Comment on "Magnetic Geometry and the Confinement of Electrically Conducting Plasmas"

Faddeev and Niemi (FN) presented in [1] some results on electrically neutral plasmas with an equal number of negative and positive charge carriers, which "challenge certain widely held views on plasma behaviour". It is frequently admitted that the virial theorem implies that there can be no equilibrium configurations of a system of charges in electromagnetic interaction in the absence of external forces. However, FN showed that simple arguments based on the virial theorem can not preclude the existence of nontrivial, nondissipative equilibrium configurations, which are "topologically stable solitons that describe knotted and linked flux tubes of helical magetic fields". Even if based on an effective microscopic field theory, their result is clearly very important, in particular for a theory of ball lightning (BL). In 1998 Rañada, Soler and Trueba [2–4] had proposed a model of BL based on a configuration of that type consisting in a macroscopic state of linked streamers coupled to a magnetic field, called "electromagnetic knot" in [5]. It can be argued that the main problem for the plausibility of an electromagnetic model of BL is to reconcile the interpretation of the virial theorem with the FN arguments or, in other words, to understand physically why this theorem does indeed allow equilibrium states or al least metastable configurations.

In the case of a plasma or gas cloud inside a volume V with border S, pressure p, density ρ, fluid velocity $\mathbf{v}$ and magnetic field $\mathbf{B}$, the virial theorem states, $\mathbf{n}$ being a unit vector normal to S, that

$$\frac{1}{2}\frac{d^2I}{dt^2} - 2T = -\int_S x_i p n_i dS + 2U + U_B + \frac{1}{\mu_0}\int_S x_i M_{ik} n_k dS, \tag{1}$$

$I = \int_V \rho r^2 d^3x/2$, $T = \int_V \rho v^2 d^3x/2$, $U = 3\int_V p\, d^3x/2$, $U_B = \int_V B^2 d^3x/2\mu_0$, $M_{ik} = B_i B_k - B^2\delta_{ik}/2$.

A usual argument runs as follows: according to the witnesses reports the magnetic energy of a BL is typically of the order of 10^4 J or even much higher, so that the term U_B in (1) would produce a very rapid increase of I, *i.e.* an explosion (nothing would balance the large magnetic pressure $B^2/2\mu_0$). The calculations indicate that this would take place in a small fraction of a second, so that the virial theorem would exclude indeed any electromagnetic model of BL. However, Chandrasekhar and Woltjer showed in 1958, that plasmas relax to minimum energy stable states, so called force-free fields (FFF), in which the electric current and the magnetic field are parallel, $\mathbf{B}\cdot\boldsymbol{\nabla}\times\mathbf{B} = 0$, so that the Lorentz force vanishes. They concluded that such states can confine large amounts of magnetic energy. What happens is that $\ddot{I}$ is much smaller if the FFF condition is verified because the sum of the two last terms in (1) is then zero, so that the argument for the instantaneous explosion fails. It is sometimes said that there are non trivial FFF inside containers but not in open space. However, note that if the FFF condition could be satisfied in open space, there could be completely stable and lasting for ever BL: there would be reports of balls lasting for hours, maybe days (there is none). A plausible possibility is a relaxation leading almost instantaneously to a localized almost FFF (in which $\mathbf{B}\cdot\boldsymbol{\nabla}\times\mathbf{B}$ is very small). In that case, an almost quiescent expansion must follow slowly after the rapid relaxation (instead of an explosion), with asymptotically decreasing Lorentz force, the lifetime being obviously much longer [2]. This resembles much more to what is seen in ball lightnings than an equilibrium state. In other words, there is no problem with FFF fields, almost FFF are enough.

Another argument against electromagnetic models of BL states that the streamers would be cut by the Lorentz force (the pinch effect) in a very short time, contrary to the FN result. But if the FFF holds, there is no pinch effect. In the case of almost FFF, the pinch effect would be small and would contribute to the decay but only after a certain much longer time.

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