}{10\varepsilon-3}, \frac{2\varepsilon-1}{10\varepsilon-3}, \frac{2\varepsilon-1}{10\varepsilon-3}, 0)$. The expected number of projects is weakly larger than $\frac{9}{7}$ and converges to $\frac{9}{7}$ as $\varepsilon \to 1$.*

Proof. Let (σ^A, σ^B) be a symmetric candidate strategy profile so $\sigma^A = \sigma^B$. Since the candidates' strategies are symmetric, voters' strategies must be such that $s^i(k,k) = \varnothing$ for $k \in \{0,1\}$, and $\{s^i(1,0) = A$ and $s^i(0,1) = B\}$ or $\{s^i(1,0) = B$ and $s^i(0,1) = A\}$ or $\{s^i(1,0) = \varnothing$ and $s^i(0,1) = \varnothing\}$ for every voter i. Suppose not $\{s^i(1,0) = A$ and $s^i(0,1) = B\}$. Then, given any strategy σ^{-J}, candidate J obtains a greater expected payoff playing s_1 than playing s_8, and a strictly greater payoff if $\sigma_8^{-J} > 0$. Therefore, in a symmetric mixed equilibrium, $\sigma_8^J = 0$. Then, it follows that for any voter i who observes $p_i^J = 1$ and $p_i^{-J} = 0$, the expected payoff for voter i is greater if J wins, thus by assumption, i votes J. Therefore, $s^i(1,0) = A$ and $s^i(0,1) = B$.

Next we prove that in any symmetric mixed strategy equilibrium, $\sigma_2^{-J} + \sigma_3^{-J} + \sigma_4^{-J} > 0$. Suppose not. Notice that given $\varepsilon > \frac{3}{4}$, $s^i(1,0) = A$ and $s^i(0,1) = B$, if $-J$ proposes one project and J proposes two projects, in expectation J wins the election more often, whereas if J proposes two and $-J$ proposes zero or three, J wins more often. So if $-J$ never proposes one project, proposing two projects in expectation defeats any other proposal with probability more than one half. Then any best response by J to σ^{-J} with $\sigma_2^{-J}+\sigma_3^{-J}+\sigma_4^{-J}=0$ must be such that $\sigma_5^J+\sigma_6^J+\sigma_7^J=1$, which in turns means that any best response by J must be such that $\sigma_2^J+\sigma_3^J+\sigma_4^J=1$, a contradiction.

Similarly, in any symmetric mixed strategy equilibrium, $\sigma_5^{-J}+\sigma_6^{-J}+\sigma_7^{-J}>0$. Suppose not. Any best response by J must be such that $\sigma_2^J+\sigma_3^J+\sigma_4^J=0$, but then the best response by $-J$ must be $\sigma_5^{-J}+\sigma_6^{-J}+\sigma_7^{-J}=1$, a contradiction.

Therefore, in any symmetric mixed strategy equilibrium, both candidates propose one project, and two projects, with positive probability. But then, it must be that $\sigma_2^J=\sigma_3^J+\sigma_4^J$ and $\sigma_5^J=\sigma_6^J=\sigma_7^J$. Given that the randomization between districts, subject to choosing a number of projects, assigns equal weight to all districts, we can reduce the strategic problem to that of assigning weights to strategies s_1, s_8, s_L, s_H. The payoff matrix is as follows:

$$\left\{\begin{array}{ccccc} & s_1 & s_L & s_H & s_8 \\ s_1 & \frac{1}{2},\frac{1}{2} & \varepsilon, 1-\varepsilon & 0,1 & \varepsilon, 1-\varepsilon \\ s_L & 1-\varepsilon, \varepsilon & \frac{1}{2},\frac{1}{2} & \frac{2}{3}\varepsilon, 1-\frac{2\varepsilon}{3} & 0,1 \\ s_H & 1,0 & 1-\frac{2\varepsilon}{3}, \frac{2\varepsilon}{3} & \frac{1}{2},\frac{1}{2} & \varepsilon, 1-\varepsilon \\ s_8 & 1-\varepsilon, \varepsilon & 1,0 & 1-\varepsilon, \varepsilon & \frac{1}{2},\frac{1}{2} \end{array}\right\}$$

An equilibrium with $\sigma_1^J>0, \sigma_L^J>0, \sigma_H^J>0$ and $\sigma_8^J=0$ must satisfy

i) $\frac{1}{2}\sigma_1^J+\varepsilon\sigma_L^J=(1-\varepsilon)\sigma_1^J+\frac{1}{2}\sigma_L^J+\frac{2}{3}\varepsilon(1-\sigma_1^J-\sigma_L^J)$

ii) $\frac{1}{2}\sigma_1^J + \varepsilon\sigma_L^J = \sigma_1^J + (1 - \frac{2\varepsilon}{3})\sigma_L^J + \frac{1}{2}(1 - \sigma_1^J - \sigma_L^J)$

iii) $\frac{1}{2}\sigma_1^J + \varepsilon\sigma_L^J \geq (1-\varepsilon)\sigma_1^J + \sigma_L^J + (1-\varepsilon)(1 - \sigma_1^J - \sigma_L^J)$.

Dropping the superindex and simplifying *i*) we obtain

$0 = \frac{2}{3}\varepsilon + \frac{1}{2}\sigma_1 + \frac{1}{2}\sigma_L - \frac{5}{3}\varepsilon\sigma_1 - \frac{5}{3}\varepsilon\sigma_L,$

$(10\varepsilon - 3)\sigma_1 = 4\varepsilon + (3 - 10\varepsilon)\sigma_L$

$$\sigma_1 = \frac{4\varepsilon}{10\varepsilon - 3} - \sigma_L.$$

Dropping the superindex and simplifying *ii*) we obtain $\varepsilon\sigma_L = \frac{1}{2}\sigma_L - \frac{2}{3}\varepsilon\sigma_L + \frac{1}{2}$ and with simple computation

$$\sigma_L = \frac{3}{10\varepsilon - 3}.$$

Thus

$$\sigma_1 = \frac{4\varepsilon - 3}{10\varepsilon - 3}$$

Then simplifying inequality *iii*):

$$\frac{1}{2}\sigma_1 + \varepsilon\sigma_L \geq \sigma_1 - \varepsilon\sigma_1 + \sigma_L + 1 - \sigma_1 - \sigma_L - \varepsilon + \varepsilon\sigma_1 + \varepsilon\sigma_L$$
$$\frac{1}{2}\sigma_1 \geq 1 - \varepsilon$$

and substituting the value of σ_1 :

$$\frac{4\varepsilon - 3}{20\varepsilon - 6} \geq 1 - \varepsilon$$

The previous expression holds as an equality for $\varepsilon = \frac{11+\sqrt{61}}{20}$ (for $\varepsilon \in (\frac{2}{3}, 1)$. So for $\varepsilon > \frac{11+\sqrt{61}}{20}$, the above mix is an equilibrium, with expected number of projects $\frac{3}{10\varepsilon-3} + 2(1 - \frac{3}{10\varepsilon-3} - \frac{4\varepsilon-3}{10\varepsilon-3}) = \frac{12\varepsilon-3}{10\varepsilon-3}$. The probability of proposing two projects is $\frac{10\varepsilon-3-4\varepsilon}{10\varepsilon-3} = \frac{6\varepsilon-3}{10\varepsilon-3}$ which converges to $\frac{9}{7}$ as ε converges to 1. The initial assumption that voters vote $s^i(1,0) = A$ and $s^i(0,1) = B$ is supported because $\sigma_8 = 0$ and $\beta > 2/3$.

If instead $\varepsilon < \frac{11+\sqrt{61}}{20}$, this equilibrium does not exist. We look then for a fully mixed equilibrium, which must satisfy:

i) $\frac{1}{2}\sigma_1 + \varepsilon\sigma_L + \varepsilon(1 - \sigma_1 - \sigma_L - \sigma_H) = (1-\varepsilon)\sigma_1 + \frac{1}{2}\sigma_L + \frac{2}{3}\varepsilon\sigma_H$

ii) $\frac{1}{2}\sigma_1 + \varepsilon\sigma_L + \varepsilon(1 - \sigma_1 - \sigma_L - \sigma_H) = \sigma_1 + (1 - \frac{2\varepsilon}{3})\sigma_L + \frac{1}{2}\sigma_H + \varepsilon(1 - \sigma_1 - \sigma_L - \sigma_H)$

iii) $\frac{1}{2}\sigma_1 + \varepsilon\sigma_L + \varepsilon(1 - \sigma_1 - \sigma_L - \sigma_H) = (1-\varepsilon)\sigma_1 + \sigma_L + (1-\varepsilon)\sigma_H + \frac{1}{2}(1 - \sigma_1 - \sigma_L - \sigma_H)$

Simplifying the first equation

$\varepsilon = \frac{1}{2}\sigma_1 + \frac{1}{2}\sigma_L + \frac{5}{3}\varepsilon\sigma_H$

i) $3\sigma_1 + 3\sigma_L + 10\varepsilon\sigma_H = 6\varepsilon$

Simplifying the second equation

$\varepsilon\sigma_L = \frac{1}{2}\sigma_1 + (1 - \frac{2\varepsilon}{3})\sigma_L + \frac{1}{2}\sigma_H$

$0 = \frac{1}{2}\sigma_1 + (1 - \frac{5\varepsilon}{3})\sigma_L + \frac{1}{2}\sigma_H$

ii) $3\sigma_1 + (6 - 10\varepsilon)\sigma_L + 3\sigma_H = 0$

Simplifying the third equation

$\varepsilon = \frac{1}{2}\sigma_L + \frac{1}{2}\sigma_H + \frac{1}{2}$

iii) $\sigma_L + \sigma_H = 2\varepsilon - 1$

From *iii)*

$$\sigma_H = 2\varepsilon - 1 - \sigma_L.$$

From $i) - ii)$

$$3\sigma_L + 10\varepsilon\sigma_H - (6-10\varepsilon)\sigma_L - 3\sigma_H = 6\varepsilon$$

$$(10\varepsilon - 3)(\sigma_H + \sigma_L) = 6\varepsilon$$

$$\sigma_H = \frac{6\varepsilon}{10\varepsilon - 3} - \sigma_L.$$

The two equalities together imply that

$$\frac{6\varepsilon}{10\varepsilon - 3} = 2\varepsilon - 1$$

$$\varepsilon = \frac{1}{20}\sqrt{61} + \frac{11}{20}.$$

Which means that the fully mixed equilibrium is non-generic. Consider equilibria such that $\sigma_1 = 0$, so that candidates mix between proposing one, two and three projects. An equilibrium with these characteristics requires:

$$\left\{\begin{array}{cccc} \frac{1}{2}, \frac{1}{2} & \varepsilon, 1-\varepsilon & 0, 1 & \varepsilon, 1-\varepsilon \\ 1-\varepsilon, \varepsilon & \frac{1}{2}, \frac{1}{2} & \frac{2}{3}\varepsilon, 1-\frac{2\varepsilon}{3} & 0, 1 \\ 1, 0 & 1-\frac{2\varepsilon}{3}, \frac{2\varepsilon}{2} & \frac{1}{2}, \frac{1}{2} & \varepsilon, 1-\varepsilon \\ 1-\varepsilon, \varepsilon & 1, 0 & 1-\varepsilon, \varepsilon & \frac{1}{2}, \frac{1}{2} \end{array}\right\}$$

$i)$ $\varepsilon\sigma_L + \varepsilon(1-\sigma_L-\sigma_H) \leq \frac{1}{2}\sigma_L + \frac{2}{3}\varepsilon\sigma_H$

$ii)$ $\frac{1}{2}\sigma_L + \frac{2}{3}\varepsilon\sigma_H = (1-\frac{2\varepsilon}{3})\sigma_L + \frac{1}{2}\sigma_H + \varepsilon(1-\sigma_L-\sigma_H)$

$iii)$ $\frac{1}{2}\sigma_L + \frac{2}{3}\varepsilon\sigma_H = \sigma_L + (1-\varepsilon)\sigma_H + \frac{1}{2}(1-\sigma_L-\sigma_H).$

Simplifying the three expressions:

i) $\varepsilon \leq \frac{1}{2}\sigma_L + \frac{5}{3}\varepsilon\sigma_H$

ii) $(\frac{5\varepsilon}{3} - \frac{1}{2})(\sigma_L + \sigma_H) = \varepsilon$

iii) $(\frac{5}{3}\varepsilon - \frac{1}{2})\sigma_H = \frac{1}{2}$.

From iii) and ii)

$(\frac{5\varepsilon}{3} - \frac{1}{2})\sigma_L = \varepsilon - \frac{1}{2}$, so $\sigma_L = \frac{6\varepsilon-3}{10\varepsilon-3}$ and $\sigma_H = \frac{3}{10\varepsilon-3}$ and thus $\sigma_8 = \frac{4\varepsilon-3}{10\varepsilon-3}$.

We then check that the first inequality is satisfied:

$$\varepsilon \leq \frac{1}{2}\frac{6\varepsilon - 3}{10\varepsilon - 3} + \frac{5}{3}\varepsilon\frac{3}{10\varepsilon - 3}$$

$$2\varepsilon(10\varepsilon - 3) \leq 6\varepsilon - 3 + 10\varepsilon$$

$$20\varepsilon^2 - 22\varepsilon + 3 \leq 0$$

which is true for $\varepsilon \in (\frac{3}{4}, \frac{11+\sqrt{61}}{20})$. The expected number of projects in this equilibrium is $\frac{18\varepsilon-6}{10\varepsilon-3}$ which converges to $\frac{5}{3}$ as $\varepsilon \to \frac{3}{4}$ and then increases but stays below $\frac{7}{4}$. Finally, there cannot be a symmetric mixed strategy equilibrium such that $\sigma_1 = \sigma_8 = 0$, because if $\sum_{k=2}^{7} \sigma_k^J = 1$, any best response by $-J$ must be such that $\sigma_5^{-J} + \sigma_6^{-J} + \sigma_7^{-J} = 1$, which, as shown above, cannot hold in a symmetric mixed strategy equilibrium with $\varepsilon > \frac{3}{4}$. ■

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7 Online Appendix

This appendix continues the Appendix section in the manuscript "On the Efficiency of Partial Information in Elections" by Jon X. Eguia and Antonio Nicolo. It uses definition, notation and results and cross-references to the Appendix section in the manuscript.

7.1 Efficiency Concerned Candidates

7.1.1 Pure strategy equilibria

Proposition 8 *For any $\beta \in \left(\frac{2}{3}, 1\right)$, the equilibrium in which candidates use the strategy pair (s_1, s_1) exists if and only if $\varepsilon \leq \frac{3-2\beta}{2}$; the equilibrium in which candidates use the strategy $(s^A, s^B) \in S_{11}$ exists if and only if $\varepsilon \leq \frac{2-\beta}{6-4\beta}$; the equilibrium in which candidates use the strategy $(s^A, s^B) \in S_{13}$ exists if and only if $\varepsilon \leq \frac{1}{8-6\beta}$; there are no other pure strategy equilibria.*

For any $\beta \in \left(\frac{1}{3}, \frac{2}{3}\right)$, the equilibrium in which candidates use the strategy pair (s_1, s_1) exists for all $\varepsilon \geq 0$; the equilibrium in which candidates use the strategy $(s^A, s^B) \in S_k$ for $k \in \{5, 6, 11\}$ exists if and only if $\varepsilon \leq \frac{1}{4-2\beta}$; the equilibrium in which candidates use the strategy $(s^A, s^B) \in S_{13}$ exists if and only $\varepsilon \leq \frac{1}{8-6\beta}$; there are no other pure strategy equilibria.

Proof. As before, to sustain equilibria, we assume off-equilibrium path beliefs such that given the equilibrium proposal s_i^J, $\omega_2^{i,J}(1 - s_i^J) = 1$ for each $i \in \{a, b, c\}$ and $J \in \{A, B\}$. That is, a voter who observes a deviation believes that the deviating candidate proposes to carry out the projects in the other two districts. We prove the high benefit case first.

S_1 : Voter strategy $s^i = (\emptyset, A, B, \emptyset)$ for each voter i and beliefs such that $\omega_0^{i,J}(0) = 1$ and $\omega_2^{i,J}(1) = 1$ for any voter i and any candidate J make the election tied and if candidate J deviates to any $s^J \neq s_1$, then J loses the election. It is also straightforward to check that the voting strategy is a best response given the strategy of the candidates and the beliefs of the voters, and that the beliefs are correct along the equilibrium path, so these strategies and beliefs are an equilibrium. Hence when $\varepsilon < \frac{1}{2}$, no hypothetical gain when full information is revealed can compensate for this loss. Since candidates are efficiency concerned and any deviation implies a welfare loss, they have not incentive to deviate. Suppose that $\varepsilon \geq \frac{1}{2}$. If candidate J deviates to $s^J \in \{s_2, s_3, s_4, s_8\}$ loses the election and makes a more inefficient proposal. If candidate J deviates to $s^J = s_5$ (or to $s^J = s_6$, or $s^J = s_7$) wins the election with probability ε (when full information is revealed) and therefore the deviation is profitable if and only if

$$\varepsilon\frac{1}{1+2(1-\beta)} > \frac{1}{2}, \text{ or}$$
$$\varepsilon > \frac{3-2\beta}{2}.$$

S_2 : Assume without loss of generality that $(s^A, s^B) = (s_1, s_2)$. Given (s_1, s_2), $v(s_1, s_2) = (B, A, A)$. By Lemma 4, this cannot occur in equilibrium.

S_3 : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_5)$. Given (s_1, s_5), $v(s_1, s_5) = (B, B, A)$. Ruled out by Lemma 4.

S_4 : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_8)$. Given beliefs such that $\omega_2^{i,B}(0) = 1$ for all $i \in \{a, b, c\}$, every voter votes for A. Ruled out by Lemma 4.

S_5 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_2)$. Given (s_2, s_2), every voter i abstains. If candidate A deviates to $s^A = s_8$, then candidate A wins the election. The deviation is profitable if only if

$$\frac{1}{1+3(1-\beta)} > \frac{1}{2}\frac{1}{1+(1-\beta)}$$

which holds for all $\beta \in (0, 1)$.

S_6 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_3)$. If candidate A deviates to $s^A = s_6$, then candidate A wins the election. The deviation is profitable since for all β, we have that

$$\frac{1}{1+2(1-\beta)} > \frac{1}{2}\frac{1}{1+(1-\beta)}$$

S_7 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_5)$. Given (s_2, s_5), $v(s_2, s_5) = (A, B, A)$. Ruled out by Lemma 4.

S_8 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_7)$. Given (s_2, s_7), $v(s_2, s_7) = (A, B, B)$. Ruled out by Lemma 4.

S_9 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_8)$. Given (s_2, s_8), $v(s_2, s_8) = (A, B, B)$. Ruled out by Lemma 4.

S_{10} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_5)$. Consider the deviation $s^J = s_8$. If information is not fully revealed candidate J wins the election because voter c votes for J. If information is fully revealed candidate J loses the election. Hence, candidate J prefers to deviate if and only if

$$(1-\varepsilon)\frac{1}{1+3(1-\beta)} > \frac{1}{2}\frac{1}{1+2(1-\beta)}$$
$$\varepsilon < \frac{2-\beta}{6-4\beta}$$

Consider the deviation $s^J = s_2$. If information is not fully revealed candidate J loses the election because voter b votes for J. If information is fully revealed candidate J wins the election. Hence, candidate J prefers to deviate if and only if

$$\varepsilon\frac{1}{1+(1-\beta)} > \frac{1}{2}\frac{1}{1+2(1-\beta)}, \text{ or}$$
$$\varepsilon > \frac{2-\beta}{6-4\beta}$$

Hence there is always profitable deviation for all $\varepsilon \neq \frac{2-\beta}{6-4\beta}$.

S_{11} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_6)$. If candidate J deviates to $s^J \in \{s_1, s_2, s_3, s_4, s_6, s_7\}$ and full information is not revealed, J loses the election. If J deviates to $s^J = s_8$ and full information is not revealed, the election is tied, but if full information is revealed, J loses the election. It follows that the best deviation for an efficiency concerned candidate is s_2 since candidate J wins the election when information is fully revealed and minimizes the number of proposed projects (if J proposes s_1 she loses the election). Candidate J prefers to deviate to $s^J = s_2$ if only if

$$\varepsilon\frac{1}{1+(1-\beta)} > \frac{1}{2}\frac{1}{1+2(1-\beta)}, \text{ or} \tag{3}$$
$$\varepsilon > \frac{2-\beta}{6-4\beta}. \tag{4}$$

S_{12} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_8)$. Given (s_5, s_8), $v(s_5, s_6) = (A, A, B)$. Ruled out by Lemma 4.

S_{13} : If candidate J deviates to any strategy $s^J \neq s_8$ and full information is not revealed, J loses the election. Hence the best deviation for an efficiency concerned candidate is s_1 because J wins the election when information is fully revealed and the proposal is efficient. Candidate J prefers to deviate to s_1 if only if

$$\varepsilon > \frac{1}{8-6\beta}. \tag{5}$$

Next we prove the low benefit case.

S_1 : Voter strategy $s^i = (\emptyset, A, B, \emptyset)$ for each voter i and beliefs such that $\omega_0^{i,J}(0) = 1$ and $\omega_2^{i,J}(1) = 1$ for any voter i and any candidate J make the election tied and if candidate J deviates to any $s^J \neq s_1$, then J loses the election. It is also straightforward to check that the voting strategy is a best response given the strategy of the candidates and the beliefs of the voters, and that the beliefs are correct along the equilibrium path, so these strategies and beliefs are an equilibrium. If full information is revealed there is not a different proposal that can defeat s_1. Hence there are no profitable deviation for all $\varepsilon \in [0,1]$.

S_2 : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_2)$. Given (s_1, s_2), $v(s_1, s_2) = (B, A, A)$. Ruled out by Lemma 4.

S_3 : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_5)$. Given (s_1, s_5), $v(s_1, s_5) = (A, A, A)$. Ruled out by Lemma 4.

S_4 : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_8)$. Given (s_1, s_5), $v(s_1, s_5) = (A, A, A)$. Ruled out by Lemma 4.

S_5 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_2)$. If candidate J deviates, she loses the election when information is not fully revealed. Hence the most profitable deviation is $s^J = s_1$ because J wins the election when information is fully revealed and the proposal is efficient. Candidate A prefers to deviate if and only if

$$\varepsilon > \frac{1}{4-2\beta}.$$

S_6 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_3)$. Given voters' beliefs, $v(s_2, s_3) = (A, B, \emptyset)$.

If J deviates and full information is not revealed, any voter who observes the deviation votes for $-J$ and J loses the election. For the same argument as above, the best deviation for an efficiency concerned candidate J is $s^J = s_1$. Candidate J prefers to deviate if and only if

$$\varepsilon > \frac{1}{4-2\beta}.$$

S_7 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_5)$. Given (s_2, s_5), $v(s_2, s_5) = (A, B, A)$. Ruled out by Lemma 4.

S_8 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_7)$. Given (s_2, s_7), $v(s_2, s_7) = (A, B, B)$. Ruled out by Lemma 4.

S_9 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_8)$. All three voters vote for A. Ruled out by Lemma 4.

S_{10} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_5)$. Given (s_5, s_5), all voters abstain. If candidate J deviates to $s^J = s_8$ and full information is not revealed, only voter c observes the deviation and $v(s_8, s_5) = (\emptyset, \emptyset, A)$. Hence by deviating candidate J wins with probability $1-\varepsilon$. Candidate J prefers to deviate if and only if

$$(1-\varepsilon)\frac{1}{1+3(1-\beta)} > \frac{1}{2}\frac{1}{1+2(1-\beta)}$$
$$\varepsilon < \frac{2+\beta}{6-2\beta}$$

Consider the deviation $s^J = s_1$. Candidate J only wins when information is fully revealed, and therefore the deviation is profitable if and only if

$$\varepsilon > \frac{1}{2}\frac{1}{1+2(1-\beta)}, \text{ or}$$
$$\varepsilon > \frac{1}{6-4\beta}$$

Since $\frac{1}{6-4\beta} < \frac{2+\beta}{6-2\beta}$ there is always a profitable deviation.

S_{11} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_6)$. Given (s_5, s_6), $v(s_5, s_6) = (\emptyset, A, B)$. If J deviates to $s^A \in \{s_1, s_2, s_3, s_4, s_6, s_7\}$ and full information is not revealed, J loses the election.

If J deviates to $s^J = s_8$ and full information is not revealed, the election is tied, but if full information is revealed, J loses the election. It follows that the most profitable deviation is $s^J = s_1$ since candidate J wins the election when information is fully revealed and the proposal is efficient. Candidate J prefers to deviate to s_1 if and only if

$$\varepsilon > \frac{1}{6 - 4\beta}.$$

S_{12} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_8)$. Given (s_5, s_8), $v(s_5, s_6) = (A, A, B)$. Ruled out by Lemma 4.

S_{13} : All voters abstain and the election is tied. If candidate J deviates and full information is not revealed, any voter who observes the deviation votes for $-J$ and J loses the election. If candidate J deviates $s^J = s_1$, she wins the election when information is fully revealed, and this proposal is efficient. Therefore candidate J prefers to deviate to s_1 if and only if

$$\varepsilon > \frac{1}{8 - 6\beta}. \tag{6}$$

■

7.1.2 Mixed strategy Equilibria

We look at the set of mixed equilibria when pure strategy equilibria do not exist. We know that if $\beta > \frac{2}{3}$ and $\varepsilon > \frac{3}{2} - \beta$. there exists no pure strategy equilibrium, and otherwise there exists at least one. Let $s_L = (0, 1/3, 1/3, 1/3, 0, 0, 0, 0)$ and $s_H = (0, 0, 0, 0, 1/3, 1/3, 1/3, 0)$ be the special mixed strategies that consist, respectively, on proposing exactly one project and randomizing which one, and proposing exactly two projects and randomizing which two. To characterize the set of mixed equilibria when candidates are efficiency concerned, is far from being trivial. The following two propositions provide a sufficiently detailed picture of the set of mixed equilibria in the area of parameters value (ε, β) where a pure strategy equilibrium does not exist.

Proposition 9 *If $\varepsilon \in (\frac{3}{2} - \beta, \frac{3}{4}\frac{2-\beta}{3-2\beta})$, candidate strategies (s_H, s_H) are supported in equilibrium. Conversely, in any symmetric equilibrium, $\sigma_5^J + \sigma_6^J + \sigma_7^J = 1$ for $J \in \{A, B\}$.*

Proof. First we prove that (s_H, s_H) can be supported in equilibrium. Given (s_H, s_H) is played, for any candidate J and voter i, if i does not observe the full proposals, beliefs are such that $\omega_1^{i,J}(1) = \omega_2^{i,J}(0) = 1$ and hence $s^i(1,0) = A$, $s^i(0,1) = B$ and $s^i(p_i^A, p_i^B) = \varnothing$ if $p_i^A = p_i^B$. Therefore, given that (s_H, s_H) is played as expected by voters, either all voters abstain if $p^A = p^B$ or one voter votes for A, one for B and one abstains if $p^A \neq p^B$; in either case the election is tied and the probability that each candidate is elected is $\frac{1}{2}$, so the expected payoff for each candidate is $\frac{1}{2}\frac{1}{3-2\beta}$.

Suppose A deviates to s_1, A loses $0-2$ if Nature does not reveal p, and A loses $1-2$ if Nature reveals p. Suppose A deviates to $s_k \in \{s_2, s_3, s_4\}$. If Nature reveals p, with probability $\frac{2}{3}$ A wins and with probability $\frac{1}{3}$ A loses; while if p is not revealed, A loses for sure. Hence, A wins with probability $\frac{2}{3}\varepsilon$, the expected utility deviating is $\frac{2}{3}\frac{\varepsilon}{2-\beta}$ and the deviation is profitable if and only if $\frac{2}{3}\frac{\varepsilon}{2-\beta} > \frac{1}{2}\frac{1}{3-2\beta}$, that is, $\varepsilon > \frac{3}{4}\frac{2-\beta}{3-2\beta}$. Suppose A deviates to $s_k \in \{s_5, s_6, s_7\}$. Then A achieves the same expected electoral outcomes and utilities as not deviating. Suppose A deviates to s_8. If Nature reveals p, A loses $1-2$. If Nature does not reveal p, then A wins $2-1$. But Nature reveals p with probability ε, hence the expected utility for A is $(1-\varepsilon)\frac{1}{4-3\beta}$ which is less than $\frac{1}{2}\frac{1}{3-2\beta}$ for any $\varepsilon > \frac{1}{2}$. Hence, there is no profitable deviation.

Second, we prove that in any symmetric equilibrium both candidates propose two projects.

Suppose (σ^J, σ^J) is part of a symmetric equilibrium. Since the equilibrium is symmetric, for any $i \in \{a, b, c\}$, if i does not observe p, then $s^i(p_i^A, p_i^B) = \varnothing$ if $p_i^A = p_i^B$ and furthermore, for $k \in \{0, 1\}$, $s^i(k, 1-k) = A$ if and only if $s^i(1-k, k) = B$. Suppose $s^i(1,0) \neq A$, so $s^i(0,1) \neq B$. Then s_8 is not a best response and it is not played in equilibrium. But if s_8 is not played, the expected payoff for i if A wins is strictly higher than if B wins given $(p_i^A, p_i^B) = (1,0)$, so by assumption $s^i(1,0) = A$, a contradiction. Thus, it must be $s^i(1,0) = A$ and $s^i(0,1) = B$. Then, given that $\varepsilon \in (\frac{3}{2} - \beta, \frac{3}{4}\frac{2-\beta}{3-2\beta})$, for any strategy σ^{-J}, a best response by J must propose two projects. Thus no strategy that proposes any other number of projects can be used in a symmetric equilibrium. ■

Claim 10 *There exists an increasing function $\varepsilon(\beta)$ such that*

if $\varepsilon \in \left(\max\left\{\frac{3}{2} - \beta, \frac{3}{4}\frac{2-\beta}{3-2\beta}\right\}, \varepsilon(\beta)\right)$, there is a unique symmetric mixed strategy equilibrium, in which $\sigma^J = (\alpha_1, \alpha_1, \alpha_1, \alpha_2, \alpha_2, \alpha_2, 1-3\alpha_1-3\alpha_2)$ and the expected number of projects *is* $\frac{18\varepsilon-6}{10\varepsilon-3}$; *and*

if $\varepsilon \in (\varepsilon(\beta), 1)$, *there is a unique symmetric mixed strategy equilibrium, in which* $\sigma^J = (0, \beta_1, \beta_1, \beta_1, \beta_2, \beta_2, \beta_2, 1-3\beta_1-3\beta_2)$ *and the expected number of projects is* $\frac{12\varepsilon-3}{10\varepsilon-3} \geq \frac{9}{7}$ *and which converges to* $\frac{9}{7}$ *as* $\varepsilon \to 1$.

We find the exact functional form of $\varepsilon(\beta)$ and the weights of the mixed strategies as part of the proof.

Proof. Let (σ^A, σ^B) be a symmetric candidate strategy profile so $\sigma^A = \sigma^B$. Since the candidates' strategies are symmetric, voters' strategies must be such that $s^i(k,k) = \varnothing$ for $k \in \{0,1\}$, and $\{s^i(1,0) = A$ and $s^i(0,1) = B\}$ or $\{s^i(1,0) = B$ and $s^i(0,1) = A\}$ or $\{s^i(1,0) = \varnothing$ and $s^i(0,1) = \varnothing\}$ for every voter i. Suppose not $\{s^i(1,0) = A$ and $s^i(0,1) = B\}$. Then given any strategy σ^{-J}, candidate J obtains a greater expected payoff playing s_1 than playing s_8, and a strictly greater payoff if $\sigma_8^{-J} > 0$. Therefore, in a symmetric mixed equilibrium, $\sigma_8^J = 0$. Then, it follows that for any voter i who observes $p_i^J = 1$ and $p_i^{-J} = 0$, the expected payoff for voter i is greater if J wins, thus by assumption, i votes J. Therefore, $s^i(1,0) = A$ and $s^i(0,1) = B$.

Next we prove that in any symmetric mixed strategy equilibrium, $\sigma_2^{-J} + \sigma_3^{-J} + \sigma_4^{-J} > 0$. Suppose not. Notice that given $\varepsilon > \frac{3}{4}$ and $s^i(1,0) = A$ and $s^i(0,1) = B$, if candidate $-J$ proposes one project and candidate J proposes two, in expectation J wins the election more often, whereas if J proposes two and $-J$ proposes zero or three, J wins more often. So if candidate $-J$ never proposes one project, proposing two projects in expectation defeats any other proposal with probability more than one half. Then, any best response by candidate J to σ^{-J} with $\sigma_2^{-J} + \sigma_3^{-J} + \sigma_4^{-J} = 0$ must be such that $\sigma_5^J + \sigma_6^J + \sigma_7^J = 1$, which in turns means that any best response by J implies $\sigma_2^J + \sigma_3^J + \sigma_4^J = 1$, a contradiction.

Similarly, in any symmetric mixed strategy equilibrium, $\sigma_5^{-J} + \sigma_6^{-J} + \sigma_7^{-J} > 0$. Suppose not. Any best response by J must be such that $\sigma_2^J + \sigma_3^J + \sigma_4^J = 0$, in which in turns implies that the best response by $-J$ is $\sigma_5^{-J} + \sigma_6^{-J} + \sigma_7^{-J} = 1$.

Therefore, in any symmetric mixed strategy equilibrium, both candidates propose one project, and two projects, with positive probability. But then, it must be that $\sigma_2^J = \sigma_3^J + \sigma_4^J$ and $\sigma_5^J = \sigma_6^J = \sigma_7^J$. Given that the randomization among districts (subject to choosing a number of projects) assigns equal weight to all districts, we can reduce the strategic problem to that of assigning weights to strategies s_1, s_8, s_L, s_H. The payoff matrix is as follows:

$$\left\{\begin{array}{ccccc} & s_1 & s_L & s_H & s_8 \\ s_1 & \frac{1}{2},\frac{1}{2} & \varepsilon, 1-\varepsilon & 0, \frac{1}{3-2\beta} & \varepsilon, \frac{1-\varepsilon}{4-3\beta} \\ s_L & \frac{1-\varepsilon}{2-\beta}, \varepsilon & \frac{1}{2(2-\beta)}, \frac{1}{2(2-\beta)} & \frac{2\varepsilon}{3(2-\beta)}, \frac{3-2\varepsilon}{3(3-2\beta)} & 0, \frac{1}{4-3\beta} \\ s_H & \frac{1}{3-2\beta}, 0 & \frac{3-2\varepsilon}{3(3-2\beta)}, \frac{2\varepsilon}{3(2-\beta)} & \frac{1}{2(3-2\beta)}, \frac{1}{2(3-2\beta)} & \frac{\varepsilon}{3-2\beta}, \frac{1-\varepsilon}{4-3\beta} \\ s_8 & \frac{1-\varepsilon}{4-3\beta}, \varepsilon & \frac{1}{4-3\beta}, 0 & \frac{1-\varepsilon}{4-3\beta}, \frac{\varepsilon}{3-2\beta} & \frac{1}{2(4-3\beta)}, \frac{1}{2(4-3\beta)} \end{array}\right\}$$

Dropping the superindex (so as to use the Maple feature of Scientific Workplace to solve the equations), a symmetric equilibrium strategy with $\sigma_1^J > 0, \sigma_L^J > 0, \sigma_H^J > 0$ and $\sigma_8^J = 0$ must satisfy

i) $\frac{1}{2}\sigma_1 + \varepsilon\sigma_L = \frac{1-\varepsilon}{2-\beta}\sigma_1 + \frac{1}{2(2-\beta)}\sigma_L + \frac{2\varepsilon}{3(2-\beta)}(1-\sigma_1-\sigma_L)$

ii) $\frac{1}{2}\sigma_1 + \varepsilon\sigma_L = \frac{1}{3-2\beta}\sigma_1 + \frac{3-2\varepsilon}{3(3-2\beta)}\sigma_L + \frac{1}{2(3-2\beta)}(1-\sigma_1-\sigma_L)$

iii) $\frac{1}{2}\sigma_1 + \varepsilon\sigma_L \geq \frac{1-\varepsilon}{4-3\beta}\sigma_1 + \frac{1}{4-3\beta}\sigma_L + \frac{1-\varepsilon}{4-3\beta}(1-\sigma_1-\sigma_L)$.

Solving we obtain

$$\sigma_1 = \frac{4\varepsilon + (3-16\varepsilon+6\beta\varepsilon)\sigma_L}{10\varepsilon - 3\beta}$$

and

$$\sigma_L = \frac{3-6(1-\beta)\sigma_1}{22\varepsilon - 12\beta\varepsilon - 3} =$$

$$\sigma_L = \frac{3-6(1-\beta)\frac{4\varepsilon+(3-16\varepsilon+6\beta\varepsilon)\sigma_L}{10\varepsilon-3\beta}}{22\varepsilon-12\beta\varepsilon-3}$$

$$\sigma_L = -\frac{-9\beta+6\varepsilon+24\beta\varepsilon}{9\beta+126\varepsilon-220\varepsilon^2-66\beta\varepsilon+120\beta\varepsilon^2-18}$$

which as $\beta \to 1$ converges to $\sigma_L = -\frac{-9+6\varepsilon+24\varepsilon}{60\varepsilon-100\varepsilon^2-9} = \frac{3(10\varepsilon-3)}{(10\varepsilon-3)^2} = \frac{3}{10\varepsilon-3}$ as it ought to.

Also,

$$\begin{aligned} \sigma_1 &= \frac{4\varepsilon - (3-16\varepsilon+6\beta\varepsilon)\frac{-9\beta+6\varepsilon+24\beta\varepsilon}{9\beta+126\varepsilon-220\varepsilon^2-66\beta\varepsilon+120\beta\varepsilon^2-18}}{10\varepsilon-3\beta} \\ \sigma_1 &= -\frac{88\varepsilon^2-60\varepsilon+18\beta\varepsilon-48\beta\varepsilon^2+9}{9\beta+126\varepsilon-220\varepsilon^2-66\beta\varepsilon+120\beta\varepsilon^2-18} \end{aligned}$$

which as $\beta \to 1$ converges to $\sigma_1 = \frac{4\varepsilon-3}{10\varepsilon-3}$ as it ought to.

Then simplifying inequality *iii*),

$$\frac{1}{2}\sigma_1 + \varepsilon\sigma_L \geq \frac{1-\varepsilon}{4-3\beta}\sigma_1 + \frac{1}{4-3\beta}\sigma_L + \frac{1-\varepsilon}{4-3\beta}(1-\sigma_1-\sigma_L)$$
$$\sigma_1 \geq \frac{6\varepsilon(1-\beta)\sigma_L - 2(1-\varepsilon)}{3\beta-4}$$
$$-\frac{88\varepsilon^2-60\varepsilon+18\beta\varepsilon-48\beta\varepsilon^2+9}{9\beta+126\varepsilon-220\varepsilon^2-66\beta\varepsilon+120\beta\varepsilon^2-18} \geq \frac{-6\varepsilon(1-\beta)\frac{-9\beta+6\varepsilon+24\beta\varepsilon}{9\beta+126\varepsilon-220\varepsilon^2-66\beta\varepsilon+120\beta\varepsilon^2-18} - 2(1-\varepsilon)}{3\beta-4}$$

$$\frac{6\varepsilon(1-\beta)\frac{-9\beta+6\varepsilon+24\beta\varepsilon}{9\beta+126\varepsilon-220\varepsilon^2-66\beta\varepsilon+120\beta\varepsilon^2-18} + 2(1-\varepsilon)}{3\beta-4} - \frac{88\varepsilon^2-60\varepsilon+18\beta\varepsilon-48\beta\varepsilon^2+9}{9\beta+126\varepsilon-220\varepsilon^2-66\beta\varepsilon+120\beta\varepsilon^2-18} = 0$$
$$\frac{48\varepsilon-9\beta-304\varepsilon^2+440\varepsilon^3+48\beta\varepsilon+24\beta\varepsilon^2-240\beta\varepsilon^3}{(3\beta-4)(10\varepsilon-3)(3\beta+22\varepsilon-12\beta\varepsilon-6)} = 0$$
$$48\varepsilon-9\beta-304\varepsilon^2+440\varepsilon^3+48\beta\varepsilon+24\beta\varepsilon^2-240\beta\varepsilon^3 = 0$$

The solution, solved by Mathematica, is a cumbersome expression that simplifies to the desired $\varepsilon > \frac{11+\sqrt{61}}{20}$ for $\beta = 1$.

The probability of proposing two projects is

$$1 + \frac{-9\beta+6\varepsilon+24\beta\varepsilon+88\varepsilon^2-60\varepsilon+18\beta\varepsilon-48\beta\varepsilon^2+9}{9\beta+126\varepsilon-220\varepsilon^2-66\beta\varepsilon+120\beta\varepsilon^2-18} = \frac{3\left(44\varepsilon^2-24\varepsilon+8\beta\varepsilon-24\beta\varepsilon^2+3\right)}{(10\varepsilon-3)(3\beta+22\varepsilon-12\beta\varepsilon-6)}$$

and the expected number of projects is $\frac{6\left(44\varepsilon^2-24\varepsilon+8\beta\varepsilon-24\beta\varepsilon^2+3\right)}{(10\varepsilon-3)(3\beta+22\varepsilon-12\beta\varepsilon-6)} - \frac{-9\beta+6\varepsilon+24\beta\varepsilon}{9\beta+126\varepsilon-220\varepsilon^2-66\beta\varepsilon+120\beta\varepsilon^2-18} = \frac{1}{10\varepsilon-3}(12\varepsilon-3) = \frac{12\varepsilon-3}{10\varepsilon-3}$, which converges to $\frac{9}{7}$ as ε converges to 1. The initial assumption that voters vote $s^i(1,0) = A$ and $s^i(0,1) = B$ is supported because $\sigma_8 = 0$ and $\beta > 2/3$.

If instead ε is below the cutoffs, candidates mix between proposing one, two and three projects. An equilibrium with these characteristics requires:

$$\left\{\begin{array}{ccccc} & s_1 & s_L & s_H & s_8 \\ s_1 & \frac{1}{2}, \frac{1}{2} & \varepsilon, 1-\varepsilon & 0, \frac{1}{3-2\beta} & \varepsilon, \frac{1-\varepsilon}{4-3\beta} \\ s_L & \frac{1-\varepsilon}{2-\beta}, \varepsilon & \frac{1}{2(2-\beta)}, \frac{1}{2(2-\beta)} & \frac{2\varepsilon}{3(2-\beta)}, \frac{3-2\varepsilon}{3(3-2\beta)} & 0, \frac{1}{4-3\beta} \\ s_H & \frac{1}{3-2\beta}, 0 & \frac{3-2\varepsilon}{3(3-2\beta)}, \frac{2\varepsilon}{3(2-\beta)} & \frac{1}{2(3-2\beta)}, \frac{1}{2(3-2\beta)} & \frac{\varepsilon}{3-2\beta}, \frac{1-\varepsilon}{4-3\beta} \\ s_8 & \frac{1-\varepsilon}{4-3\beta}, \varepsilon & \frac{1}{4-3\beta}, 0 & \frac{1-\varepsilon}{4-3\beta}, \frac{\varepsilon}{3-2\beta} & \frac{1}{2(4-3\beta)}, \frac{1}{2(4-3\beta)} \end{array}\right\}$$

$i)\ \varepsilon\sigma_L + \varepsilon(1-\sigma_L-\sigma_H) \leq \frac{1}{2(2-\beta)}\sigma_L + \frac{2\varepsilon}{3(2-\beta)}\sigma_H$

$ii)\ \frac{1}{2(2-\beta)}\sigma_L + \frac{2\varepsilon}{3(2-\beta)}\sigma_H = \frac{3-2\varepsilon}{3(3-2\beta)}\sigma_L + \frac{1}{2(3-2\beta)}\sigma_H + \frac{\varepsilon}{3-2\beta}(1-\sigma_L-\sigma_H)$

$iii)\frac{1}{2(2-\beta)}\sigma_L + \frac{2\varepsilon}{3(2-\beta)}\sigma_H = \frac{1}{4-3\beta}\sigma_L + \frac{1-\varepsilon}{4-3\beta}\sigma_H + \frac{1}{2(4-3\beta)}(1-\sigma_L-\sigma_H).$

Simplifying the first inequality expressions, we get $\varepsilon \leq \frac{1}{2(2-\beta)}\sigma_L + \frac{(8-3\beta)}{3(2-\beta)}\varepsilon\sigma_H$.

From ii)

$$\sigma_L = \frac{12\varepsilon - 6\beta\varepsilon + (14\beta\varepsilon + 6 - 3\beta - 24\varepsilon)\sigma_H}{20\varepsilon - 10\beta\varepsilon - 3}$$

From iii)

$$\sigma_H = -\frac{(3\beta + 6\sigma_L - 6\beta\sigma_L - 6)}{3\beta + 28\varepsilon - 18\beta\varepsilon - 6}$$

So,

$$\begin{aligned}
\sigma_L &= \frac{12\varepsilon - 6\beta\varepsilon - (14\beta\varepsilon + 6 - 3\beta - 24\varepsilon)\frac{(3\beta+6\sigma_L-6\beta\sigma_L-6)}{3\beta+28\varepsilon-18\beta\varepsilon-6}}{20\varepsilon - 10\beta\varepsilon - 3} \\
\sigma_L &= \frac{9\beta + 108\varepsilon - 168\varepsilon^2 - 60\beta\varepsilon + 108\beta\varepsilon^2 - 18}{18\beta + 174\varepsilon - 280\varepsilon^2 - 114\beta\varepsilon + 180\beta\varepsilon^2 - 27}
\end{aligned}$$

which simplifies to

$$\sigma_L = \frac{9 + 108\varepsilon - 168\varepsilon^2 - 60\varepsilon + 108\varepsilon^2 - 18}{18 + 174\varepsilon - 280\varepsilon^2 - 114\varepsilon + 180\varepsilon^2 - 27} = \frac{6\varepsilon - 3}{10\varepsilon - 3}$$

when $\beta = 1$ as desired. Also,

$$\begin{aligned}
\sigma_H &= -\frac{3\beta - 6 + 6(1-\beta)\sigma_L}{3\beta + 28\varepsilon - 18\beta\varepsilon - 6} = -\frac{3\beta - 6 + 6(1-\beta)\frac{9\beta+108\varepsilon-168\varepsilon^2-60\beta\varepsilon+108\beta\varepsilon^2-18}{18\beta+174\varepsilon-280\varepsilon^2-114\beta\varepsilon+180\beta\varepsilon^2-27}}{3\beta + 28\varepsilon - 18\beta\varepsilon - 6} \\
\sigma_H &= \frac{1}{10\varepsilon - 3}\frac{24\varepsilon + 6\beta\varepsilon - 9}{6\beta + 28\varepsilon - 18\beta\varepsilon - 9}
\end{aligned}$$

which simplifies to

$$\frac{1}{10\varepsilon - 3}\frac{24\varepsilon + 6\varepsilon - 9}{6 + 28\varepsilon - 18\varepsilon - 9} = \frac{3}{10\varepsilon - 3}$$

and, the expected number of projects in this equilibrium is:

$$\begin{aligned}
& 2\frac{1}{10\varepsilon-3}\frac{24\varepsilon+6\beta\varepsilon-9}{6\beta+28\varepsilon-18\beta\varepsilon-9}+\frac{9\beta+108\varepsilon-168\varepsilon^2-60\beta\varepsilon+108\beta\varepsilon^2-18}{18\beta+174\varepsilon-280\varepsilon^2-114\beta\varepsilon+180\beta\varepsilon^2-27} \\
& +3(1-\frac{24\varepsilon+6\beta\varepsilon-9-(9\beta+108\varepsilon-168\varepsilon^2-60\beta\varepsilon+108\beta\varepsilon^2-18)}{(10\varepsilon-3)(6\beta+28\varepsilon-18\beta\varepsilon-9)}) \\
= \; & \frac{18\varepsilon-6}{10\varepsilon-3}.
\end{aligned}$$

■

7.2 One Office Motivated and one Efficiency Concerned candidate

We characterize the set of pure equilibria when one candidate is office motivated and the other one is efficiency concerned.

Proposition 11 *If projects are not very inefficient* $\beta \in (\frac{2}{3}, 1)$*, there exist a cutoff function* $\varepsilon_1(\beta)$ *such* $0 < \varepsilon_1(\beta) < \frac{1}{2}$ *and:*

If $\varepsilon \in [0, \varepsilon_1(\beta)]$*, there exist multiple pure equilibria;*

If $\varepsilon \in (\varepsilon_1(\beta), \frac{1}{2})$*, there is a unique pure strategy equilibrium in which both candidates propose the efficient policy; and*

If $\varepsilon > \frac{1}{2}$ *there is no pure strategy equilibrium.*

If projects are very inefficient $\beta \in (\frac{1}{3}, \frac{2}{3})$*, there exists a cutoff function* $\varepsilon_1(\beta)$ *such* $0 < \varepsilon_1(\beta) < \frac{1}{2}$ *and:*

If $\varepsilon \in [0, \varepsilon_1(\beta)]$*, there exist multiple pure equilibria;*

If $\varepsilon \in (\varepsilon_1(\beta), 1]$ *in the unique pure equilibrium both candidates propose the efficient policy.*

Proof. Without loss of generality, let assume that candidate A is office motivated. We prove the high benefit case first.

S_1 : Voter strategy $s^i = (\emptyset, A, B, \emptyset)$ for each voter i and beliefs such that $\omega_0^{i,J}(0) = 1$ and $\omega_2^{i,J}(1) = 1$ for any voter i and any candidate J make the election tied and if candidate J deviates to any $s^J \neq s_1$, then J loses the election. It is also straightforward to check that the voting strategy is a best response given the strategy of the candidates and the beliefs of the voters, and that the beliefs are correct along the equilibrium path, so these strategies and beliefs are an equilibrium. If $\varepsilon \leq \frac{1}{2}$, no hypothetical gain when full information is revealed can compensate for this loss. If a candidate is efficiency concerned, deviations give her a lower payoff so there is not incentive to deviate. Suppose now that $\varepsilon > \frac{1}{2}$. By assumption candidate A is office motivated. If A deviates to $s^A = s_5$ she wins the election with probability ε (when full information is revealed) and therefore such deviation is profitable for the office motivated candidate.

Candidate A is purely office motivated and candidate B is efficiency concerned. The set S_2 contains profiles which are not strategically equivalent because candidate B is not indifferent, for instance, between winning with proposals s_1 or s_2. We must partition the set S_2 into the following subsets of profiles which are equivalent for both players, $S_2' = \{(s_1, s_2), (s_1, s_3), (s_1, s_4), \}$ and $S_2'' = \{(s_2, s_1), (s_3, s_1), (s_4, s_1)\}$

S_2' : Assume without loss of generality that $(s^A, s^B) = (s_1, s_2)$. Given (s_1, s_2), $v(s_1, s_2) = (B, A, A)$. Ruled out by Lemma 4.

S_2'' = Assume without loss of generality that $(s^A, s^B) = (s_2, s_1)$. Given (s_2, s_1), $v(s_2, s_1) = (A, B, B)$. Ruled out by Lemma 4.

Similary as before we partition S_3 in two subsets $S_3' = \{(s_1, s_5), (s_1, s_6), (s_1, s_7)\}$ and $S_3'' = \{(s_5, s_1), (s_6, s_1), (s_7, s_1)\}$.

S_3' : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_5)$. Given (s_1, s_5), $v(s_1, s_5) = (B, B, A)$. Ruled out by Lemma 4.

S_3'' : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_1)$. Given (s_5, s_1), $v(s_5, s_1) = (A, A, B)$. Ruled out by Lemma 4.

(s_1, s_8) : Every voter votes for A. Ruled out by Lemma 4.

(s_8, s_1) : Every voter votes for B. Ruled out by Lemma 4.

S_5 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_2)$. Given (s_2, s_2), every voter i abstains. If candidate A deviates to $s^J = s_8$ wins the election both in case the information is revealed and in case it is not.

S_6 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_3)$. Given (s_2, s_3), $v(s_2, s_3) = (A, B, \emptyset)$. If A deviates to $s^A = s_6$ both in case full information is revealed and in case it not revealed, A

wins the election. Hence the deviation is profitable for all $\varepsilon \geq 0$.

Let $S_7' = \{(s_2, s_5), (s_2, s_6), (s_3, s_5), (s_3, s_7), (s_4, s_6), (s_4, s_7), \}$

and $S_7'' = \{(s_5, s_2), (s_6, s_2), (s_5, s_3), (s_7, s_3), (s_6, s_4), (s_7, s_4)\}$.

S_7' : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_5)$. Given (s_2, s_5), $v(s_2, s_5) = (A, B, A)$. Ruled out by Lemma 4.

S_7'' : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_2)$. Given (s_5, s_2), $v(s_5, s_2) = (B, A, B)$. Ruled out by Lemma 4.

Let $S_8' = \{(s_2, s_7), (s_3, s_6), (s_4, s_5), \}$ and $S_8'' = \{(s_7, s_2), (s_6, s_3), (s_5, s_4)\}$

S_8' : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_7)$. Given (s_2, s_7), $v(s_2, s_7) = (A, B, B)$. Ruled out by Lemma 4.

S_8'' : Assume w.l.o.g. that $(s^A, s^B) = (s_7, s_2)$. Given (s_7, s_2), $v(s_7, s_2) = (B, A, A)$. Ruled out by Lemma 4.

Let $S_9' = \{(s_2, s_8), (s_3, s_8), (s_4, s_8)\}$ and $S_9'' = \{(s_8, s_2), (s_8, s_3), (s_8, s_4)\}$;

S_9' : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_8)$. Given (s_2, s_8), $v(s_2, s_8) = (A, B, B)$. Ruled out by Lemma 4.

S_9'' : Assume w.l.o.g. that $(s^A, s^B) = (s_8, s_2)$. Given (s_8, s_2), $v(s_8, s_2) = (B, B, A)$. Ruled out by Lemma 4.

S_{10} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_5)$. Given (s_5, s_5), all voters abstain. Suppose first that $\varepsilon < \frac{1}{2}$. If candidate A deviates to $s^A = s_8$ and full information is not revealed, only voter c observes the deviation and $v(s_8, s_5) = (\emptyset, \emptyset, A)$. Hence by deviating candidate A wins with probability at least $1 - \varepsilon > \frac{1}{2}$. Suppose that $\varepsilon > \frac{1}{2}$. If A deviates to $s^A = s_2$ then A wins the election when information is fully revealed. Hence the deviation is profitable.

S_{11} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_6)$. Suppose first that $\varepsilon \leq \frac{1}{2}$. Given $(s^A, s^B) = (s_5, s_6)$, beliefs such that $\omega_2^{i,J}(1 - s_i^J) = 1$ for each $i \in \{a, b, c\}$ and $J \in \{A, B\}$ support an equilibrium in which $v(s_5, s_6) = (\emptyset, A, B)$ and each candidate wins with equal probability. Consider first the office motivated candidate A. It suffices to check that A has no incentives to deviate. If A deviates to $s^A \in \{s_1, s_2, s_3, s_4, s_6, s_7\}$ and full information is not revealed, A loses the election. If A deviates to $s^A = s_8$ and full information is not revealed, the election is tied, but if full information is revealed, A loses the election. In any case, after a deviation A wins the election with probability less than $\frac{1}{2}$. Consider now candidate B who, by assumption, is efficiency concerned. Deviating to $s^B = s^8$ is clearly unprofitable. Playing any other deviation candidate B loses the election when information is not fully revealed. So the best deviation is $s^B = s_2$ because it minimizes the inefficiency and candidate B wins the election when information is fully revealed.(playing $s^B = s_4$ gives the same payoff as s_2 when information is fully revealed, but if B plays s_4 she looses 3-0 when information is not fully revealed while if B plays s_1 she looses $2-1$) Candidate B prefers to deviate to s_2 if and only if

$$\varepsilon \frac{1}{1+(1-\beta)} > \frac{1}{2}\frac{1}{1+2(1-\beta)}, \text{ or} \tag{7}$$

$$\varepsilon > \frac{2-\beta}{6-4\beta} \tag{8}$$

Hence there is a profitable deviation for candidate B if the previous condition holds. Suppose now that $\varepsilon > \frac{1}{2}$. If candidate A deviates to $s^A = s_2$, then A wins the election when full information is revealed. Hence the deviation is profitable.

Let $S'_{12} = \{(s_5, s_8), (s_6, s_8), (s_7, s_8)\}$ and $S''_{12} = \{(s_8, s_5), (s_8, s_6), (s_8, s_7)\}$;

S'_{12} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_8)$. Given (s_5, s_8), $v(s_5, s_8) = (A, A, B)$. Ruled out by Lemma 4.

S''_{12} : Assume w.l.o.g. that $(s^A, s^B) = (s_8, s_5)$. Given (s_8, s_5), $v(s_8, s_5) = (A, B, B)$. Ruled out by Lemma 4.

S_{13} : Let voter strategy $s^i = (\emptyset, B, A, \emptyset)$ for each voter i and beliefs such that $\omega_2^{i,J}(0) = \omega_2^{i,J}(1) = 1$ for any voter i. Suppose first that $\varepsilon \leq \frac{1}{2}$. In equilibrium the election is tied; consider the office motivated candidate A. If candidate A deviates to any strategy $s^A \neq s_8$ and full information is not revealed, A loses the election. Consider the efficiency concerned candidate B. If information is not fully revealed, candidate B looses the election if she deviates. If information is fully revealed, the unique profitable deviation is $s^B = s_1$. Candidate B prefers to deviate if and only if

$$\varepsilon > \frac{1}{8-6\beta} \tag{9}$$

Hence there is a profitable deviation for B if the previous condition holds. Suppose that $\varepsilon > \frac{1}{2}$. If candidate A deviates to $s^A = s_0$, then A wins the election when full information is revealed. Hence the deviation is profitable.

Next we prove the low benefit case. To sustain equilibria, assume that off-equilibrium path beliefs such that given the equilibrium proposal s_i^J, $\omega_2^{i,J}(1 - s_i^J) = 1$ for each $i \in \{a, b, c\}$ and $J \in \{A, B\}$. That is, a voter who observe a deviation believes that the deviating candidate proposes to carry out the projects in the other two districts.

S_1 : Voter strategy $s^i = (\emptyset, A, B, \emptyset)$ for each voter i and beliefs such that $\omega_0^{i,J}(0) = 1$ and $\omega_2^{i,J}(1) = 1$ for any voter i and any candidate J make the election tied. There is no a different proposal that can defeat s_1 both in case the information is revealed and in case it is not. The efficiency concerned candidate has lower incentive to deviate because s_1 is the efficient proposal. It is also straightforward to check that the voting strategy is a best response given the strategy of the candidates and the beliefs of the voters, and that the beliefs are correct along the equilibrium path, so these strategies and beliefs are an equilibrium. Hence no candidate has profitable deviations for all $\varepsilon \in [0, 1]$.

S_2' : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_2)$. Given (s_1, s_2), $v(s_1, s_2) = (B, A, A)$. Ruled out by Lemma 4.

$S_2'' =$ Assume without loss of generality that $(s^A, s^B) = (s_2, s_1)$. Given (s_2, s_1), $v(s_2, s_1) = (A, B, B)$. Ruled out by Lemma 4.

S_3' : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_5)$. Given (s_1, s_5), $v(s_1, s_5) = (A, A, A)$. Ruled out by Lemma 4.

S_3'' : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_1)$. Given (s_5, s_1), $v(s_5, s_1) = (B, B, B)$. Ruled out by Lemma 4.

(s_1, s_8) : Every voter votes for A. Ruled out by Lemma 4.

(s_8, s_1) : Every voter votes for B. Ruled out by Lemma 4

S_5 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_2)$. Suppose first that $\varepsilon \leq \frac{1}{2}$. Given (s_2, s_2), every voter abstains. If the office motivated candidate A deviates and full information is not revealed, any voter who observes the deviation votes for B and A loses the election. Consider candidate B. If candidate B deviates, B looses the election when information is not fully revealed. When information is revealed the most profitable deviation for candidate B is $s^B = s_1$, since B wins the election and the proposal is efficient. The deviation $s^B = s_1$ is profitable if and only if

$$\varepsilon > \frac{1}{4 - 2\beta}.$$

Suppose now $\varepsilon > \frac{1}{2}$. If candidate A deviates to $s^A = s_1$, A wins the election when full information is revealed. Hence the deviation is profitable.

S_6 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_3)$. Suppose $\varepsilon \leq \frac{1}{2}$. Given (s_2, s_3), $v(s_2, s_3) = (A, B, \emptyset)$. If the office motivated candidate A deviates and full information is not revealed, any voter who observes the deviation votes for B and A loses the election. Any deviation makes the candidate looses the election when information is not fully revealed. When information is revealed the most profitable deviation for candidate B is $s^B = s_1$, since she wins the election and the proposal is efficient. The deviation $s^B = s_1$ is profitable if and only if

$$\varepsilon > \frac{1}{4 - 2\beta}.$$

Suppose now $\varepsilon > \frac{1}{2}$. If candidate A deviates to $s^A = s_1$, A wins the election when full information is revealed. Hence the deviation is profitable.

S_7' : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_5)$. Given (s_2, s_5), $v(s_2, s_5) = (A, B, A)$. Ruled out by Lemma 4.

S_7'' : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_2)$. Given (s_5, s_2), $v(s_5, s_2) = (B, A, B)$. Ruled out by Lemma 4.

S_8' : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_7)$. Given (s_2, s_7), $v(s_2, s_7) = (A, B, B)$. Ruled out by Lemma 4.

S_8'' : Assume w.l.o.g. that $(s^A, s^B) = (s_7, s_2)$. Given (s_7, s_2), $v(s_7, s_2) = (B, A, A)$. Ruled out by Lemma 4.

S_9' : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_8)$. All three voters vote for A. Ruled out by Lemma 4.

S_9'' : Assume w.l.o.g. that $(s^A, s^B) = (s_8, s_2)$. Given (s_8, s_2), $v(s_8, s_2) = (B, B, B)$. Ruled out by Lemma 4.

S_{10} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_5)$. Suppose first that $\varepsilon \leq \frac{1}{2}$. Given (s_5, s_5), all voters abstain. If the office motivated candidate A deviates to $s^A = s_8$ and full information is not revealed, only voter c observes the deviation and $v(s_8, s_5) = (\emptyset, \emptyset, A)$. Hence by deviating candidate A wins with probability at least $1 - \varepsilon > \frac{1}{2}$. Suppose now $\varepsilon > \frac{1}{2}$. If candidate A deviates to $s^A = s_1$, A wins the election when full information is revealed. Hence the deviation is profitable.

S_{11} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_6)$. Suppose first that $\varepsilon \leq \frac{1}{2}$. Given (s_5, s_6), $v(s_5, s_6) = (\emptyset, A, B)$. The office motivated candidate A has no incentives to deviate. If A deviates to $s^A \in \{s_1, s_2, s_3, s_4, s_6, s_7\}$ and full information is not revealed, A loses the election. If A deviates to $s^A = s_8$ and full information is not revealed, the election is tied, but if full information is revealed, A loses the election. In any case, after a deviation A wins the election with probability less than $\frac{1}{2}$. Consider candidate B who is efficiency concerned. Deviating to $s^B = s^8$ is clearly unprofitable. By deviating candidate B loses the election when information is not fully revealed. So the best deviation is $s^B = s_1$ because it minimizes the inefficiency and candidate B wins the election when information is fully revealed. Candidate B prefers to deviate to s_1 if and only if

$$\varepsilon > \frac{1}{6 - 4\beta}$$

Suppose now $\varepsilon > \frac{1}{2}$. If candidate A deviates to $s^A = s_1$, A wins the election when full information is revealed. Hence the deviation is profitable

S'_{12} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_8)$. Given (s_5, s_8), $v(s_5, s_6) = (A, A, B)$. Ruled out by Lemma 4.

S''_{12} : Assume w.l.o.g. that $(s^A, s^B) = (s_8, s_5)$. Given (s_8, s_5), $v(s_8, s_5) = (A, B, B)$. Ruled out by Lemma 4.

S_{13} : All voters abstain and the election is tied. Suppose $\varepsilon \leq \frac{1}{2}$. If the office motivated candidate A deviates and full information is not revealed, any voter who observes the deviation votes for B and A loses the election. Consider candidate B who is efficiency concerned. By deviating candidate B loses the election when information is not fully revealed. So the best deviation is $s^B = s_1$ because it minimizes the inefficiency and candidate B wins the election when information is fully revealed. Candidate B prefers to deviate to $s^B = s_1$ if and only if

$$\varepsilon > \frac{1}{8-6\beta} \tag{10}$$

Hence there is a profitable deviation for candidate B if the previous condition holds. Suppose now $\varepsilon > \frac{1}{2}$. If candidate A deviates to $s^A = s_1$, A wins the election when full information is revealed. Hence the deviation is profitable. ■

The proofs of Proposition 3 and of Theorem 2 follow by the previous results. To conclude the proof of Proposition 1 we show in the following lemma that there is no equilibrium in mixed strategies when $\beta \in (\frac{1}{3}, \frac{2}{3})$ and $\varepsilon > \frac{1}{2}$.

Lemma 12 *For any $(\alpha_i, \alpha_j) \in \{0,1\}^2$ and any $\beta \in (\frac{1}{3}, \frac{2}{3})$, if $\varepsilon > \frac{1}{2}$ there is no equilibrium in which a candidate proposes to provide a public good with positive probability.*

Proof. Suppose candidate J plays with positive probability any strategy different than s_1. If candidate $-J$ plays s_1, then candidate J looses the election when full information is revealed. If candidate $-J$ is playing any strategy $s_k^{-J} \neq s_1^{-J}$ then candidate J wins with probability one by playing s_1 when full information is revealed. Therefore if candidate J replace in the mixed strategy any $s_k^J \neq s_1^J$ with strategy s^J increases her probability of winning the election. ■

7.3 Equilibria with $\varepsilon = 0$

In this subsection we characterize the set of Bayesian equilibria when $\varepsilon = 0$ and voters do not play weakly dominated strategies.

Lemma 13 *Assume $\varepsilon = 0$. Every strategy is undominated.*

Proof. No candidate strategy is weakly dominated, because the payoffs to candidates depend on the strategies of the voters.

For the voters, consider the generic information set $(p_i^A, p_i^B) = (x, y)$ with $x, y \in \{0, 1\}$. If $s^b(x, y) = s^c(x, y) = \emptyset$, s^J consists of proposing $(x, 0, 0)$, and s^{-J} consists of proposing $(y, 1, 1)$, then a is strictly better off voting for J, while if s^J consists of $(x, 1, 1)$ and s^{-J} consists of $(y, 0, 0)$, then a is strictly better off voting for $-J$. Thus, any strategy is undominated. ■

7.3.1 Office Motivated Candidates

Proposition 14 *Assume $\varepsilon = 0$, candidates are office motivated and $\beta \in \left(\frac{2}{3}, 1\right)$. An equilibrium in which candidates use the strategy pair (s^A, s^B) exists if and only if $(s^A, s^B) \in S_k$ for some $k \in \{1, 2, 3, 4, 8, 11, 13\}$.*

Proof. For each strategy pair class, we find whether an element of the class can be sustained in equilibrium. To sustain equilibria, we assume that off-equilibrium path beliefs such that given the equilibrium proposal s_i^J, $\omega_2^{i,J}(1 - s_i^J) = 1$ for each $i \in \{a, b, c\}$ and $J \in \{A, B\}$. That is, a voter who observes a deviation believes that the deviating candidate proposes to carry out the projects in the other two districts.

S_1 : Voter strategy $s^i = (\emptyset, A, B, \emptyset)$ and the election is tied. If candidate J deviates to any $s^J \neq s_1$, then J loses the election. It is also straightforward to check that the voting strategy is a best response given the strategy of the candidates and the beliefs of the voters, and that the beliefs are correct along the equilibrium path, so these strategies and beliefs are an equilibrium.

S_2 : Assume without loss of generality that $(s^A, s^B) = (s_1, s_2)$. In equilibrium, $v(s_1, s_2) = (B, A, A)$. Given voters' beliefs, candidate B cannot win the election by deviating and candidate A cannot increase her vote margin by deviating.

S_3 : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_5)$. In equilibrium, $v(s_1, s_5) = (B, B, A)$. Given the beliefs, neither candidate can improve her electoral outcome by deviating.

S_4 : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_8)$. Given beliefs, in equilibrium every voter votes for A and continues to vote for A after any deviation by B.

S_5 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_2)$. Given (s_2, s_2), every voter i abstains. If A deviates to $s^A = s_6$,only voter c observes the deviation so $v(s_6, s_2) = (\emptyset, \emptyset, A)$ and candidate A wins the election.

S_6 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_3)$. Given (s_2, s_3), $v(s_2, s_3) = (A, B, \emptyset)$. If A deviates to $s^A = s_6$, only voter c observes the deviation, so $v(s_6, s_2) = (A, B, A)$ and candidate A wins the election.

S_7 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_5)$. Given (s_2, s_5), $v(s_2, s_5) = (A, B, A)$. If B deviates to $s^B = s_7$, only voter c observes the deviation, so $v(s_2, s_7) = (A, B, B)$ and candidate B finds the deviation profitable.

S_8 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_7)$. In equilibrium in which $v(s_2, s_7) = (A, B, B)$. It is easy to check that no candidate can improve her electoral outcome with any deviation.

S_9 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_8)$. Given (s_2, s_8), $v(s_2, s_8) = (A, B, B)$. If candidate A deviates to s_8, voters b and c either vote for candidate A or abstains, depending upon their beliefs, and therefore candidate A wins the election (voter $a's$ beliefs do not change).

S_{10} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_5)$. Given (s_5, s_5), all voters abstain. If candidate A deviates and proposes s_8, beliefs of voters a and b are unaffected, while voter c votes for candidate A for all possible beliefs over candidate $A's$ strategy. Therefore candidate A wins the election.

S_{11} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_6)$. In equilibrium $v(s_5, s_6) = (\emptyset, A, B)$ and it is again easy to check that no candidate can gain any vote by deviating.

S_{12} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_8)$. Given (s_5, s_8), $v(s_5, s_8) = (A, A, B)$. If A deviates to s_8, voter c either votes for A as well, or abstains, hence A is better off deviating because A increases the vote margin.

S_{13} : In equilibrium the election is tied, and if candidate J deviates to any strategy $s^J \neq s_8$, J loses the election. ■

Proposition 15 *Assume $\varepsilon = 0$, candidates are office motivated and $\beta \in \left(\frac{1}{3}, \frac{2}{3}\right)$. An equilibrium in which candidates use the strategy pair (s^A, s^B) exists if and only if $(s^A, s^B) \notin S_{10} \cup S_{12}$.*

Proof. First note that $(s^A, s^B) \in S_{10}$ cannot be supported in equilibrium. Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_5)$. Given (s_5, s_5), all voters abstain. If candidate J deviates and proposes s_8 voters a and b beliefs are unaffected, while voter c votes for candidate J. Therefore candidate J wins the election.

Similarly, $(s^A, s^B) \in S_{12}$ cannot be supported in equilibrium. Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_8)$. Given (s_5, s_8), voters a and b vote A, while voter c votes B. Suppose A

deviates to s_8. Voters a and b do not observe the deviation, and continue to vote A, while voter c now abstains. Hence now A wins the election by a greater margin.

All the other strategy profiles are sustained in equilibria by the following beliefs. Beliefs' over equilibrium strategies are correct. Out-of-equilibrium beliefs are such that given the equilibrium proposal s_i^J, then $\omega_2^{i,J}(1 - s_i^J) = 1$ for both $J \in \{A, B\}$ and for $i \in \{a, b, c\}$. These are most pessimistic beliefs that a voter can have regarding candidates' strategy when she observes a deviation.

S_1 : All voters abstain and the election is tied. Any voter who observes a deviation votes against the candidate who deviates.

S_2 : Assume without loss of generality that $(s^A, s^B) = (s_1, s_2)$. In equilibrium, $v(s_1, s_2) = (A, B, B)$, and any voter who observes a deviation votes against the candidate who deviates.

S_3 : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_5)$. In equilibrium, every voter votes for A and continues to vote for A after any deviation by B.

S_4 : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_8)$. In equilibrium every voter votes for A and continues to vote for A after any deviation by B.

S_5 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_2)$. Given (s_2, s_2), every voter abstains, and votes against any candidate who deviates.

S_6 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_3)$. Given (s_2, s_3), $v(s_2, s_3) = (A, B, \emptyset)$. Every voter votes against any deviating candidate.

S_7 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_5)$. Given (s_2, s_5), $v(s_2, s_5) = (A, B, A)$. Voters a and c do not vote for B after any deviation by B, and voter b does not vote for A after any deviation.

S_8 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_7)$. Given (s_2, s_7), $v(s_2, s_7) = (A, B, B)$, and given any deviation by J, no voter changes her vote from voting for $-J$ to abstention or voting for J.

S_9 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_8)$. All three voters vote for A and continue to do so after any deviation by B.

S_{10} : Not an equilibrium as shown above.

S_{11} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_6)$. Given (s_5, s_6), $v(s_5, s_g) = (\emptyset, A, B)$, and it is again easy to check that no candidate can gain any vote by deviating.

S_{12} : Not an equilibrium as shown above.

S_{13} : All voters abstain and the election is tied. Any voter who observes a deviation votes against the candidate who deviates. ■

7.3.2 Efficiency Concerned Candidates

Proposition 16 *Assume $\varepsilon = 0$,candidates are efficiency concerned and $\beta \in \left(\frac{2}{3}, 1\right)$. An equilibrium in which candidates use the strategy pair (s^A, s^B) exists if and only if $(s^A, s^B) \in S_k$ for some $k \in \{1, 2, 3, 4, 8, 11, 12, 13\}$.*

Proof. To sustain equilibria, we assume that off-equilibrium path beliefs such that given the equilibrium proposal s_i^J, $\omega_2^{i,J}(1 - s_i^J) = 1$ for each $i \in \{a, b, c\}$ and $J \in \{A, B\}$. That is, a voter who observes a deviation believes that the deviating candidate proposes to carry out the projects in the other two districts.

S_1 : Voter strategy $s^i = (\emptyset, A, B, \emptyset)$ and the election is tied. If candidate J deviates to any $s^J \neq s_1$, then J loses the election. It is also straightforward to check that the voting strategy is a best response given the strategy of the candidates and the beliefs of the voters, and that the beliefs are correct along the equilibrium path, so these strategies and beliefs are an equilibrium.

S_2 : Assume without loss of generality that $(s^A, s^B) = (s_1, s_2)$. In equilibrium, $v(s_1, s_2) = (B, A, A)$. Given voters' beliefs, candidate B cannot win the election by deviating and candidate A cannot increase her vote margin by deviating.

S_3 : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_5)$. In equilibrium, $v(s_1, s_5) = (B, B, A)$. Given the beliefs, neither candidate can improve her electoral outcome by deviating.

S_4 : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_8)$. Given beliefs, in equilibrium every voter votes for A and continues to vote for A after any deviation by B.

S_5 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_2)$. Given (s_2, s_2), every voter i abstains. If A deviates to $s^A = s_6$,only voter c observes the deviation so $v(s_6, s_2) = (\emptyset, \emptyset, A)$ and candidate

A wins the election. Candidate A prefers to deviate if and only if $\frac{1}{1+(1-\beta)2} > \frac{1}{2}\frac{1}{1+(1-\beta)}$ which holds for all $\beta < 1$.

S_6 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_3)$. Given (s_2, s_3), $v(s_2, s_3) = (A, B, \emptyset)$. If A deviates to $s^A = s_6$, only voter c observes the deviation, so $v(s_6, s_2) = (A, B, A)$ and candidate A wins the election. As proved above, all candidate candidate A finds the deviation profitable.

S_7 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_5)$. Given (s_2, s_5), $v(s_2, s_5) = (A, B, A)$. If B deviates to $s^B = s_7$, only voter c observes the deviation, so $v(s_2, s_7) = (A, B, B)$ and candidate B finds the deviation profitable.

S_8 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_7)$. In equilibrium in which $v(s_2, s_7) = (A, B, B)$. It is easy to check that no candidate can improve her electoral outcome with any deviation.

S_9 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_8)$. Given (s_2, s_8), $v(s_2, s_8) = (A, B, B)$. If candidate A deviates to s_8, voters b and c abstain and therefore candidate A wins the election (voter $a's$ beliefs do not change), so candidate $A's$ finds profitable to deviate.

S_{10} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_5)$. Given (s_5, s_5), all voters abstain. If candidate A deviates and proposes s_8, beliefs of voters a and b are unaffected, while voter c votes for candidate A for all possible beliefs over candidate $A's$ strategy. Therefore candidate A wins the election. Candidate A finds profitable to deviate if and only if $\frac{1}{1+(1-\beta)3} > \frac{1}{2}\frac{1}{1+2(1-\beta)}$ which holds for all $\beta < 1$.

S_{11} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_6)$. In equilibrium $v(s_5, s_6) = (\emptyset, A, B)$ and it is again easy to check that no candidate can gain any vote by deviating.

S_{12} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_8)$. Given (s_5, s_8), $v(s_5, s_8) = (A, A, B)$. By deviating candidate B cannot increase the number of votes he gets, due to voters' beliefs. For candidate A, the unique strategy that increases A's vote margin is s_8 because voter c would abstain, but an office motivated candidate does not find this deviation profitable.

S_{13} : In equilibrium the election is tied, and if candidate J deviates to any strategy $s^J \neq s_8$, J loses the election. ■

Proposition 17 *Assume $\varepsilon = 0$, candidates are efficiency concerned and $\beta \in \left(\frac{1}{3}, \frac{2}{3}\right)$. An equilibrium in which candidates use the strategy pair (s^A, s^B) exists if and only if $(s^A, s^B) \notin$*

S_{10}.

Proof. First note that $(s^A, s^B) \in S_{10}$ cannot be supported in equilibrium. Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_5)$. Given (s_5, s_5), all voters abstain. If candidate J deviates and proposes s_8 voters a and b beliefs are unaffected, while voter c votes for candidate J for all possible beliefs over candidate J strategy. Therefore candidate J wins the election. This deviation is profitable if and only if $\frac{1}{1+3(1-\beta)} > \frac{1}{2}\frac{1}{1+2(1-\beta)}$ which holds for all $\beta < 1$.

All the other strategy profiles are sustained in equilibria by the following beliefs. Beliefs' over equilibrium strategies are correct. Out-of-equilibrium beliefs are such that given the equilibrium proposal s_i^J, then $\omega_2^{i,J}(1 - s_i^J) = 1$ for both $J \in \{A, B\}$ and for $i \in \{a, b, c\}$.

S_1 : All voters abstain and the election is tied. Any voter who observes a deviation votes against the candidate who deviates.

S_2 : Assume without loss of generality that $(s^A, s^B) = (s_1, s_2)$. In equilibrium, $v(s_1, s_2) = (A, B, B)$, and any voter who observes a deviation votes against the candidate who deviates.

S_3 : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_5)$. In equilibrium, every voter votes for A and continues to vote for A after any deviation by B.

S_4 : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_8)$. In equilibrium every voter votes for A and continues to vote for A after any deviation by B.

S_5 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_2)$. Given (s_2, s_2), every voter abstains, and votes against any candidate who deviates.

S_6 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_3)$. Given (s_2, s_3), $v(s_2, s_3) = (A, B, \emptyset)$. Every voter votes against any deviating candidate.

S_7 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_5)$. Given (s_2, s_5), $v(s_2, s_5) = (A, B, A)$. Voters a and c do not vote for B after any deviation by B, and voter b does not vote for A after any deviation.

S_8 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_7)$. Given (s_2, s_7), $v(s_2, s_7) = (A, B, B)$, and given any deviation by J, no voter changes her vote from voting for $-J$ to abstention or voting for J.

S_9 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_8)$. All three voters vote for A and continue to do so after any deviation by B.

S_{10} : Not an equilibrium as shown above.

S_{11} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_6)$. Given (s_5, s_6), $v(s_5, s_g) = (\emptyset, A, B)$, and it is again easy to check that no candidate can gain any vote by deviating.

S_{12} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_8)$. Given (s_5, s_8), $v(s_5, s_8) = (A, A, B)$. By deviating candidate B cannot increase the number of votes he gets, due to voters' beliefs. For candidate A, the unique strategy that increases A's vote margin is s_8 because voter c would abstain, but an office motivated candidate does not find this deviation profitable.

S_{13} : All voters abstain and the election is tied. Any voter who observes a deviation votes against the candidate who deviates. ■

7.3.3 One efficiency concerned, one office motivated candidate

Proposition 18 *Assume* $\varepsilon = 0$, *one candidate is efficiency concerned and the other is office motivated, and* $\beta \in \left(\frac{2}{3}, 1\right)$. *An equilibrium in which candidates use the strategy pair* (s^A, s^B) *exists if and only if* $(s^A, s^B) \in S_k$ *for some* $k \in \{1, 2, 3, 4, 8, 11, 13\}$ *and if* $(s^A, s^B) \in S_{12}''$ *where* $S_{12}'' = \{(s_8, s_5), (s_8, s_6), (s_8, s_7)\}$

Proof. Supose without loss of generality that candidate A is office motivated. We assume that off-equilibrium path beliefs such that given the equilibrium proposal s_i^J, $\omega_2^{i,J}(1 - s_i^J) = 1$ for each $i \in \{a, b, c\}$ and $J \in \{A, B\}$. That is, a voter who observes a deviation believes that the deviating candidate proposes to carry out the projects in the other two districts.

S_1 : Voter strategy $s^i = (\emptyset, A, B, \emptyset)$ and the election is tied. If candidate J deviates to any $s^J \neq s_1$, then J loses the election. It is also straightforward to check that the voting strategy is a best response given the strategy of the candidates and the beliefs of the voters, and that the beliefs are correct along the equilibrium path, so these strategies and beliefs are an equilibrium.

S_2' : Assume without loss of generality that $(s^A, s^B) = (s_1, s_2)$. Given (s_1, s_2), $v(s_1, s_2) = (B, A, A)$. Given voters' beliefs, candidate B cannot win the election by deviating and candidate A cannot increase her vote margin by deviating.

S_2'' = Assume without loss of generality that $(s^A, s^B) = (s_2, s_1)$. Given (s_2, s_1), $v(s_2, s_1) = (A, B, B)$. Given voters' beliefs, candidate A cannot win the election by deviating and candidate B cannot increase her vote margin by deviating.

S'_3 : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_5)$. Given (s_1, s_5), $v(s_1, s_5) = (B, B, A)$. Candidate A cannot win the election deviating to any strategy $s_A \neq s_1$. Candidate B is efficiency concerned so he might be interested in deviating to reduce the number of projects he proposes. By deviating to s_2, the election is tied: candidate B prefers to deviate if and only if $\frac{1}{2}\frac{1}{1+(1-\beta)} > \frac{1}{1+2(1-\beta)}$, but this condition never occurs. By deviating to s_1 candidate B looses the election so this deviation is never profitable.

S''_3 : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_1)$. Given (s_5, s_1), $v(s_5, s_1) = (A, A, B)$. No candidate has a profitable deviation given voters' beliefs.

(s_1, s_8) : Every voter votes for A. No candidate has a profitable deviation given voters' beliefs.

(s_8, s_1) : Every voter votes for B. No candidate has a profitable deviation given voters' beliefs.

S_5 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_2)$. Given (s_2, s_2), every voter i abstains. If candidate A deviates to s_8 wins the election.

S_6 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_3)$. Given (s_2, s_3), $v(s_2, s_3) = (A, B, \emptyset)$. If A deviates to $s^A = s_6$ both in case full information is revealed and in case it not revealed, A wins the election.

Let $S'_7 = \{(s_2, s_5), (s_2, s_6), (s_3, s_5), (s_3, s_7), (s_4, s_6), (s_4, s_7), \}$

and $S''_7 = \{(s_5, s_2), (s_6, s_2), (s_5, s_3), (s_7, s_3), (s_6, s_4), (s_7, s_4)\}$.

S'_7 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_5)$. Given (s_2, s_5), $v(s_2, s_5) = (A, B, A)$. No candidate has a profitable deviation given voters' beliefs.

S''_7 : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_2)$. Given (s_5, s_2), $v(s_5, s_2) = (B, A, B)$. No candidate has a profitable deviation given voters' beliefs.

Let $S'_8 = \{(s_2, s_7), (s_3, s_6), (s_4, s_5), \}$ and $S''_8 = \{(s_7, s_2), (s_6, s_3), (s_5, s_4)\}$

S'_8 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_7)$. Given (s_2, s_7), $v(s_2, s_7) = (A, B, B)$. If candidate B deviates to any strategy with less projects looses the election.

S''_8 : Assume w.l.o.g. that $(s^A, s^B) = (s_7, s_2)$. Given (s_7, s_2), $v(s_7, s_2) = (B, A, A)$. No candidate has a profitable deviation given voters' beliefs.

Let $S_9' = \{(s_2, s_8), (s_3, s_8), (s_4, s_8)\}$ and $S_9'' = \{(s_8, s_2), (s_8, s_3), (s_8, s_4)\}$;

S_9' : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_8)$. Given (s_2, s_8), $v(s_2, s_8) = (A, B, B)$. If candidate B deviates to any strategy with less projects looses the election.

S_9'' : Assume w.l.o.g. that $(s^A, s^B) = (s_8, s_2)$. Given (s_8, s_2), $v(s_8, s_2) = (B, A, A)$. No candidate has a profitable deviation given voters' beliefs.

S_{10} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_5)$. Given (s_5, s_5), all voters abstain. If candidate A deviates to s_8 wins the election.

S_{11} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_6)$. Then $v(s_5, s_6) = (\emptyset, A, B)$ and each candidate wins with equal probability. No candidate has a profitable deviation given voters' beliefs.

Let $S_{12}' = \{(s_5, s_8), (s_6, s_8), (s_7, s_8)\}$ and $S_{12}'' = \{(s_8, s_5), (s_8, s_6), (s_8, s_7)\}$;

S_{12}' : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_8)$. Given (s_5, s_8), $v(s_5, s_8) = (A, A, B)$. If candidate A deviates to s_8, then voter c abstains and therefore candidate A wins with greater margin.

S_{12}'' : Assume w.l.o.g. that $(s^A, s^B) = (s_8, s_5)$. Given (s_8, s_5), $v(s_8, s_5) = (A, B, B)$. No candidate has a profitable deviation given voters' beliefs, because candidate B is efficiency concerned and does not find deviation s_8 profitable even if he would win with a greater vote margin.

S_{13} : In equilibrium the election is tied; if a candidate deviates to any strategy different than s_8 looses the election. ■

Proposition 19 *Assume $\varepsilon = 0$, one candidates efficiency concerned and the other is office motivated, and $\beta \in \left(\frac{1}{3}, \frac{2}{3}\right)$. An equilibrium in which candidates use the strategy pair (s^A, s^B) exists if and only if $(s^A, s^B) \notin S_{10} \cup S_{12'}$ where $S_{12}' = \{(s_5, s_8), (s_6, s_8), (s_7, s_8)\}$.*

Proof. Supose without loss of generality that candidate A is office motivated. We assume that off-equilibrium path beliefs such that given the equilibrium proposal s_i^J, $\omega_2^{i,J}(1 - s_i^J) = 1$ for each $i \in \{a, b, c\}$ and $J \in \{A, B\}$. That is, a voter who observes a deviation believes that the deviating candidate proposes to carry out the projects in the other two districts.

S_1 : In equilibrium the election is tied. If candidate J deviates to any $s^J \neq s_1$, then J loses the election. It is also straightforward to check that the voting strategy is a best response given the strategy of the candidates and the beliefs of the voters, and that the beliefs are correct along the equilibrium path, so these strategies and beliefs are an equilibrium.

S_2' : Assume without loss of generality that $(s^A, s^B) = (s_1, s_2)$. Given (s_1, s_2), $v(s_1, s_2) = (B, A, A)$. Given voters' beliefs, candidate B cannot win the election by deviating and candidate A cannot increase her vote margin by deviating.

$S_2'' =$ Assume without loss of generality that $(s^A, s^B) = (s_2, s_1)$. Given (s_2, s_1), $v(s_2, s_1) = (A, B, B)$. Given voters' beliefs, candidate A cannot win the election by deviating and candidate B cannot increase her vote margin by deviating.

S_3' : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_5)$. Given (s_1, s_5), $v(s_1, s_5) = (A, A, A)$. No candidate has a profitable deviation given voters' beliefs.

S_3'' : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_1)$. Given (s_5, s_1), $v(s_5, s_1) = (B, B, B)$. No candidate has a profitable deviation given voters' beliefs.

(s_1, s_8) : Every voter votes for A. No candidate has a profitable deviation given voters' beliefs.

(s_8, s_1) : Every voter votes for B. No candidate has a profitable deviation given voters' beliefs.

S_5 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_2)$. Given (s_2, s_2), every voter i abstains. No candidate has a profitable deviation given voters' beliefs.

S_6 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_3)$. Given (s_2, s_3), $v(s_2, s_3) = (A, B, \emptyset)$. No candidate has a profitable deviation given voters' beliefs.

Let $S_7' = \{(s_2, s_5), (s_2, s_6), (s_3, s_5), (s_3, s_7), (s_4, s_6), (s_4, s_7), \}$

and $S_7'' = \{(s_5, s_2), (s_6, s_2), (s_5, s_3), (s_7, s_3), (s_6, s_4), (s_7, s_4)\}$.

S_7' : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_5)$. Given (s_2, s_5), $v(s_2, s_5) = (A, B, A)$. No candidate has a profitable deviation given voters' beliefs.

S_7'' : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_2)$. Given (s_5, s_2), $v(s_5, s_2) = (B, A, B)$. No candidate has a profitable deviation given voters' beliefs.

Let $S_8' = \{(s_2, s_7), (s_3, s_6), (s_4, s_5), \}$ and $S_8'' = \{(s_7, s_2), (s_6, s_3), (s_5, s_4)\}$

S_8' : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_7)$. Given (s_2, s_7), $v(s_2, s_7) = (A, B, B)$. If candidate B deviates to any strategy with less projects looses the election.

S_8'' : Assume w.l.o.g. that $(s^A, s^B) = (s_7, s_2)$. Given (s_7, s_2), $v(s_7, s_2) = (B, A, A)$. No candidate has a profitable deviation given voters' beliefs.

Let $S_9' = \{(s_2, s_8), (s_3, s_8), (s_4, s_8)\}$ and $S_9'' = \{(s_8, s_2), (s_8, s_3), (s_8, s_4)\}$;

S_9' : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_8)$. Given (s_2, s_8), $v(s_2, s_8) = (A, B, B)$. If candidate B deviates to any strategy with less projects looses the election.

S_9'' : Assume w.l.o.g. that $(s^A, s^B) = (s_8, s_2)$. Given (s_8, s_2), $v(s_8, s_2) = (B, A, A)$. No candidate has a profitable deviation given voters' beliefs.

S_{10} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_5)$. Given (s_5, s_5), all voters abstain. If candidate A deviates to s_8 wins the election.

S_{11} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_6)$. Then $v(s_5, s_6) = (\emptyset, A, B)$ and each candidate wins with equal probability. No candidate has a profitable deviation given voters' beliefs.

Let $S_{12}' = \{(s_5, s_8), (s_6, s_8), (s_7, s_8)\}$ and $S_{12}'' = \{(s_8, s_5), (s_8, s_6), (s_8, s_7)\}$;

S_{12}' : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_8)$. Given (s_5, s_8), $v(s_5, s_8) = (A, A, B)$. If candidate A deviates to s_8, then voter c abstains and therefore candidate A wins with greater margin.

S_{12}'' : Assume w.l.o.g. that $(s^A, s^B) = (s_8, s_5)$. Given (s_8, s_5), $v(s_8, s_5) = (A, B, B)$. No candidate has a profitable deviation given voters' beliefs.

S_{13} : In equilibrium the election is tied; if a candidate deviates to any strategy different than s_8 looses the election. ■