

Dynamic Collective Choice with Endogenous Status Quo*

Wioletta Dziuda[†] Antoine Loeper[‡]

December 2011
First version: April 2010

Abstract

This paper analyzes an ongoing bargaining situation in which preferences evolve over time and the previous agreement becomes the next status quo and determines the payoffs until a new agreement is reached. We show that the endogeneity of the status quo exacerbates the players' conflict of interest and decreases the responsiveness of the bargaining outcome to the environment. In some cases, it can lead the negotiations to a complete gridlock. As compared to a bargaining protocol with an exogenous status quo, the status quo stays more often in place and equilibrium welfare is lower.

In a legislative setting, this model shows that the inertial effect of the endogenous the status quo can be mitigated by concentrating decision power, and can be eliminated by sunset provisions.

JEL Classification Numbers: C73, D72, D78

Keywords: Dynamic voting, endogenous status quo, partisanship, polarization, policy inertia, sunset provision, checks and balances.

*Previously circulated under the title "Ongoing Negotiation with an Endogenous Status Quo." We thank David Austen-Smith, David Baron, Daniel Diermeier, Bard Harstad, and the participants of the seminars at Bonn University, Leuven University, Northwestern University, Nottingham University, Rice University, the University of Chicago, and Paris Game Theory Seminar.

[†]Kellogg School of Management, Northwestern University. Email: wdziuda@northwestern.edu

[‡]Universidad Carlos III de Madrid. Email: aloeper@eco.uc3m.es

1 Introduction

This paper analyzes an ongoing collective decision problem in which i) there are shocks to the environment that affect individual preferences, and hence call for renegotiation of the past agreement, and ii) agreements are determined using an endogenous status-quo protocol: the previous agreement stays in place and determines the payoffs until a new agreement is reached.

A prominent example of negotiations in a changing environment with an endogenous status quo is legislative bargaining. For instance, legislators' preferences over fiscal policies reflect heterogeneous ideologies and constituencies, but are also affected by shocks such as business cycles, changes in the country's credit rating, or the vagaries of public opinion. At the same time, in most democracies, a vast array of fiscal policies are set using an endogenous status quo: once enacted, the law or program continues in effect until further legislative action is taken. For example, about two third of the U.S. federal budget—called mandatory spending—continues year after year by default. Outside of the fiscal sphere, many ideologically charged issues such as immigration, financial regulation, minimum wage, civil liberties and national security are also affected by shocks (e.g., demographic transitions, financial innovation, national security threat) and are typically regulated by permanent legislation.¹

The starting point of our analysis is the observation that the endogenous status quo creates a *dynamic linkage* between bargaining periods. In a changing environment, this linkage presents the negotiating parties with a trade-off between responding to the current shock and securing a favorable position for future bargaining. To illustrate this trade-off, consider the case of legislators in the U.S. congress negotiating the size of mandatory spending. During a recession, generous deficit spending may be favored by all parties to stimulate a short-term economic growth. During a boom, all parties may agree to use the extra tax revenues to bring the public debt under control. In normal times, however, legislators may genuinely disagree on the optimal level of public spending. Anticipating this disagreement, fiscal conservatives may be reluctant to increase public spending during a recession, out of fear that their liberal counterparts will veto a return to fiscal discipline when the economy improves.

¹Temporary legislation and sunset provisions—provisions attached to a legislation that set its expiration day—are the exception rather than the norm in most democracies (Tsebelis 2002, Rasch 2000). Even in the U.S., which is probably one of the country whose local and federal legislatures rely most on temporary legislation (Gersen 2007), the most important laws are typically permanent. For instance, in the fiscal shpere, the Social Security Act of 1935 and its latter expansion by the Johnson administration's Great Society programs were permanent provisions. Likewise, the U.S. Earned Income Tax Credit and its subsequent expansions in 1986, 1990, 1993, and 2001 did not have a sunset provision. See Section 6 for a discussion of temporary legislations.

Likewise, liberals may refuse to lower spending in times of economic prosperity, out of fear that conservatives will oppose a fiscal expansion when the boom is over.

In this paper, we analyze the trade-off created by the combination of a changing environment and the endogenous status quo. We show that it results in large and detrimental distortions in players' behavior, and study ways of mitigating these distortions.

In the basic model, two players engage in an infinite sequence of collective choices over two policies, called left and right. Players' preferences are unambiguously ordered along the ideological line: one player has a stronger preference for left than her opponent. Both players, however, can prefer either alternative with positive probability. In each period, the state of the economy changes and affects players' preferences. At the beginning of each period one policy, called the current status quo, is in place. If both players agree to move away from the status quo, the new policy is implemented. Otherwise, the status quo stays in place. In both cases, the implemented policy determines the players' payoffs in this period and becomes the new status quo. We are looking for the stationary equilibria of this game.

Consistent with the motivating example, we show that the endogeneity of the status quo distorts players' behavior. Each player is willing to sacrifice her current payoff to secure a favorable status quo for the next period: in any period, she votes for her preferred status quo unless the relative payoff benefit from the other policy exceeds a certain positive cutoff. In other words, each player's vote is strictly biased in favor of one alternative.

The equilibrium analysis reveals that a player's preferred status quo—and hence her voting bias—is determined not by her expected preferences, but by her expected preferences *conditional on disagreement*. For this reason, even if both players on average prefer right, in equilibrium, the most leftist player biases her vote in favor of left. In other words, players' voting biases depend not on their absolute but on their relative ideology.

This leads us to the central finding of the paper: the endogenous status quo exacerbates the ideological differences between players. To make this statement more formal, consider the alternative bargaining protocol in which the status quo is exogenously set in each period. With an exogenous status quo, today's policy has no impact on tomorrow's status quo, so players vote for their most preferred policy in every period. Therefore, they disagree only when their preferences disagree. Since under the endogenous status quo, the most leftist player biases her vote in favor of left and the more rightist player biases her vote in favor of right, as compared to the exogenous status quo protocol, the endogenous status quo protocol induces players to disagree more often. As a result, the status quo stays in place more often, and the bargaining outcome is less responsive to the environment.

We show that the polarizing effect of the endogenous status quo can be quite dramatic. Arbitrarily similar players may become very biased for opposite alternatives and behave

as if their interest were highly discordant. Moreover, if players are patient enough, the negotiations may come to a gridlock in which players vote solely along ideological lines. Despite the fact that players' preferences agree with positive probability in every period, the bargaining outcome is completely unresponsive to the preference realization. It is worth noting that this result is not a direct consequence of players' patience, but stems from the fact that players' biases reinforce each other. More patient players care more about tomorrow's status quo, which increases players' voting biases and leads to more disagreement. A greater probability of disagreement increases the importance of the status quo further, which further increases the voting biases.

In legislative bargaining, the behavior described above reminds us of what is commonly referred to as *partisanship*: each legislator votes for one particular alternative more often than is favored by her current preferences, and this bias results in more polarization. Although partisanship is often defined as a blind allegiance to a party or ideology, this paper shows that when the status quo is endogenous, a similar behavior can be generated by strategic considerations.

To assess the welfare effect of the endogeneity of the status quo, we compare the equilibrium welfare under the endogenous and exogenous status quo protocol. Our analysis shows that with an exogenous status-quo, players do not display any partisanship, the policy is more reactive to shocks, and welfare is higher.

In the legislative bargaining context, the last result provides a rationale for sunset provisions. A sunset provision is a clause that repeals a law, a tax change, or a program after a specific date, unless further legislative action is taken. Hence, an automatic sunset provision is strategically equivalent to an exogenous status quo. Sunset provisions have usually been advocated to improve parliamentary control of executive agencies, or to evaluate the efficiency of new laws. The rationale advanced by this paper has a more strategic flavor: sunset provisions sever the link between today's agreement and tomorrow's status quo, which mitigates the conflict of interest between legislators, and makes policies more responsive to the environment.

Our results extend to an N -player game with an arbitrary voting rule. Within this framework, we show that the inertial effect of the endogenous status quo depends on the dispersion of power implied by the voting rule. If a voting rule requires the approval of a larger set of players, the probability of disagreement increases; and it does so for two reasons. As in a static game, disagreement becomes more likely because more players have to agree. However, since disagreement becomes more likely, players become more partisan, which increases further the probability of disagreement. When the preference distribution is not too skewed, we show that increasing the dispersion of power is socially detrimental.

By principle, the decision process in a democracy is based on the simple majority rule, which, according to our definition, has a high concentration of power. However, in most modern democracies, legislative proposals have to pass several additional institutional hurdles to be enacted. These hurdles can take many forms, such as presidential veto, supermajority requirements, bicameralism, judicial review by a constitutional court. Any such checks and balances increase the set of players whose agreement is necessary to change the policy, and thus increase partisanship. Our analysis implies therefore that the endogenous status quo exacerbates the inertial effect of checks and balances.

The conclusions of our analysis are presented in the legislative bargaining context, but we want to stress that they apply to other environments such as renegotiation of labor or financial contracts, trade agreements, and international treaties (e.g., WTO, EU). In particular, they have implications for monetary policy institutions. In some countries, the monetary policy is set by a committee with heterogeneous preferences and beliefs, and the interest rate stays the same until the committee agrees to change it according to its internal voting rule.² Our results show that the endogeneity of the status quo, and the voting rule used in monetary policy making can greatly affect the ability of the committee to respond to economic shocks and to dampen the business cycle.

Despite its pervasiveness, the impact of the endogenous status quo in a changing environment has received little attention in the literature. This is likely due to the complexity of the strategic interactions that it generates. This paper simplifies the analysis by restricting the choice set in each period to two alternatives, which eliminates the need to specify the details of the bargaining stage game, such as the determination of the proposer or the sequence of offers. However, the central result of this paper does not rely on this restriction. As long as the bargaining stage game is such that a more rightist status quo gives a relatively more favorable bargaining position to the relatively more rightist player, the endogenous status quo will induce a rightist player to favor a more rightist agreement than what her current preferences suggest. This simple intuition suggests that the polarizing effect of the endogenous status quo should hold in richer environments.

The paper is organized as follows. Section 2 discusses the related literature. Section 3 describes the basic model. In Section 4, we solve a simple example that illustrates the main findings of the model. Section 5 formalizes these findings. Section 6 compares the equilibrium welfare with an endogenous and an exogenous status quo. Section 7 extends the model to N players. Section 8 discusses how the results extend to more general preference distributions. Section 9 concludes. All proofs are in the appendix.

²See Riboni and Ruge-Murcia (2008) for more on the role of the status quo in monetary policy institutions.

2 Related literature

The formal literature on dynamic bargaining with an endogenous status quo started with the seminal paper of Baron (1996).³ His model has been extended in various settings by Baron and Herron (2003), Kalandrakis (2004, 2007), Cho (2005), Fong (2006), Bernheim et al. (2006), Battaglini and Palfrey (2007), Anesi (2010), Diermeier and Fong (2011), Baron, Diermeier, and Fong (2011), and Zapal (2011a).⁴ These models, however, consider static environments: policies evolve over time not because preferences change, but because the set of actions available to each player varies across voting stages. Most of these papers focus on the impact of the bargaining protocol on the proposer power. We abstract away from the distributional issue of proposal power and focus instead on the efficiency and responsiveness of the policy-making process to preference shocks.⁵

It is interesting to note, though, that in our model the endogenous status quo exacerbates the conflict between players, while in some of the aforementioned models, the endogenous status quo has a moderating effect: the endogenous status quo forces the proposer to get closer to the ideal point of the median voter (Baron 1996, Barron and Herron 2003, Zapal 2011a) and reduces the incentives of voters to expropriate each other (Diermeier and Fong 2011). The following observation explains the apparent contradiction between these results and ours. In these papers, players' behavior is driven by the fear of being expropriated by the winning coalitions in the next period. With an endogenous status quo, players can minimize the cost of being excluded from the winning coalition tomorrow by implementing a moderate policy today. In our model, players behavior is driven by the fear of a status quo that is not in line with their future preferences. With an endogenous status quo, players can minimize the probability of an unfavorable status quo tomorrow by voting today for a policy that is more in line with their own ideology.

Even though dynamic bargaining with an endogenous status quo in a stochastic environment is at the center of many economically relevant situations, the existing literature on this topic is scarce. This may be a consequence of the relative intractability of these games. As Romer and Rosenthal (1978) showed in a static setup with single-peaked preferences, the induced preferences over the status quo are typically not convex, which makes the

³Epple and Riordan (1987) study a similar model but consider nonstationary equilibria. The principle of an evolving status quo was first introduced in a cooperative bargaining literature by Kalai (1977).

⁴The models of Bernheim et al. (2006) and Diermeier and Fong (2011) are originally cast in a single policy period, but they can be extended or interpreted as dynamic legislative bargaining games.

⁵Because most of these models consider the division of a pie of exogenous size or single-peaked preferences, equilibrium outcomes are always efficient in a static sense and can be inefficient in a dynamic sense only when citizens are risk-averse. In contrast, when preferences vary as in our model, equilibrium outcomes are typically Pareto inefficient even in a static sense, independently of risk aversion.

multi-period extension technically hard to analyze. With a continuum of alternatives and an infinite horizon, the existence of the stationary equilibrium is not guaranteed even under standard preference specifications.⁶ To the best of our knowledge, only Diermeier and Fong (2008), Riboni and Ruge-Murcia (2008), Duggan and Kalandrakis (2009), and Zapal (2011b) make progress on this front. Adding noise to the status quo, Duggan and Kalandrakis (2009) establish the existence of an equilibrium. The generality of their model does not allow an analytical characterization of the equilibrium, so they resort instead to numerical methods. Riboni and Ruge-Murcia (2008) analyze a game with quadratic utility functions and a finite state space. They analytically solve a two-period two-state example, but use numerical solutions for the general model. Diermeier and Fong (2008) analyze a two-period three-state model with a richer institutional framework. Finally, Zapal (2011b) characterizes the infinite horizon equilibrium in a symmetric two-state case.⁷ Our paper differs from these contributions in that we simplify the space of alternatives, but fully characterize the policy dynamics with an infinite bargaining horizon for any preference distributions. Moreover, our institutionally sparse model allows us to isolate the effect of the endogeneity of the status quo in a transparent way.

Montagnes (2010) looks at a two-period financial contracting environment in which the current contract serves as the default option in future negotiations. He shows that both contracting parties may prefer to commit ex ante to ceding a future decision power. Such a commitment breaks the dynamic linkage and avoids ex-ante inefficiencies.

Fernandez and Rodrik (1991) and Alesina and Drazen (1991) have emphasized that the distributional uncertainty of policy reforms can lead to status quo inertia. In our model, it is not the uncertainty but the evolution of preferences over time that drives the result.

Our results on policy responsiveness to shocks are related to the political economy literature on growth and the dynamics of welfare policies.⁸ In this literature, the current policy affects future preferences (via private or public investment decisions). This dynamic linkage can generate policy persistence. In contrast, in our paper, the implemented policy does not affect future preferences, but inertia emerges because today's policy affect players' position for future bargaining.

Our results on the effect of the concentration of voting power contrast with the literature on distributive politics. As Buchanan and Tullock (1962) and Riker (1962) first argued,

⁶See, e.g., Kalandrakis (2004, 2007) or Duggan and Kalandrakis (2009) for more on this issue.

⁷In line with our results, Zapal (2011b) shows that under the endogenous status quo protocol, the policy may remain constant even though preference evolve over time. However, contrary to our setup, for the particular reference distribution he considers, a constant policy is socially optimal.

⁸See, among others, Glomm and Ravikumar (1995), Krussell and Rios-Rull (1996,1999), Coate and Morris (1999), Saint Paul and Verdier (1997), Benabou (2000), Saint Paul (2001), Hassler et al. (2003, 2005), Battaglini and Coate (2007, 2008), and Prato (2011).

majoritarian decision making allows the concentration of benefits and the collectivization of costs, and thus leads to the adoption of inefficient pork-barrel programs.⁹ Contrary to our model, the concentration of power exacerbates these perverse incentives, and efficiency is restored only when unanimity rule is used. Battaglini and Coate (2007, 2008) extend this framework to a dynamic legislative model of public finance. The availability of pork-barrel programs leads the minimal winning coalition to pass inefficient budgets and be present-biased, more so the lower the supermajority requirement. In our model, the continuing nature of policies lead voters to be future-biased, more so the larger the supermajority requirement.

Finally, Casella (2005) shows that linking voting decisions across time allows voters to express their preference intensity, which can be socially beneficial. Our results suggests that the endogeneity of the status quo, despite the pervasiveness of this institution, is not an efficient way to elicit preference intensity. Barbera and Jackson (2010) let ex ante identical voters choose the group decision rule after having learned their first period preferences. As in our framework, bundling the current and the future decision rules generate inefficiencies. But since the dynamic linkage is only between the first and the subsequent periods, sufficiently patient players always select the optimal voting rule.

3 The model

Two players, i and j , are in a relationship that lasts for infinitely many periods. In each period t , players adopt one of two alternatives, $y^t \in \{L, R\}$. The utility of player $k \in \{i, j\}$ in period t depends on the alternative adopted in period t and is given by

$$u(\theta_k^t, y^t) = \begin{cases} \theta_k^t & \text{if } y^t = R \\ -\theta_k^t & \text{if } y^t = L \end{cases} . \quad (1)$$

Hence, if θ_k^t is positive (negative), player k prefers alternatives R (L) to be implemented in period t . The realization of (θ_i^t, θ_j^t) summarizes players' preferences over the current policy, so we refer to θ_k^t as player k 's *current preference* in period t .

In each period, a state s is drawn from a finite set S . The process $(s^t)_{t \geq 1}$ is Markov with a stationary and irreducible transition matrix.¹⁰ The probability of moving from state $s \in S$ to state $s' \in S$ is denoted by $\pi(s, s')$. In each period, if the state is s , the preference

⁹Ferejohn, Fiorina, and McKelvey (1987), Baron and Ferejohn (1989), and Baron (1991, 1993) first formalized this prediction in models of legislative bargaining.

¹⁰A Markov process is irreducible if the probability of going from any state to any other state in a finite number of periods is strictly positive. It is stationary if the transition probabilities do not depend on time.

equilibria in stage-undominated strategies (henceforth equilibria) as defined in Baron and Kalai (1993).¹¹ As shown in Baron and Kalai (1993), these equilibria have a focal point property that derives from their simplicity. In the legislative sphere, the stationarity assumption can be justified on the grounds that the game is played by a sequence of legislators who are never certain to be reelected. In such cases, the institutional memory required for more sophisticated nonstationary equilibria involving infinitely nested punishment strategies may be inappropriate. Stage-undomination is a standard equilibrium refinement in voting games, which basically amounts to assuming that in every period, players cast their votes as if they were pivotal. This refinement rules out pathological equilibria such as both players always voting for the status quo.¹²

A few comments on the modeling assumptions are in order. First, our setup allows the preferences to be correlated across players and across time. The stationarity of the preference distribution is a simplifying assumption which is consistent with the recurring nature of the shocks that affect issues such as taxation, public spending, immigration, or civil liberties (e.g., economic cycles, demographic transitions, public opinion swings, or national security threats). Second, we analyze a two-player game with a unanimity requirement to change the status quo, but in section 7 we show that our results extend to an N -player game with a large class of voting rules. Third, restricting attention to two alternatives allows us to abstract away from the details of the stage game and the issue of proposal power.¹³ It thereby allows us to isolate the effect of the endogeneity of the status quo on the equilibrium outcomes in a transparent way. Fourth, what players know about each other's current preferences is immaterial. Finally, we assume that today's action has no impact on tomorrow's preferences (that is, π does not depend on the status quo q) because this dynamic linkage has already received some attention in the dynamic political economy literature (see the literature review in section 2). Ruling it out allows us to isolate the effect of the endogenous status quo.

¹¹Stage undominated stationary equilibria, or variants of it, are used in almost all of the infinite horizon models cited in this paper. The only exception that we are aware of is Epple and Riordan (1987), and Baron and Ferejohn (1989). Both papers prove results that have the flavor of the folk theorem in repeated game theory.

¹²Moreover, the equilibria eliminated by this refinement hinge on details of the bargaining protocols which are difficult to map to reality. For instance, they would disappear if we assumed instead that players vote sequentially. See, e.g., Acemoglu, Egorov, and Sorin (2009).

¹³With two alternatives, many static bargaining protocols are equivalent. In particular, using standard equilibrium concepts, equilibrium outcomes are the same when players vote simultaneously or sequentially, when they make take-it-or-leave-it offers, or when we allow for n rounds of bargaining within each period with either a random or alternating proposer.

4 An example

We start by solving a simple example that illustrates the workings of the model. We formalize all observations of this example later in the paper.

Assume that $|S| = 1$, $\theta_i^t = \bar{\theta}_i + \varepsilon^t$ and $\theta_j^t = \bar{\theta}_j + \varepsilon^t$, where the sequence of random variables $(\varepsilon^t)_{t \geq 1}$ is i.i.d. over time, and for each t , $\varepsilon^t \sim N(0, 1)$. Hence, players' preferences are perfectly correlated, ε^t is the common shock, and $\bar{\theta}_i$ and $\bar{\theta}_j$ are the expected preference of player i and j , respectively. Since $|S| = 1$, the initial conditions boils down to the initial status quo q , so we can simplify the notations and denote the game with an endogenous and an exogenous status quo by Γ_q^{en} and Γ_q^{ex} , respectively.

Let us first derive the equilibrium characterization of Γ_q^{en} with a simple heuristic reasoning. For any player $k \in \{i, j\}$, the policy implemented in period t impacts player k 's payoff via two channels. First, it affects her current payoff θ_k^t . Second, it determines the future status quo. Let $V_k(q)$ be the continuation value for player $k \in \{i, j\}$ when the status quo is q . Since player k votes as if she were pivotal, in period t she votes for R if

$$\theta_k^t + \delta V_k(R) > -\theta_k^t + \delta V_k(L),$$

and for L if the reverse inequality holds. Therefore, she uses a cutoff strategy with the cutoff

$$c_k = \frac{\delta}{2} (V_k(L) - V_k(R)). \quad (2)$$

Observe that future payoffs depend on the current status quo only if players disagree in the next period. Disagreement in turn happens when players' preferences θ_i^t and θ_j^t are on opposite sides of their respective cutoffs c_i and c_j . Hence, we can rewrite the right-hand side of (2) as follows:

$$c_k = \frac{\delta}{2} \left(\int_{-\infty}^{c_j} \int_{c_i}^{\infty} (-\theta_k + \delta V_k(L) - (\theta_k + \delta V_k(R))) f(\boldsymbol{\theta}) d\theta_i d\theta_j + \int_{c_j}^{\infty} \int_{-\infty}^{c_i} (-\theta_k + \delta V_k(L) - (\theta_k + \delta V_k(R))) f(\boldsymbol{\theta}) d\theta_i d\theta_j \right).$$

Substituting (2) inside the integral in the above equation, we obtain that the equilibrium cutoffs solve the following fixed point problem:

$$c_k = \delta \left(\int_{-\infty}^{c_j} \int_{c_i}^{\infty} (c_k - \theta_k) f(\boldsymbol{\theta}) d\theta_i d\theta_j + \int_{c_j}^{\infty} \int_{-\infty}^{c_i} (c_k - \theta_k) f(\boldsymbol{\theta}) d\theta_i d\theta_j \right). \quad (3)$$

One can interpret the equilibrium cutoffs as *voting biases*. With positive probability, a player with a positive (negative) cutoff votes for L (R) even though her current preferences favor R (L). Players' voting behavior is biased because they trade off implementing a policy that is optimal according to their current preferences against securing a favorable status quo given their expected future preferences. From (2), we see that the sign of c_k determines whether player k prefers the status quo to be R (c_k negative) or L (c_k positive), and the absolute value of c_k measures the intensity of this preference. Hence, $\theta_k - c_k$ measures player k 's intertemporal preferences over the current policy. Equation (3) shows that the voting bias of each player is given by her expected intertemporal preferences in the next period *conditional on disagreement*.

By comparison, with an exogenous status quo, the only equilibrium would have both players vote myopically for their most preferred policy in every period, because the policy in a given period has no effect on future periods. This means that in Γ_q^{ex} , the equilibrium cutoffs would be 0. Therefore, the sign and magnitude of the equilibrium cutoffs in Γ_q^{en} completely capture the effect of the endogeneity of the status quo on players' voting behavior.

We solve Equation (3) numerically for $\delta = 0.9$ and $\bar{\theta}_i = 0.5$, while varying $\bar{\theta}_j$. When $\bar{\theta}_j = -0.5$, then the resulting cutoffs are $c_j \approx 4.49$ and $c_i \approx -4.49$. When $\bar{\theta}_j = 0.1$, players are less polarized, and the resulting cutoffs are $c_j \approx 0.04$ and $c_i \approx -0.038$.

We would like to point out a few features of this example. First, with both preference distributions, $c_i < 0 < c_j$. Hence, player i is biased for R while player j is biased for L . Notice that this happens even though in the case $(\bar{\theta}_i, \bar{\theta}_j) = (0.5, 0.1)$, both players prefer R on average. The reason for this is that players' voting biases reflect their preferences over next period's status quo, and the status quo matters only in case of disagreement. In both cases, $\bar{\theta}_i > \bar{\theta}_j$, so when players disagree, i prefers R while j prefers L . Hence, the direction of players' voting biases is determined not by their absolute but by their relative ideology.

Second, $c_i < 0 < c_j$ means that the relatively more rightist player i behaves as if she was even more rightist while the relatively more leftist player j behaves as if she was even more leftist. Hence, the voting cutoffs act as a polarization-magnifying preference shift. This implies that players disagree more often than their actual preferences do. More precisely, players' current preferences disagree when $\varepsilon^t \in (-\bar{\theta}_i, -\bar{\theta}_j)$, while at the equilibrium of Γ_q^{en} , players vote for opposite alternatives when $\varepsilon^t \in (-\bar{\theta}_i + c_i, -\bar{\theta}_j + c_j)$. Since in Γ_q^{ex} , players disagree only when their current preferences disagree, this example shows that the endogeneity of the status quo increases the probability that players disagree in equilibrium.

Third, note that this polarizing effect can be large: for example when $(\bar{\theta}_i, \bar{\theta}_j) = (0.5, 0.1)$, in any period, the probability that players' preferences disagree is 0.15, while the probability that players vote for opposite alternatives in Γ_q^{en} is 0.3. When $(\bar{\theta}_i, \bar{\theta}_j) = (0.5, -0.5)$, these

probabilities are 0.38 and ≈ 1 respectively.

Finally, by comparing the equilibrium at the two preference distributions, we see that the voting biases increase with the preference polarization: as we make the leftist players even more leftist ($\bar{\theta}_j$ decreases), i 's bias in favor of right ($-c_i$) and j 's bias in favor of left (c_j) increase. Hence, more polarized players disagree more often not only because their preferences are farther apart, but also because their preference polarization is magnified by their voting behavior. In fact, as we increase δ to 1, for $(\bar{\theta}_i, \bar{\theta}_j) = (0.5, -0.5)$, the voting cutoffs increase to $c_i = -\infty$ and $c_j = \infty$. That is, as players become very patient, the negotiations come to a complete gridlock: players always vote for opposite alternatives even though their preferences agree with positive probability in every period.

The following observation is key to understanding the magnitude of the equilibrium cutoffs in Γ_q^{en} . Even if players expect that their opponent will use a 0 cutoff, they still expect some disagreement. Hence, player j prefers to use a positive cutoff to defend L as a status quo, and player i prefers to use a negative cutoff to defend R as a status quo. But these non-zero cutoffs increase the probability of disagreement, and thus increase the probability that the status quo will stay in place, which in turn makes defending the status quo even more important. Realizing that, each player has an incentive to become even more biased, which increases the probability of disagreement, which again increases the incentive to become more biased, and so on and so forth. In other words, the polarizing effect of the endogenous status quo feeds on itself because players' biases reinforce each other.

Whenever the voting cutoffs are nonzero, players implement Pareto dominated alternatives with positive probability. For example, when $q^t = R$, $\theta_j^t < 0$, and $c_i < \theta_i^t < 0$, player i vetoes the Pareto optimal alternative L . This suggests that the endogeneity of the status quo is socially detrimental. To confirm that intuition, we compare the equilibrium welfare in Γ_q^{ex} and in Γ_q^{en} . For example, when $(\bar{\theta}_i, \bar{\theta}_j) = (0.5, -0.5)$ and δ is sufficiently large, both players prefer to bargain with an exogenous status quo. Another way to mitigate the effect of the endogenous status quo is to let one player be the dictator, because under dictatorship, the status quo is irrelevant. It turns out that when $\bar{\theta}_j = -0.5$, both players would prefer to cede their veto power and let their opponent be the dictator instead of playing Γ_q^{en} .¹⁴

5 The equilibrium

The following proposition characterizes the equilibria of the game Γ_{q^S, π_0}^{en} for general preference distributions.

¹⁴See Example 5 in Dziuda and Loeper (2010) for a formal proof of this result in that example.

Proposition 1 *In any equilibrium of Γ_{q^s, π^0}^{en} , the players use state-dependent, status quo-independent cutoff strategies: there exists $\mathbf{c}^S \in \mathbb{R}^{2S}$ such that in state $s \in S$, player $k \in \{i, j\}$ votes for R if $\theta_k > c_k^s$ and for L if $\theta_k < c_k^s$.*

The equilibrium cutoffs are the fixed points of the mapping \mathbf{H}^S defined as follows: for all $s \in S$ and all $\mathbf{c}^S \in \mathbb{R}^{2S}$,

$$\begin{aligned} \mathbf{H}^s(\mathbf{c}^S) = \delta \sum_{s' \in S} \pi(s, s') & \left(\int_{-\infty}^{c_j^{s'}} \int_{c_i^{s'}}^{\infty} (\mathbf{c}^{s'} - \boldsymbol{\theta}) f^{s'}(\boldsymbol{\theta}) d\theta_i d\theta_j \right. \\ & \left. + \int_{c_j^{s'}}^{\infty} \int_{-\infty}^{c_i^{s'}} (\mathbf{c}^{s'} - \boldsymbol{\theta}) f^{s'}(\boldsymbol{\theta}) d\theta_i d\theta_j \right). \end{aligned} \quad (4)$$

The set of equilibrium cutoffs is a complete lattice for the partial order $(\leq, \geq)^S$ defined as follows: for all $\mathbf{c}^S, \mathbf{d}^S \in \mathbb{R}^{2S}$, $\mathbf{c}^S (\leq, \geq)^S \mathbf{d}^S$ if for all $s \in S$, $c_i^s \leq d_i^s$ and $c_j^s \geq d_j^s$.

Consistent with our example, players use cutoff strategies. However, since the continuation of the game in general depends on the state, cutoffs are state-dependent. Equation (4) says that the cutoffs of player k are given by the expected intertemporal preference of player k , $c_k^s - \theta_k$, conditional on players disagreeing in the next period, i.e., conditional on $c_i^s - \theta_i$ and $c_j^s - \theta_j$ being of opposite sign.

The cutoff structure of the equilibria holds for any preference distribution. However, the properties of the voting cutoffs depend on the latter. For the clarity of exposition, in this section, we will make the following assumption:

Assumption 1 *In all states $s \in S$, $\theta_i \geq \theta_j$ with probability 1, and in some state $s' \in S$, θ_i and θ_j are of opposite sign with positive probability.*

Assumption 1 has a natural interpretation in political economy or monetary policy applications: players can be unambiguously ranked on the ideological spectrum. Player i is always more rightist than player j (there is no preference reversal), and players disagree with positive probability.¹⁵ Note, however, that this assumption imposes no restriction on the preference distribution of a single player nor on the severity of the conflict of interest between players: both players might prefer policy L arbitrarily often in some state s and policy R arbitrarily often in another state s' . Assumption 1 is relaxed in Section 8.

¹⁵If players disagreed with probability 0, there would be no conflict of interest, and the bargaining situation would be trivial. All our results, however, would still hold, but the strict inequalities would have to be replaced by weak ones.

5.1 Voting biases and disagreement

The following proposition states that the more leftist player is always biased in favor of L and the more rightist player is always biased in favor of R .

Proposition 2 *In all equilibria of Γ_{q^S, π^0}^{en} , for all $s \in S$, $c_i^s < 0$ and $c_j^s > 0$.*

To understand the consequences of the endogeneity of the status quo, we compare players' behavior under the bargaining protocol Γ_{q^S, π^0}^{en} to players' behavior under its natural alternative: the bargaining protocol Γ_{q^S, π^0}^{ex} in which the (possibly state dependent) status quo is exogenously fixed at q^S in every period. As in the example in Section 4, the exogenous status quo protocol is of particular interest because it severs the link between today's policy and tomorrow's status quo. Hence, in the game Γ_{q^S, π^0}^{ex} , the players consider each period in isolation and vote according to their current preferences. This observation is summarized in the following remark.

Remark 1 *The game Γ_{q^S, π^0}^{ex} has a unique equilibrium. In that equilibrium, the players use voting cutoffs $c_i^s = c_j^s = 0$ in all states and all periods.*

As stated in the following corollary, the comparison of Γ_{q^S, π^0}^{en} with Γ_{q^S, π^0}^{ex} delivers the main qualitative insight of this paper: the endogenous status quo amplifies the ideological differences between players and leads to more disagreement.

Corollary 1 *The endogenous status quo increases the probability of disagreement and hence the status quo inertia: for all π^0 , q^S , and q^{tS} , at any equilibrium, the probability that players vote for opposite alternatives in some period t with some state s is higher in Γ_{q^S, π^0}^{en} than in $\Gamma_{q^{tS}, \pi^0}^{ex}$.*

To understand the above corollary, note that Proposition 2 and Remark 1 imply the following. If the status quo is exogenous, in a given period t , players disagree when

$$\theta_j^t \leq 0 \leq \theta_i^t.$$

If instead the status quo is endogenous, and if the current state in period t is s , players disagree when

$$\theta_j^t - c_j^s \leq 0 \leq \theta_i^t - c_i^s.$$

Since $c_i^s \leq 0$ and $c_j^s \geq 0$, these two expressions imply that the set of preference realizations for which players disagree and the status quo stays in place is greater under the endogenous status quo.

The equilibrium behavior of the players in Γ_{q^S, π^0}^{en} reminds us of what is commonly referred to as *partisanship*. Oxford Dictionaries define partisanship as prejudice in favour of a particular cause; a bias. In multi-party systems, this term carries a negative connotation—it refers to those who wholly support their party’s policies and are reluctant to acknowledge any common ground with their political opponents. This definition resonates with the players’ voting behavior: each player favors a distinct alternative for which she votes more often than her current preferences justify, and this in turn leads to more disagreement. This model shows that when the status quo is endogenous, partisanship can be generated by strategic considerations.

Definition 1 *The partisanship of player $k \in \{i, j\}$ in state $s \in S$ is $|c_k^s|$.*

5.2 The magnitude of partisanship

The game Γ_{q^S, π^0}^{en} might have multiple equilibria. Multiplicity is driven by the fact that partisanship feeds on itself. If players expect their opponent to be very partisan, they expect to disagree often. Therefore, defending the correct status quo becomes very important, which means that players will vote in a very partisan way. By the same token, if players expect a low degree of partisanship, the status quo matters less, and players do not need to be very partisan.

As Proposition 1 shows, this strategic complementarity implies that the equilibrium cutoffs have a lattice structure. From Proposition 2, for each player, the sign of cutoffs is the same across equilibria. Therefore, there exists a least and a most partisan equilibria. The following proposition further shows that the partisanship ranking coincides with the Pareto order: more partisan equilibria are Pareto worse.

Proposition 3 *Let \mathbf{c}^S and \mathbf{d}^S be the cutoffs of two equilibria of Γ_{q^S, π^0}^{en} . Under Assumption 1, if $\mathbf{c}^S (\leq, \geq)^S \mathbf{d}^S$, then \mathbf{d}^S Pareto dominates \mathbf{c}^S . In particular, the least and the most partisan equilibria, i.e., the least and the greatest equilibria for the order $(\leq, \geq)^S$, are the Pareto best and worst equilibria, respectively.*

When deriving comparative statics and determining the magnitude of partisanship, we use Pareto efficiency as a selection criterion and focus on the least partisan equilibrium.¹⁶ We want to stress, however, that the exact same comparative statics holds for the most partisan equilibrium.

¹⁶An additional support for this equilibrium selection can be found in Dziuda and Loeper (2010, proposition 2), where it is shown that the least partisan equilibrium is the limit of the finite horizon version of the game Γ_{q^S, π^0}^{en} as the bargaining horizon goes to infinity.

The following definition will be helpful when deriving comparative statics with respect to the preference distribution f^S :

Definition 2 *Let f^S and g^S be two preference distributions. The distribution f^S is more polarized than g^S if there exists a random variable ε^S with support on $(\mathbb{R}_+ \times \mathbb{R}_-)^S$ and a random variable θ^S such that the p.d.f. of θ^S and $\theta^S + \varepsilon^S$ is g^S and f^S , respectively.*

We use the terminology “more polarized” because if g^S satisfies Assumption 1, the preference distribution f^S is more polarized than g^S if f^S can be obtained from g^S by shifting the preferences of the rightist player farther to the right and the preferences of the leftist player farther to the left.

Denote by $\mathbf{c}^S(\delta, f^S)$ the cutoffs in the least partisan equilibrium of Γ_{q^S, π_0}^{en} with a discount factor δ and a distribution of preference f^S . The next proposition shows how partisanship varies with the main preference parameters:

Proposition 4 *In the least partisan equilibrium,*

- a) *partisanship increases with patience: $\mathbf{c}^S(\delta, f^S)$ is increasing in δ in the order $(\leq, \geq)^S$;*
- b) *partisanship increases with the polarization of preferences: if f^S is more polarized than g^S , then $\mathbf{c}^S(\delta, f^S) (\leq, \geq)^S \mathbf{c}^S(\delta, g^S)$.*

The intuition for part (a) is that when players trade off the adequacy of the policy to the current environment versus securing a favorable status quo for tomorrow, more patient players put more weight on the latter, and thus are more partisan. As for part (b), the preferences of more polarized players are more likely to disagree, which makes the status quo more important, and thus increases partisanship. This result reinforces the findings of Corollary 1 in that it shows that status-quo endogeneity exacerbates the ideological differences between players, more so, the more polarized the players.

The next proposition further shows that the polarizing effect of the endogenous status quo can be dramatic.

Proposition 5 *There exists a preference distribution g^S such that for all f^S which are more polarized than g^S , $\lim_{\delta \rightarrow 1} \mathbf{c}^s(\delta, f^S) = (-\infty, +\infty)$, for all $s \in S$.*

Proposition 5 states that when players are sufficiently polarized and patient, their partisanship can lead to complete gridlock. Even though in all periods, players agree with

positive probability, they always vote for opposite alternatives. As a result, the policy is totally unresponsive to the shocks to the environment.¹⁷

Observe that this result is not a mechanical consequence of increasing patience. The alternative adopted in period t impacts players' payoff in some subsequent period t' only if players' preferences disagree for all periods between $t + 1$ and t' , which for any finite level of partisanship happens with a probability smaller than 1. Hence, the difference in continuation value induced by different status quos stays finite even as $\delta \rightarrow 1$. For this reason, irrespective of the players' patience, the best response of a player to a finite level of partisanship of her opponent is also a finite level of partisanship. What drives the completely unresponsive behavior of patient players is the vicious cycle in which patience increases partisanship, partisanship then increases the life expectancy of the status quo, and this in turn increases partisanship.

Proposition 5 does not state how polarized the preference distribution g^S must be. The following corollary shows that gridlock can arise with quite modest degree of preference polarization.

Corollary 2 *Let $|S| = 1$, $\theta_i = \bar{\theta} + \varepsilon$ and $\theta_j = -\bar{\theta} + \varepsilon$, where $\bar{\theta} \in \mathbb{R}^+$ measures the players' ideological polarization and ε is a random shock with distribution symmetric around 0. If $\bar{\theta} \geq E(|\varepsilon|)$, then $\lim_{\delta \rightarrow 1} \mathbf{c}(\delta, f^S) = (-\infty, +\infty)$.*

The condition in Corollary 2 is only sufficient, and one can relax it by looking at particular classes of preference distribution. For instance, if we assume, as in the example solved in Section 4, that $\varepsilon \sim N(0, 1)$, then gridlock occurs when $\bar{\theta} \geq 0.35$. At $\bar{\theta} = 0.35$, even though players' preferences agree with probability 0.73, as they become very patient their voting behavior almost always disagree.

6 Welfare Analysis

In this section, we compare the expected level of the utilitarian welfare $W\left(\Gamma_{q^S, \pi^0}^{en}\right)$ in the endogenous status quo game Γ_{q^S, π^0}^{en} to the expected level of the utilitarian welfare $W\left(\Gamma_{q^S, \pi^0}^{ex}\right)$ in the game Γ_{q^S, π^0}^{ex} in which the status quo is set exogenously in each period (see Section 3 for a formal definition of Γ_{q^S, π^0}^{ex}).

This comparison is relevant for two reasons. First, from a theoretical point of view, the exogenous status quo is the natural alternative to the endogenous status quo in the sense

¹⁷Assumption 1 is not needed for Proposition 5, and g^S can have full support. More precisely, we show in the appendix that for any preference distribution g^S , there exists $\mathbf{m}^o \in \mathbb{R}^2$ such that for all $\mathbf{m}(\geq, \leq) \mathbf{m}^o$, the preference distribution $g_{\mathbf{m}}^S$ defined by $g_{\mathbf{m}}^S(\boldsymbol{\theta} + \mathbf{m}) = g^S(\boldsymbol{\theta})$ is such that for all $s \in S$ $\lim_{\delta \rightarrow 1} \mathbf{c}^s(\delta, g_{\mathbf{m}}^S) = (-\infty, +\infty)$.

that it is the simplest protocol that breaks the linkage between today’s policy and tomorrow’s status quo. Second, even though the protocol of the endogenous status quo is prevalent in many dynamic bargaining settings (legislative bargaining, trade agreements at the W.T.O., monetary policy in the U.S.), the exogenous status quo is the most common alternative.¹⁸ For instance, the permanent provisions of the Agricultural Adjustment Act of 1938 and the Agriculture Act of 1949 serve as a fixed status quo for U.S. farm bills (Kwan 2009). Bilateral international agreements implicitly have an exogenous status quo of no agreement because either country can unilaterally opt out. In the U.S. budget process, federal spending is divided into two categories. One—called mandatory spending—continues year after year by default. The other one—called discretionary spending—requires annual appropriation bills, which means that the status quo is exogenously fixed at zero.¹⁹

In the legislative sphere, breaking the link between today’s policy and tomorrow’s status quo can also be achieved via sunset provision. A sunset provision is a clause that specifies a duration after which an act is repealed, unless further legislative action is taken. When it is automatic, a sunset provision is strategically equivalent to the exogenous status quo. An example of automatic sunset provisions are the sunset legislations in 24 U.S. states that require automatic termination of a state agency, board, commission, or committee.²⁰

The utilitarian welfare under the endogenous and the exogenous status quo protocols differ for two reasons. First, the endogenous status quo creates partisanship, while the exogenous status quo does not (see Remark 1). Partisanship, in turn, is detrimental to welfare as Pareto dominated alternatives are implemented with positive probability; for example, when $q^t = R$, $\theta_j^t < 0$, and $c_i^s < \theta_i^t < 0$, player i vetoes the Pareto optimal alternative L . Second, these two protocols induce a different distribution over the status quo in each period. However, an exogenous status quo—if carefully set in each state—can do at least as good as the endogenous status quo protocol. The next proposition formalizes this observation.

Proposition 6 *There exists $q^{tS} \in \{R, L\}^S$ such that for all $\pi^0 \in \Delta(S)$, $q^S \in \{R, L\}^S$, and $\delta \in (0, 1)$, $W\left(\Gamma_{q^S, \pi^0}^{en}\right) < W\left(\Gamma_{q^{tS}, \pi^0}^{en}\right)$.*

¹⁸See Lowi (1969), Weaver (1985, 1988), Hird (1991), and Gersen (2007) for more on the ongoing and temporary nature of the policies enacted by the U.S. congress. @@@Add Tsebelis@@@

¹⁹Mandatory spending, also called direct spending, consists almost entirely of entitlement programs such as Social Security benefits, Medicare and Medicaid. Discretionary spending includes the budgets of federal agencies (defense, national parks...) and pork barrel projects. Mandatory spending currently represents about two thirds of the federal budget.

²⁰See The Book of the States, 2011, Council of State Governments. See Kearney (1990) for more on the use of sunset provisions by US state legislatures. In the U.S., automatic sunset clauses are less common at the federal level, although there has been attempts to introduce them systematically in Congress (the Federal Sunset Act). In the budget process, the Byrd rule is equivalent to imposing an automatic sunset clause on any provision that increases the deficit and that does not garner a filibuster proof majority.

Proposition 6 provides an argument in favor of the exogenous status quo protocol and automatic sunset provisions. Though we are not the first ones to advocate the use of sunset provisions, the rationale behind our recommendation is novel. Sunset legislations have traditionally been advocated for two reasons: to improve the legislative oversight of executive agencies and regulations through periodic reviews, and to ensure ex-post evaluation of policies with uncertain effects. The argument advanced by our model has instead a more strategic underpinning: by severing the link between today's decision and tomorrow's status quo, sunset provisions decrease the partisanship of the supporters and detractors of a policy and make its enactment and repeal more responsive to the current situation.

In light of Proposition 6, it may seem surprising that the exogenous status quo is not more commonly used. A natural explanation for this disconnect is political economy considerations. Proposition 6 provides a normative result but is mute on individual preferences over bargaining protocols. In fact, there might not exist an exogenous status that Pareto improves on the endogenous status quo. Moreover, even when a Pareto improving exogenous status quo exists, for many ideologically charged policies such as income taxation, immigration, or hand-weapon regulation, different political actors will favor different exogenous status quos. A careful analysis of the negotiations over bargaining protocols is left for future research, but it should be clear that players' disagreement over what the ideal exogenous status is might prevent them from changing the bargaining protocol.

Finally, we would like to emphasize that Proposition 6 requires the exogenous status quo to be state dependent. This means that in order to improve over the endogenous status quo, the exogenous status quo may need to depend on the variables that affect players' preferences. For instance, in the case of monetary policy, an optimal status quo should depend on the unemployment rate and the inflation rate.²¹ In the case of welfare policies, an optimal exogenous status quo may need to be tied to the level of fiscal revenues, the number of recipients, the cost of living, or to business cycle indicators (e.g., the growth rate, the capacity utilization rate). Since the aforementioned variables are verifiable, and many countries already use them explicitly in the policy making process, the requirement that the status quo be state dependent is not unrealistic in these cases.²²

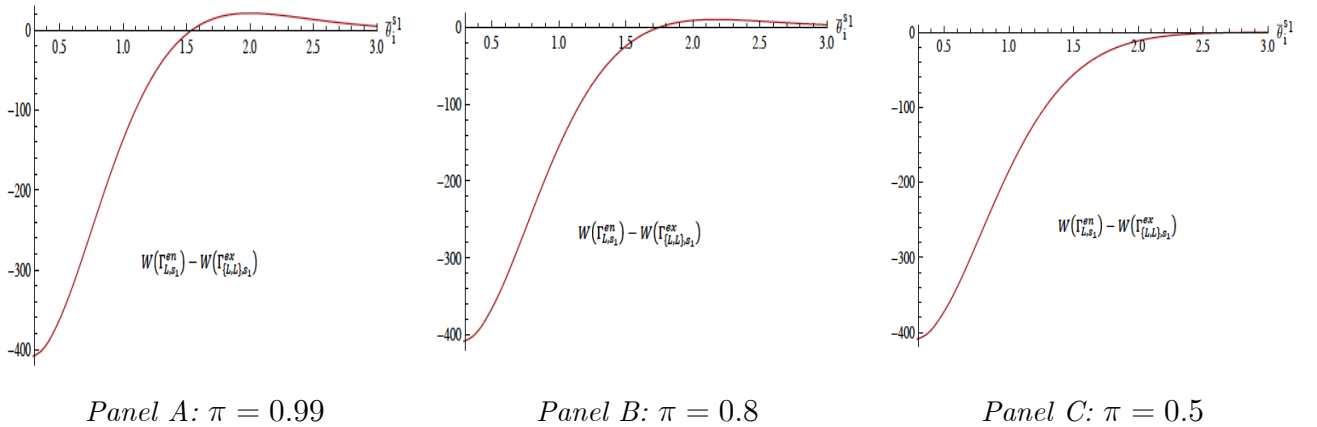
²¹The Taylor rule provides an example of a monetary policy that depends mechanically on observable variables. This rule, first introduced by Taylor (1993), ties the change in the nominal interest rate to the variations of the output, unemployment, and inflation rate.

²²For instance, welfare and retirement policies are typically set in terms of the level of individual benefits and the eligibility requirements, so the aggregate level of spending mechanically depends on the unemployment rate, the income distribution, and the age distribution. Most countries also index the benefits' level to the cost of living. Some countries tie pension benefits to the return of the pension fund (Netherlands), and other countries tie the eligibility criteria to the life expectancy (Sweden and France). See Bikker and Vlaar (2007).

However, for some policies, the relevant state variables are not verifiable. Consider for instance the case of national security. The need to restrict civil liberties to protect national security depends on the likelihood of future threats, but the latter can hardly be measured in an impartial way. When it is impossible to implement a state-dependent status quo, the welfare comparison with the endogenous status quo becomes ambiguous. Example 1 compares the welfare under the endogenous status quo with welfare under the state-independent exogenous status quo. It shows that when it is optimal for the status quo to vary across states, the endogenous status quo may dominate the exogenous status quo if the latter needs to be state independent.

Example 1 We extend the example in Section 4 to two states as follows: $S = (s_1, s_2)$, $\pi_{s_1, s_1} = \pi_{s_2, s_2} \equiv \pi \geq \frac{1}{2}$ and $\theta_i^s = \bar{\theta}_i^s + \varepsilon$ and $\theta_j^s = \bar{\theta}_j^s + \varepsilon$, where $\varepsilon \sim N(0, 1)$. That is, the two states are somewhat persistent and players' preferences are perfectly correlated. Players preference in s_1 are such that $\bar{\theta}_i^{s_1} > \bar{\theta}_j^{s_1}$, $\bar{\theta}_i^{s_1} + \bar{\theta}_j^{s_1} \geq 0$, and s_2 is the symmetric of s_1 : $\bar{\theta}_i^{s_2} = -\bar{\theta}_j^{s_1}$ and $\bar{\theta}_j^{s_2} = -\bar{\theta}_i^{s_1}$. Hence, player i is more rightist than player j in both states, but in state s_1 , when players' preferences disagree, R is socially better, so R is the socially optimal status quo in s_1 . Conversely, L is the socially optimal status quo in s_2 .

The following figure compares utilitarian welfare in the least partisan equilibrium of Γ_{q^s, π^0}^{en} and Γ_{q^s, π^0}^{ex} , where $\pi^0(s_1) = 1$ and $q^s = L$ in both states. We fix players' initial polarization $\bar{\theta}_i^{s_1} - \bar{\theta}_j^{s_1}$ at 0.5 and let $\bar{\theta}_i^{s_1}$ vary. Note that given our assumptions, when $\bar{\theta}_i^{s_1} = 0.25$, both states are identical. As $\bar{\theta}_i^{s_1}$ increases, the preferences of both players move to the right in s_1 and to the left in s_2 : the states becomes less similar, but in each state, the probability of preference disagreement decreases. Each panel depicts $W(\Gamma_{L, s_1}^{en}) - W(\Gamma_{\{L, L\}, s_1}^{ex})$ as a function of $\bar{\theta}_i^{s_1}$ for $\pi = 0.99, 0.8$, and 0.5 .



All panels show that the exogenous status quo dominates for small $\bar{\theta}_i^{s_1}$, but this can reverse for large $\bar{\theta}_i^{s_1}$. A comparison of panels A, B and C reveal that the endogenous status quo is more likely to dominate for as π increases. The intuition for this is as follows. When $\bar{\theta}_i^{s_1} = 0.25$, both states are identical, and the exogenous status quo dominates trivially by Proposition 6. As $\bar{\theta}_i^{s_1}$ increases, the probability of disagreement in each state decreases; hence, defending the status quo becomes less important, and the degree of partisanship in Γ_{L,s_1}^{en} decreases. At the same time, both players become more likely to vote for L in s_1 and R in s_2 , which guarantee an optimal status quo in the next period if the state does not change. This effect is more beneficial the more persistent the states.

7 N-player game

In this section, we extend the model to $N > 2$ players. Abusing notation, $N = \{1, \dots, n, \dots, N\}$ will also refer to the set of players. For any generic parameter p , the bold symbol \mathbf{p} now refers to the vector $(p_n)_{n \in N}$. As in the two player case, the payoff of each player is given by Equation (1), where $(\boldsymbol{\theta}^t)_{t \geq 1}$ follows a stationary and irreducible Markovian process on the finite state space S , with a probability density function f^S and a transition matrix π . In line with Assumption 1, we assume that f^S is such that in all states, $\theta_1 \geq \dots \geq \theta_N$ with probability one, and for any two distinct voters $n, m \in N$, there exists a state in which θ_n and θ_m are of opposite sign with positive probability.

The game proceeds exactly like in the two-player game, but we allow for a broader class of voting rules. A voting rule is characterized by a pair of collections of winning coalitions (Ω_L, Ω_R) , which determine the voting outcome as follows: if the status quo is L (R) in a given period, then it is replaced by R (L) if and only if the set of players who vote for R (L) in this period is an element of Ω_L (Ω_R). We impose the following conditions on the voting rules.

Definition 3 A voting rule is a pair of collection of coalitions $\Omega = (\Omega_L, \Omega_R)$ where for all $q \in \{L, R\}$, $\Omega_q \in 2^N$ satisfies the following conditions:

- (i) *Monotonicity*: if $C \in \Omega_q$ and $C \subseteq C'$, then $C' \in \Omega_q$,
- (ii) *Properness*: if $C \in \Omega_q$, then $N \setminus C \notin \Omega_q$
- (iii) *Nonemptiness*: $\{1..N\} \in \Omega_q$
- (iv) *Joint properness*: for $q' \neq q$, if $C \in \Omega_q$, then $N \setminus C \notin \Omega_{q'}$.

Conditions (i) to (iii) are standard in the voting literature (see, e.g., Austen Smith and Banks 2000). Monotonicity ensures that having more votes in favor of R cannot change the outcome to L ; properness ensures that the outcome of the vote is unique; nonemptiness

ensures that the voting rule is Paretian. Condition (iv) means that if a coalition can change the status quo, then the players outside this coalition cannot reverse this change. The class of voting rules defined by conditions (i) – (iv) encompass nonunanimous rules such as majoritarian voting rules, but also other nonanonymous, and nonneutral voting rules.²³

The N –player game that uses a voting rule Ω and begins with an initial state distribution π^0 and an initial status quo distribution q^S is denoted by $\Gamma_{q^S, \pi^0}^{en}(\Omega)$.

7.1 The equilibrium

Suppose that all players vote myopically for their most preferred policy. Then since $\theta_1 \geq \dots \geq \theta_N$, if player n votes for R , then all players $i \leq n$ also vote for R . Conditions (i) – (iii) in Definition 3 imply then that there exists a player n_R such that when the status quo is R , L is implemented if and only if that player votes for R . By the same token, there exists a player n_L such that if the status quo is L , R is implemented if and only if that player votes for R . We will call these players *pivotal*. Formally:

Definition 4 *The pivotal players for the voting rule Ω are (n_L, n_R) such that*

$$\begin{aligned} \{1, \dots, n_L\} &\in \Omega_L \text{ and } \{1, \dots, n_L - 1\} \notin \Omega_L, \\ \{n_R, \dots, N\} &\in \Omega_R \text{ and } \{n_R + 1, \dots, N\} \notin \Omega_R. \end{aligned}$$

Note that condition (iv) in Definition 3 implies that $n_R \leq n_L$ (equivalently, $\theta_{n_R} \geq \theta_{n_L}$): the player pivotal to implement a change to L is more rightist than the player pivotal to implement a change to R . For instance, in the case of unanimity rule, $n_R = 1$ and $n_L = N$, while in the case of simple majority rule, $n_R = \lfloor \frac{N+1}{2} \rfloor$ and $n_L = \lceil \frac{N+1}{2} \rceil$.²⁴

The following proposition characterizes the equilibria of $\Gamma_{q^S, \pi^0}^{en}(\Omega)$ for any voting rule. It shows that all players are partisan. Moreover, in equilibrium, the players (n_L, n_R) characterized in Definition 4 are pivotal in every period in that the outcome depends solely on their

²³An example of a nonanonymous voting rule is the combination of simple majority rule and a veto player $n \in N$, i.e.,

$$\Omega_R = \Omega_L = \left\{ C \subseteq N : |C| > \frac{N}{2} \text{ and } n \in C \right\}.$$

An example of a nonneutral voting rule is simple majority rule when the status quo is L and unanimity rule when the status quo is R , i.e.,

$$\Omega_L = \left\{ C \subseteq N : |C| > \frac{N}{2} \right\} \text{ and } \Omega_R = \{N\}.$$

²⁴For any real number x , $\lfloor x \rfloor$ is the largest integer smaller or equal to x , and $\lceil x \rceil$ is the smallest integer greater or equal to x . When N is odd, $\frac{N+1}{2} = \lfloor \frac{N+1}{2} \rfloor = \lceil \frac{N+1}{2} \rceil$.

vote.

Proposition 7 *In all equilibria of $\Gamma_{q^S, \pi^0}^{en}(\Omega)$, the players use state dependent but status-quo independent cutoff strategies: there exists $\mathbf{c}^S \in \mathbb{R}^{N \times S}$ such that in state $s \in S$, player $n \in N$ votes for R if $\theta_n > c_n^s$ and for L if $\theta_n < c_n^s$. The equilibrium cutoffs are given by the fixed point of the mapping \mathbf{H}^S defined as follows: for all $\mathbf{c}^S \in \mathbb{R}^{N \times S}$,*

$$H_n^s(\mathbf{c}^S) = \delta \sum_{s'} \pi(s, s') \left(\int_{\{\boldsymbol{\theta} \in \mathbb{R}^N : \theta_{n_L} \leq c_{n_L}^s \text{ and } \theta_{n_R} \geq c_{n_R}^s\}} (c_n^{s'} - \theta_n) f^{s'}(\boldsymbol{\theta}) d\boldsymbol{\theta} \right). \quad (5)$$

Moreover,

- (i) for all $s \in S$, $c_1^s \leq \dots \leq c_N^s$, so in any period, the status quo changes if and only if players n_L and n_R vote against it;
- (ii) for all $s \in S$, $c_{n_R}^s \leq 0 \leq c_{n_L}^s$;
- (iii) the set of equilibrium cutoffs of the pivotal players $(c_{n_L}^S, c_{n_R}^S)$ is a complete lattice for the partial order $(\leq, \geq)^S$;
- (iv) if \mathbf{c}^S and \mathbf{d}^S are two equilibria such that both pivotal players are more partisan with \mathbf{d}^S than with \mathbf{c}^S (i.e., $(d_{n_L}^S, d_{n_R}^S) (\leq, \geq)^S (c_{n_L}^S, c_{n_R}^S)$), then all players $n \in \{n_R, \dots, n_L\}$ are better off with \mathbf{c}^S than with \mathbf{d}^S . In particular, there exists a Pareto worse and Pareto best equilibrium for those players.

The proof of Proposition 7 proceeds by showing that since θ_n^t is increasing in n , stage dominance implies that partisanship is also monotonic in n . Therefore, the status quo changes if and only if players n_L and n_R vote against it. Thus, analyzing $\Gamma_{q^S, \pi^0}^{en}(\Omega)$ boils down to analyzing the 2–player game Γ_{q^S, π^0}^{en} with the preference distribution equal to the marginal distribution of $(\theta_{n_R}^t, \theta_{n_L}^t)$, and all results from the 2–player game follow. In particular, the pivotal voters are partisan in directions that exacerbate their conflict of interests, $c_{n_R}^s \leq 0 \leq c_{n_L}^s$, and thus the endogeneity of the status quo increases the probability that pivotal players disagree and decreases the responsiveness of the agreements to the shocks.

7.2 Concentration of power and welfare

Proposition 7 shows that the results from the 2–player game fully extend to the N –player game. However, this more general setup allows us to investigate how different voting rules affect the equilibrium behavior. In what follows, we will say that a rule has a *greater concentration of power* if a smaller set of voters is required to change the status quo. Formally:

Definition 5 *The concentration of power is greater (the dispersion of power is lower) under Ω than under Ω' if $\Omega_L \subseteq \Omega'_L$ and $\Omega_R \subseteq \Omega'_R$. The concentration of power under Ω is maximal if $n_R = n_L$.*

In words, the concentration of power increases when the approval of a smaller set of players is required to change the status quo. We show in the appendix (see the proof of Proposition 8) that when the concentration of power under Ω is greater than under Ω' , then the pivotal voters are more moderate under Ω : $n'_R \leq n_R \leq n_L \leq n'_L$.²⁵

The following proposition shows that the concentration of power mitigates players' partisanship.

Proposition 8 *Let $\mathbf{c}^S(\Omega)$ denote the cutoffs from the least partisan equilibrium of $\Gamma_{q^S, \pi^0}^{en}(\Omega)$. If the concentration of power is greater under Ω than under Ω' , then for all $s \in S$,*

$$c_{n'_R}^s(\Omega') \leq c_{n'_L}^s(\Omega) \leq c_{n_R}^s(\Omega) \leq 0 \leq c_{n_L}^s(\Omega) \leq c_{n'_L}^s(\Omega) \leq c_{n'_R}^s(\Omega'), \quad (6)$$

where (n_L, n_R) and (n'_L, n'_R) denote the pivotal players under Ω and Ω' , respectively.

There exists a nonpartisan equilibrium (i.e. $\mathbf{c}^S = \mathbf{0}^S$) under the voting rule Ω if and only if the concentration of power under Ω is maximal.

The intuition behind Proposition 8 is as follows. As argued before, dispersion of power makes more extreme players pivotal:

$$n'_R \leq n_R \leq n_L \leq n'_L.$$

This has two consequences. From Proposition 7, we know that for a given voting rule Ω , more extreme players are more partisan, which explains the four inner inequalities in (6). But since the players determining the agreements are now n'_L and n'_R instead of n_L and n_R , their disagreement is more likely than the disagreement of n_R and n_L . This increases the inertia of the status quo. As a result, players care more about the identity of the status quo and are thus more partisan. This effect explains the two outer inequalities in (6). In other words, dispersion of power increases the status quo inertia not only because more players have to agree, but also because the pivotal players become more partisan. In this sense, the endogenous status quo exacerbates the inertial effect of a dispersion of power.

The second part of Proposition 8 states that partisanship disappears when power is maximally concentrated. This is the case for instance under dictatorship, but also under

²⁵The condition $n_L = n_R$ is satisfied when the voting rule is strong (see, e.g., Austen Smith and Banks 2000): for all $q \in \{R, L\}$ and all $C \subseteq N$, if $C \notin \Omega_q$, then $N \setminus C \in \Omega_q$.

more equitable rules such as a simple majority rule. The reason is that in these two cases, a single player (the dictator and the median player, respectively) is always pivotal, and hence, votes according to her preferences.

The effect of the dispersion of power on players' welfare may depend on their ideological position. The increase in partisanship generated by the dispersion of power is clearly detrimental to all players. However, an extreme player may benefit from having pivotal players that are more extreme. This is because in all periods in which her most preferred status quo is in place, the policy will be chosen by someone with preferences more similar to hers. As shown in the following proposition, the latter effect is absent for players who are more moderate than the pivotal players; hence, the dispersion of power clearly hurts these players. It may benefit the more extreme players, but when the average preferences are between those of the pivotal players, utilitarian welfare must decrease with the dispersion of power.

Proposition 9 *If the concentration of power under Ω is greater than under Ω' , and if (n_L, n_R) denote the pivotal players under Ω , then all players $n \in \{n_L, \dots, n_R\}$ are better off under Ω than under Ω' . Moreover, if $\theta_{n_R} \leq \frac{1}{N} \sum_n \theta_n \leq \theta_{n_L}$ with probability 1, then utilitarian welfare is greater under Ω than under Ω' .*

Note that the condition $\theta_{n_R} \leq \frac{1}{N} \sum_n \theta_n \leq \theta_{n_L}$ is satisfied under standard specifications. For instance, it is satisfied if Ω is a supermajority rule; i.e., the approval of $M > \frac{N+1}{2}$ players is required to change the status quo, and the preference distribution across players is relatively symmetric around $\theta_{\frac{N+1}{2}}$; i.e., $\frac{1}{N} \sum_n \theta_n \approx \theta_{\frac{N+1}{2}} \in [\theta_{n_{N-M+1}}, \theta_M]$.

Propositions 8 and 9 have important consequences for constitutional design. There exists no modern democracy in which a single decision maker is pivotal in every decision, even when majority rule is used at all stages of the decision process. For instance, short of a strong party discipline and a sufficient majority in both chambers, bicameralism implies the existence of two distinct pivotal voters. Moreover, in most constitutions, majoritarian decision making is complemented by other rules and institutions, such as the presidential veto power, judicial review by the constitutional court, the possibility of public initiative, or supramajoritarian requirements such as the filibuster tradition in the U.S. Congress.²⁶

Admittedly, these checks and balances are not meant to smooth the decision process. Their role is to limit agency costs and abuses of power between voters and their representatives. Our model shows, however, that when checks and balances are introduced in a

²⁶It should be noted that if we allow for preference reversal, for instance if θ has full support at least in some state, then even under simple majority rule, all players can be pivotal in every period. Therefore, whenever a player is pivotal, she would consider how her future preferences might conflict with that of the next pivotal players, and she would bias her vote accordingly. This means that partisanship would emerge even under simple majority rule.

decision process that uses the endogenous status-quo protocol, they tend to make legislators more partisan, and this strategic effect can greatly exacerbate the inertial effect of checks and balances.

Our previous results on gridlock and the negative welfare effects of partisanship (see Propositions 5 and 6) suggest that in order to avoid a welfare decreasing partisan behavior, a system of checks and balances should be complemented with the use of exogenous status quos or sunset provisions. Interestingly, a very similar argument was made by Thomas Jefferson when he famously argued in favor of laws of limited durations in his correspondence with James Madison.²⁷

7.3 Biased voting rules

It is not uncommon for a voting rule to require the approval of a set of voters of different size to pass different policies; i.e., $|\Omega_R| \neq |\Omega_L|$. An example of such a biased rule can be found in the U.S. budget process. The U.S. budget process is governed by the Congressional Budget Act, which prevents the use of the filibuster against budget resolutions. At the same time, the Byrd Rule, which was adopted in 1985 and amended in 1990, modifies the Congressional Budget Act to allow the use of filibuster against any provision that would increase the deficit for a fiscal year beyond those covered by the reconciliation measure. Effectively, the Byrd Rule requires a higher majority to raise the budget deficit than to lower it, and curtailing the latter was one of its rationales.²⁸ However, the game-theoretic logic highlighted in our model suggests that this rule might have unintended consequences. Fiscally expansionist legislators may be unwilling to reduce the budget deficit in good times, realizing that it will be very difficult to increase it in the future. As a result, deficit reduction may become more difficult than if a filibuster-proof majority were required for all or none of the budget resolutions. The example below demonstrates this possibility.

Example 2 *We extend the example from Section 4 to three players as follows: $|S| = 1$, $N = 3$, for all $n \in \{1, 2, 3\}$, $\theta_n = \bar{\theta}_n + \varepsilon$, with $\bar{\theta}_1 > \bar{\theta}_2 > \bar{\theta}_3$ and $\varepsilon \sim N(0, 1)$. Consider the following two voting rules: the simple majority rule, under which a policy replaces the status*

²⁷ “[T]he power of repeal is not an equivalent [to mandatory expiration]. It might indeed be if [...] the will of the majority could always be obtained fairly and without impediment. But this is true of no form. [...] Various checks are opposed to every legislative Proposition [...] and other impediments arise so as to prove to every practical man that a law of limited duration is much more manageable than one which needs a repeal.” (The Letters of Thomas Jefferson, To James Madison, Sept. 6. 1789).

²⁸ More precisely, to pass a provision that increases the deficit, the Byrd rule requires a filibuster proof majority or a simple majority together with a sunset clause on that provision. We leave that latter possibility aside for the sake of simplicity, since our goal here is to illustrate the incentives generated by biased voting rules rather than to propose a detailed and realistic model of the U.S. budget process.

quo if it is approved by two players, and the R -biased rule, under which a simple majority is needed to replace L , and unanimity is required to replace R .

Under the simple majority rule, $n_R = n_L = 2$ so player 2 is always pivotal, while under the R -biased rule, $n_R = 1$ and $n_L = 2$, so player 1 is pivotal when the status quo is R and player 2 is pivotal when the status quo is L . From Proposition 7, under the simple majority rule players vote myopically for their most preferred policy, so R wins when $\varepsilon^t > -\bar{\theta}_2$ and L wins when $\varepsilon^t < -\bar{\theta}_2$. Under the R -biased rule players are partisan, and R wins when $\varepsilon^t > c_1 - \bar{\theta}_1$, L wins when $\varepsilon^t < c_2 - \bar{\theta}_2$, and the status quo stays in place when $\varepsilon^t \in (c_1 - \bar{\theta}_1, c_2 - \bar{\theta}_2)$. Proposition 7 implies that $c_1 - \bar{\theta}_1 < -\bar{\theta}_2 < c_2 - \bar{\theta}_2$, so compared to the simple majority rule, R stays in place more often, but so does L . The latter effect may dominate if players' partisanship is strong. Assume that $\bar{\theta}_1 = 0.3$ and $\delta = 0.9$. We show numerically that for $\bar{\theta}_2 \leq -0.5$, the probability of L being implemented at the invariant distribution is higher under the R -biased rule. For example, when $\bar{\theta}_2 = -0.5$, then in the long run, the probability that L is implemented under the majority rule is 0.69, while under the R -biased rule, the probability increases to 0.9996.²⁹

8 Preference Reversal

We believe that Assumption 1 excluding preference reversals is satisfied in a vast array of environments. However, in this section we discuss when and how the results of Section 5 change when we relax this assumption. That is, we assume now that f^S can have full support.

Note first that the equilibrium characterization in Proposition 1 was derived without Assumption 1, so for any f^S players use cutoff strategies in any equilibrium. Moreover, the partisanship generated by the endogeneity of the status quo is a rule, not an exception. To see that, observe that from Proposition 1, an equilibrium with zero cutoffs (i.e., $c_i^s = c_j^s = 0$ for all s) exists if and only if for all $s \in S$,

$$\sum_{s' \in S} \pi(s, s') \left(\int_{-\infty}^0 \int_0^{\infty} \boldsymbol{\theta} f^{s'}(\boldsymbol{\theta}) d\theta_i d\theta_j + \int_0^{\infty} \int_{-\infty}^0 \boldsymbol{\theta} f^{s'}(\boldsymbol{\theta}) d\theta_i d\theta_j \right) = (0, 0). \quad (7)$$

Clearly, condition (7) is satisfied only in special cases. For instance, if $|S| = 1$ and the distribution of $\boldsymbol{\theta}$ is bivariate normal, condition (7) holds if and only if $\bar{\boldsymbol{\theta}} = (0, 0)$.³⁰

²⁹Observe that with an exogenous status quo, players would vote according to their preferences under both rules, and it should be clear that the R -biased rule would favor R since R would be implemented more often than under simple majority rule.

³⁰For the formal proof, see Dziuda and Loeper (2010, Example 5 in the appendix). In that paper, we also show (see Example 1) that the symmetry of the preference distribution across players and the symmetry of

Second, the polarizing and inertial effect of the endogeneity of the status quo are robust phenomena. Proposition 4 part b) and Proposition 5 hold for any preference distribution. Hence, even if we allow for preference reversal, it is always true that more polarized players are more partisan, which makes them disagree even more often. And when they are sufficiently polarized and patient, the endogenous status quo leads to a complete gridlock.

The main change in the analysis is the determination of the sign of the equilibrium cutoffs. Since one cannot unambiguously order players' preferences, Proposition 2 does not extend automatically. Moreover, as we shall see in Example 3 below, the signs of the cutoffs may even change across equilibria. It turns out, however, that if one player is rightist and the other is leftist in a sense defined below, then there exists an intuitive equilibrium in which the former is partisan for R and the latter is partisan for L . Proposition 10 formalizes this and provides a reason for why such equilibria may be more plausible.

Before stating the proposition, however, we need to introduce a necessary notation. Let $\Gamma_{q^S, \pi^0}^{en}(t)$ denote the finite horizon game which proceeds as Γ_{q^S, π^0}^{en} but ends after t periods. As shown in Dziuda and Loeper (2010), this game admits a unique stage-undominated equilibrium, which is in cutoff strategies, and we denote by $\mathbf{c}^S(t)$ the equilibrium cutoffs in period t . The definition below links a player's ideological position to her preferences over different exogenous status quo. For example, $H_k^s(0, \dots, 0) \leq 0$ means that conditional on her preference disagreeing with her opponent's preferences, player k prefers alternative R in state s . Therefore, from Remark 1, she would prefer the exogenous status quo in state s to be R , and we call her a rightist in state s . Formally:

Definition 6 *Player k is rightist (leftist) if for all $s \in S$, $H_k^s(0, \dots, 0) \leq 0$ (≥ 0).*

Proposition 10 *Assume that f^S is such that player i is rightist and player j is leftist. Then, there exists equilibria \mathbf{c}^S such that $\mathbf{c}^S(\leq, \geq)^S \mathbf{0}^S$. The set of such equilibria forms a complete lattice for the order $(\leq, \geq)^S$. The least partisan of these equilibria in the order $(\leq, \geq)^S$ is equal to $\lim_{t \rightarrow \infty} \mathbf{c}^S(t)$. The comparative statics in Proposition 4 hold for that equilibrium selection.*

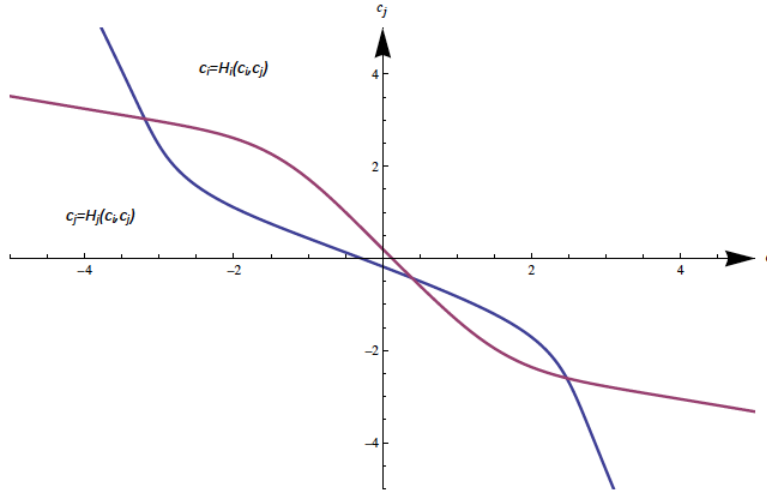
If the players do not have opposed ideologies in the sense of Definition 6, however, general results on the direction of players' partisanship are more elusive. To understand why, consider the following case: $|S| = 1$, $H_i(0, 0) > 0$ and $H_j(0, 0) > 0$. In that case, the players' expected preferences conditional on disagreement are congruent: they both prefer L to be the status quo. Hence, one could conjecture that with an endogenous status quo, there is an equilibrium

the marginal distribution of each player's preferences across alternatives is not a sufficient condition for (7) to hold.

in which both players are partisan for L . However, this might not be true. If i votes often for L , then the disagreement in which i prefers R and j prefers L happens rarely. The reverse disagreement may be more likely, so j may prefer to defend R as the status quo. As a result, j may end up partisan for R .³¹

One could conjecture that allowing for preference reversal should decrease partisanship, as no player can be sure which alternative she will prefer when players' disagree. As the following example demonstrates, this is not necessarily true. Moreover, arbitrarily similar players can behave as if their interest were highly discordant. Hence, if anything, the possibility of preference reversal exacerbates the polarizing effect of the endogenous status quo.

Example 3 Suppose that $|S| = 1$ and $\delta = 0.9$. Let $\theta_i = \bar{\theta}_i + \varepsilon_i$ and $\theta_j = \bar{\theta}_j + \varepsilon_j$. Let ε_i and ε_j be i.i.d. with ε_k drawn from $N(0, 1)$ with probability $\frac{1}{2}$ and from $N(0, 10)$ with the remaining probability. The figure below depicts the locus points of H_i and H_j for $\bar{\theta}_i = -\bar{\theta}_j = 0.1$; the intersections are the equilibria. We see that Γ_q^{en} has two equilibria with $c_i > 0 > c_j$ and one with $c_i < 0 < c_j$.



In the intuitive equilibrium, $c_i = -c_j \simeq -3.2$. As $\bar{\theta}_i = -\bar{\theta}_j \rightarrow 0$, all three equilibria remain. Only in the middle one players' partisanship vanishes, but as argued in Proposition 10, the equilibrium in which $c_i < 0 < c_j$ is the most plausible. Hence, the endogenous status quo leads arbitrarily similar players to be very partisan for opposite alternatives and behave as if their interest were highly discordant.³² Moreover, if we restored Assumption 1 by assuming instead that ε_i and ε_j are perfectly correlated with $N(0, 1)$ being the marginal distribution,

³¹This possibility is proved formally in Dziuda and Loeper 2010.

³²This phenomenon cannot occur in the case of no preference reversal: for $|S| = 1$ and for any sequence

then in equilibrium, $c_i = -c_j = 0.0042$. Hence, allowing for preference reversal can have a dramatic effect on partisanship.

Allowing for preference reversal complicates the equilibrium welfare comparison between the exogenous and endogenous status quo. The reason is that besides the two effects of partisanship on welfare outlined in Section 6, a third beneficial effect arises. A partisan player, while voting for her preferred status quo, may defer to her opponent's preferences: if $c_i < \theta_i < 0 < \theta_j$, player i will vote for the alternative preferred by player j . This may be socially beneficial if the opponent's preferences are relatively more intense. In Dziuda and Loeper 2010 (Proposition 7) we show that under some regularity conditions which basically require that the probability of a preference reversal is not too large, the welfare results in Proposition 6 hold.

9 Conclusion

Negotiations in a changing environment with an endogenous status quo are at the center of many economically relevant situations. They present the negotiating parties with a fundamental trade-off between responding adequately to the current environment and securing a favorable bargaining position for the future. In this paper, we show that this trade-off has a detrimental impact on the efficiency of agreements and their responsiveness to political and economic shocks. Bundling the vote on today's policy and tomorrow's status quo exacerbates the players' conflict of interest and increases the probability of a disagreement, which in turn increases status quo inertia. Even if some agreements are commonly known to be mutually beneficial, they may not be adopted.

Our analysis sheds light on the effect of some important rules governing legislative institutions: we provide a new argument in favor of sunset provisions and we show that checks and balances exacerbate the partisanship and the inertia generated by the endogenous status quo.

This parsimonious model lends itself to many extensions. First, our paper is mute on the selection of the bargaining protocol. Our analysis suggests that both bargaining protocols (the exogenous and the endogenous status quo) that we considered could be selected by the negotiating parties. However, a formal analysis is needed. Ideally, such analysis should also

of preference distribution $(\boldsymbol{\theta}^k)_{k \geq 1}$ such that $\bar{\boldsymbol{\theta}}_i^k - \bar{\boldsymbol{\theta}}_j^k$ tends to 0, all equilibrium thresholds tend to $(0, 0)$. To see this, observe that since $\theta_i^k - \theta_j^k > 0$ with probability 1, $E\left(\left|\theta_i^k - \theta_j^k\right|\right)$ must tend to 0. So $|H_i^k - H_j^k|$ tends to 0 uniformly over \mathbb{R}^2 . Using proposition 2, the fixed point of \mathbf{H} must all tend to $(0, 0)$.

extend the endogenous status quo protocol by allowing the negotiating parties to attach endogenous sunset provisions at any stage.

Second, adding transfers—interpreted as pork-barrel spending—to the N –player model would allow us to analyze the trade-off between their positive role as a lubricant for passing efficient policies and the perverse incentives they generate by concentrating benefits and collectivizing costs. Such a model would better approximate the U.S. budget process, which distinguishes between two expenditure categories: discretionary spending and direct spending. The former can be targeted and requires an annual appropriation bill while the latter is not easily targeted and is continuing in nature. The model, hence, could shed also some light on the dynamics of these two types of spending.

And finally, in many situations, implemented policies affect the future state of the economy, which introduces an additional dynamic linkage. For example, an expansionary fiscal policy increases public debt, leading all players to adopt a more fiscally conservative stand in the future. Technically, this amounts to introducing a state variable in the model.

10 Appendix

Throughout the appendix, we will use the following notations:

Notation 1 For any preference distribution f^S , all $\delta \in [0, 1]$, all $s \in S$, and all $\mathbf{c}^S \in \mathbb{R}^{2S}$, we denote by $\mathbf{G}^s(\delta, f^s, \mathbf{c})$ the map defined by:

$$\mathbf{G}^s(\delta, f^s, \mathbf{c}) = \delta \left(\int_{-\infty}^{c_j} \int_{c_i}^{\infty} (\mathbf{c} - \boldsymbol{\theta}) f^s(\boldsymbol{\theta}) d\theta_i d\theta_j + \int_{c_j}^{\infty} \int_{-\infty}^{c_i} (\mathbf{c} - \boldsymbol{\theta}) f^s(\boldsymbol{\theta}) d\theta_i d\theta_j \right). \quad (8)$$

We denote by $\mathbf{H}^s(\delta, f^S, \mathbf{c}^S)$ the map defined by:

$$\mathbf{H}^s(\delta, f^S, \mathbf{c}^S) = \sum_{s' \in S} \pi(s, s') \mathbf{G}^{s'}(\delta, f^{s'}, \mathbf{c}^{s'}). \quad (9)$$

We denote by $\mathbf{c}^S(\delta, f^S)$ the smallest fixed point of $\mathbf{H}^S(\delta, f^S, \mathbf{c}^S)$ for the order $(\leq, \geq)^S$, when it exists. Finally, $\mathbf{0}$ and $\mathbf{0}^S$ are the null element of \mathbb{R}^2 and \mathbb{R}^{2S} , respectively.

The map $\mathbf{H}^S(\delta, f^S, \mathbf{c}^S)$ is simply the map $\mathbf{H}^S(\mathbf{c}^S)$ defined in the main text in (4) with an explicit reference to the preference distribution f^S and discount factor δ . The next two lemmas derive important properties of \mathbf{G}^S and \mathbf{H}^S .

Lemma 1 Using the conventions of Notation 1, for all $s \in S$, all $\mathbf{c}^s \in \mathbb{R}^2$, and all $k \neq k'$,

$$0 \leq \frac{\partial G_k^s(\delta, f^s, \mathbf{c})}{\partial c_k} \leq \delta, \text{ and } \frac{\partial G_k^s(\delta, f^s, \mathbf{c})}{\partial c_{k'}} \leq 0,$$

and if g^s is more polarized than f^s (see definition 2),

$$G_i^s(\delta, f^s, \mathbf{c}) \leq G_i^s(\delta, g^s, \mathbf{c}) \text{ and } G_j^s(\delta, f^s, \mathbf{c}) \geq G_j^s(\delta, g^s, \mathbf{c}).$$

Proof. Using the Leibnitz integral rule on (8), we get

$$\begin{aligned} \frac{\partial G_i^s(\delta, f^s, \mathbf{c})}{\partial c_i} &= \delta \left(\int_{-\infty}^{c_j} \int_{c_i}^{\infty} f^s(\boldsymbol{\theta}) d\theta_i d\theta_j + \int_{c_j}^{\infty} \int_{-\infty}^{c_i} f^s(\boldsymbol{\theta}) d\theta_i d\theta_j \right), \\ \frac{\partial G_i^s(\delta, f^s, \mathbf{c})}{\partial c_j} &= -\delta \int_{-\infty}^{+\infty} |\theta_i - c_i| f^s(\theta_i, c_j) d\theta_i, \end{aligned}$$

which proves the first part.

To prove the second part, for all $s \in S$, using the notations of definition 2, let us denote by f_α^s the probability density function of the random variable $\theta^s + \alpha \varepsilon^s$. Therefore, $f_0^s = f^s$ and $f_1^s = g^s$. Moreover, if we denote by h^s the joint probability density function of θ^s and ε^s ,

$$G_i^s(\delta, f_\alpha^s, \mathbf{c}^s) = \delta \left(\int_{\boldsymbol{\theta}} \int_{-\infty}^{\frac{c_j - \theta_j}{\alpha}} \int_{\frac{c_i - \theta_i}{\alpha}}^{\infty} (c_i - \theta_i - \alpha \varepsilon_i) h^s(\boldsymbol{\theta}, \boldsymbol{\varepsilon}) d\varepsilon_i d\varepsilon_j d\theta_i d\theta_j \right. \\ \left. + \int_{\boldsymbol{\theta}} \int_{\frac{c_j - \theta_j}{\alpha}}^{\infty} \int_{-\infty}^{\frac{c_i - \theta_i}{\alpha}} (c_i - \theta_i - \alpha \varepsilon_i) h^s(\boldsymbol{\theta}, \boldsymbol{\varepsilon}) d\varepsilon_i d\varepsilon_j d\theta_i d\theta_j \right),$$

so using the Leibnitz integral rule, we obtain

$$\begin{aligned} \frac{\partial G_i^s(\delta, f_\alpha^s, \mathbf{c})}{\partial \alpha} &= - \int_{\boldsymbol{\theta}} \int_{\boldsymbol{\varepsilon} \in]-\infty, \frac{c_i - \theta_i}{\alpha} [\times] \frac{c_j - \theta_j}{\alpha}, +\infty [\cup]-\infty, \frac{c_i - \theta_i}{\alpha} [\times] \frac{c_j - \theta_j}{\alpha}, +\infty [\varepsilon_i^s h^s(\boldsymbol{\theta}, \boldsymbol{\varepsilon}) d\varepsilon_i d\varepsilon_j d\theta_i d\theta_j \\ &\quad - \int_{\boldsymbol{\theta}} \int_{\frac{c_i - \theta_i}{\alpha}}^{\infty} \frac{c_j - \theta_j}{\alpha^2} (c_i - \theta_i - \alpha \varepsilon_i) h^s\left(\boldsymbol{\theta}, \varepsilon_i, \varepsilon_j = \frac{c_j - \theta_j}{\alpha}\right) d\varepsilon_i d\theta_i d\theta_j \\ &\quad + \int_{\boldsymbol{\theta}} \int_{-\infty}^{\frac{c_i - \theta_i}{\alpha}} \frac{c_j - \theta_j}{\alpha^2} (c_i - \theta_i - \alpha \varepsilon_i) f^s\left(\boldsymbol{\theta}, \varepsilon_i, \varepsilon_j = \frac{c_j - \theta_j}{\alpha}\right) d\varepsilon_i d\theta_i d\theta_j \\ &= - \int_{\boldsymbol{\theta}} \int_{\boldsymbol{\varepsilon} \in]-\infty, \frac{c_i - \theta_i}{\alpha} [\times] \frac{c_j - \theta_j}{\alpha}, +\infty [\cup]-\infty, \frac{c_i - \theta_i}{\alpha} [\times] \frac{c_j - \theta_j}{\alpha}, +\infty [\varepsilon_i^s h^s(\boldsymbol{\theta}, \boldsymbol{\varepsilon}) d\varepsilon_i d\varepsilon_j d\boldsymbol{\theta} \\ &\quad + \int_{\boldsymbol{\theta}} \left(\int_{-\infty}^{\infty} \frac{c_j - \theta_j}{\alpha^2} |c_i - \theta_i - \alpha \varepsilon_i| f^s\left(\boldsymbol{\theta}, \varepsilon_i, \varepsilon_j = \frac{c_j - \theta_j}{\alpha}\right) d\varepsilon_i \right) d\theta_i d\theta_j \end{aligned} \quad (11)$$

By assumption, $\varepsilon_i \geq 0$ with probability 1, so (10) is negative, and $\varepsilon_j \leq 0$ with probability 1, so (11) is negative also. Therefore, $G_i^s(\delta, f_0^s, \mathbf{c}) \leq G_i^s(\delta, f_1^s, \mathbf{c})$. A similar arguments shows

that $G_j^s(\delta, f_0^s, \mathbf{c}) \geq G_j^s(\delta, f_1^s, \mathbf{c})$. ■

Lemma 2 *Using the conventions of Notation 1, $\mathbf{H}^S(\delta, f^S, \mathbf{c}^S)$ is isotone in \mathbf{c}^S the order $(\leq, \geq)^S$, and for all $k \in \{i, j\}$, $H_k^S(\delta, f^S, \mathbf{c}^S)$ is δ -Lipschitz continuous in c_k^S for the sup norm on \mathbb{R}^S . If we denote the set $\left(\left[-\frac{\delta\|\theta\|}{1-\delta}, \frac{\delta\|\theta\|}{1-\delta} \right]^2 \right)^S$ by A , where $\|\theta\|$ is $\max_{s \in S, k \in \{i, j\}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |\theta_k| f^s(\boldsymbol{\theta}) d\boldsymbol{\theta}$, then all fixed points \mathbf{c}^S of $\mathbf{H}^S(\delta, f^S, \mathbf{c}^S)$ are in A and $\mathbf{H}^S(A) \subseteq A$.*

Proof. That $\mathbf{H}^S(\delta, f^S, \mathbf{c}^S)$ is isotone is immediate from Lemma 1 and (9). To show Lipschitz continuity, from (9), for all $s, s' \in S$, $\partial H_k^s / \partial c_k^{s'} = \pi(s, s') \partial G_k^{s'} / \partial c_k^{s'}$, so from lemma 1, $\sum_{s'} |\partial H_k^s / \partial c_k^{s'}| < \delta \sum_{s'} \pi(s, s') < \delta$.

To show the last point, for all $\mathbf{c}^S \in \mathbb{R}^{2S}$, let us denote $\max_{s \in S, k \in \{i, j\}} c_k^s$ by $\|\mathbf{c}^S\|$. From (8), we see that for all $\mathbf{c} \in \mathbb{R}^2$ and all $s \in S$, $|G_k^s(\delta, f^S \mathbf{c})|$ is bounded by $\delta(\|\theta\| + |c_k|)$, so from (9), $|H_k^s(\delta, f^S, \mathbf{c}^S)|$ is bounded by $\delta(\|\theta\| + \|\mathbf{c}^S\|)$. Therefore, for all $\mathbf{c}^S \in A$,

$$|H_k^s(\delta, f^S, \mathbf{c}^S)| \leq \delta \left(\|\theta\| + \frac{\delta\|\theta\|}{1-\delta} \right) = \frac{\delta\|\theta\|}{1-\delta}$$

so $\mathbf{H}^S(\delta, f^S, \mathbf{c}^S) \in A$. If \mathbf{c}^S is a fixed point of \mathbf{H}^S , $\|\mathbf{c}^S\| \leq \delta(\|\theta\| + \|\mathbf{c}^S\|)$, which implies that $\mathbf{c}^S \in A$. ■

Definition 7 *In the game Γ_{q^s, π^0}^{en} , a stationary strategy σ_k^S is an element of $[0, 1]^{S \times \{R, L\} \times \mathbb{R}^2}$ where for all $s \in S$, $q \in \{R, L\}$ and $\boldsymbol{\theta} \in \mathbb{R}^2$, $\sigma_k^S(q, \boldsymbol{\theta})$ is the probability that player k votes for R in any period t in which the current state is s , the status quo is q , and the preference realization is $\boldsymbol{\theta}$.³³*

The next two lemmas characterize properties of the best response correspondence of the game Γ_{q^s, π^0}^{en} which are key to characterizing its equilibria.

Lemma 3 *Let σ_j^S be a stationary strategy of player j in the game Γ_{q^s, π^0}^{en} (see Definition 7).*

- (i) *There exists a unique cutoff strategy for player i that is a best response to σ_j^S .*
- (ii) *This cutoff strategy is also the unique stage-undominated one given σ_j^S .*
- (iii) *These voting cutoffs are stationary and independent of the current status quo.*
- (iv) *If we denote c_i^S this cutoff strategy and $V_i^s(q)$ the continuation value at the strategy profile (c_i^S, σ_j^S) for player i at the beginning of a period in which the status quo is*

³³If players do not know each other's preferences, $\sigma_k^S(q, \cdot)$ is a function of θ_k only.

$q \in \{R, L\}$, the previous state was $s \in S$, but the current state and preferences have not been realized, then

$$c_i^s = \frac{\delta}{2} (V_i^s(L) - V_i^s(R)). \quad (12)$$

The same results hold by switching the role of i and j .

Proof. Observe first that because σ_j is stationary, any two best response to σ_j must generate the same (stationary) continuation value.

Now let σ_i be a best response to σ_j^S (not necessarily stationary). For any history leading to period t , player i cannot do better than voting for the alternative that gives him the greatest intertemporal payoff from date t onward. So if the state and preferences in that period are s and θ_i , player i cannot do better than voting for R when

$$\theta_i + \delta V_i^s(R) > -\theta_i + \delta V_i^s(L), \quad (13)$$

and for L when the reverse inequality holds. Therefore, given the continuation play prescribed by (σ_i, σ_j^S) , an optimal action in period t is to use a voting cutoff is given by (13) and since we have assumed that the marginal distribution of θ_i has full support in every state s , any other cutoff strategy in that period would not be optimal. For the same reason, any undominated action of the stage game induced by the continuation value $V_i^s(\cdot)$ should also be a cutoff strategy and should satisfy (13).

Since $V_i^S(\cdot)$ is the same for all best responses, changing the best response strategy σ_i in any period t for this cutoff strategy should not change the continuation value before period t . Therefore, by doing so in every period, we obtain a status quo independent, stationary cutoff strategy which is a best response to σ_j^S . Since any cutoff best response, or any stage undominated strategy, must satisfy (13) in every period, they must coincide with the strategy we have constructed. ■

Lemma 4 *If player j plays a stationary, status quo independent cutoff strategy $c_j^S \in \mathbb{R}^S$, then the optimal cutoff strategy of player i is the unique solution of $c_i^S = H_i(c_i^S, c_j^S)$. The same results hold by switching the role of i and j .*

Proof. Suppose that player j plays a stationary, status quo independent cutoff strategy c_j^S , and let c_i^S be the best response of player i characterized in Lemma 3. Since the status quo in a given period affects the payoffs only when players vote for opposite alternatives in

that period, we have that for all $s \in S$,

$$V_i^s(L) - V_i^s(R) = \delta \sum_{s' \in S} \pi(s, s') \left(\int_{-\infty}^{c_j^{s'}} \int_{c_i^{s'}}^{\infty} (-\theta_i + \delta V_i^{s'}(L) - (\theta_i + \delta V_i^{s'}(R))) f^{s'}(\boldsymbol{\theta}) d\theta_i d\theta_j \right. \\ \left. + \int_{c_i^{s'}}^{\infty} \int_{-\infty}^{c_j^{s'}} (-\theta_i + \delta V_i^{s'}(L) - (\theta_i + \delta V_i^{s'}(R))) f^{s'}(\boldsymbol{\theta}) d\theta_i d\theta_j \right). \quad (14)$$

Substituting (12) on both sides of (14), we get $c_i^S = H_i(c_i^S, c_j^S)$, which is simply the Bellman equation of the maximization problem of player i . From lemma 2 and the Banach fixed point theorem, this equation has a unique solution in c_i^S , so this solution must be the the best response we are looking for. ■

Proof. As shown in the proof of Lemma 3, if $V_i^{S,n}$ are the continuation values of player i at the strategy profile $(\sigma_i^{S,n}, \sigma_j^{S,n})$, $V_i^{S,n}$ are also the continuation values of player i at the strategy profile $(c_i^{S,n}, \sigma_j^{S,n})$ where $c_i^{S,n}$ is the unique cutoff best response to $\sigma_j^{S,n}$. Moreover, it should be clear that since $\sigma_j^{S,n}$ is totally mixed, player i is always pivotal with a positive probability, so $\sigma_i^{S,n}$ can only be the cutoff strategy $c_i^{S,n}$.

Since $\sigma_j^{S,n}$ tends to σ_j^S and since player i is best responding, the maximum theorem on the (compact) space of cutoff strategies implies that $V_i^{S,n}$ must tend to the continuation values V_i^S given by the strategy profile (c_i^S, σ_j^S) . From (12), this implies that $c_i^{S,n}$ tends towards c_i^S . ■

Proof of proposition 1. From Lemma 3, stationary, stage undominated equilibria must be (status quo independent) cutoff strategies. From Lemma 4, the equilibrium cutoffs are given by the fixed points of the map H . Using the notations of Lemma 2, A is a complete lattice for the order $(\leq, \geq)^S$, so Lemma 2 together with Tarski's fixed point theorem imply that the set of fixed points of the restriction of \mathbf{H}^S on A (and hence the set of fixed points of \mathbf{H}^S on \mathbb{R}^{2S}) is a complete lattice in the order $(\leq, \geq)^S$. ■

Proof of Proposition 2.

If f^S satisfies Assumption 1, for all $\mathbf{c} \in \mathbb{R}^2$ and $s \in S$, $G_i^s(\delta, f^S, \mathbf{c}) - \delta c_i^s \leq G_j^s(\delta, f^S, \mathbf{c}) - \delta c_j^s$, so

$$H_i^s(\delta, f^S, \mathbf{c}) - \delta \sum_{s' \in S} \pi(s, s') c_i^{s'} \leq H_j^s(\delta, f^S, \mathbf{c}) - \delta \sum_{s' \in S} \pi(s, s') c_j^{s'}.$$

If \mathbf{c}^S is an equilibrium, from Proposition 1, it is a fixed point of \mathbf{H}^S , so the above inequality implies that for all $s \in S$,

$$c_i^s - \delta \sum_{s' \in S} \pi(s, s') c_i^{s'} \leq c_j^s - \delta \sum_{s' \in S} \pi(s, s') c_j^{s'},$$

which, in matrix form, can be rewritten as $(I - \delta\pi)(c_i - c_j) \leq 0$ where I is the $|S| \times |S|$ identity matrix and \leq is the product order on $\mathbb{R}^{|S|}$. The inverse of the matrix $I - \delta\pi$ is $\sum_{n \geq 0} \delta^n \pi^n$, which has all its entries positive. Therefore, $(I - \delta\pi)(c_i - c_j) \leq 0$ implies that $(c_i - c_j) \leq 0$, that is, for all s , $c_i^s \leq c_j^s$. Therefore, the event $\theta_i \leq c_i^s$ and $\theta_j \geq c_j^s$ has probability 0, so when players vote for opposite alternatives, player i always votes for R while player j always votes for L . From (12), this means that when players' intertemporal preferences disagree, player i always prefers R while player j always prefers L . Since the status quo matters only in case of disagreement, in any period, player i prefers status quo R while player j prefers status quo L , which means that $\mathbf{c}^S (\leq, \geq)^S \mathbf{0}^S$.

To conclude the proof, we show that at any equilibrium, $\mathbf{c}^S (<, >)^S \mathbf{0}^S$. Using Notation 1 (we drop the reference to δ and f^S for simplicity), Assumption 1 implies that

$$\begin{aligned} & \text{for all } s \in S, \mathbf{G}^s(\mathbf{c}^s) (\leq, \geq) \mathbf{G}^s(\mathbf{0}) (\leq, \geq) \mathbf{0}, \\ & \text{for some } s^0 \in S, \mathbf{G}^{s^0}(\mathbf{c}^{s^0}) (\leq, \geq) \mathbf{G}^{s^0}(\mathbf{0}) (<, >) \mathbf{0}. \end{aligned}$$

Since the Markov process is irreducible, for some $s^1 \in S$, $\pi(s^1, s^0) > 0$, so using the above inequalities,

$$\mathbf{c}^{s^1} = \mathbf{H}^{s^1}(\mathbf{c}^S) (\leq, \geq) \mathbf{H}^{s^1}(\mathbf{0}^S) = \delta \sum_{s' \in S} \pi(s, s') \mathbf{G}^{s'}(\mathbf{0}^S) (<, >) \mathbf{0}.$$

Since $\theta_i \geq \theta_j$ almost surely, for almost all $c_j^s > 0$, conditional on $\theta_j = c_j^s$, $\theta_i \geq 0$ almost surely, so for all $c_i < 0$, $|\theta_i - c_i| \geq |c_i|$ almost surely. Therefore, for almost all $\mathbf{c}(<, >) \mathbf{0}$,

$$\frac{\partial G_i^s(\mathbf{c})}{\partial c_j} = -\delta \int_{-\infty}^{+\infty} |\theta_i - c_i| f^s(\theta_i, c_j) d\theta_i < -\delta f_j^s(c_j) |c_i| < 0.$$

Since $G_i^s(\mathbf{c})$ is non decreasing in c_i , this shows that $G_i^{s^1}(\mathbf{c}^{s^1}) < 0$. A similar reasoning shows that $G_j^{s^1}(\mathbf{c}^{s^1}) > 0$. Therefore, for all $s^2 \in S$, $\pi(s^2, s^1) > 0$, $\mathbf{c}^{s^2} (<, >) \mathbf{0}$, and with a simple induction, for all $n \geq 2$ and all $s^n \in S$ such that $\pi(s^n, s^{n-1}) > 0, \dots, \pi(s^2, s^1) > 0$ for some $s^n, \dots, s^2 \in S$, we must have $\mathbf{c}^{s^n} (<, >) \mathbf{0}$. Since the Markov process is irreducible, this shows that $\mathbf{c}^S (<, >) \mathbf{0}^S$. ■

Definition 8 For all $k, k' \in \{i, j\}$, Let σ_k and $\sigma_{k'}$ be two strategy of player k and k' for the game Γ_{q^S, π^0}^{en} , we say that $\sigma_{k'}$ is more rightist (leftist) than σ_k is for all histories, in all periods, with probability 1, if σ_k votes R (L), so does $\sigma_{k'}$.

Lemma 5 Let \mathbf{c}^S be an equilibrium of Γ_{q^S, π^0}^{en} such that c_i^S is more rightist than c_j^S (see Definition 8), and let \mathbf{d}^S be a strategy profile such that d_i^S is more rightist than c_i^S and d_j^S is

more leftist than c_j^S , then both players are better-off at \mathbf{c}^S than at \mathbf{d}^S .

Proof. Consider the strategy profile \mathbf{c}^S . Suppose that players deviates from \mathbf{c}^S to \mathbf{d}^S only in the first period of Γ_{q^S, π^0}^{en} . This deviation has an impact on players' welfare only at the preference profiles for which the outcome in the first period is not the same under \mathbf{c}^S and \mathbf{d}^S . This can arise only when $d_i \leq \theta_i^1 \leq c_i$, or when $c_j \leq \theta_j^1 \leq c_j$. Since c_i is more rightist than c_j , $d_i \leq \theta_i \leq c_i$ and $c_j \leq \theta_j \leq c_j$ are mutually incompatible.

Let s denote the state realization in period 1, and suppose first that $d_i^s \leq \theta_i^1 \leq c_i^s$, i.e., the deviation makes player i vote for R instead of L . Since c_i^S is more rightist than c_j^S , $\theta_i^1 \leq c_i^s$ implies that $\theta_j^1 \leq c_j^s$, which in turns imply $\theta_j^1 \leq d_j^s$, so in that case, player j votes for L . Therefore, if the status quo is R in period 1, the deviation from \mathbf{c}^S to \mathbf{d}^S changes the outcome from L to R , which increases the payoff of player $k \in \{i, j\}$ in period 1 by $2\theta_k$. Since players play their equilibrium strategy in the subgame starting from period 2 onwards, using the notations of Lemma 3, if period 1's outcome is y^1 , the continuation value for player $k \in \{i, j\}$ after period 1 is simply $V_k^s(y^1)$. Therefore, if $d_i^s \leq \theta_i^1 \leq c_i^s$ and of $q^1 = R$, the net effect of the deviation for player k is $2\theta_k + \delta (V_i^s(R) - V_i^s(L))$. From Lemma 3, this net effect is equal to $2(\theta_k^1 - c_k^s)$. Since $\theta_i^1 \leq c_i^s$ implies $\theta_j^1 \leq c_j^s$, this effect is negative for both players.

To conclude the argument, consider the above the strategy profile in which players play \mathbf{d}^S in period 1 and \mathbf{c}^S afterwards, and let players deviate from \mathbf{c}^S to \mathbf{d}^S in the second period. The same reasoning shows that the net effect from period 2 onwards is negative irrespective of the status quo distribution at the beginning of period 2, and that deviation has no effect in period 1. By induction on the number of periods in which players deviates from \mathbf{c}^S to \mathbf{d}^S , the lemma follows. ■

Proof of Proposition 3. Since \mathbf{c}^S is an equilibrium, from Proposition 2, $\mathbf{c}^S (\leq, \geq)^S \mathbf{0}^S$. Together with Assumption 1, this implies that c_i^S is more rightist than c_j^S . Likewise, $\mathbf{d}^S (\leq, \geq)^S \mathbf{c}^S$ implies that d_i^S is more rightist than c_i^S and d_j^S is more leftist than c_j^S . Lemma 5 completes the proof. ■

The following Proposition implies Proposition 4.³⁴

Proposition 11 *Using the conventions in Notation 1,*

- a) *If for some $\delta_o > 0$, $\mathbf{H}^S(\delta_o, f^S, \mathbf{0}^S) (\leq, \geq)^S \mathbf{0}^S$, and for all $\delta \in [\delta_o, 1[$, $\mathbf{c}^S(\delta, f^S) (\leq, \geq)^S \mathbf{0}^S$, then $\mathbf{c}^S(\delta, f^S)$ is increasing in δ on $[\delta_o, 1[$ in the order $(\leq, \geq)^S$;*
- b) *If g^S is more polarized than f^S (see definition 2), $\mathbf{c}^S(\delta, g^S) (\leq, \geq)^S \mathbf{c}^S(\delta, f^S)$.*

³⁴To see why it implies Proposition 4 part a), notice that from Proposition 2, for any f^S which satisfies Assumption 1, for all $\delta \in (0, 1)$, $\mathbf{c}^S(\delta, f^S) (\leq, \geq)^S \mathbf{0}^S$ and $\mathbf{H}^S(\mathbf{0}^S) (\leq, \geq)^S \mathbf{0}^S$.

Proof. Part (a): Clearly, if $\mathbf{H}^S(\delta_o, f^S, \mathbf{0}^S) (\leq, \geq)^S \mathbf{0}^S$ then for all $\delta \in [\delta_o, 1[$, $\mathbf{H}^S(\delta, f^S, \mathbf{0}^S) (\leq, \geq)^S \mathbf{0}^S$. Moreover, from Lemma 2, \mathbf{H}^S is isotone in \mathbf{c}^S in the order $(\leq, \geq)^S$, so $\mathbf{H}^S(\delta_o, f^S, (\mathbb{R}_- \times \mathbb{R}_+)^S) \subseteq (\mathbb{R}_- \times \mathbb{R}_+)^S$. From (4), for all $\mathbf{c}^S (\leq, \geq)^S \mathbf{0}^S$, $\frac{\partial \mathbf{H}^S(\delta_o, f^S, \mathbf{c}^S)}{\partial \delta} = \frac{\mathbf{H}^S(\delta_o, f^S, \mathbf{c}^S)}{\delta} (\leq, \geq)^S \mathbf{0}^S$. The result follows from Corollary 1 in Villas Boas (1997) applied to the restriction of $\mathbf{c}^S \rightarrow \mathbf{H}^S(\delta, f^S, \mathbf{c}^S)$ on $(\mathbb{R}_- \times \mathbb{R}_+)^S$ and to the order $(\leq, \geq)^S$.

Part (b): From Lemma 1 and (4), for all $s \in S$ and all $k \neq l$, $\frac{\partial H_k^s}{\partial m_k} \geq 0$ and $\frac{\partial H_l^s}{\partial m_k} \leq 0$. The result follows from Corollary 1 in Villas Boas (1997) applied to $\mathbf{c}^S \rightarrow \mathbf{H}^S(\delta, f^S, \mathbf{c}^S)$ and the order $(\leq, \geq)^S$. ■

Proof of Proposition 5. To prove Proposition 5, we will show that in the case $|S| = 1$, there exists a p.d.f. f on \mathbb{R}^2 such that, if $\mathbf{c}(\delta, f)$ denotes the smallest fixed point of $\mathbf{c} \rightarrow \mathbf{G}(\delta, f, \mathbf{c})$ where \mathbf{G} is defined in (8), then $\lim_{\delta \rightarrow 1} \mathbf{c}(\delta, f) = (-\infty, +\infty)$.³⁵ From Proposition 11 part b), for any finite state space S and any preference distribution g^S which is more polarized than f^S where for all $s \in S$, $f^s = f$, we must have $\lim_{\delta \rightarrow 1} \mathbf{c}^s(\delta, g^S) = (-\infty, +\infty)$.

Throughout this proof, $(\mathbf{m}^n)_{n \geq 0}$ is an arbitrary sequence which tends to $(+\infty, -\infty)$, f is an arbitrary p.d.f., and for all $\mathbf{m} \in \mathbb{R}^2$, $f_{\mathbf{m}}$ is defined by $f_{\mathbf{m}}(\boldsymbol{\theta}) = f(\boldsymbol{\theta} - \mathbf{m})$. With a simple change of variable, for all $\mathbf{c} \in \mathbb{R}^2$,

$$\mathbf{G}(\delta, f_{\mathbf{m}}, \mathbf{c}) = \delta \left(\int_{-\infty}^{c_j - m_j} \int_{c_i - m_i}^{\infty} (\mathbf{c} - \boldsymbol{\theta} - \mathbf{m}) f(\boldsymbol{\theta}) d\theta_i d\theta_j + \int_{c_j - m_j}^{\infty} \int_{-\infty}^{c_i - m_i} (\mathbf{c} - \boldsymbol{\theta} - \mathbf{m}) f(\boldsymbol{\theta}) d\theta_i d\theta_j \right). \quad (15)$$

We will show that for n sufficiently large, $\lim_{\delta \rightarrow 1} \mathbf{c}(\delta, f_{\mathbf{m}^n}) = (-\infty, +\infty)$.

Step 1: For n sufficiently large, $\mathbf{c} \rightarrow \mathbf{G}(1, f_{\mathbf{m}^n}, \mathbf{c})$ has no fixed point.

Let $s \in S$. From (15), for all $\mathbf{m}, \mathbf{c} \in \mathbb{R}^2$,

$$\begin{aligned} c_i - G_i(1, f_{\mathbf{m}}, \mathbf{c}) &= \int_{-\infty}^{c_j - m_j} \int_{-\infty}^{c_i - m_i} c_i f(\boldsymbol{\theta}) d\theta_i d\theta_j + \int_{c_j - m_j}^{\infty} \int_{c_i - m_i}^{\infty} c_i f(\boldsymbol{\theta}) d\theta_i d\theta_j \\ &\quad + \int_{c_j - m_j}^{\infty} \int_{-\infty}^{c_i - m_i} (m_i + \theta_i) f(\boldsymbol{\theta}) d\theta_i d\theta_j + \int_{-\infty}^{c_j - m_j} \int_{c_i - m_i}^{\infty} (m_i + \theta_i) f(\boldsymbol{\theta}) d\theta_i d\theta_j. \end{aligned} \quad (16)$$

We denote by $A(\mathbf{m}, \mathbf{c})$, $B(\mathbf{m}, \mathbf{c})$, $C(\mathbf{m}, \mathbf{c})$ and $D(\mathbf{m}, \mathbf{c})$ the four integrals in the order they appear on the right-hand side of (16). Suppose by contradiction that for all n , $\mathbf{G}(1, \mathbf{m}^n, \mathbf{c})$ has a fixed point, and let $(\mathbf{c}^n)_{n \in \mathbb{N}}$ be selection of them. From Corollary 1 of Villas-Boas 1997, we can choose $(\mathbf{c}^n)_{n \in \mathbb{N}}$ to be increasing in n in the order (\leq, \geq) . In particular, $\mathbf{c}^n - \mathbf{m}^n$ tends to $(-\infty, +\infty)$. Moreover, if g is the p.d.f. of an integrable real random variable,

³⁵The reader can check that Lemma 2 and its proof hold unchanged for \mathbf{G} (set π equal to the identity matrix), which shows that $\mathbf{c}(\delta, f)$ exists for all $\delta < 1$.

$\int_{-\infty}^x |xg(u)| du \rightarrow 0$ as $x \rightarrow -\infty$, so

$$\begin{aligned} |A(\mathbf{m}^n, \mathbf{c}^n)| &\leq \int_{-\infty}^{c_i^n - m_i^n} |c_i^n| f_i(\theta_i) d\theta_i \rightarrow 0, \\ |C^s(\mathbf{m}^n, \mathbf{c}^n)| &\leq \int_{-\infty}^{c_i^n - m_i^n} (|m_i^n| + |\theta_i|) f_i(\theta_i) d\theta_i \rightarrow 0, \end{aligned} \quad (17)$$

and $D^s(\mathbf{m}^n, \mathbf{c}^n) \rightarrow +\infty$. Substituting $c_i^n = G_i(1, f_{\mathbf{m}^n}, \mathbf{c}^n)$, (17), and $D^s(\mathbf{m}^n, \mathbf{c}^n) \rightarrow +\infty$ in (16), we get that that $B(\mathbf{m}^n, \mathbf{c}^n) \rightarrow \infty$. However,

$$|B(\mathbf{m}^n, \mathbf{c}^n)| \leq |c_i^n| \int_{c_j^n}^{\infty} f_j(\theta_j) d\theta_j = \frac{|c_i^n|}{|c_j^n|} \times |c_j^n| \int_{c_j^n}^{\infty} f_j(\theta_j) d\theta_j,$$

so $B(\mathbf{m}^n, \mathbf{c}^n) \rightarrow \infty$ implies that $|c_i^n| / |c_j^n| \rightarrow +\infty$. The symmetric argument for j implies that $|c_j^n| / |c_i^n| \rightarrow +\infty$, a contradiction.

Step 2: For n sufficiently large, $\mathbf{c}(\delta, f_{\mathbf{m}^n}) \rightarrow_{\delta \rightarrow 1} \mathbf{c}(1, f_{\mathbf{m}^n})$, where $c_i(1, f_{\mathbf{m}^n}) \in [-\infty, 0] \times [0, +\infty]$.

From Proposition 4 part b), for all n , $\mathbf{c}(\delta, f_{\mathbf{m}^n}) (\leq, \geq) \mathbf{c}(\delta, f_{\mathbf{m}^0})$. One can see from (15) that as $\mathbf{m} \rightarrow (+\infty, -\infty)$, $\mathbf{G}(\delta, f_{\mathbf{m}}, \mathbf{c}) \rightarrow (-\infty, +\infty)$ uniformly on $\delta \in [0, 1]$ and $\mathbf{c} \in [-\infty, c_i(\delta, f_{\mathbf{m}^0})] \times [c_j(\delta, f_{\mathbf{m}^0}), +\infty]$. So for n sufficiently large, $\mathbf{c}(\delta, f_{\mathbf{m}^n}) (\leq, \geq) \mathbf{0}$, and $\mathbf{G}(\delta, f_{\mathbf{m}^n}, \mathbf{0}) (\leq, \geq) \mathbf{0}$. Therefore, from Proposition 4 part a), $\mathbf{c}(\delta, f_{\mathbf{m}^n})$ is monotonic in δ in the order $(\leq, \geq)^S$, which shows the existence of $\mathbf{c}^S(1, f_{\mathbf{m}^n})$.

Step 3: For some $k \in \{i, j\}$, $c_k(1, f_{\mathbf{m}^n})$ is infinite.

If $\mathbf{c}(1, \mathbf{m}^n)$ was finite, then by continuity of $\mathbf{G}(\delta, f_{\mathbf{m}}, \mathbf{c})$ in δ and in \mathbf{c} , $\mathbf{c}(1, \mathbf{m}^n)$ would be a fixed point of $\mathbf{H}(1, \mathbf{m}^n, \mathbf{c})$, which is impossible from step 1.

Step 4: for n sufficiently, large, $\mathbf{c}(1, f_{\mathbf{m}^n}) = (-\infty, +\infty)$.

Suppose that $c_k(1, \mathbf{m}^n)$ is finite for all n and for some k . To fix ideas, let $k = j$ (the proof in the case $k = i$ is identical). From step 3, for n sufficiently large, $c_i(1, f_{\mathbf{m}^n}) = -\infty$. By continuity, for all n , $c_j(1, f_{\mathbf{m}^n})$ must be a fixed point the map $G_j^n(-\infty, c_j)$ is defined by:

$$G_j^n(-\infty, c_j) = \lim_{c_i \rightarrow -\infty} G_j(1, \mathbf{m}^n, c_i, c_j) = \int_{-\infty}^{c_j - m_j^n} (c_j - \theta_j - m_j^n) f_j(\theta_j) d\theta_j.$$

Observe that

$$G_j^n(-\infty, c_j) - c_j = -m_j^n + (c_j - m_j^n) \int_{c_j - m_j^n}^{+\infty} f_j(\theta_j) d\theta_j - \int_{-\infty}^{c_j - m_j^n} \theta_j f_j(\theta_j) d\theta_j. \quad (18)$$

The last two terms of the right-hand side of (18) are bounded for all $m_j^n \leq 0$ and $c_j \geq 0$. Therefore, for n sufficiently large, $G_j^n(-\infty, c_j) - c_j$ is bounded above 0 as c_j tends to $+\infty$. Moreover, simple calculus shows that $\frac{dG_j^n(-\infty, c_j)}{dc_j} \leq 1$. Therefore, $G_j^n(-\infty, c_j)$ has no fixed point, a contradiction. ■

Proof of Corollary 2. Using the notations in the text of the Proposition, since ε is symmetric, $\mathbf{c}(\delta) = (-c(\delta), c(\delta))$. From Proposition 2, $c(\delta) \geq 0$, and from Proposition 1, $c(\delta)$ is the smallest fixed point of $H(\delta, c) = \delta \int_{-c-\bar{\theta}}^{c+\bar{\theta}} (c + \bar{\theta} - \varepsilon) f(\varepsilon) d\varepsilon$. From the proof of Proposition 5, to show that $\lim_{\delta \rightarrow 1} \mathbf{c}(\delta) = (-\infty, +\infty)$, we need to show that $H(1, c)$ has no fixed point for $\bar{\theta} \geq \sigma$. Using successively the symmetry of ε and Holder inequality, for all $c > 0$,

$$\begin{aligned} H(1, c) - c &= \int_{-c-\bar{\theta}}^{c+\bar{\theta}} (c + \bar{\theta}) f(\varepsilon) d\varepsilon - \int_{-\infty}^{\infty} c f(\varepsilon) d\varepsilon \\ &= - \int_{-\infty}^{-c-\bar{\theta}} (c + \bar{\theta}) f(\varepsilon) d\varepsilon - \int_{c+\bar{\theta}}^{+\infty} (c + \bar{\theta}) f(\varepsilon) d\varepsilon + \bar{\theta} \\ &> - \int_{-\infty}^{+\infty} |\varepsilon| f(\varepsilon) d\varepsilon + \bar{\theta}. \end{aligned}$$

■

Proof of Propostion 6. Let \mathbf{c}^s be an equilibrium of Γ_{q^s, π^0}^{en} . We shall compare the equilibrium payoffs in every period t in Γ_{q^s, π^0}^{en} and in Γ_{q^s, π^0}^{ex} . Let s the state in period t . There are 5 possible cases to consider:

Case 1: $\theta_i^t < c_i^s$ In this case, necessarily, $\theta_j^t < \theta_i^t < c_i^s < 0 < c_j^s$ so both players vote for L in Γ_{q^s, π^0}^{en} and in Γ_{q^s, π^0}^{ex} . Therefore, the payoffs in the two games are the same.

Case 2: $\theta_j^t > c_j^s$ In this case, necessarily, $c_i^s < 0 < c_j^s < \theta_j^t < \theta_i^t$ so both players vote for R in Γ_{q^s, π^0}^{en} and in Γ_{q^s, π^0}^{ex} . Therefore, the payoffs in the two games are the same.

Case 3: $c_i^s < \theta_i^t < 0$ In this case, necessarily, $\theta_j^t < \theta_i^t < 0 < c_j^s$ so both players vote for L in Γ_{q^s, π^0}^{ex} but they disagree in Γ_{q^s, π^0}^{en} . Since $\theta_i^t + \theta_j^t < 0$, the sum of players' payoff is weakly higher in Γ_{q^s, π^0}^{ex} than in Γ_{q^s, π^0}^{en} .

Case 4: $0 < \theta_j^t < c_j^s$ In this case, necessarily, $c_i^s < 0 < \theta_j^t < \theta_i^t$ so both players vote for R in Γ_{q^s, π^0}^{ex} but they disagree in Γ_{q^s, π^0}^{en} . Since $\theta_i^t + \theta_j^t > 0$, the sum of players' payoff is weakly higher in Γ_{q^s, π^0}^{ex} than in Γ_{q^s, π^0}^{en} .

Case 5: $\theta_j^t < 0 < \theta_i^t$ In this case, necessarily, $c_i^s < \theta_i^t$ and $\theta_j^t < c_j^s$ players disagree in both games, so which game yields the highest social welfare depends on the distribution of the status quo in the period t of Γ_{q^s, π^0}^{ex} and Γ_{q^s, π^0}^{en} .

In the first four cases, the sum of players' payoff is higher in Γ_{q^S, π^0}^{ex} than in Γ_{q^S, π^0}^{en} . In the last case, it depends on q^S . Assume for a moment that the exogenous status quo is allowed to depend on the preference and state realization in all previous periods. Then since strategies in Γ_{q^S, π^0}^{en} depend only on the preference and state realization in all previous periods, the distribution of the exogenous status quo can be chosen to replicate the distribution of the status quo in the game Γ_{q^S, π^0}^{en} in all histories. Therefore, in case 5, the sum of player's payoff will be the same in both games.

To conclude the argument, it should be clear that the optimal exogenous status quo q^{*S} in some period t (i.e., the state dependent status quo that maximizes the sum of player's payoff in a given period) is independent of t , δ , and π^0 , and from what precedes, the sum of player's expected payoff is weakly higher in every period of $\Gamma_{q^{*S}, \pi^0}^{ex}$ than of Γ_{q^S, π^0}^{en} . To see why it is strictly higher, observe that since the marginal distribution of θ_i and θ_j are assumed to have full support, Proposition 2 implies that cases 3 and 4 occur with strictly positive probability. Depending on the identity of the status quo, in at least one of the two cases, welfare is strictly higher in $\Gamma_{q^{*S}, \pi^0}^{ex}$. ■

Proof of Proposition 7. One can easily check that Lemma 3 and its proof hold unchanged for the game $\Gamma_{q^S, \pi^0}^{en}(\Omega)$ if we replace player i and j by player $n \in N$ and all the other players, respectively. This shows that a stationary, stage undominated equilibrium must be a cutoff strategy \mathbf{c}^S , and that c_n^s must satisfy (12) for all $n \in N$ and all $s \in S$.

Let $\mathbf{V}^S(L)$ and $\mathbf{V}^S(R)$ denote the continuation values for the strategy profile \mathbf{c}^S . For all $\mathbf{c} \in \mathbb{R}^N$, let $D(\mathbf{c}) \subseteq \mathbb{R}^N$ be the set of preference realizations $\boldsymbol{\theta}$ such that if players vote according to the strategy profile \mathbf{c} , the outcome of the vote is different with the voting rule Ω_L and status quo L than with the voting rule Ω_R and status quo R . The status quo matters in some period t with state s only if $\boldsymbol{\theta}^t \in D(\mathbf{c}^s)$, so

$$V_n^s(L) - V_n^s(R) = \delta \sum_{s' \in S} \pi(s, s') \left(\int_{D(\mathbf{c}^{s'})} \left(-\theta_n + \delta V_n^{s'}(L) - \left(\theta_n + \delta V_n^{s'}(R) \right) \right) f^{s'}(\boldsymbol{\theta}) d\boldsymbol{\theta} \right).$$

If we substitute (12) on both sides of the above equation, we get

$$c_n^s = \delta \sum_{s' \in S} \pi(s, s') \int_{D(\mathbf{c}^{s'})} \left(-\theta_n + c_n^{s'} \right) f^{s'}(\boldsymbol{\theta}) d\boldsymbol{\theta}. \quad (19)$$

Since $\theta_1 \geq \dots \geq \theta_N$ with probability one, for all $s \in S$, $\int_{D(\mathbf{c}^s)} \theta_n f^s(\boldsymbol{\theta}) d\boldsymbol{\theta}$ is weakly decreasing in n . Together with (19), this implies that $c_n^s - \delta \sum_{s' \in S} \pi(s, s') c_n^{s'}$ is weakly decreasing in n . As shown in the proof of Proposition 2, this implies in turn that c_n^s is weakly increasing in n .

For all $\mathbf{c} \in \mathbb{R}^N$, $D(\mathbf{c})$ can be rewritten as the union of $D'(\mathbf{c})$ and $D''(\mathbf{c})$, where:

$$\begin{aligned} D'(\mathbf{c}) &= \{\boldsymbol{\theta} \in \mathbb{R}^N : \{i \in N : \theta_i \geq c_i\} \notin \Omega_L \text{ and } \{i \in N : \theta_i \leq c_i\} \notin \Omega_R\}, \\ D''(\mathbf{c}) &= \{\boldsymbol{\theta} \in \mathbb{R}^N : \{i \in N : \theta_i \geq c_i\} \in \Omega_L \text{ and } \{i \in N : \theta_i \leq c_i\} \in \Omega_R\}. \end{aligned}$$

Condition (iv) in Definition 3 implies that $D''(\mathbf{c}) = \emptyset$ when c_n^s is weakly increasing in n . From what precedes, for all state s , $\theta_1 - c_1^s \geq \dots \geq \theta_N - c_N^s$ with probability one. So with probability 1, there exists $n \in \{0, \dots, N\}$ such that

$$\begin{aligned} \{i \in N : \theta_i \geq c_i^s\} &= \{1, \dots, n\}, \\ \{i \in N : \theta_i \leq c_i^s\} &= \{n+1, \dots, N\}. \end{aligned}$$

Therefore, up to a zero measure set,

$$D'(\mathbf{c}^s) = \{\boldsymbol{\theta} \in \mathbb{R}^N : \theta_{n_L} \leq c_{n_L}^s \text{ and } \theta_{n_R} \geq c_{n_R}^s\},$$

which proves (5) and shows that players n_L and n_R are always pivotal, and $\Gamma_{q^s, \pi^0}^{en}(\Omega)$ boils down to the two player game Γ_{q^s, π^0}^{en} played by the two pivotal players. Since $n_R \leq n_L$, with probability 1, $\theta_R \geq \theta_L$ so from Proposition 2, for all $s \in S$, $c_{n_R}^s \leq c_{n_L}^s$. The lattice structure follows from Propositions 1.

To conclude the proof, we show that the order on the lattice also coincide with the Pareto ranking for all the players $n \in \{n_R, \dots, n_L\}$. To do so, we will show that the welfare result established in Proposition 3 for the two players of Γ_{q^s, π^0}^{en} (i.e., n_R and n_L in this case) also applies to any individual whose preference distribution $(\theta^t)_{t \geq 1}$ over the outcome of Γ_{q^s, π^0}^{en} is in between that of i and j (i.e., for all states, $\theta_i \geq \theta^t \geq \theta_j$ with probability 1). Since that the proof of Proposition 3 follows directly from Lemma 5, it suffices to check that Lemma 5 also holds for that intermediate individual. This is the case because for all $s \in S$, if $V^s(y)$ denotes the continuation value for that player at the end of a period in which the state is s and the outcome is y (see Lemma 3), and if we denote $c^s = \frac{\delta}{2}(V^s(L) - V^s(R))$, then $c_i^s \leq c^s \leq c_j^s$.³⁶ So with probability one, if $\theta_i - c_i^s$ and $\theta_j - c_j^s$ are negative, $\theta - d^s$ must also be negative. Therefore, using the notation of Lemma 5, the intermediate individual is also better-off at \mathbf{c}^s than at \mathbf{d}^s . ■

Proof of Proposition 8. The inequalities $c_{n_R}(\Omega) \leq 0 \leq c_{n_L}(\Omega)$ are established in Proposition 7.

³⁶To see that, simply consider the three player game with players i , j , and the intermediate player, and the unanimity rule. Proposition 7 implies that $c_i^s \leq c^s \leq c_j^s$.

Let (n_L, n_R) and (n'_L, n'_R) be the pivotal players of $\Gamma_{q^S, \pi^0}^{en}(\Omega)$ and $\Gamma_{q^S, \pi^0}^{en}(\Omega')$, respectively. From Definition 4,

$$\{1, \dots, n'_L\} \in \Omega'_L \text{ and } \{n'_R, \dots, N\} \in \Omega'_R.$$

Since the concentration of power is greater under Ω than under Ω' ,

$$\{1, \dots, n'_L\} \in \Omega_L \text{ and } \{n'_R, \dots, N\} \in \Omega_R.$$

It follows from Definition 4 and 3 that $n'_R \leq n_R$ and $n_L \leq n'_L$. Proposition 7 implies then that $c_{n'_R}(\Omega) \leq c_{n_R}(\Omega)$ and $c_{n_L}(\Omega) \leq c_{n'_L}(\Omega)$.

To complete the proof, it remains to show that $c_{n'_R}(\Omega') \leq c_{n'_R}(\Omega)$ and $c_{n'_L}(\Omega) \leq c_{n'_L}(\Omega')$. Since $\theta_1 \geq \dots \geq \theta_N$ with probability 1, the distribution of $(\theta_{n'_L}, \theta_{n'_R})$ is more polarized than the distribution of $(\theta_{n_L}, \theta_{n_R})$ in the sense of Definition 2. Proposition 4 implies then that for all $s \in S$, at the least partisan equilibria of $\Gamma_{q^S, \pi^0}^{en}(\Omega)$ and $\Gamma_{q^S, \pi^0}^{en}(\Omega')$,

$$c_{n'_R}^s(\Omega') \leq c_{n_R}^s(\Omega) \leq \text{and } c_{n_L}^s(\Omega) \leq c_{n'_L}^s(\Omega'). \quad (20)$$

From Proposition 7,

$$c_{n'_L}^s(\Omega') = \delta \sum_{r \in S} \pi(s, r) \int_{\left\{ \theta \in \mathbb{R}^N : \theta_{n'_R} \geq c_{n'_R}^r(\Omega') \text{ and } \theta_{n'_L} \leq c_{n'_L}^r(\Omega') \right\}} \left(c_{n'_L}^r(\Omega') - \theta_{n'_L} \right) f^r(\theta) d\theta.$$

Hence, $c_{n'_L}^s(\Omega')$ is a sum of integrals whose integrands are nonnegative on their respective domains. Moreover, (20) together with $n'_R \leq n_R$ and $n_L \leq n'_L$ imply that for all $r \in S$, with probability one,

$$\left\{ \theta_{n'_R} \geq c_{n'_R}^r(\Omega') \text{ and } \theta_{n'_L} \leq c_{n'_L}^r(\Omega') \right\} \Rightarrow \left\{ \theta_{n_R} \geq c_{n_R}^r(\Omega) \text{ and } \theta_{n_L} \leq c_{n_L}^r(\Omega) \right\}.$$

Therefore,

$$c_{n'_L}^s(\Omega') \leq \delta \sum_{r \in S} \pi(s, s') \int_{\left\{ \theta \in \mathbb{R}^N : \theta_{n_R} \geq c_{n_R}^r(\Omega) \text{ and } \theta_{n_L} \leq c_{n_L}^r(\Omega) \right\}} \left(c_{n'_L}^{s'}(\Omega') - \theta_{n'_L} \right) f^{s'}(\theta) d\theta.$$

From Proposition 7, the right hand-side of the above equation is simply $c_{n'_L}^s(\Omega)$. A similar proof shows that $c_{n'_R}^s(\Omega) \leq c_{n'_R}^s(\Omega')$.

That $c_{n_R}^s(\Omega) < 0 < c_{n_L}^s(\Omega)$ when $n_R > n_L$ follows directly from Proposition 2, and from our assumption on the distribution of θ . ■

Lemma 6 *Let σ and σ' be two strategy profile of Γ_{q^S, π^0}^{en} such that for all $k \in \{i, j\}$, σ'_k is more rightist than σ_k in the sense of Definition 8, and let $(\zeta^t)_{t \geq 1}$ and $(\zeta'^t)_{t \geq 1}$ be two*

preference distributions such that $(\zeta^t)_{t \geq 1}$ is more rightist than $(\zeta^t)_{t \geq 1}$ (i.e., for all t , with probability 1, $\zeta^t \leq \zeta^t$). If the expected payoff of σ' for $(\zeta^t)_{t \geq 1}$ is greater than that of σ , then the expected payoff of σ' for $(\zeta^t)_{t \geq 1}$ is also greater than that of σ . The same result holds if σ' is more leftist than σ and $(\zeta^t)_{t \geq 1}$ is more leftist than $(\zeta^t)_{t \geq 1}$.

Proof. With a slight abuse of notation, let $(\zeta^t)_{t \geq 1}$ and $(\zeta^t)_{t \geq 1}$ be some realization of the preference distributions, and let $(y^t)_{t \geq 1}$ and $(y^t)_{t \geq 1}$ be the outcome of Γ_{q^S, π^0}^{en} for some realization of σ and σ' , respectively. For convenience, R will be denoted by 1 and L by -1 , so that for all t , y^t and y^t are in $\{-1, 1\}$. Almost surely, for all t , $\zeta^t \leq \zeta^t$. Moreover, since σ is more rightist than σ' , a straightforward induction argument shows that almost surely, in all periods, $y^t \leq y^t$. Therefore,

$$\sum_n \zeta^t (y^t - y^t) \leq \sum_n \zeta^t (y^t - y^t).$$

The expectation of the left-hand side of that inequality is nonnegative by assumption, so the expectation of the right hand-side must also be positive. ■

Proof of Proposition 9. Let (n_R, n_L) and (n'_R, n'_L) be the pivotal players of $\Gamma_{q^S, \pi^0}^{en}(\Omega)$ and $\Gamma_{q^S, \pi^0}^{en}(\Omega')$, respectively. As shown in the proof of Proposition 8, with probability one,

$$\theta_{n'_R} \geq \theta_{n_R} \geq \theta_{n_L} \geq \theta_{n'_L}. \quad (21)$$

For all $n, m \in N$, let $\Gamma_{q^S, \pi^0}^{en}(n, m)$ denote the 2-player game Γ_{q^S, π^0}^{en} defined in section 3 in which player i and j have the same preference distribution as player n and m in $\Gamma_{q^S, \pi^0}^{en}(\Omega)$. Let σ^0 denote a stationary equilibrium of $\Gamma_{q^S, \pi^0}^{en}(n'_R, n'_L)$ (implicitly, the strategy of each player in every period t is allowed to depend on the whole profile of preference $(\theta_n^t)_{n \in N}$). Consider the sequence of (stationary) strategy profile $(\sigma^n)_{n \geq 0}$ such that for all $n \geq 0$,

- if n is odd, $\sigma_j^{n+1} = \sigma_j^n$ and σ_i^{n+1} is player i 's best response to σ_j^n in $\Gamma_{q^S, \pi^0}^{en}(n_R, n_L)$ as described in Lemma 3,
- if n is even, $\sigma_i^{n+1} = \sigma_i^n$ and σ_j^{n+1} is player j 's best response to σ_i^n in $\Gamma_{q^S, \pi^0}^{en}(n_R, n_L)$ as described in Lemma 3.

We will show by induction on n that for all $n \geq 0$, σ_i^{n+1} is more leftist than σ_i^n while σ_j^{n+1} is more leftist than σ_j^n in the sense of 8, and, using the terminology of Lemma 6, that any individual whose preference distribution $(\zeta^t)_{t \geq 1}$ is more leftist than player i and more rightist than player j in the game $\Gamma_{q^S, \pi^0}^{en}(n_R, n_L)$ prefers the outcome of the the strategy profile σ^{n+1} to the outcome of the strategy profile σ^n in that game.

From (21), the best response of player i to σ_j^0 in $\Gamma_{q^S, \pi^0}^{en}(n_R, n_L)$ is more leftist than the best response of player i to σ_j^0 in $\Gamma_{q^S, \pi^0}^{en}(n'_R, n'_L)$. This means that σ_i^1 is more leftist than σ_i^0 . Since σ_i^1 is her best response to σ_j^0 , player i in $\Gamma_{q^S, \pi^0}^{en}(n_R, n_L)$ is better off at $(\sigma_i^1, \sigma_j^0) = \sigma^1$ than at σ^0 . From Lemma 6, any individual whose preference distribution $(\zeta^t)_{t \geq 1}$ is more leftist than player i in the game $\Gamma_{q^S, \pi^0}^{en}(n_R, n_L)$ prefers the outcome of the the strategy profile σ^1 to that of σ^0 in that game.

A symmetric reasoning shows that σ_j^2 is more rightist than $\sigma_j^1 = \sigma_j^0$, and that player j , as well as any individual whose preference distribution $(\zeta^t)_{t \geq 1}$ is more rightist than player j is better off at σ^2 than at σ^1 in the game $\Gamma_{q^S, \pi^0}^{en}(n_R, n_L)$. Therefore, we have shown the induction property at $n = 0$ and $n = 1$.

Suppose now that the induction property holds at n and $n + 1$ for some even $n \geq 0$. From the induction hypothesis, σ_j^{n+2} is more rightist than σ_j^{n+1} , so the best response of player of i to σ_j^{n+2} must be more leftist than her best response to $\sigma_j^{n+1} = \sigma_j^n$.³⁷ This means that σ_i^{n+3} is more leftist than $\sigma_i^{n+1} = \sigma_i^{n+2}$. Since σ_i^{n+3} is player i 's best response to σ_j^{n+2} , she is better off at $(\sigma_i^{n+3}, \sigma_j^{n+2}) = \sigma^{n+3}$ than at σ^{n+2} in $\Gamma_{q^S, \pi^0}^{en}(n_R, n_L)$. From Lemma 6, any individual whose preference distribution $(\zeta^t)_{t \geq 1}$ is more leftist than player i prefers the outcome of the strategy profile σ^{n+3} than that of σ^{n+2} in the game $\Gamma_{q^S, \pi^0}^{en}(n_R, n_L)$, which shows the induction property at $n + 2$. A symmetric reasoning shows that the induction property holds at $n + 3$.

By construction, $(\sigma^n)_{n \geq 0}$ is a sequence of cutoff strategies for the players of $\Gamma_{q^S, \pi^0}^{en}(n_R, n_L)$, so for all n , σ^n can be viewed as an element of \mathbb{R}^{2S} . From what precedes, for all n ,

$$\mathbf{0}^S (\leq, \geq)^S \sigma^{n+1} (\leq, \geq)^S \sigma^n.$$

Therefore, $(\sigma^n)_{n \geq 0}$ has a limit σ^∞ . By continuity, σ^∞ must be a cutoff equilibrium of $\Gamma_{q^S, \pi^0}^{en}(n_R, n_L)$. The induction property implies that any individual whose preference distribution $(\zeta^t)_{t \geq 1}$ is more leftist than player i and more rightist than player j must prefer the outcome of σ^∞ to that of σ^0 . This can be rephrased as follows: any player $n \in \{n_R, n_L\}$ prefers the equilibrium of $\Gamma_{q^S, \pi^0}^{en}(n_R, n_L)$ in which pivotal players play σ^∞ to the equilibrium of $\Gamma_{q^S, \pi^0}^{en}(n'_R, n'_L)$ in which pivotal players play σ^0 . From Proposition 7, any player $n \in \{n_R, n_L\}$ prefers the least partisan equilibrium of $\Gamma_{q^S, \pi^0}^{en}(n_R, n_L)$ to σ^∞ , which concludes the proof. ■

³⁷To see this, observe σ_j is a cutoff strategy of player j , and from Lemma 4, the cutoff best response of player i is the unique solution of $c_i^S = H_i(c_i^S, c_j^S)$. From Lemma 2, $H_i(c_i^S, c_j^S) - c_i^S$ is decreasing in c_i^S and in c_j^S for the product order on \mathbb{R}^S (which coincides with the order “more rightist than” on the space of cutoff strategies).

References

- [1] Daron Acemoglu, Georgy Egorov, Konstantin Sonin. 2008. Coalition Formation in Non Democracies. *Review of Economic Studies*, 75(4), pp. 987-1009
- [2] Daron Acemoglu, Georgy Egorov, Konstantin Sonin. 2011. Dynamics and Stability of Constitutions, Coalitions, and Clubs. Forthcoming, *American Economic Review*.
- [3] Alesina, A. and A. Drazen, 1991. Why Are Stabilizations Delayed? *American Economic Review*, 81(5), pp. 1170-88. Alesina, A. and A. Drazen, 1991. Why Are Stabilizations Delayed? *American Economic Review*, 81(5), pp. 1170-88.
- [4] Anesi, Vincent. 2010. Noncooperative foundations of stable sets in voting games. *Games and Economic Behavior*. Volume 70, Issue 2, November 2010, Pages 488-493.
- [5] Austen-Smith, D., and J. Banks, 2000. *Positive Political Theory I: Collective Preferences*. Ann Arbor, Michigan: University of Michigan Press.
- [6] Barbera, S. and M. Jackson, 2010. The Role of a Status Quo and Compromise in Dynamic Voting. Unpublished manuscript.
- [7] Baron, D., 1991. Majoritarian Incentives, Pork Barrel Programs, and Procedural Control. *American Journal of Political Science*, 35(1), pp. 57–90.
- [8] Baron, D., 1993. A Theory of Collective Choice for Government Programs. Unpublished manuscript.
- [9] Baron, D., 1996. A Dynamic Theory of Collective Goods Programs. *The American Political Science Review*, 90 (2), pp. 316-330
- [10] David P. Baron, and John A. Ferejohn. Bargaining in Legislatures. *The American Political Science Review*, Vol. 83, No. 4. (Dec., 1989), pp 1181-1206.
- [11] Baron, D., D. Diermeier, and P. Fong, 2007. A Dynamic Theory of Parliamentary Democracy. Forthcoming, *Economic Theory*.
- [12] Baron, D. and M. Herron, 2003. A Dynamic Model of Multidimensional Collective Choice. *Computational Models of Political Economy*, Ken Kollman, John H. Miller, and Scott E. Page, eds., MIT Press, 2003, pp. 13-47.
- [13] Baron D. and E. Kalai, 1993. The Simplest Equilibrium of a Majority-Rule Division Game. *Journal of Economic Theory*, 61(2), pp. 290-301.

- [14] Battaglini, M. and S. Coate, 2007. Inefficiency in Legislative Policymaking: A Dynamic Analysis. *American Economic Review*, 97(1), pp. 118-149.
- [15] Battaglini, M. and S. Coate, 2008. A Dynamic Theory of Public Spending, Taxation, and Debt. *American Economic Review*, 98(1), pp. 201–236.
- [16] Battaglini, M. and T. Palfrey, 2007. The Dynamics of Distributive Politics. Unpublished manuscript.
- [17] Benabou, R., 2000. Unequal Societies: Income Distribution and the Social Contract. *American Economic Review*, 90(1), pp. 96–129.
- [18] Bernheim, D., A. Rangel, and L. Rayo, 2006. The Power of the Last Word in Legislative Policy Making. *Econometrica*, 74(5), pp. 1161-90.
- [19] Bikker, J. A. and P.J.G Vlaar, 2007. Conditional Indexation in Defined Benefit Pension Plans in the Netherlands. *The Geneva Papers on Risk and Insurance—Issues and Practice*, 32, pp. 494–515.
- [20] Buchanan, J., and G. Tullock, 1962. *The Calculus of Consent*. Ann Arbor: University of Michigan Press.
- [21] Casella, A., 2005. Storable votes. *Games and Economic Behavior* 51, pp. 391–419.
- [22] Cho, S., 2005. A Dynamic Model of Parliamentary Democracy. Unpublished manuscript.
- [23] Coate, S. and S. Morris, 1999. Policy Persistence. *American Economic Review*, 89(5), pp. 1327-1336.
- [24] Diermeier, D., and P. Fong, 2011. Legislative Bargaining with Reconsideration. *The Quarterly Journal of Economics* 126(2): 947-985.
- [25] Diermeier, D. and P. Fong. 2008. Policy Persistence in Multi-Party Parliamentary Democracies. *Institutions and Economic Performance*, ed. Elhanan Helpman.
- [26] Duggan, J. and T. Kalandrakis, 2009. Dynamic Legislative Policy Making. Unpublished manuscript.
- [27] Dziuda, W. and A. Loeper, 2010. Dynamic Collective Choice with Endogenous Status Quo. Northwestern University, Discussion Paper 1514.
- [28] Epple, D. and M. Riordan, 1987. Cooperation and Punishment Under Repeated Majority Voting. *Public Choice*, 55, pp. 41-73.

- [29] Ferejohn, J. A., M. P. Fiorina, and R. D. McKelvey, 1987. Sophisticated Voting and Agenda Independence in the Distributive Politics Setting. *American Journal of Political Science*, 31, pp.169-93.
- [30] Fernandez, R. and D. Rodrik, 1991. Resistance to Reform: Status Quo Bias in the Presence of Individual-Specific Uncertainty. *American Economic Review*, 81(5), pp. 1146-55.
- [31] Fong, P., 2006. Dynamics of Government and Policy Choice. Unpublished manuscript.
- [32] Jacob E. Gersen. 2007. Temporary Legislation, University of Chicago Law Review pp. 247-298.
- [33] Glomm, G. and B. Ravikumar, Endogenous Public Policy and Multiple Equilibria. *European Journal of Political Economy*, December 1995, 11(4), pp. 653–62.
- [34] Hassler, J., J. V. Rodriguez Mora, K. Storesletten, and F. Zilibotti, 2001. The Survival of the Welfare State. *American Economic Review*, 93, pp. 87-112.
- [35] Hassler J., P. Krusell, K. Storesletten, and F. Zilibotti, 2005. The Dynamics of Government. *Journal of Monetary Economics*, 52, pp. 1331-1358.
- [36] Hird, J. A., 1991. The Political Economy of Pork: Project Selection at the U.S. Army Corps of Engineers. *American Political Science Review*,85, pp. 430-56.
- [37] Kalai, E., 1977. Proportional Solutions to Bargaining Situations: Interpersonal Utility Comparisons. *Econometrica*, 45 (7), pp. 1623-1630.
- [38] Kalandrakis, T., 2004. A three-player dynamic majoritarian bargaining game. *Journal of Economic Theory*, 116, 294–322.
- [39] Kalandrakis, T., 2006. Proposal Rights and Political Power. *American Journal of Political Science*, 50(2), pp. 441-448.
- [40] Kalandrakis, T., 2007. Majority Rule Dynamics with Endogenous Status Quo. Unpublished manuscript.
- [41] Kearney, R., 1990. Sunset: A Survey and Analysis of the State Experience. *Public Administration Review*, 50(1), pp. 49-57.
- [42] Krusell, P. and J. V. Rios-Rull, 1996. Vested Interests in a Positive Theory of Stagnation and Growth. *Review of Economic Studies*, 63(2), pp. 301–329.

- [43] Krusell, P. and J. V. Rios-Rull, 1999. On the Size of the U.S. Government: Political Economy in the Neoclassical Growth Model.” *American Economic Review*, 89(5), pp. 1156–81.
- [44] Charlene C. Kwan. 2009. Fixing the Farm Bill: Using the "Permanent Provisions" in Agricultural Law to Achieve WTO Compliance. *Boston College Environmental Affairs Law Review*, Volume 36, Issue 2, Article 11
- [45] Montagnes, P., 2010. Voting Rules and Contractual Defaults. Unpublished manuscript.
- [46] Lowi, T.J., 1969. *The End of Liberalism: Ideology, Policy, and the Crisis of Public Authority*. Norton.
- [47] Rasch, Bjorn Eric. 2000. Parliamentary Floor Procedures and Agenda Setting in Europe. *Legislative Studies Quarterly* 25, pp. 3-23.
- [48] Riboni, A. and F. Ruge-Murcia, 2008. The Dynamic (In)efficiency of Monetary Policy by Committee. *Journal of Money, Credit, and Banking*, 40, pp. 1001-1032.
- [49] Riker, W. H., 1962. *The Theory of Political Coalitions*. New Haven: Yale University Press.
- [50] Romer, T. and H. Rosenthal, 1978. Political Resource Allocation, Controlled Agendas, and the Status Quo. *Public Choice*, 33, pp. 27-43.
- [51] Saint Paul, G., 2001. The Dynamics of Exclusion and Fiscal Conservatism. *Review of Economic Dynamics*, 4(2), pp. 275–302.
- [52] Saint Paul, G. and T. Verdier, 1997. Power, Distributive Conflicts, and Multiple Growth Paths. *Journal of Economic Growth*, 2(2), pp. 155–68.
- [53] Taylor, John, B., 1993. Discretion versus Policy Rules in Practice. *Carnegie-Rochester Conference Series on Public Policy*, 39, pp. 195-214.
- [54] Tsebelis, George, 2002. *Veto Players: How Political Institutions Work*, Princeton: Princeton University Press.
- [55] Villas-Boas, J.M., 1997. Comparative Statics of Fixed Points. *Journal of Economic Theory*, 73, pp. 183-198.
- [56] Weaver, R. K., 1985. Controlling Entitlements. *The New Directions in American Politics*, eds. John E. Chub and Paul E. Peterson. Washington, D.C.: Brookings Institution.

- [57] Weaver, R. K., 1988. *Automatic Government: The Politics of Indexation*. Washington, D.C.: Brookings Institution.
- [58] Zapal, Jan, 2011a. Simple equilibria in dynamic bargaining games over policies. Unpublished manuscript.
- [59] Zapal, Jan, 2011b. Explicit and implicit status-quo determination in dynamic bargaining: Theory and application to FOMC directive. Unpublished manuscript.