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Time, Clocks and Parametric Invariance

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Abstract In the context of a parametric theory (with the time being a dynamical variable) we consider here the coupling between the quantum vacuum and the background gravitation that pervades the universe (unavoidable because of the universality and long range of gravity). We show that this coupling, combined with the fourth Heisenberg relation, would break the parametric invariance of the gravitational equations, introducing thus a difference between the marches of the atomic and the astronomical clocks. More precisely, they would be progressively and adiabatically desynchronized with respect to one another in such a way that the latter would lag behind the former. This would produce a discrepancy between gravitational theory and observations, which use astronomical and atomic time respectively. It turns out that this result, surprising at it might be, is fully compatible with current physics, since it does not conflict with any known physical law or principle. We argue that this phenomenon must be studied, since it could have cosmological consequences.

Keywords Parametric invariance · Time · Quantum vacuum · Background gravity

1 Introduction

The problem of time is one of the most obscure and controversial in the history of knowledge, and has obvious implications in all the fields of thought. Sticking to physics, the problem is initially posed in the dynamical description of the systems

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(i.e. equations of motion). In this regard, the starting point is the *a priori* existence of a parameter t, called from now onwards “parametric time”, which describes the Newtonian concept of time. It is a fundamental part of a structure of reality constituted by an inert background in which dynamics takes place, but which, paradoxically, lacks dynamical character on its turn. It must be emphasized that the equations of physics do not contain magnitudes in themselves but rather their measurements. It is thus more in the scope of physics to speak about “clocks” or “clock-time” than about time. In more intuitive terms, the problem can be posed as a question on the dynamical character or not of the time variable (parametric and deparametrized theories).

2 The Transit from Astronomical to Atomic Time

Two clock-times, respectively measured with astronomical and atomic clocks, are relevant in physics. For most of the past, the motion of astronomical bodies has served as basis for timekeeping, specially the rotation of Earth and its revolution around the Sun [1]. In the XX century and after several different modes of defining the astronomical time, a conference organized in Paris in 1950 by the International Astronomical Union (IAU) proposed to use a new standard time, baptized as Ephemeris Time (ET). Not being based on the day, it is not measured by observing when a star crosses the meridian, as was done up to then. Instead, the motion of the Moon and the planets against the background of stars is observed, after which the time is deduced from the dynamics of the Solar System, which becomes thus a clock with many hands. More precisely, the conference recommended to determine ET by means of a formula devised by the astronomer Simon Newcomb, which allowed to list in a table the position of the Sun, *i.e.* the ephemeris, at regular time intervals. The ephemeris of the Moon and the planets can be referred to the same time as those of the Sun. The change was approved by the IAU General Assembly in 1952. Four years later, the Comité International des Poids et Mesures recommended that the SI second should be defined as the fraction $1/31\,556\,925.9747$ of the tropical year for 1900 January 0 at 12 hours Ephemeris time.

The first atomic clock, built in 1949 in the US National Bureau of Standards, was based on an ammonia maser, but, unfortunately, was less precise and stable than expected. A decisive advance was the development by a team lead by L. Essen in 1955 of a more precise Cæsium standard of frequencies in the National Physical Laboratory (NPL), the British standard laboratory [2]. It was not properly a clock since, having an oscillator but not a counter, it did not tell the time. However, it served to find that an astronomical second was equal to 9192 631 830 cycles of the caesium frequency with an uncertainty of several cycles. In the words of Essen, this meant “the death of the astronomical second and the birth of atomic time”. The director of NPL, Sir Edward Bullard, proposed then to define a ‘physical second’ in terms of the period of the Cæsium atom, choosing its numerical value so that it agrees as well as may with the current estimates of the second of Ephemeris time [3]. The time was ripe to consider the timekeeping as a task more adequate for physicists than for astronomers. In the General Assembly of the IAU, Essen suggested that the timekeeping should be based on the atomic clocks, but the Assembly voted to go on with ET. Finally, in the thirteenth General Assembly of the Conférence Générale des Poids et Mesures in 1967 the following definition was approved "The second is the duration of 9192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the Cæsium-133 atom".

The atomic time was possible thanks to groundbreakings in physics due to very talented scientists in universities, laboratories and astronomical observatories, no less than 13 of which received the Nobel prize in Physics [1]. It must be stressed that, during its development, it was always assumed as a matter of fact, as something evident, that both types of clocks measure the same time; the atomic ones were preferred for many purposes because they are more precise and the determination of time is less laborious with them. The same implicit and standard assumption is made today. However, this is not necessarily true since the two types of clocks are based on different physical laws.

The importance of this point must be stressed. The *astronomical clock-time*, say $t_{\rm astr}$, is defined by the trajectories of the planets and other celestial bodies. It is measured with *classical and gravitational clocks*, as the solar system. On the other side, the *atomic clock-time*, say $t_{\rm atomic}$, is founded on the oscillations of atomic systems. It is measured using *quantum and electromagnetic systems as clocks*, in particular the oscillations of atoms or masers. Note that, contrary to the concept of time, which is subtle and difficult, the idea of "clock" is clearly defined from the operational point of view. This is done by means of certain dynamical systems, the clocks, in such a way that the time measured by each one is a dynamical variable, the angular position of a pointer for instance. The measured time could be different from one kind of clock to the other since, at least in principle, they could tick at different rates and give inequivalent times, even at the same place and having the same velocity. It is clear that these two clock-times are very close at least but, since we lack a unified theory of gravitation and quantum physics, the future quantum gravity, the assumption that they are exactly the same $t_{\rm astr} = t_{\rm atomic}$ must not be taken for granted. It needs to be confirmed.

3 Time and Clocks

In classical dynamics the physical time appears as a non-dynamical variable that allows the expression of the action integral with the form $S = \int L\mathrm{d}t$. As a consequence of this non-dynamical character, does not exist a canonical momentum conjugate to t. From the Hamiltonian

$$H(p,q) = \sum p\dot{q} - L, \tag{1}$$

the equations of motion in the standard form are $\dot{q} = \partial H/\partial p$, $\dot{p} = -\partial H/\partial q$. To translate this formal machinery to a scheme in which the time acquires the character of a dynamical variable, there exists a canonical approach which allows an interpretation in terms of "clock-time" with its specific dynamical variables [4–6]. Instead of the deparametrized action $S = \int [p\dot{q} - H(p,q)]\mathrm{d}t$, we can use the alternative form

$$S = \int \{\Pi(t)\dot{\sigma}(t) + p(t)\dot{q}(t) - u(t)[\Pi(t) + H(p(t),q(t))]\}\mathrm{d}t, \tag{2}$$

(overdot means derivation with respect to the parameter t) where $H(p(t), q(t))$ has the same functional form as the Hamiltonian in (1), t is introduced in such a way that the theory becomes invariant with respect to reparametrizations and $\sigma(t)$, $\Pi(t)$ are conjugate dynamical variables that describe the clock. Note that Π_u, the momentum conjugate to $u(t)$, weakly vanishes.

The corresponding Hamiltonian writes

$$\hat{H} = u[\Pi + H(p, q)] + \lambda\Pi_u, \tag{3}$$

where λ is a Lagrangian multiplier. The stability of the weak condition $\Pi_u = 0$ implies $\Pi + H(p, q) = 0$. Both are first-class constraints (*i.e.* symmetries). The transformations induced by Π_u allow then to interpret $u(t)$ as an arbitrary function which can be considered as non-dynamical. The extended Hamiltonian is then

$$H^E = u[\Pi + H(p, q)], \tag{4}$$

being hence singular as far as it is proportional to the scalar constraint. Simple algebra allows to verify that $\Pi + H(p, q)$ is the reparametrization generator, as requested by the invariance properties of the action (2). The Hamilton equations of motion are then

$$\dot{q} = u\frac{\partial H}{\partial p}, \qquad \dot{p} = -u\frac{\partial H}{\partial q}, \qquad u = \dot{\sigma}, \qquad \dot{\Pi} = -\dot{H} = 0. \tag{5}$$

From these equations it follows that

$$\frac{\mathrm{d}q}{\mathrm{d}\sigma} = \frac{\partial H}{\partial p}, \qquad \frac{\mathrm{d}p}{\mathrm{d}\sigma} = -\frac{\partial H}{\partial q}, \qquad u = \frac{\mathrm{d}\sigma}{\mathrm{d}t}, \qquad \frac{\mathrm{d}H}{\mathrm{d}\sigma} = 0. \tag{6}$$

The first two are the canonical equations of motion with σ as the time variable, in such a way that the evolution becomes a correlation between dynamical variables. The third one expresses what we call the "march" of the clock u with respect to the parameter t. This theory being invariant under reparametrization, we may fix, for instance, the gauge by the condition $\sigma = t$ (i.e. $u = 1$), so that we recover the ordinary canonical formalism with t being the Newtonian time. We see that with this choice σ is the time measured by an ideal clock, defined as one which can be made to run with the Newtonian time. As σ and Π (which is weakly equal to $-H$) are canonically conjugate variables, the fourth Heisenberg relation (involving now a dynamical time variable and the energy) acquires clearly a dynamical meaning. The extension of this formalism to more complex functional dependence on the time variables gives essentially the same result, although at the expense of some complexity, unnecessary for our purpose [5, 6].

The Hamiltonian (3) is the sum of two terms, describing, respectively, the physical system and the clock. *The equation of motion of the second term,* $H_{\mathrm{clock}} = u\Pi + \lambda\Pi_u$, *is precisely that of a clock* $u = \mathrm{d}\sigma/\mathrm{d}t$. The situation has to be understood as the arbitrary definition of standard clock as one that verifies the relationship $u = \mathrm{d}\sigma_0/\mathrm{d}t = 1$, denoting the time of the standard clock as σ_0. Notice that the definition of a standard clock refers precisely to its march. No change of units is involved as it happens when scale transformations are present [7].

As long as the observations make use of only the standard clock, the scheme is nothing else than the Hamiltonian equations. This may not occur, however, if there is another clock with a different march. In the latter case, the motion equations are (6), but with σ_0 instead of t

$$\frac{\mathrm{d}q}{\mathrm{d}\sigma}=\frac{\partial H}{\partial p};\qquad \frac{\mathrm{d}p}{\mathrm{d}\sigma}=-\frac{\partial H}{\partial q};\qquad \frac{\mathrm{d}\sigma}{\mathrm{d}\sigma_0}=u, \tag{7}$$

which describe the physics of a system in operationally realistic terms. This means that they do not refer to any unobservable parametric time but to σ_0 and σ, which are times really observed by real clocks. The novelty is here the presence of the third equation (7), which is the dynamic equation of the second clock with respect to the standard one. As a consequence of the previous arguments, classical dynamics can be formulated in two equivalent versions, deparametrized and parametric. As we have showed, it is always possible to construct the parametric theory from the deparametrized one. The converse, however, is not true in general. General Relativity is a good example of a theory conceived in an essentially parametric version, a fact on which we will insist later on. On the other hand, classical dynamics reveals itself as a very realistic description, as long as it is able to include the real time (clock) as a dynamical degree of freedom. It must be emphasized that, at the end, we measure a motion using another one or any other natural rhythm as a clock.

It must be underscored that there is no criterion to determine the march u, other than to refer to the internal properties of the clock or the pure observation, particularly if they are based on different phenomena. Physics has accepted traditionally without any discussion the implicit "principle" that all kinds of clocks have the same march and measure the same time. However, *if two clocks are based on different phenomena, they are not necessarily equivalent, specially in the case where the parametric invariance is broken*, in the sense that the march $\mathrm{d}\sigma/\mathrm{d}\sigma_0$ may not be a constant (if it were, the two clocks would measure the same time with different units). In the following, we will use as standard clock-time t_{atomic}, *i.e.* the time measured by the atomic clocks. Notice the close analogy between the third equation (6) with the very concept of proper time in general relativity $\mathrm{d}\tau=\sqrt{g_{00}}\mathrm{d}t$, where $\sqrt{g_{00}}$ is the march of a proper clock with respect to the parametric time.

It may be reminded that there is no clock in Einstein-Hilbert formulation of General Relativity. However, the constancy of the light speed provides us with a dynamical time: the proper time. In fact the geometric structure of space-time induces a relative permittivity ε_r and permeability μ_r of empty space different from 1, their common value being $\varepsilon_\mathrm{r}=\mu_\mathrm{r}=(g_{00})^{1/2}$ [8]. As a consequence of parametric invariance, the choice of a proper time defined as $\mathrm{d}\tau=\sqrt{g_{00}}\mathrm{d}t$ restores the constancy of the light speed.

4 Coupling between the Background Gravitation and the Quantum Vacuum

It is more and more evident that the relevance of the quantum structures associated to the vacuum will play an essential role in the near future the general understanding of the cosmological problem. The case of the cosmological constant is probably the most manifest. Following then the previous arguments, we assume here a classical parametric invariant description, which allows us to choose t as a non-relativistic Newtonian time. Because gravitation is a long range universal interaction that affects all the matter and energy, there must be necessarily a coupling between the background potential due to the entire universe $\Psi(t)$ and the quantum vacuum. Consequently, the existence must be admitted of some kind of adiabatic progressive modification of the properties of the quantum vacuum in the expanding universe. Such modification must comply with the fourth Heisenberg relation.

Traditionally, quantum physics has stated that the sea of virtual pairs that pop-up and disappear constantly in empty space, with their charges and spins, *i.e.* the quantum vacuum, has infinite energy density as follows from the simple application of its basic principles. However, there is now some evidence that it may be finite. In fact, it is not understood why this density seems to be so small as is shown by its possible cosmological manifestations. The question is important since the quantum vacuum fixes the values of some observable quantities, as the electron charge and the light speed, or gives rise to observable phenomena, as the Casimir effect or the Lamb shift. On the average and phenomenologically, a virtual pair created with energy E lives during a time $\tau_0 = \hbar/E$, according to the fourth Heisenberg relation. This has an important consequence: the optical density of empty space must depend on the gravitational field. Indeed, at a spacetime point with gravitational potential $\Psi(\mathbf{r}, t)$, the pairs have an extra energy $E\Psi$. Their lifetime and number density must depend on Ψ, therefore, as

$$\tau_\Psi = \hbar/(E + E\Psi) = \tau_0/(1 + \Psi), \tag{8}$$

[9–11]. The consequence is clear if unexpected: the number density of pairs $\mathcal{N}_\Psi$ depends on Ψ as

$$\mathcal{N}_\Psi = \mathcal{N}_0/(1 + \Psi). \tag{9}$$

If Ψ decreases, the quantum vacuum becomes denser, since the density of charges and spins becomes higher; if Ψ increases, it becomes thinner. Consequently, the gravity created by mass or energy thickens the quantum vacuum, while the gravity created by the cosmological constant or the dark energy attenuates it. It must be stressed that this is not an *ad hoc* hypothesis, but a necessary consequence of the fourth Heisenberg relation and the universality of gravitation.

Let us consider now the background gravitational potential that pervades the universe and let us do it from a phenomenological viewpoint. The potential near Earth can be written approximately as $\Psi_\mathrm{E}(\mathbf{r}, t) = \Psi_\mathrm{loc}(\mathbf{r}) + \Psi(t)$ (here $\Psi = \Phi/c^2$, Φ being the dimensional Newtonian potential). The first term Ψ_loc is the part of the local inhomogeneities, as the Sun, the Earth and the Milky Way, which are not expanding so that it is time independent. The potentials of these three systems at Earth surface are about -10^{-8}, -7×10^{-10} and -6×10^{-7}, respectively, approximating the Milky Way as a mass of $\simeq 10^{11}$ $\mathrm{M}_\odot$ at its center. The second $\Psi(t)$ is due to all the mass-energy in the universe assuming that it is uniformly distributed. Contrary to the first, it depends on time because of the expansion. The former has a nonvanishing gradient but is small, the latter is larger but its gradient vanishes. In the following, $\Psi(t)$ will be called the *background potential*. It is easy to prove that $|\Psi| \gg |\Psi_\mathrm{loc}|$ (see [9] for the details) and that $\Psi_{\rm loc}$ goes up and down across the universe, while $\Psi(t)$ increases secularly, so that its temporal derivative is positive at present time $\dot{\Psi}_0 > 0$. This is because the background potential Ψ can be taken to be the sum of two terms, one due to the matter, either ordinary and dark, the other to the cosmological constant or the dark energy. The first is negative and increases with time because the galaxies are separating; the second is positive and increases with the radius of the universe. Indeed they are proportional to $-1/S$ and $+S^2$, respectively, where S is the scale factor.

It is thus possible to neglect $\Psi_{\rm loc}$, as will be done here, specially since we are interested in the time evolution. Since the gravity is weak and the geometry of the universe has approximately flat space sections, we can take the Newtonian approximation. The previous arguments suggest that the quantum vacuum can be considered as a substratum, a transparent optical medium characterized by a relative permittivity $\varepsilon_{\rm r}(\Psi)$ and a relative permeability $\mu_{\rm r}(\Psi)$, which are decreasing functions of $\Psi(t)$. As $\Psi(t)$ increases, the optical density of the medium decreases (since there are less charges and spins) and *vice versa*. Therefore, the permittivity and the permeability of empty space can be written as $\varepsilon = \varepsilon_{\rm r}\varepsilon_0$ and $\mu = \mu_{\rm r}\mu_0$, where the first factors express the effect of the gravitational potential, *i.e.*, the thickening or thinning of empty space. We can write, at first order in the variation of $\Psi(t)$,

$$\varepsilon_{\rm r}(t) = 1 - \beta[\Psi(t) - \Psi(t_0)], \qquad \mu_{\rm r}(t) = 1 - \gamma[\Psi(t) - \Psi(t_0)], \tag{10}$$

where $\Psi(t_0)$ is the average potential at present time t_0 and β and γ are certain coefficients, necessarily positive since the quantum vacuum must be dielectric but paramagnetic (its effect on the magnetic field is due to the magnetic moments of the virtual pairs). The results of this paper will depend only on the semisum $\eta = (\beta + \gamma)/2$. Not surprisingly, it turns out that $\Psi(t)$ varies adiabatically (because of the expansion) and that its time derivative at present time $\dot{\Psi}_0 = \dot{\Psi}(t_0)$ is positive and very small, of the order of H_0 [9]. It must be underscored that (10) express the adiabatic modification of the properties of the quantum vacuum as a natural consequence of its universal coupling with the background gravitation in the expanding universe.

It is easy to show that if $\varepsilon_{\rm r}$ and $\mu_{\rm r}$ decrease adiabatically as (10) (and the optical density of empty space, therefore), the frequency and the speed of an electromagnetic wave increase adiabatically as $\dot{\nu}/\nu = \dot{c}/c = -(\dot{\varepsilon}/\varepsilon + \dot{\mu}/\mu)/2 = \eta\dot{\Psi}_0$, so that near time t_0

$$\nu = \nu_0[1 + \eta\dot{\Psi}_0(t - t_0)], \qquad c = c_0[1 + \eta\dot{\Psi}_0(t - t_0)]. \tag{11}$$

The proof is very simple [9]: just write the Maxwell's equations with ε and μ decreasing adiabatically with time as in (10) to find that (11) are satisfied at first order in $\dot{\Psi}_0$. This is not dissimilar to what happens to light in a medium, say diamond, where speed is different from c because the quantum effects of the lattice add to those of empty space. As a consequence, the quantum vacuum can be characterized by a refractive index $n(t) = 1 + \eta\dot{\Psi}_0(t - t_0)$ depending on the Newtonian time. In other words, the quantum vacuum would have time dependent optical properties as a result of the expansion of the universe. A warning may be necessary: this is not a variable light speed theory, as will be clear soon.

In principle we could attempt to estimate the value of η using gravity as an effective quantum field theory, for instance to compute the quantum corrections to the photon propagator. Nevertheless, on very general grounds, one realizes that, when diff-invariance is present as it is the case in perturbative quantum gravity, parametric invariance must be necessarily maintained. Therefore, it is reasonable to assume that the logical result for the value of η in such a scheme must be zero. However, this does nor affect this work. Our claim is that the effect here considered requires new physics beyond perturbative quantum gravity and related to the phenomenological properties of the quantum vacuum.

We can interpret this result in two ways: (i) the first and obvious one is that light accelerates with the Newtonian time; however, attention must be paid to the fact that this statement depends on the particular choice of the Newtonian clock. (ii) Nevertheless, since the dynamical equation of a clock is its march, it suffices to take a clock with march relative to the Newtonian one equal to the refractive index

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t} = n(t), \tag{12}$$

for the frequencies and the speed of light to be constant (11). It is clear that such a clock is precisely an atomic clock since the periods of an electromagnetic wave are its basic units. In fact, if one uses the time t_{atomic}, defined through

$$\mathrm{d}t_{\text{atomic}} = [1 + \eta\dot{\Psi}_0(t - t_0)]\mathrm{d}t, \tag{13}$$

it follows from (11) that both the frequency and the light speed *are constant with this time*. Therefore, t_{atomic} *is the time measured by the atomic clocks*, since the periods of the atomic oscillations are obviously constant with respect to it. In fact they are its basic units. The light speed is thus a universal constant, as it must be, if defined with respect to t_{atomic}. Note that the derivatives with respect to the two times are equal at t_0 because $\mathrm{d}t_{\text{atomic}}/\mathrm{d}t = 1$. The same symbol may be used for both. Since $\dot{\Psi}_0 > 0$, the march verifies $u = 1 + \eta\dot{\Psi}_0(t - t_0) > 1$ for $t > t_0$.

Being a quantum effect, the coupling with the quantum vacuum is an alien element outside of General Relativity. Therefore it cannot be included in the definition of proper time. Furthermore it can be a source for a breaking of the parametric invariance.

Hence, three times can be considered. Including for the rest of this section the effect of the local potential, they are

(i) the *coordinate parametric time* t, for instance the ephemeris time. It will be called *astronomical time* since it is the time in Einstein equations. In the following we will note it t_{astr}.
(ii) The *proper time* τ of General Relativity, such that $\mathrm{d}\tau = \sqrt{g_{00}}\mathrm{d}t = [1 + \Psi_{\text{loc}}(\mathbf{r})]\mathrm{d}t$.
(iii) The *atomic time* or *time of the atomic clocks* t_{atomic}, defined as $\mathrm{d}t_{\text{atomic}} = [1 + \eta\dot{\Psi}_0(t - t_0)]\mathrm{d}\tau$. It must be stressed that the relation between t_{atomic} and t_{astr} is not integrable. However, if the local potential can be neglected, as in this work, it can be approximated by (13), this relation being then integrable.

5 Desynchronization of the Astronomical and the Atomic Clocks

It follows from (13) that, at any space point, the astronomical and the atomic clocks are desynchronized with respect to one another, in such a way that the astronomical time decelerates with respect to the atomic time, the deceleration a being

$$\mathrm{d}^2 t_{\mathrm{astr}}/\mathrm{d}t_{\mathrm{atomic}}^2 = -\eta\dot{\Psi}_0 = -a \tag{14}$$

(remind that the observations are made with atomic clocks). In fact, after synchronizing the two clocks at present time so that $t_{\mathrm{astr},0} = t_{\mathrm{atomic},0} = t_0 = 0$, they get progressively out of synchronization since (13) leads to

$$t_{\mathrm{atomic}} = t_{\mathrm{astr}} + \frac{1}{2}\eta\dot{\Psi}_0 t_{\mathrm{astr}}^2, \qquad t_{\mathrm{astr}} = t_{\mathrm{atomic}} - \frac{1}{2}\eta\dot{\Psi}_0 t_{\mathrm{atomic}}^2, \tag{15}$$

where Ψ_{loc} is neglected (it will be reintroduced later to consider the gravitational redshift). This deceleration would be an effect of the coupling background gravity-quantum vacuum.

The acceleration $a = \dot{\Psi}_0$ must be very small, so that it could have been ignored up to now. It corresponds to what Anderson *et al.* introduced in a purely phenomenology way as the *acceleration of the clocks*, when attempting to give an explanation of the Pioneer anomaly in their first paper [12, 13]. It could be said that the atoms tick progressively and adiabatically faster than the celestial bodies. Note also that the proper and the atomic times are different here.

If the march $u = \mathrm{d}t_{\mathrm{atomic}}/\mathrm{d}t$ is not constant, the speed of a spaceship measured with Doppler effect and devices sensible to the quantum time, say $v_{\mathrm{atomic}} = \mathrm{d}\ell/\mathrm{d}t_{\mathrm{atomic}}$, would be different from the astronomical speed $v_{\mathrm{astr}} = \mathrm{d}\ell/\mathrm{d}t_{\mathrm{astr}}$. Indeed

$$v_{\mathrm{atomic}} = v_{\mathrm{astr}}\mathrm{d}t/\mathrm{d}t_{\mathrm{atomic}} = v_{\mathrm{astr}}/u. \tag{16}$$

This could pose a problem, since the gravitation theory gives v_{astr} while the observers measure v_{atomic}. *This should must cause a discrepancy between theory and observation*. An apparent but unreal violation of standard gravity would be detected.

6 Agreement with the Gravitational Redshift and other Observations

The coupling background gravity-quantum vacuum affects the astronomical time but does not change the atomic time. For this reason, this model predicts the same standard values for the frequencies of all the spectral lines because it does not affect the measurements made with atomic clock-time. Any one of these lines, for instance the 1,420 MHz line of the hyperfine structure of Hydrogen, known with an accuracy of 10^{-14}, has in this model exactly the same value as in the tables of physical constants or data. The reason is simple: this frequency is calculated in atomic theory and measured with devices based on quantum physics, which use therefore the atomic time t_{atomic}, not the astronomical time t_{astr}. These include lasers, masers, transponders, prisms, diffraction gratings, spectrographs and so on. What is new in this work is that the frequencies defined with respect to $t_{\rm astr}$ are different but these are not usually measured, probably never.

This model obviously complies with the experiments on gravitational redshift, because they are performed with atomic clocks, which are exactly the same thing in this work as in standard physics. In particular, it agrees with Einstein formula

$$\Delta\nu/\nu = -\Delta\Psi_{\rm loc}. \tag{17}$$

However, since the standard analysis is based on the equality of proper and atomic times, it can be clarifying to compare the two approaches. In this work and taking into account the local potential by means of $\mathrm{d}\tau = (1+\Psi_{\rm loc})\mathrm{d}t$, one has $\mathrm{d}t_{\rm atomic} = [1+\eta\dot{\Psi}_0(\tau-\tau_0)]\mathrm{d}\tau$ at first order. The relative difference between the two approaches is therefore of order $\eta\dot{\Psi}_0\tau_{\rm flight}$, where $\tau_{\rm flight}$ is the flight time of the light beam. In the very precise observations by Levine and Vessot, with accuracy $\simeq 2\times 10^{-4}$, this time is $\simeq 3.3\times 10^{-2}$ s [14, 15]. The condition on the clock acceleration is then $a = \eta\dot{\Psi}_0 < 6\times 10^{-3}\ \mathrm{s}^{-1}$; a is surely much smaller that this bound, probably of the order of H_0 (*i.e.* 14 orders of magnitude smaller), otherwise the effect would have been detected before. The difference between τ and $t_{\rm atomic}$ is thus too small to de detected in experiments of this kind.

It might be argued that the deceleration of $t_{\rm astr}$ with respect to $t_{\rm atomic}$ predicted by this work could conflict with the well-known cartography of the Solar System, particularly with the observed periods of the planets. It is not so, however. The dominant potential, as indicated before, is that of the Milky Way, which can be taken as constant at the scale of the Solar System and equal to $\simeq 6\times 10^{-7}$. The march is then $u = \mathrm{d}t_{\rm atomic}/\mathrm{d}t_{\rm astr} = 1 - 6\times 10^{-7}$, which is constant. This means that the equations of the Solar System in Newtonian physics

$$\frac{\mathrm{d}^2\mathbf{r}_i}{\mathrm{d}t_{\rm astr}^2} = -\sum_{j\neq i} Gm_j\frac{(\mathbf{r}_j-\mathbf{r}_i)}{|\mathbf{r}_j-\mathbf{r}_i|^3}, \tag{18}$$

are the same with the two times $t_{\rm astr}$ and $t_{\rm atomic}$. The reason is clear: These equations are invariant under the transformation $t_{\rm astr}\to t_{\rm atomic} = ut_{\rm astr}$, $G\to G/u^2$ (provided that u can be taken as constant). The third Kepler law, for instance, is equally valid with $t_{\rm astr}$ and G as with $t_{\rm atomic}$ and G/u^2. Indeed, the best known value of G, obtained with atomic clocks, must be in fact G/u^2. The relativistic corrections, suffer a relative change of order 10^{-6} and the relative difference between the duration of the year with the two times is about 9×10^{-11}. The effect is thus too small to cause any conflict with the cartography of the Solar system.

7 Looking for the Effect

The arguments of this work seem compelling: there must be a discrepancy between the atomic and the astronomical times, consisting in a progressive desynchronication of the astronomical and the atomic clocks. However, a precise theoretical analysis of the intensity of the effect is currently not possible, since the coefficient η cannot be calculated without a theory of quantum gravity. Taking the experimental stance, therefore, one must look for observable consequences of the discrepancy. They can be expected in situations in which the main interaction is Einstein gravity but the observations are made with Doppler effect, performed with atomic clocks. At this purpose, a special attention must be paid to the Pioneer anomaly [12, 13], as long as it corresponds to a body moving along a trajectory in a gravitational field. Gravitational theory gives its velocity as v_{astr}, while the observers measure the value v_{atomic}. The latter is smaller than the former since $u > 1$ after the initial time t_0 because of the expansion of the universe, as explained before. The body would seem, therefore, to travel too slowly. Its position, as determined from its velocity by measuring Doppler effects with atomic clocks, would seem to lag behind the predictions. If the body recedes from the Sun, it would be easy to interpret this effect as due to an unexpected constant acceleration towards the Sun of unknown origin. However, it must be emphasized that the Pioneer flight is a two clock system. One, the astronomical time, is related to the trajectory $\mathbf{r}(t_{\text{astr}})$ as calculated with gravitation theory, which establishes a relation between astronomical time and position. The other, the atomic time, is used to determine the observed position through the Doppler effect. The relative acceleration between the two clocks produces then a discrepancy between theory and observation. Another case could be the Hubble law, which can be studied theoretically with the Friedmann equation but is observed by measuring frequencies with the Doppler effect. A third interesting case could be the application of the virial theorem to the determination of the mass of galaxy clusters. These topics will be analyzed in future work.

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References

1. Jones, T.: Splitting the Second. The Story of Atomic Time. Institute of Physics Publishing, Bristol (2000)
2. Essen, L.: An atomic standard of frequency and time interval: A Cæsium resonator. Nature **176**, 280–282 (1955)
3. Bullard, E.: Definition of the second of time. Nature **176**, 282 (1955)
4. Hanson, A., Regge, T., Teitelboim, C.: Constrained Hamiltonian Systems. Accademia dei Lincei, Roma (1976)
5. Tiemblo, A., Tresguerres, R.: Internal time and gravity theories. Gen. Relativ. Gravit. **34**, 31–47 (2002)
6. Barbero, J.F., Tiemblo, A., Tresguerres, R.: Husain-Kuchar model: Time variables and nondegenerate metrics. Phys. Rev. D **57**, 6104–6112 (1998)
7. Canuto, V., Adams, P.J., Hsieh, S.-H., Tsieng, E.: Scale-covariant theory of gravitation and astrophysical applications. Phys. Rev. **16**, 1643–1663 (1977)
8. Landau, L.D., Lifshitz, E.M.: The Classical Theory of Fields, 4th revised English edn. Pergamon Press, Oxford (1975). Chap. 10
9. Rañada, A.F.: The Pioneer riddle, the quantum vacuum and the variation of the light velocity. Europhys. Lett. **63**, 653–659 (2003)
10. Rañada, A.F.: The light speed and the interplay of the quantum vacuum, the gravitation of all the universe and the fourth Heisenberg relation. Int. J. Mod. Phys. D **12**, 1755–1762 (2003)
11. Rañada, A.F.: The Pioneer anomaly as acceleration of the clocks. Found. Phys. **34**, 1955–1971 (2004)
12. Anderson, J.D., Laing, Ph.A., Lau, E.L., Liu, A.S., Martin Nieto, M., Turyshev, S.G.: Indication, from Pioneer 10/11, Galileo and Ulysses data, of an apparent anomalous, weak, long-range acceleration. Phys. Rev. Lett. **81**, 2858 (1998)

13. Anderson, J.D., Laing, Ph.A., Lau, E.L., Liu, A.S., Martin Nieto, M., Turyshev, S.G.: Study of the anomalous acceleration of Pioneer 10 and 11. Phys. Rev. D **65**, 082004 (2002)
14. Will, C.: Theory and Experiment in Gravitational Physics. Cambridge University Press, Cambridge (1993)
15. Vessot, R.F.C., et al.: Test of relativistic gravitation with a space-borne hydrogen maser. Phys. Rev. Lett. **45**, 2081–2084 (1980)