The fine structure constant at infinite energy equal to $1/4\pi$?

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ABSTRACT. A proposed topological mechanism for the quantization of the electric charge is considered, which gives the value $e_0 = \sqrt{\hbar c}$ for both the fundamental electric and magnetic charges. It is argued here that the corresponding fine structure constant $\alpha_0 = 1/4\pi$ could be interpreted as its value at infinite energy.

1 Introduction

A topological mechanism of electromagnetism proposed by the author in 1989 [1, 2] implies a topological mechanism for the quantization of the electric charge that gives the value $e_0 = \sqrt{\hbar c}$, about 3.3 times the electron charge, for the fundamental unit of charge (in the system of units in which $\alpha = e^2/4\pi\hbar c$). One prediction of that model is therefore that the fine structure constant is equal to $\alpha_0 = 1/4\pi$. The same mechanism applies also to the hypothetical magnetic charge with the same value of the fundamental unit [2, 3, 4]. The result is intriguing; to determine if it is sensible, it will be combined in this work with the appealing and plausible idea that, in the limit of very high energies, the interactions of charged particles could be determined by their bare charges (this meaning the value that their charges would have if they were not renormalized by the quantum vacuum, see for instance section 11.8 of [5]). To be precise, the expression "bare charge" will be used here as equivalent and synonymous to "infinite energy limit of the charge" or, more correctly, "charge at infinite momentum transfer", defined as $e_\infty = \sqrt{4\pi\hbar c\alpha_\infty}$, where $\alpha_\infty = \lim \alpha(Q^2)$ when $Q^2 \to \infty$. This paper explores the possibility that the previous given value of the fine structure constant is equal to the bare fine structure constant or, in other words that $\alpha_\infty = \alpha_0 = 1/4\pi$

The possibility of a finite value for α_∞ is perhaps contrary to widely held views but is, nevertheless, worth of study. Indeed, it was discussed very early by Gell-Mann and Low in their classic and seminal paper "QED at small distances" [6], in which they showed that it is something to be seriously considered. However, they could not decide with their analysis whether e_∞ is finite or infinite. The standard QED statement that it is infinite was established later on the basis of perturbative calculations. Nevertheless and contrary to an extended belief, the alternative presented by Gell-Mann and Low has not been really settled. It is still open, in spite of the many attempts to clarify this question. This work proposed an idea based on a topological model.

As is known, the infinite energy charge e_∞ of an electron is partially screened by the sea of virtual pairs that are continuously being created and destroyed in empty space. It is hence said that it is renormalized. As the pairs are polarized, they generate a cloud of polarization charge near any charged particle, with the result that the observed value of the charge is smaller than e_∞. Moreover, the apparent electron charge increases as any probe goes deeper into the polarization cloud and is therefore less screened. This effect is difficult to measure, as it can only be appreciated at extremely short distances, but it has been observed indeed in experiments of electron-positron scattering at high energies [7]. In other words: the vacuum is dielectric. But it is paramagnetic, since its effect on the magnetic field is due to the spin of the pairs and, as a consequence, the quantum vacuum makes smaller the hypothetical magnetic charge, so that its observed value at low energy would be higher than at very high energy.

It is easy to understand intuitively that the name bare charge is appropriate for e_∞. When two electrons interact with very high momentum transfer, each one is so deeply inside the polarization cloud around the other that no space is left between them to screen their charges, so that the bare values, i.e. e_∞, interact directly. As unification is assumed to occur at very high energy, it is an appealing idea that $\alpha_\infty = \alpha_{\rm GUT}$. Indeed, although this possibility is neglected almost always, it is certainly worth of careful consideration. This suggests that a unified theory could be a theory of bare particles (in the sense of neglecting the effect of the vacuum). If this were the case, nature would have provided us with a natural cutoff, in such a way that $\alpha_{\rm GUT} = \alpha_\infty$.

2 Summary of the topological quantization of the charge

Let us summarize now the topological model on which the charge quantization mechanism given in [2] is based, the reader being referred from now on to [8], a review where the previous work is discussed in detail (see also [9, 10]). That model makes use of the idea of force line, either magnetic or electric, as the basic element. As any other family of curves, the electric (resp. magnetic) lines can be represented as the level curves of a complex scalar field $\theta(\mathbf{r}, t)$ (resp. $\phi(\mathbf{r}, t)$). This means that any force line is labelled by a complex number, the corresponding value of the scalar. After identifying, via stereographic projection, the physical space R^3 with the sphere S^3, and the complex plane with the sphere S^2, these two scalars represent maps $S^3 \mapsto S^2$, and the Faraday 2-form $\mathcal{F} = \frac{1}{2} F_{\mu\nu} dx^\mu dx^\nu$ and its dual are the pull-backs of the area 2-form in S^2 by the scalars ϕ and θ respectively.

The scalars ϕ, θ obey highly nonlinear equations. Surprisingly however these nonlinear equations are transformed exactly into Maxwell equations by the transformation $T : \phi, \theta \mapsto F_{\mu\nu}$. Consequently, the $F_{\mu\nu}$ of the model are standard Maxwell fields (although behaving in a particular way around the infinity), so that it is equivalent to Maxwell standard theory in any bounded spacetime domain.

Let Σ be a closed surface around an electric charge. As a consequence of the topological structure, the charge inside Σ is equal to $n\sqrt{\hbar c}$, n being the degree of the map $\theta : \Sigma \mapsto S^2$, an integer number. This means that there are n units of fundamental charge $\sqrt{\hbar c}$. Furthermore, it turns out that among the electric lines which converge to or diverge from a point charge, there are exactly $|n|$ of them with the same value of the label, taking into account the orientation of the map (the same would apply to a magnetic charge, with ϕ instead of θ). As this topological mechanism operates at the classical level and since the charge is necessarily affected by the quantum vacuum [5] to give the dressed observed value, the fundamental charge $e_0 = \sqrt{\hbar c}$ must be interpreted as the infinite energy value of both the electric and magnetic charges e_∞ and g_∞. In other words, the model predicts that $e_\infty = g_\infty = e_0$.

A remark is in order. According to this topological model, any charge must be equal to an integer multiple of $q_0 = \sqrt{\hbar c}$. Consequently, the charge renormalization should be understood as giving the value $q = n(t) q_0$ for the charge inside any volume around an electron, $n(t) + 1$ being the number of vacuum virtual charges $+e_0$ inside that volume at time t, an integer number which fluctuates very fast. The observed charge is the time average of this rapidly changing value. To take a simple example, if $n = -1$ for a fraction 0.3028 of the time and $n = 0$ for the rest, then $q = -e$.

(It is perhaps worth mentioning that, in a different context, these topological ideas have inspired a model of ball lightning in which this phenomenon is assumed be a magnetic knot coupled to a plasma [11, 12, 4]. The linking of magnetic lines turns out to have a stabilizing effect which allow the fireballs to last for much more time than expected.)

3 An interpretation of the topological quantization of the charge

As a consequence of these considerations, it can be argued that the topological model implies the equalities $\alpha_{\text{GUT}} = \alpha_\infty = 1/4\pi$. The argument goes through the following stages.

1. The value of the fundamental charge $e_0 = \sqrt{\hbar c}$ implied by the topological mechanism [2] is "right" to be intrepreted as the bare charge $e_0 = e_\infty = g_\infty$, that is to be equal to the common value of both the fundamental electric and magnetic infinite energy charges. This is so because, as the quantum vacuum is dielectric but paramagnetic, the following inequalities must be satisfied: $e < e_0 < g$, as they are indeed, since $e = 0.3028$, $e_0 = 1$, $g = e/2\alpha = 20.75$, in natural units.

Note that it is impossible to have a complete symmetry between electricity and magnetism simultaneously at low and high energy. The lack of symmetry between the electron and the Dirac monopole charges would be due, in this view, to the vacuum polarization: according to the topological model, the electric and magnetic infinite energy charges are equal and verify $e_\infty g_\infty = e_0^2 = 1$, but they would be decreased and increased, respectively, by the sea of virtual pairs, until the electron and the monopole charge values verifying the Dirac relation $eg = 2\pi$ [14]. The qualitative picture seems nice and appealing.

2. Let us admit as a working hypothesis that two charged particles interact with their bare charges in the limit of very high energies (as explained above). There could be then a conflict between (i) a unified theory of electroweak and strong forces, in which $\alpha = \alpha_s$ at very high energies, and (ii) an infinite value of α_∞. This is so because unification implies that the curves of the running constants $\alpha(Q^2)$ and $\alpha_s(Q^2)$ must converge asymptotically to the same value α_{GUT}. It could be argued that, to have unification at a certain scale, it would be enough that these two curves be close in an energy interval, even if they cross and separate afterwards. But, in that case, the unified theory would be just an approximate accident at certain energy interval. On the other hand, the assumption that both running constants go asymptotically to the same finite value $\alpha_{\rm GUT}$ gives a much deeper meaning to the idea of unified theory, and is therefore much more appealing. In that case, e_∞ must be expected to be finite, and the equality $\alpha_{\rm GUT} = \alpha_\infty$ must be satisfied.

3. The value $\alpha_0 = e_0^2/4\pi\hbar c = 1/4\pi = 0.0796$ for the infinite energy fine structure constant α_∞ is thought provoking and fitting, since $\alpha_{\rm GUT}$ is believed to be in the interval (0.05, 0.1). This reaffirms the assert that the fundamental value of the charge given by the topological mechanism e_0 could be equal to e_∞, the infinite energy electron charge (and the infinite energy monopole charge also), and supports the statement that $\alpha_{\rm GUT}$ must be equal to α_0 and to $1/4\pi$. All this is certainly curious and intriguing since the topological mechanism for the quantization of the charge [2] is obtained just by putting some topology in elementary classical low energy electrodynamics [9].

4 Conclusion

The conclusion of this letter is that the following three ideas must be studied carefully: (i) that there is a complete symmetry between electricity and magnetism at the level of the bare charges, both being equal to $\sqrt{\hbar c}$, the symmetry being broken by the dielectric and paramagnetic quantum vacuum; (ii) that the topological model on which the topological mechanism of quantization is based gives a theory of high energy electromagnetism at the unification scale; and (iii) that the value which it predicts for the fine structure constant $\alpha_0 = 1/4\pi$ could be equal to the infinite energy limit α_∞ and also to $\alpha_{\rm GUT}$, the constant of the unified theory of strong and electroweak interactions.

If this is so, the electric and the magnetic fine structure constants at infinite momentum transfer and $\alpha_{\rm GUT}$) would be equal and there would be a complete symmetry between electricity, magnetism and strong force at the level of bare particles (i.e. at $Q^2 = \infty$), this symmetry being broken by the effect of the quantum vacuum.

Acknowledgements. I acknowledge many stimulating discussions with the late prof. M. Soler. I am also indebted to Dr. J. L. Trueba and Profs. A. Aroca, E. López and A. Ibort.

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(Manuscrit reçu le 30 janvier 2002)