The Rise and Fall of S&P500 Variance Futures*

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1. Volatility Indexes

(1) The volatility of an asset of composite index can be traded by using derivatives, such as volatility swaps and variance swaps.

(2) However, swaps are traded over the counter rather than exchange traded, thereby leading to much lower liquidity.

(3) Chicago Board Options Exchange (CBOE) introduced the Volatility Index (VIX) futures on 26 March 2004.

(4) CBOE introduced VIX options on 24 February 2006.

(5) VIX options and futures are highly traded.

2. VIX and VXO Volatility Indexes

(1) VIX is a key measure of market expectations of volatility, and hence is a key barometer of investor sentiment and market volatility.

(2) VIX is based on S&P500 call and put options over a wide range of strike prices, and hence is not model based.

(3) The original CBOE volatility index, VXO, is based on the Black-Scholes implied volatilities from S&P100 index, and hence is model based.

(4) The Black-Scholes model assumes normality, which is typically unrealistic for financial market data.

3. Volatility Indexes

(1) Speculators can trade on volatility risk with VIX derivatives, with views on whether volatility will increase or decrease in the future.

(2) Hedgers can use volatility derivatives to avoid exposure to volatility risk.

(3) Volatility risk can occur for a long trading position, which is exposed to the risk of falling market prices, or for a short trading position, which is exposed to the risk of rising market prices.

(4) Value-at-Risk (VaR) forecasts typically focus on losses due to falling market prices, whereby investors are assumed to have long positions.

4. Volatility Indexes for Hedging

(1) VIX is not traded, but VIX futures and options lead to indirect trading in VIX.

(2) VIX futures can be hedged using VIX futures of different maturities.

(3) VIX options can be hedged using VIX futures.

(4) Optimal hedge ratios can be calculated using consistently estimated dynamic conditional correlations.

5. Rise and Fall

(1) CBOE Futures Exchange (CFE) introduced the S&P500 3-month variance futures on 18 May 2004.

(2) CFE introduced the S&P500 12-month variance futures on 23 March 2006.

(3) S&P500 12-month variance futures were delisted as of 17 March 2011.

(4) As contract values are available until 18 March 2011, S&P500 12-month variance futures did not reach its Fifth Anniversary.

6. What went wrong? The chicken or the egg?

(1) Investors clearly understood the meaning and value of VIX as a "fear" index, and of VIX options and VIX futures as derivatives of a "fear" index.

(2) S&P500 3-month and 12-month variance futures were not clearly understood as derivative measures of market volatility or risk.

(3) S&P500 3-month and 12-month variance options have not been created, and hence have not been listed.

(4) The S&P500 variance futures are not model based, so the assumptions underlying the index were not clearly understood.

(5) As these two variance futures were thinly traded, their returns could not be modelled accurately using a variety of risk models.

(6) As standard risk models could not be used to model the risks and dynamic correlations of these two S&P500 variance futures, optimal hedge ratios would be difficult to calculate.

(7) Therefore, S&P500 variance futures could not be used for hedging purposes.

7. Models of Daily Volatility

McAleer et al. (2010b) and Chang et al. (2011), among others, discuss how Authorized Deposit-taking Institutions (ADIs) can use internal models to determine their Value-at-Risk (VaR) thresholds by using alternative time series models for estimating conditional volatility. In what follows, we present several well-known conditional volatility models that can be used to evaluate strategic market risk disclosure, namely GARCH, GJR and EGARCH, with Gaussian, Student-*t* (with estimated degrees of freedom), and Generalized Normal distribution errors, where the parameters are estimated.

These conditional volatility models are chosen as they are widely used in the literature. For an extensive discussion of the theoretical properties of several of these models (see Ling and McAleer (2002a, 2002b, 2003a) and Caporin and McAleer (2010b)). We include a section on these models to present them in a unified framework and notation, and to make explicit the specific versions we are using.

7.1 GARCH

For a wide range of financial data series, time-varying conditional variances can be explained empirically through the autoregressive conditional heteroskedasticity (ARCH) model, which was proposed by Engle (1982). When the time-varying conditional variance has both autoregressive and moving average components, this leads to the generalized ARCH(p,q), or GARCH(p,q), model of Bollerslev (1986). It is very common in practice to impose the widely estimated GARCH(1,1) specification in advance.

Consider the stationary AR(1)-GARCH(1,1) model for daily returns, y_t:

$$y_t = \varphi_1 + \varphi_2 y_{t-1} + \varepsilon_t, \qquad |\varphi_2| < 1 \tag{1}$$

for $t = 1,...,n$, where the shocks to returns are given by:

$$\begin{aligned} \varepsilon_t &= \eta_t \sqrt{h_t}, \quad \eta_t \sim iid(0,1) \\ h_t &= \omega + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1}, \end{aligned} \tag{2}$$

and $\omega > 0, \alpha \geq 0, \beta \geq 0$ are sufficient conditions to ensure that the conditional variance $h_t > 0$, while $\alpha + \beta < 1$ is sufficient for a finite unconditional variance. The stationary AR(1)-GARCH(1,1) model can be modified to incorporate a non-stationary ARMA(p,q) conditional mean and a stationary GARCH(r,s) conditional variance, as in Ling and McAleer (2003b).

7.2 GJR

In the symmetric GARCH model, the effects of positive shocks (or upward movements in daily returns) on the conditional variance, h_t, are assumed to be the same as the effect of negative shocks (or downward movements in daily returns) of equal magnitude. In order to accommodate asymmetric behaviour, Glosten, Jagannathan and Runkle (1992) proposed a model (hereafter GJR), for which GJR(1,1) is defined as follows:

$$h_t = \omega + (\alpha + \gamma I(\eta_{t-1}))\varepsilon_{t-1}^2 + \beta h_{t-1}, \qquad (3)$$

where $\omega > 0, \alpha \geq 0, \alpha + \gamma \geq 0, \beta \geq 0$ are sufficient conditions for $h_t > 0$, $\alpha + \beta + \gamma/2 < 1$ is sufficient for a finite unconditional variance, and $I(\eta_t)$ is an indicator variable defined by:

$$I(\eta_t) = \begin{cases} 1, & \varepsilon_t < 0 \\ 0, & \varepsilon_t \geq 0 \end{cases} \qquad (4)$$

as η_t has the same sign as ε_t. The indicator variable differentiates between positive and negative shocks, so that asymmetric effects in the data are captured by the coefficient γ. For financial data, it is expected that $\gamma \geq 0$ because negative shocks have a greater impact on risk than do positive shocks of similar magnitude. The asymmetric effect, γ, measures the contribution of shocks to both short run persistence, $\alpha + \gamma/2$, and to long run persistence, $\alpha + \beta + \gamma/2$.

Although GJR permits asymmetric effects of positive and negative shocks of equal magnitude on conditional volatility, the special case of leverage, whereby negative shocks increase volatility while positive shocks decrease volatility (see Black (1976) for an argument using the debt/equity ratio), cannot be accommodated, in practice (for further details on asymmetry versus leverage in the GJR model, see Caporin and McAleer (2010b)). The reason why leverage does not exist in the GJR model is that restriction on the ARCH parameter arising from positive shocks, namely $\alpha < 0$, is not consistent with the interpretation of the model. Moreover, a negative and significant estímate of α is not found in practice.

7.3 EGARCH

An alternative model to capture asymmetric behaviour in the conditional variance is the Exponential GARCH, or EGARCH(1,1), model of Nelson (1991), namely:

$$log h_t = \omega + \alpha \left| \frac{\varepsilon_{t-1}}{h_{t-1}} \right| + \gamma \frac{\varepsilon_{t-1}}{h_{t-1}} + \beta log h_{t-1}, \quad |\beta| < 1 \tag{5}$$

where the parameters α, β and γ have different interpretations from those in the GARCH(1,1) and GJR(1,1) models discussed above.

EGARCH captures asymmetries differently from GJR. The parameters α and γ in EGARCH(1,1) represent the magnitude (or size) and sign effects of the standardized residuals, respectively, on the conditional variance, whereas α and $\alpha + \gamma$ represent the effects of positive and negative shocks, respectively, on the conditional variance in GJR(1,1). Unlike GJR, EGARCH can accommodate leverage, namely $\gamma < 0$ and $\gamma < \alpha < -\gamma$, depending on the restrictions imposed on the size and sign parameters, though leverage is not guaranteed (for further details, see Caporin and McAleer (2010b)).

As noted in McAleer et al. (2007), there are some important differences between EGARCH and the previous two models, as follows: (i) EGARCH is a model of the logarithm of the conditional variance, which implies that no restrictions on the parameters are required to ensure $h_t > 0$; (ii) moment conditions are required for the GARCH and GJR models as they are dependent on lagged unconditional shocks, whereas EGARCH does not require moment conditions to be established as it depends on lagged conditional shocks (or standardized residuals); (iii) Shephard (1996) observed that $|\beta| < 1$ is likely to be a sufficient condition for consistency of QMLE for EGARCH(1,1); (iv) as the standardized residuals appear in equation (7), $|\beta| < 1$ would seem to be a sufficient condition for the existence of moments; and (v) in addition to being a sufficient condition for consistency, $|\beta| < 1$ is also likely to be sufficient for asymptotic normality of the QMLE of EGARCH(1,1).

8. Data and Empirical Results

(1) Daily data on S&P500 3-month variance futures, with 3 month maturity, are obtained for the period 18 May 2004 to 1 April 2011.

(2) Daily data on S&P500 12-month variance futures, with 3 month maturity, are obtained for the period 24 March 2006 to 17 March 2011.

(3) The three conditional volatility models given above are estimated under the following distributional assumptions on the conditional shocks: (1) Gaussian, (2) Student-*t*, with estimated degrees of freedom, and (3) Generalized Normal. As the models that incorporate the t distributed errors are estimated by QMLE, the resulting estimators are consistent and asymptotically normal, so they can be used for estimation, inference and forecasting.

8.1 Figures 1-4

In Figure 1, there is little evidence of volatility in 3-month variance futures until the impact of the GFC in the third quarter of 2008, with a substantial reduction in 2009.

In Figure 2, the high volatility for 12-month variance futures persists toward the end of 2009, well after the GFC had been presumed to have ended, after which there is much lower volatility.

The 3-month variance futures returns in Figure 3 show that positive returns were far more numerous, and of greater magnitude, than negative returns.

The 12-month variance futures returns in Figure 4 also show that positive returns were far more numerous than negative returns, but the most extreme return is a single negative return toward the end of 2009.

For these reasons, only the 3-month variance futures returns will be used to estimate volatility.

8.2 Tables 1-2

The 3-month and 12-month variance futures prices are more highly correlated at 0.64 than are the corresponding 3-month and 12-month variance futures returns correlations at 0.52.

8.3 Table 3

The GARCH volatility estimates for the 3-month variance futures are presented in Table 3 for three probability densities for the full sample ("All"), as well as the subsamples given as before, during and after the GFC.

The estimates for All and Before GFC are very similar, with the After GFC estimates being quite different from remaining estimates, especially for α and β, and hence $\alpha + \beta$.

Negative estimates of α are obtained for the normal and Student-t distributions, which is uncommon for financial data.

The estimates of α and β are similar across the three distributions only for the During GFC subperiod.

The estimate of $\alpha + \beta$ exceeds unity for the All and Before GFC subperiod under the Student-t density.

8.4 Table 4

The GJR volatility estimates for the 3-month variance futures are presented in Table 4 for three probability densities for the full sample and the three subsamples.

Depending on the probability density and the sample period, negative estimates of α, γ and β are obtained, which is uncommon for financial data.

Asymmetry seems to be significant, but whether it is positive or negative, as well as its magnitude, depends on the probability density and the sample period considered.

Apart from the results for the normal density, the estimates seem closest for the All and Before GFC subperiod.

The estimate of $\alpha + \beta + \gamma/2$ exceeds unity for all three densities for at least one subperiod.

8.5 Table 5

The EGARCH volatility estimates for the 3-month variance futures are presented in Table 5 for three probability densities for the full sample and the three subsamples.

The estimates of α, γ and β are substantially different between the normal density, on the one hand, and the Student-t and generalized normal densities, on the other.

As the estímate of γ is negative, and the estímate of α is bounded by γ, there is leverage for All, as well as Before and After GFC subperiods for the normal density, but there is no leverage for the Student-t and generalized normal densities.

For the normal density and After GFC subperiod, the estímate of β exceeds unity.

8.5 Figures 5-7

Recursive estimates of the parameters for the full simple period are given in Figures 5, 6 and 7 for GARCH, GJR and EGARCH, respectively.

Consistent with the results presented in Tables 3-5, the estimates are highly variable, and differ according to the probability density.

For the GARCH and GJR models, the results for the Student-t density seem to be the least variable, with some semblance of persistence rather than randomness.

The estimates for the EGARCH model display some similarity under the Student-t and generalized normal densities.

Figure 1

S&P500 3-Month Variance Futures
(18/05/2004 – 01/04/2011)

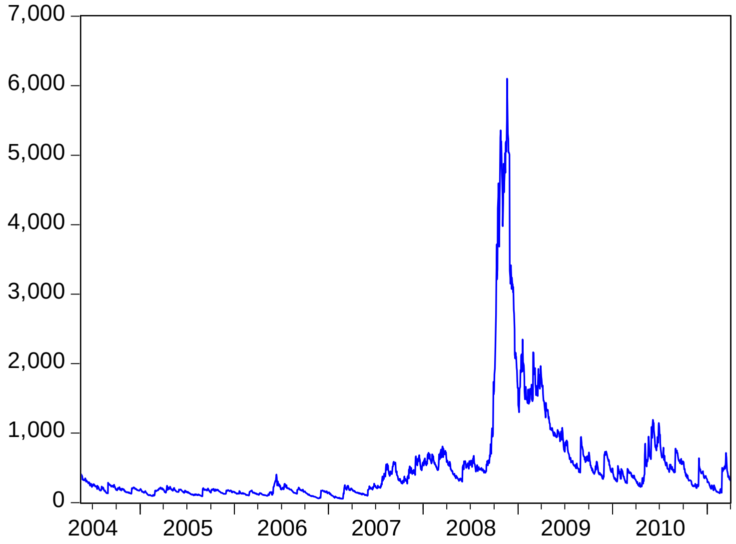


Figure 2

S&P500 12-Month Variance Futures
(24/03/2006 – 01/04/2011)

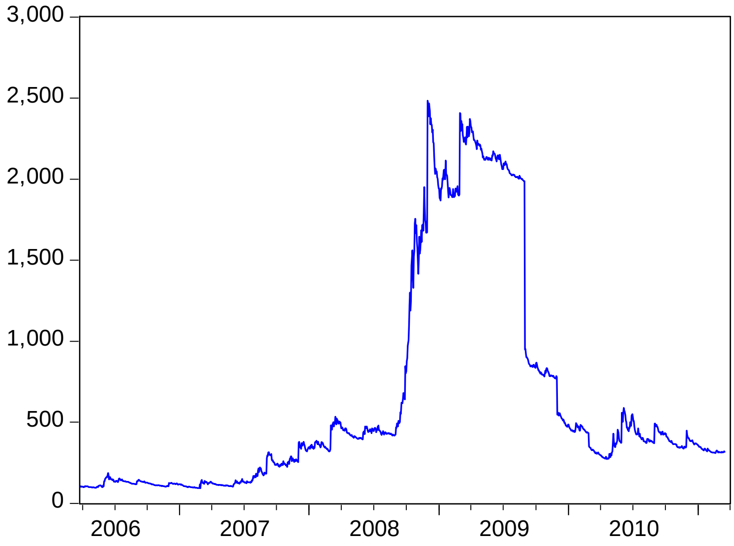


Figure 3

S&P500 3-Month Variance Futures Returns
(18/05/2004 – 01/04/2011)

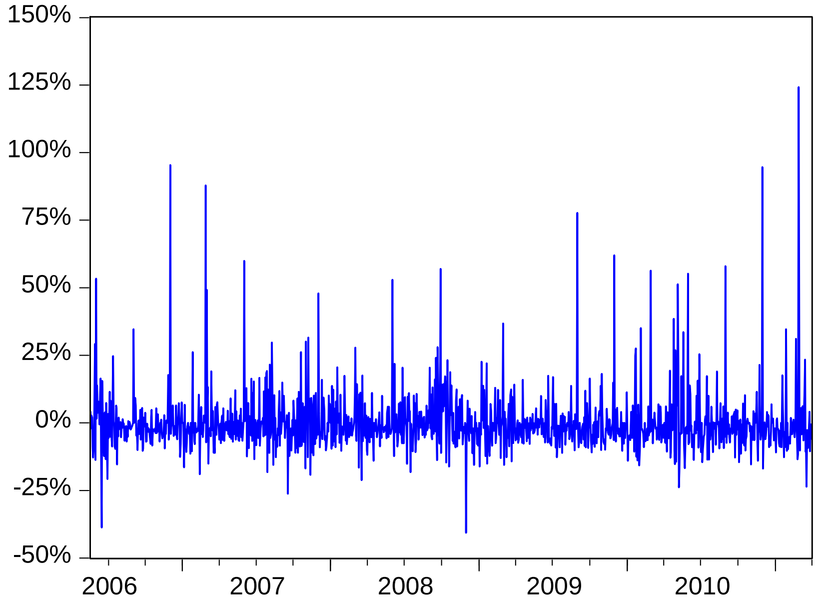


Figure 4

S&P500 12-Month Variance Futures Returns
(24/03/2006 – 01/04/2011)

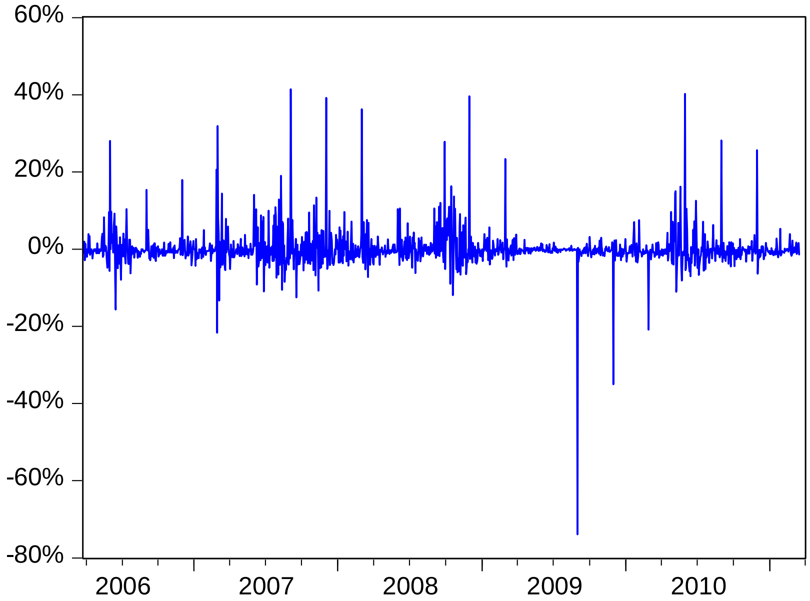


Table 1

Price Correlations

Variable	3-Month VF	12-Month VF
3-Month VF	1	0.64
12-Month VF	0.64	1

Note: VF denotes variance futures.

Table 2

Returns Correlations

Variable	3-Month VF returns	12-Month VF returns
3-Month VF returns	1	0.52
12-Month VF returns	0.52	1

Note: VF denotes variance futures.

Table 3

GARCH Estimates for 3-month VF Before, During and After GFC

Density	Parameter	All	Before	During	After
Normal	α	-0.0030	-0.0044	0.0099	-0.0110
	β	0.978	0.9805	0.9224	0.6822
	$\alpha + \beta$	0.9754	0.9761	0.9321	0.6712

Density	Parameter	All (2.16)	Before (2.15)	During (2.5)	After (11)
Student-t	α	0.1252	0.1697	0.0906	-0.0128
	β	0.8952	0.8838	0.9049	0.4147
	$\alpha + \beta$	1.0204	1.0535	0.9954	0.4018

Density	Parameter	All	Before	During	After
Generalized Normal	α	0.0358	0.0329	0.0397	0.0694
	β	0.8360	0.8512	0.9102	0.4115
	$\alpha + \beta$	0.8718	0.8841	0.9499	0.4809

Notes: All denotes the full sample period. The entries in parentheses for the Student-t distribution are the estimated degrees of freedom.

Table 4

GJR Estimates for 3-month VF Before, During and After GFC

Density	Parameter	All	Before	During	After
Normal	α	-0.0065	0.0726	0.0591**	-0.0111
	γ	0.3488	-0.1642	-0.1396	0.3843
	β	-0.1726	0.5808	0.5808**	0.9594
	$\alpha + \beta + \gamma/2$	-0.0047	0.5715	0.5701	1.1404

Density	Parameter	All (2.19)	Before (2.27)	During (17.54)	After (2.16)
Student-t	α	0.0971**	0.0772	0.0301**	0.5375**
	γ	0.5382**	0.4568	-0.1269**	0.9899**
	β	0.8750	0.8677	0.4826**	0.3151**
	$\alpha + \beta + \gamma/2$	1.2412	1.1733	0.4492**	1.3475**

Density	Parameter	All	Before	During	After
Generalized Normal	α	0.0146	0.0150	0.0514**	-0.0012
	γ	0.1689	0.1942	-0.1191**	0.2790
	β	0.8738	0.8468	0.7041**	0.9083
	$\alpha + \beta + \gamma/2$	0.9729	0.9589	0.6960**	1.04664

Notes: All denotes the full sample period. The entries in parentheses for the Student-t distribution are the estimated degrees of freedom.
** These parameters are not statistically significant.

Table 5

EGARCH Estimates for 3-month VF Before, During and After GFC

Density	Parameter	All	Before	During	After
Normal	α	0.1192	-0.0321	-0.5012	0.0070
	γ	-0.1674	-0.1730	0.2580	-0.1318
	β	-0.8083	-0.8941	-0.4134**	1.0020

Density	Parameter	All (2.29)	Before (2.36)	During (2.9)	After (2.21)
Student-t	α	0.2315	0.2514	-0.1908	0.2685
	γ	-0.0314	-0.0571	0.2615	0.0307
	β	0.9477	0.9500	0.8921	0.6978

Density	Parameter	All	Before	During	After
Generalized Normal	α	0.1272	0.1284	-0.1859**	0.1647
	γ	-0.0161	-0.0207	0.2142	0.0013
	β	0.9282	0.9241	0.9019	0.7463

Notes: All denotes the full sample period. The entries in parenthesis for the Student-t distribution are the estimated degrees of freedom.

Figure 5

α and β Estimates: GARCH

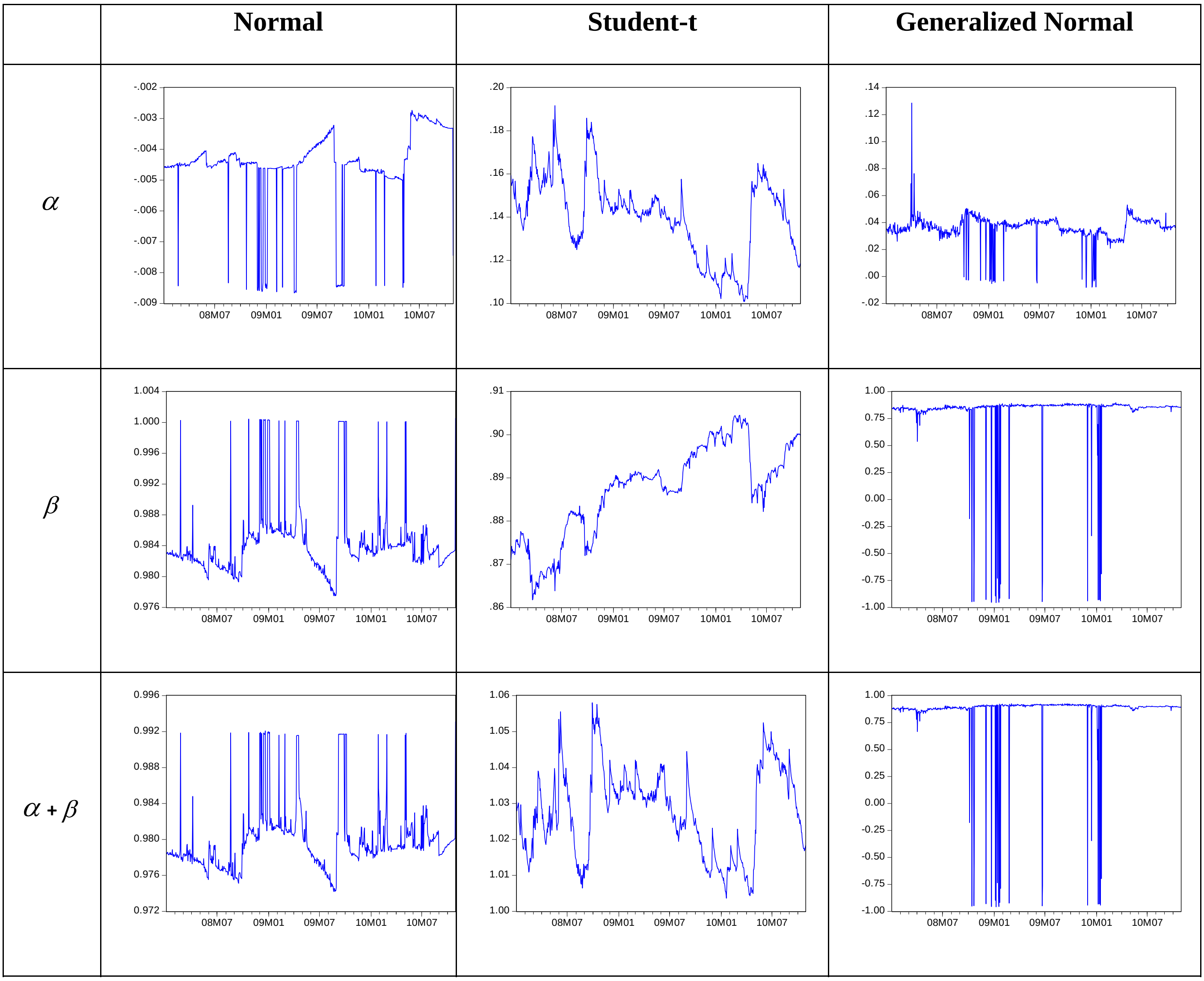


Figure 6

α , β and γ Estimates: GJR

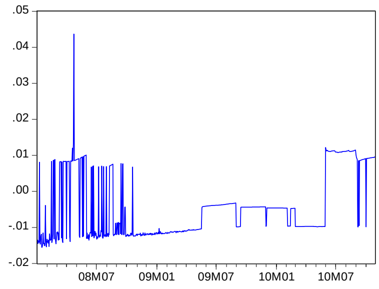
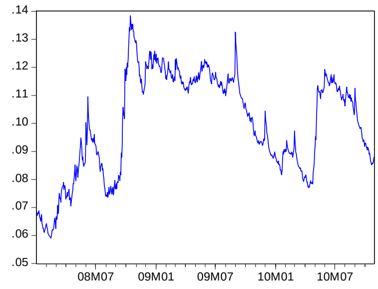
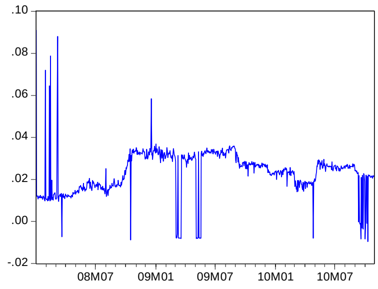
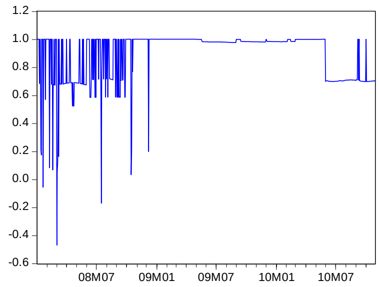
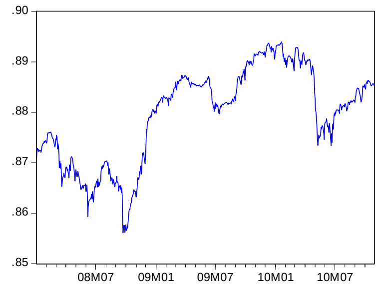
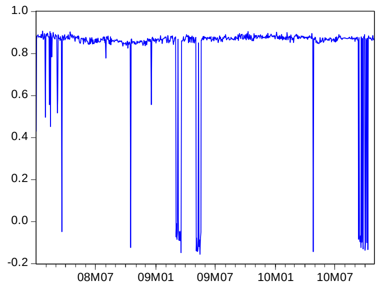
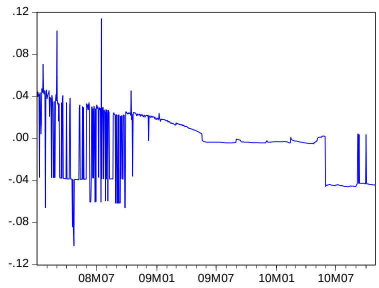
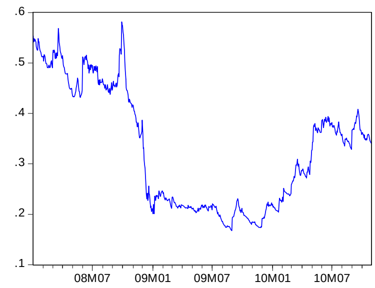
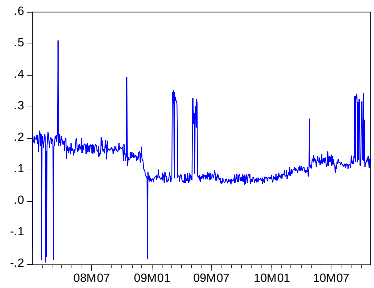
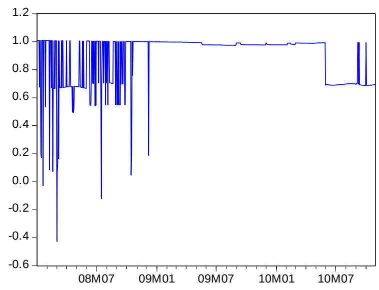
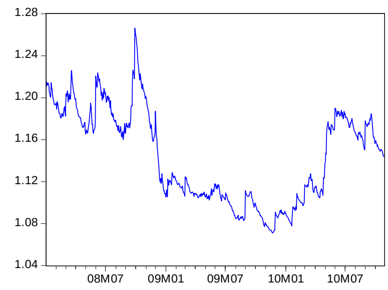
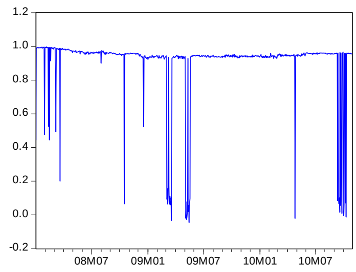
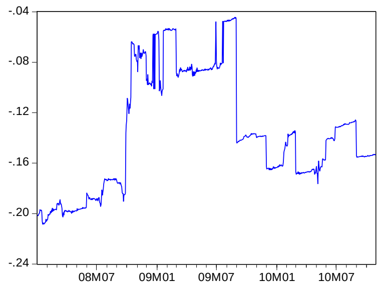
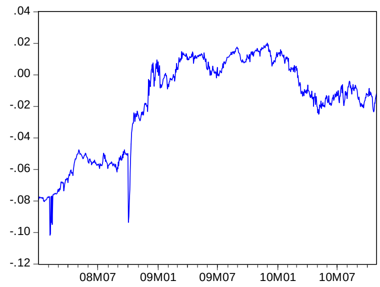
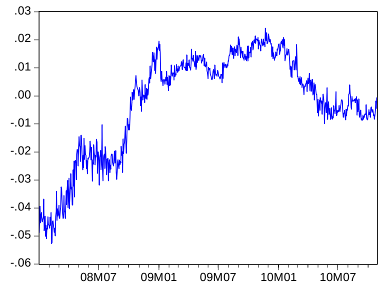
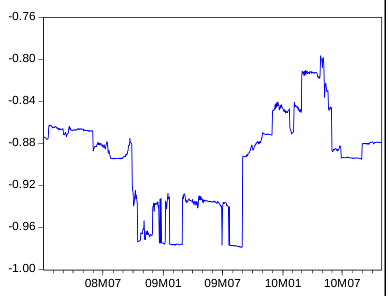
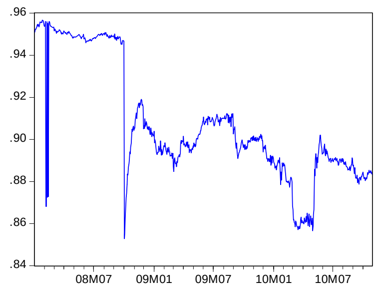
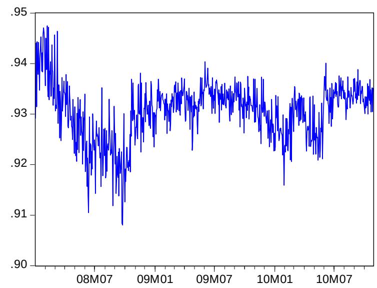
	Normal	Student-t	Generalized Normal
α			
β			
γ			
$\alpha + \beta + \gamma /2$			

Figure 7

α , β and γ Estimates: EGARCH

	Normal	Student-t	Generalized Normal
α			
β			
γ