

INCOME TAXES, PUBLIC INVESTMENT AND WELFARE IN A GROWING ECONOMY*

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ABSTRACT:

In a one sector endogenous growth model with public capital and a spillover externality from aggregate private capital, we find that raising taxes on factor incomes as part of an optimal fiscal policy may be a more common result than usually thought. This comes about in spite of the fact that the two technological externalities in the economy tend to support subsidies on the production factors. The interaction between the technological externalities and a fiscal constraint that arises when the government must finance an exogenous stream of public expenditures is central for our result. Factor taxes can be optimal even in cases when the technological externalities are sizeable, if the type of expenditures mentioned is sufficiently high, as a percentage of output. **This result is robust to depreciation rates and to the elasticity of intertemporal substitution.** We also show that welfare losses from deviations from the optimal policy are always smaller when compensated with changes in income taxes than when adjusting lump-sum taxes. [QUITAR... This result would justify an active role for income taxation, while lump-sum taxes remain essentially unchanged.]

Keywords: Endogenous growth, public investment, income taxes, policy deviations, public expenditure constraint.

JEL Classification: E13, E62, H21, O41

1 Introduction

Clarifying whether factor rents should be taxed or subsidized continues to be a central issue when characterizing optimal fiscal policy. In a Ramsey-type setting with no externalities, taxing production factor incomes will generally undermine growth and welfare. In such a setup, Judd (1985), Chamley (1986) and Lucas (1990) emphasized the negative incidence of income taxes on welfare, proposing a zero tax rate on capital rents, at least in the long-run. Jones *et al.* (1997) and Milesi-Ferretti and Roubini (1998) extend the zero tax rate result to labor and consumption taxes in models with human capital. A government may even want to subsidize capital income when the presence of some externalities may lead the competitive mechanism to an underaccumulation of physical capital [Turnovsky (2000) and Cassou and Lansing (1997)].

However, taxes on factor rents represent more than 50% of total tax receipts in developed countries. An answer to this apparent *puzzle* can be found in the literature on economies with heterogenous agents.¹ In a representative agent setting, the existence of externalities inducing over-accumulation of physical capital can also lead to the optimality of factor income taxes. Often, externalities of this kind have been introduced in the production function [Turnovsky (1996), Fisher and Turnovsky (1998) and Corsetti and Roubini (1996)] or as a credit market imperfection in an stochastic environment [Chamley (2001)].² We attempt to contribute along this line of research. In a simple growing economy with infinitely-lived agents, we find that raising taxes on factor incomes as part of an optimal fiscal policy may be a more common result than usually thought. This comes about in spite of the fact that a spillover externality from aggregate private capital [Romer (1986)] and the presence of public capital in the production function [Glomm and Ravikumar (1994, 1999) and Turnovsky (2000, 2004)] both tend to support subsidies on the production factors.

The public revenue and expenditure decisions are clearly interdependent, and considering this simultaneous choice is an important feature of our analysis. Resources to finance total public expenditures can be raised either through lump-sum taxes, with a strong crowding-out effect on current consumption and a relatively small loss in capital accumulation and growth, or through income taxes, with a bigger effect on growth and long-term utility but a smaller loss in current utility. Essentially, the economy faces a very simple trade-off between current consumption versus future consumption and growth. We show that the need to pay for a given amount of public expenditures, exogenously given as a fraction of total output, produces an important budget distortion in the form of a lower bound for government

¹Aiyagari (1995) and Domeij and Heathcote (2004), among others, show that the optimal income tax rate is positive and far from zero in an heterogenous agents framework with asset market incompleteness. Jones and Manuelli (1992) found a similar result in an overlapping generations framework.

²Caballé (1998) also found the optimality of capital income taxes in a framework with altruistic preferences and low elasticity of intertemporal substitution, without intergenerational transfers.

revenues. Above certain levels of this requirement on public expenditures, a lump-sum tax financing mechanism might reduce current consumption and utility excessively so that collecting additional resources from distortionary taxation may turn out to be preferable.³ This simple but important effect has not been seriously considered when characterizing optimal tax policy in dynamic settings. Obviously, the optimal tax/subsidy result depends on the interaction and relative strength of the spillover and the public capital externalities, on the one hand, and the requirement on public expenditures on the other, but we show that taxing factor incomes may be part of an optimal fiscal policy for standard parameterizations of the model economy.

Finally, we show that welfare losses from deviations from the optimal expenditure policy (i.e., due to fiscal shocks) are always smaller when compensated with changes in income taxes than when adjusting lump-sum taxes. This important result would justify an active role for income taxation, while lump-sum taxes remain essentially unchanged. This finding fits well into the related line of enquiry by Baxter and King (1993), who focus on fiscal policy as a potential major source of macroeconomic fluctuations.

A broad classification of public expenditures distinguishes between productive and non-productive concepts. Roughly, we consider as *public investment* the productive type of public expenditure components, like infrastructures and education. Among the non-productive concepts, it seems safe to assume that some of them (defense, health, police services, etc.) increase consumers' welfare and can be endogenously determined each period. However, some other concepts, such as public wages, the payment of interest on public debt or bureaucratic costs, can be seen as a type of public expenditures that were previously committed and that any *future* government should take as exogenous. The number of public servants ultimately in charge of implementing government policy is mostly given, as it is the need to provide current funding to pay for maintenance of infrastructures for transportation and other public services, like education and health. In addition, the political cost of cutting down on most public consumption items is very high, even if their current levels are clearly above their *optimum* values. Hence, there is a variety of reasons suggesting the interest and convenience of assuming that part of total public expenditures could be seen as being exogenous, having been previously committed by policy decisions made over a period of time.

The total money amount will depend on the efficiency of the public sector, but also on decisions taken by previous governments. As a percentage of output, these public expenditure concepts are far from zero and have remained fairly constant over the last decades in actual economies.⁴ This suggests the existence of some restrictions in actual economies that lead

³This result is in line with Jones et al. (1997), who pointed out that certain *public policy constraints* could imply that taxing productive factors positively in the long-run might be optimal.

⁴Over the last decades, public wages and interest payments on outstanding debt represent about 10% of total GDP (OECD statistics), while total public consumption and public investment amount to about 20% and 3% of GDP, respectively [see also Easterly and Rebelo (1993)]. In addition, the standard deviation of

governments into a second-best choice of public investment and consumption, as well as of their appropriate financing scheme. The main contribution of the paper is to analyze the effects of this realistic public expenditure restriction in a simple growth economy, and emphasize that its size may be determinant for optimal tax design.

The paper is organized as follows. The basic framework is described in section 2. In section 3 we analyze the centralized resource allocation mechanism, leaving the analysis of the competitive equilibrium for section 4. In this section we obtain the welfare maximizing ratios of public consumption and investment to output, and characterize the optimal tax mix financing government expenditures. Section 5 discusses policy implications of previous results. Section 6 illustrates the main conclusions of the paper with numerical simulations. Finally, section 7 concludes.

2 The economy

The model draws on work by Glomm and Ravikumar (1994), Cassou and Lansing (1998), Turnovsky (2004) and Marrero and Novales (2005). The economy consists of a government, a continuum of identical firms and a representative household.

2.1 Firms

Each firm rents every period physical capital k_t and labor l_t from households, and produces y_t units of the consumption commodity. The capital stock used on the aggregate by all firms, K_t , is taken as a proxy for the index of knowledge available to each single firm, as in Romer (1986). Finally, public capital, G_t , is an exogenous factor in the private production process and affects all individual firms in the same way, as in Glomm and Ravikumar (1994, 1999) and Turnovsky (1996, 2000, 2004).⁵

Except for these two *technological externalities*, output is produced according to a standard *Cobb-Douglas* function,

$$y_t = f(l_t, k_t, K_t, G_t) = Al_t^{1-\alpha} k_t^\alpha K_t^\phi G_t^\theta, \quad (1)$$

where α is the share of private capital in output, θ and ϕ are the constant elasticities of output with respect to public capital and the knowledge index, and A is a technological

public investment is similar to its mean, the ratio being half as much for public consumption and salaries. This observation supports the idea that the time path of some public expenditure concepts can be much more easily altered than some others.

⁵Empirical work by Ratner (1983), Aschauer (1989) and Munnell (1990), among others, emphasizes the positive link between public expenses and the private production process.

scale factor. Since all firms are identical, we can aggregate on (1) to obtain total output, Y_t ,

$$Y_t = AL_t^{1-\alpha} K_t^{(\alpha+\phi+\theta)} \left(\frac{G_t}{K_t} \right)^\theta, \quad (2)$$

where L_t , K_t are the total amounts of labor and physical capital used by all firms in the economy. In what follows, we restrict our attention to economies with $\alpha + \theta + \phi = 1$, a condition needed for endogenous growth. Under this condition, total output will grow at a constant rate γ_y in the long-run.

2.2 The Government

The public sector collects taxes to finance its expenditures. We consider as *public investment*, I^g , the productive type of public expenditures. As it is standard in the related literature, public investment is assumed to be a constant and positive fraction, \varkappa_i , of total output,

$$I_t^g = \varkappa_i Y_t. \quad (3)$$

Public capital accumulates according to

$$G_{t+1} = I_t^g + (1 - \delta^g)G_t, \quad (4)$$

where G_{t+1} denotes the stock of public capital at the end of time t , and $\delta^g \in (0, 1)$ being the depreciation rate of this type of capital.

Government expenditures include also purchases of non-productive goods and services, that we refer to as *public consumption*. We assume that some of these concepts contribute to consumers' welfare, C^g , and its amount could be decided by any *new* government. However, there also exist some *rigid* public consumption items, C^w , which are pre-committed and whose amount should be taken as exogenous by any new government. For simplicity, we assume that C^w does not affect consumers' welfare. The relevant assumption here is that this item of public expenditure is taken as exogenous when deciding on public investment, on the endogenous part of public consumption, and on the optimal tax-mix. To simplify the exposition, we will refer them as to *endogenous* and *exogenous* public consumption, respectively.

We also assume the government claims a constant fraction \varkappa_c of output for the endogenous public consumption,

$$C_t^g = \varkappa_c Y_t, \quad (5)$$

while the exogenous part of public consumption is assumed to follow the process

$$C_t^w = C_0^w (1 + \gamma_w)^t, \quad (6)$$

where C_0^w is its initial level and γ_w its constant growth rate. Since total output will grow at a constant rate γ_y in the long-run, we assume that $\gamma_w = \gamma_y$, which implies that C_w is also a constant ratio \varkappa_w of total output in the long-run.⁶

We consider taxes on total income at a rate τ , as well as *lump-sum taxes*, X . For analytical convenience, we denote by v the ratio of X to output, although it should be clear that it is X , rather than v , that is chosen each period. Debt issuing is not allowed in the economy, so that the government's budget balances every period,⁷

$$I_t^g + C_t^g + C_t^w = \tau_t Y_t + X_t. \quad (7)$$

2.3 Households

We assume zero population growth and population size is normalized to one. The representative consumer is endowed with a unit of time every period, which is supplied inelastically. She is the owner of physical capital, and allocates her resources between consumption, C_t , and investment in physical capital, I_t . Private physical capital accumulates over time according to,

$$K_{t+1} = I_t + (1 - \delta)K_t, \quad (8)$$

where K_{t+1} denotes the stock of private physical capital at the end of time t , and δ denotes the depreciation rate for physical capital.

The consumer obtains utility from private and public consumption according to,

$$U(C_t, C_t^g) = \ln C_t + \varphi \ln C_t^g, \quad (9)$$

which discounts at a constant rate of β , $0 < \beta < 1$, and $\varphi > 0$ determines the relative appreciation for public and private consumption. She faces the budget constraint

$$C_t + K_{t+1} - (1 - \delta)K_t + X_t \leq (1 - \tau_t)(w_t L_t + r_t K_t), \quad (10)$$

each period, where w_t, r_t are real wages and interest rates.

In what follows, we will restrict our attention to the case of full depreciation of both types of capital every period, $\delta = \delta^g = 1$ [as in Barro (1990) and Glomm and Ravikumar (1994), among many others], and to *stationary* fiscal policies, which will enable us to obtain

⁶If the growth rate γ_w was bigger than γ_y , the ratio C^w/Y would rise without any bound. On the other hand, if γ_w was lower than γ_y , that ratio would go to zero, preventing us from analyzing the long-run limitations that public consumption impose on the choice of optimal fiscal policy. On the other hand, since our economy will display an ongoing growth process, it has no sense to assume a constant path for C^w .

⁷As in Barro(1990), Glomm y Ravikumar (1994) and Barro and Sala-i-Martin (1992). Cassou and Lansing (1997, 1998) point out that this might be a closer description of actual fiscal constraints than the alternative of allowing the government to borrow or lend arbitrary amounts of resources every period.

an analytical characterization of the planner and the competitive equilibrium allocations.⁸ Under these set of assumptions, our economy is just a version of the AK-model, not displaying any transitional dynamics.

3 Centralized resource allocation

We begin by considering the optimum allocation of resources by a central planner in the economy described above. In the next section, we will show that this allocation of resources can be replicated by the competitive equilibrium mechanism under the appropriate public expenditure policy and the right mixture of income and lump-sum taxes.

The Pareto-optimum allocation in an economy with the structure described above is the solution to a planner's problem in which, given $K_0, G_0 > 0$ and $\varkappa_w \geq 0$, a decision is made on the public investment and consumption ratios, \varkappa_i, \varkappa_c , to maximize

$$\sum_{t=0}^{\infty} \beta^t U(C_t, C_t^g), \quad (11)$$

under the global constraint of resources

$$Y_t = K_{t+1} + C_t + \varkappa_i Y_t + \varkappa_c Y_t + \varkappa_w Y_t, \quad (12)$$

given the available technology (2), non-negativity constraints, $C_t, K_{t+1}, G_{t+1} \geq 0$, and the transversality conditions on the accumulation of private and public capital,

$$\lim_{t \rightarrow \infty} \beta^t K_{t+1} \frac{\partial U(\cdot)}{\partial C_t} = 0, \quad (13)$$

$$\lim_{t \rightarrow \infty} \beta^t G_{t+1} \frac{\partial U(\cdot)}{\partial C_t} = 0. \quad (14)$$

The simplicity of the model under full depreciation allows for an analytical characterization of the solution to the planner's problem. First, we show in Part 1.a in the Appendix that the optimal sequences of private consumption and private and public capital are:

$$C_t = (1 - \beta) \frac{1 - \varkappa_i - \varkappa_c - \varkappa_w}{1 - \beta(\theta - \varphi + \varphi\theta)} AG_t^\theta K_t^{1-\theta} = a^p Y_t, \quad (15)$$

$$K_{t+1} = \beta(1 + \varphi)(1 - \theta) \frac{1 - \varkappa_i - \varkappa_c - \varkappa_w}{1 - \beta(\theta - \varphi + \varphi\theta)} AG_t^\theta K_t^{1-\theta} = b^p Y_t, \quad (16)$$

$$G_{t+1} = \varkappa_i AG_t^\theta K_t^{1-\theta} = \varkappa_i Y_t, \quad (17)$$

⁸Considering stationary policies when solving the welfare maximization problem is consistent with the tax smoothing argument in Barro(1979). In fact, Cassou and Lansing (1998) considers a simple endogenous growth model with public capital, and show that the welfare maximizing policy calls for constant tax rates over time.

so that along the optimal path, private consumption as well as both types of capital are every period a fixed proportion of output. As a consequence, transversality conditions are clearly satisfied. Since the economy is of the AK-type, it jumps from initial conditions K_0, G_0 to values for C_0, K_1 and G_1 , the three variables growing from that time on at the common rate γ^p characterized below.

3.1 The balanced growth path

A balanced growth path (BGP) is a trajectory along which aggregate variables grow at a zero or positive constant rate. With $\alpha + \theta + \phi = 1$, a standard argument⁹ shows that, under a stationary fiscal policy, the paths for $Y_t, C_t, K_t, G_t, C_t^g, C_t^w$ solving the previous planner's problem all grow at the same constant rate, γ^p . This growth rate is obtained by combining (2) with (16)-(17),

$$1 + \gamma^p = Y_{t+1}/Y_t = A\kappa_i^\theta \left[\frac{\beta(1+\varphi)(1-\theta)(1-\kappa_i - \kappa_c - \kappa_w)}{1 - \beta(\theta - \varphi + \varphi\theta)} \right]^{1-\theta}, \quad (18)$$

which decreases as κ_c or κ_w increase, since both take resources away from investment. On the other hand, the effect on growth of a higher public investment ratio κ_i has the same sign as $(1 - \kappa_c - \kappa_w)\theta - \kappa_i$. The interpretation is that even though a higher κ_i takes again resources away from investment and is growth reducing, this expenditure component is productive, leading to higher growth so long as the additional output is not claimed by the government as nonproductive expenditures.

3.2 The optimal allocation

Given $\kappa_w \geq 0$ and $K_0, G_0 > 0$, the absence of transition under full depreciation of both types of capital reduces the characterization of optimal public expenditures to searching for a vector (κ_i, κ_c) solving the problem

$$\max V(C_0, C_0^g, \gamma^p) = \left[\frac{1}{1-\beta} (\ln C_0 + \varphi \ln C_0^g) + \frac{\beta(1+\varphi)}{(1-\beta)^2} \ln(1 + \gamma^p) \right]. \quad (19)$$

The next proposition presents the solution to this problem:

Proposition 1 *The optimal government expenditure policy is given by $\kappa_c^{p+} = \varphi(1 - \kappa_w) \frac{1-\beta}{1+\varphi}$ and $\kappa_i^{p+} = \theta\beta(1 - \kappa_w)$.*

Proof. *See appendix, part 1.b* ■

⁹See Barro (1990), Rebelo (1991) or Jones and Manuelli (1997), among many others.

From (15)-(17), this policy produces optimal paths for consumption, private and public capital,

$$C_t = (1 - \varkappa_w) \frac{1 - \beta}{1 + \varphi} Y_t = a^{p+} Y_t, \quad (20)$$

$$K_{t+1} = \beta (1 - \varkappa_w) (1 - \theta) Y_t = b^{p+} Y_t, \quad (21)$$

$$G_{t+1} = \beta \theta (1 - \varkappa_w) Y_t = \varkappa_i^{p+} Y_t. \quad (22)$$

Since public capital becomes productive in the next period, the optimal public investment-to-output ratio in the centralized solution falls below the one needed to maximize growth by a factor of β .¹⁰

4 Decentralized resource allocation

Under the competitive equilibrium mechanism, consumers and firms take prices and fiscal policy as given, maximizing time aggregate utility and profits, respectively. The representative consumer chooses $\{C_t, K_{t+1}\}_{t=0}^{\infty}$ to maximize the discounted time-aggregate utility (11), subject to the budget constraint (10), the transversality condition (13) and $K_{t+1} \geq 0$, $C_t \geq 0$, for any period t . Marginal utility equal to infinity at the origin guarantees that strict inequalities will hold for $K_{t+1} > 0$, $C_t > 0$ at all time periods, so long as there are some resources available for consumption and investment. The consumer offers her unit endowment of time inelastically every period, since she does not receive any utility from leisure. The solution to this problem leads to the standard intertemporal optimality condition,¹¹

$$\frac{C_{t+1}}{C_t} = \beta(1 - \tau_{t+1})r_{t+1}. \quad (23)$$

The profit maximizing problem of the firm turns out to be static, leading to the usual marginal product conditions,

$$r_t = f'_{k_t} = \alpha A l_t^{1-\alpha} k_t^{\alpha-1} K_t^\phi G_t^\theta = \alpha \frac{y_t}{k_t} = \alpha \frac{Y_t}{K_t}, \quad (24)$$

$$w_t = f'_{l_t} = (1 - \alpha) A l_t^{-\alpha} k_t^\alpha K_t^\phi G_t^\theta = (1 - \alpha) \frac{y_t}{l_t} = (1 - \alpha) \frac{Y_t}{L_t}, \quad (25)$$

¹⁰To maximize growth, we set $\varkappa_c = 0$ in the expression for $1 + \gamma^p$ and take derivatives with respect to \varkappa_i . It is easy to show that the growth maximizing ratio of public investment to output is given by $\varkappa_i^{p*} = \theta(1 - \varkappa_w)$. All these results are in line with Barro (1990), Futagami *et al.* (1993), Glomm and Ravikumar (1994) and the related literature afterwards.

¹¹The optimal time allocation of private consumption does not depend upon public consumption because of the separability of the utility function, making our results somewhat limited. In addition to the convenience of starting with a simple model to better grasp the intuitions behind our results, separability is needed for the competitive equilibrium being analytically tractable. Modelling nontrivial interactions between private and public consumption in preferences is left for future research.

where we have used the fact that each firm treats its own contribution to the aggregate capital stock as given. In period t , each firm would then pay the competitively determined real wage w_t on the labor it hires and the real interest rate r_t on the capital it rents. From these optimality conditions we have the standard result on income distribution,

$$Y_t = r_t K_t + w_t L_t. \quad (26)$$

4.1 The competitive equilibrium

Definition 2 *Given a feasible policy $\Pi = \{\varkappa_i, \varkappa_c, \varkappa_w, v_t, \tau_t\}_{t=0}^{\infty}$ satisfying (7) and $K_0, G_0 > 0$, a Π -competitive equilibrium is a vector of allocations $\{C_t, C_t^g, C_t^w, K_{t+1}, G_{t+1}, I_t, I_t^g, L_t, Y_t\}_{t=0}^{\infty}$ and prices $\{r_t, w_t\}_{t=0}^{\infty}$ such that, given the vector of prices: i) $\{L_t, K_{t+1}\}_{t=0}^{\infty}$ solve the profit maximizing problem of firms [i.e., (24)-(25) hold], ii) $\{C_t, L_t, K_{t+1}\}_{t=0}^{\infty}$ maximize the utility of households, i.e., (10), (13) and (23) hold, together with $C_t, K_{t+1} \geq 0$, iii) the technology constraints (2), (8), (4) hold and iv) markets clear every period:*

$$L_t = 1, \quad (27)$$

$$Y_t = I_t + I_t^g + C_t + C_t^g + C_t^w. \quad (28)$$

The simplicity of the model allows for the Π -competitive equilibrium to be analytically characterized under full depreciation of capital and a stationary, feasible fiscal policy Π . Plugging into the equilibrium conditions (23), (10), (27), (7), (24), (25), (2) a linear guess for the dependence of C_t and K_{t+1} on output, as it is done in Part 1.a in the Appendix to solve the planner's problem, we get,

$$C_t = [(1 - \varkappa_i - \varkappa_c - \varkappa_w) - \beta\alpha(1 - \tau)]AG_t^\theta K_t^{1-\theta} = aY_t, \quad (29)$$

$$K_{t+1} = \alpha\beta(1 - \tau)AG_t^\theta K_t^{1-\theta} = bY_t, \quad (30)$$

$$G_{t+1} = \varkappa_i AG_t^\theta K_t^{1-\theta} = \varkappa_i Y_t. \quad (31)$$

Using (2), (30), (31) and (7), the growth rate that emerges along the BGP under a Π -CE, γ , is

$$1 + \gamma = Y_{t+1}/Y_t = A\varkappa_i^\theta [\alpha\beta(1 - \tau)]^{1-\theta}. \quad (32)$$

As in the planner's solution, the economy jumps from initial conditions K_0, G_0 , to equilibrium values C_0, K_1 and G_1 , the three variables growing from that time on at the common rate γ .

From (29), it is clear that increasing public expenditures crowds out private consumption, this negative effect on consumption being more intense when the increased expenditures are financed through lump-sum taxation.¹² When that is the case, the crowding-out effect

¹² $C_0 > 0$ so long as $\varkappa_i + \varkappa_c + \varkappa_w + \beta\alpha(1 - \tau) < 1$.

on consumption increases with the fraction \varkappa_w of public spending. On the other hand, (30) and (32) show that capital accumulation and economic growth are affected only if the increase in expenditures is financed through income taxation. The trade-off between lump-sum and income taxes is clear: taxes on factor incomes will reduce growth, with a lower negative impact on initial consumption, the opposite being the case under lump-sum taxes. An excessive crowding-out on private consumption due to large lump-sum taxes is the key argument leading to the possibility that taxes on factor incomes may be part of an optimal fiscal policy, as we will see in the next section.

4.2 The Ramsey problem

Each choice of government policy has a competitive equilibria and BGP path associated, and the Ramsey problem chooses the Π -competitive equilibrium allocation maximizing (11). Given $K_0, G_0 > 0$ and $\varkappa_w \geq 0$, the Ramsey problem reduces to searching among the set of Π -CE allocations, for the vector $(\varkappa_i, \varkappa_c, v, \tau)$, maximizing

$$V(C_0, C_0^g, \gamma) = \left[\frac{1}{1-\beta} (\ln C_0 + \varphi \ln C_0^g) + \frac{\beta(1+\varphi)}{(1-\beta)^2} \ln(1+\gamma) \right]. \quad (33)$$

Proposition 3 characterizes optimal fiscal policy under the competitive equilibrium mechanism, $\varkappa_i^+, \varkappa_c^+, v^+$ and τ^+ . The optimal public expenditure ratios are equal to those obtained under the planner's solution: $\varkappa_i^+ = \varkappa_i^{p+}$ and $\varkappa_c^+ = \varkappa_c^{p+}$. Moreover, the full intertemporal allocation of resources is the same under the Ramsey solution than under the planner's problem, so long as we allow for subsidies and choose the right mixture of tax instruments (Proposition 4). The results concerning optimal fiscal policy are interpreted in the next section.

Proposition 3 *The welfare-maximizing fiscal policy in the competitive equilibrium resource allocation is given by: $\varkappa_c^+ = \frac{\varphi}{1+\varphi} (1-\beta) (1-\varkappa_w)$, $\varkappa_i^+ = \beta\theta (1-\varkappa_w)$, $v^+ = \left(\frac{(1-\theta)(1-\alpha\beta)}{\alpha} - \frac{1-\beta}{1+\varphi} \right) (1-\varkappa_w)$, $\tau^+ = 1 - (1-\varkappa_w) \frac{1-\theta}{\alpha}$ and $v^+ + \tau^+ = 1 - \left((1-\theta)\beta + \frac{1-\beta}{1+\varphi} \right) (1-\varkappa_w)$.*

Proof. See part 1.c in the appendix ■

Proposition 4 *The resource allocation emerging from the fiscal policy that solves the Ramsey problem is the optimal allocation of resources.*

Proof. Taking $(\varkappa_i^+, \varkappa_c^+, \tau^+, v^+)$ to (29) and (30) we get: $C_t = a^+ Y_t$, $K_t = b^+ Y_t$, with

$$\begin{aligned} a^+ &= [(1-\varkappa_i^+ - \varkappa_c^+ - \varkappa_w) - \beta\alpha(1-\tau^+)] = (1-\varkappa_w) \frac{1-\beta}{1+\varphi} = a^{p+}, \\ b^+ &= \alpha\beta(1-\tau^+) = \beta(1-x_w)(1-\theta) = b^{p+}, \end{aligned}$$

the same time paths we got under the planner's solution. Hence, the fiscal policy characterized in Proposition 3 is optimal ■

5 Policy implications

Decisions on revenue raising and spending are taken simultaneously by the government so as to maximize consumers' welfare. On the revenue side, the government decides on which percentage of resources to raise through income and lump-sum taxation. On the other side of the public budget, the government determines the percentage of output devoted to public investment and public consumption, given a committed and exogenous stream of public services. Finally, since the government has the chance to collect funds from two alternative sources, it could decide to use a fraction of lump-sum revenues to subsidize factor rents.

We address three important issues in this section. First, we discuss whether factor rents should be taxed or subsidized. Second, we characterize in detail how the optimal mixture of income and lump-sum taxation, as well as the optimal way to allocate the resultant revenues among different public expenditure concepts, depend on the economic environment. Third, we assess the welfare consequences of deviating from the optimal policy and the best way to adjust that deviation.

5.1 Taxing or subsidizing factor rents?

Not perceiving the spillover externality or the fact that the stock of public capital next period depends on current private investment, the consumer takes into account a private marginal productivity of capital below its social product, leading to underinvestment in the competitive equilibrium allocation. Because of these externalities, a government will generally be interested in subsidizing rents from production factors, financing these subsidies through lump-sum taxation.

A third relevant externality arising in our economy comes from the need to finance an exogenous fraction of public consumption. As mentioned above, since the crowding-out impact on private consumption of raising tax revenues is larger under lump-sum than under income taxes, and this difference is more important for a larger \varkappa_w , the requirement to finance an exogenous fraction of public consumption tends to reduce subsidies on productive factors and may even lead to income taxation being optimal.

The optimal fiscal policy attempts to reduce the effects of all the externalities. Choosing the ratio of public investment to output appropriately, the government mitigates the underinvestment produced by the public capital externality [Barro (1990)]. However, the government lacks a direct instrument to correct the other two externalities, so it will have to introduce either taxes or subsidies on production factors. The spillover externality suggests subsidizing production factors, while the constraint imposed by the exogenous component of public expenditures points to taxing factor incomes. Which option prevails depends on their relative strength.

The following proposition shows that, whenever the exogenous stream of public con-

sumption goes beyond a given proportion of output, taxing factor incomes becomes optimal, against the alternative of subsidizing them. This critical share depends on the two technological externalities.

Proposition 5 *There is a threshold level \varkappa_w^+ for the ratio of exogenous public consumption to output, above which taxing factor incomes is part of an optimal fiscal policy,*

$$\varkappa_w^+ = 1 - \frac{\alpha}{1 - \theta} = \frac{\phi}{1 - \theta}. \quad (34)$$

Proof. *It comes directly from the expression for τ^+ in Proposition 3* ■

A strong spillover of aggregate private capital, or a high elasticity of public capital will each lead to subsidizing factor incomes as part of an optimal fiscal policy.¹³ Next, we exploit the constant returns to scale assumption $\alpha + \theta + \phi = 1$ to characterize the levels of each externality below which taxing factor incomes becomes optimum.

Corollary 6 *Taxing factor incomes is part of an optimal fiscal policy whenever $\theta + \phi / \varkappa_w < 1$.*

The range of values for the spillover parameter which are consistent with taxing production factor incomes increases with \varkappa_w , and the same can be said for the range of values of the elasticity of public capital which is consistent with factor income taxes. More importantly, the proposition also shows that taxing factor incomes will be optimal whenever the aggregate strength of both externalities is not too large.

From (34), several remarks are in order. First, the spillover externality is necessary for subsidies on factor incomes to be part of an optimal fiscal policy: when $\phi = 0$, taxing factor incomes is always optimal for any positive level of \varkappa_w . Second, the presence of an externality from public capital is not enough, by itself, to imply the optimality of subsidizing production factor incomes: when $\theta = 0$, taxes on factor incomes would be optimal for any ratio of public waste spending to output above $1 - \alpha$. Third, a positive and exogenous fraction of public consumption is necessary for factor income taxes to be part of an optimal fiscal policy.¹⁴ Finally, optimality of factor taxes does not rely on having a public expenditures target in the form of a given proportion of output. Let us suppose C^w is constant over time, as opposed to all other aggregate variables in the economy. If this level of spending was large enough, lump-sum financing might bring down consumption to a very low level, preventing the possibility of subsidizing private production inputs at least along the initial periods, and income taxes would therefore be claimed.

¹³This can be seen by comparing economies with the same value of ϕ and different combinations of values (α, θ) satisfying $\alpha + \theta = 1 - \phi$, as well as economies sharing the same value of θ and (α, ϕ) -pairs, with $\alpha + \phi = 1 - \theta$. These results can be easily derived from the expressions characterizing optimal policy in proposition (3).

¹⁴The standard AK economy emerges as a special case when $\theta = \phi = 0$, and income taxes are optimal for any $\varkappa_w > 0$.

5.2 The optimal tax mix and the composition of public expenditures

Our analysis reveals important aspects regarding the relationship between the tax mix and the optimal composition of public expenditures. Proposition 5 shows that taxing factor incomes is optimal whenever $\varkappa_w > \varkappa_w^+$. However, revenues from income taxes are never greater than the exogenous public expenditure items and consequently, optimal lump-sum taxes are always positive. These results are summarized in the following propositions:

Proposition 7 *Along the optimal tax policy, income tax revenues are never above the exogenous public consumption stream.*

Proof. *It comes directly from the expression for τ^+ in Proposition 3 ■*

Proposition 8 *It is never optimal to implement negative lump-sum taxes.*

Proof. *It comes directly from the expression for v^+ in Proposition 3 ■*

Table 1 shows the effects on optimal fiscal policy of changes in the ratio of exogenous public consumption to output, \varkappa_w , as well as in either the elasticity of public capital, θ , or the spillover parameter, ϕ . As a percentage of output, income tax revenues move in the same direction as \varkappa_w , while lump-sum taxes and optimal public investment and consumption move in the opposite direction. The ratio of total tax revenues to output increases. To maintain the constant returns to scale assumption, changes in θ and ϕ are compensated by a change of equal size and opposite sign in α .

A higher elasticity of public capital leads to a higher public investment ratio, without affecting public consumption. The optimal tax mix changes, with a higher ratio for lump-sum taxes and a lower one for income taxes, and higher tax revenues overall, as a percentage of output. The optimal tax mix would change in the same direction with a higher spillover of aggregate private capital in production, in this case without any effect on optimal public investment and consumption, or total tax revenues, as a percentage of output.¹⁵

[INSERT TABLE 1 ABOUT HERE]

A final issue concerns the relationship between different types of revenues and expenditures. Proposition 7 shows that the optimal fiscal structure is such that lump-sum taxation allows for financing endogenous public consumption and public investment, as well as for a fraction of the exogenous public consumption item. On the revenue side, Proposition 3 shows that the optimal income tax rate depends only on technology parameters, while lump-sum

¹⁵To see this, substitute $(1 - \phi - \alpha)$ for θ and $(\phi + \alpha)$ for $(1 - \theta)$ in proposition 4, and take derivatives with respect to ϕ .

taxes depend on preference parameters as well. On the expenditure side, the optimal choice of the public investment ratio, given \varkappa_w , depends only on technology parameters, while public consumption expenditures depend only on parameters in preferences. This double duality is produced mainly by the separability of the utility function, and has some incidence on the optimal structure of government financing: for a given size of the exogenous component of public consumption, any desired increase in other public consumption concepts will generally be financed through lump-sum taxes, while an increase in public investment will be financed through a combination of lump-sum and income taxation.

5.3 Deviations from optimal policy

The relevance of factor income taxes goes beyond the optimal structure of fiscal policy. In the standard time evolution of actual economies, perturbations of different kinds will produce deviations between observed tax revenue and expenditure ratios and their optimal values. We show in this section that such deviations should better be accommodated using factor income taxes than lump-sum taxes. Hence, this result would justify a very active role for distortionary taxation, while lump-sum taxes remain essentially constant.

Starting at the optimal policy, $\pi^+ = \{\varkappa_i^+, \varkappa_c^+, \varkappa_w, \tau^+, v^+\}$, we change \varkappa_i by an amount ε , $\varkappa_i = \varkappa_i^+ + \varepsilon$, which we alternatively assume to be financed either by a change in τ , to $\tau = \tau^+ + \varepsilon$, or by a change in v , to $v = v^+ + \varepsilon$. Values of ε in the interval $-\varkappa_i^+ < \varepsilon < \frac{(1-\varkappa_w)(1-\beta)}{1+\varphi}$ guarantee nonnegativity of public investment and private consumption. A deviation in the public investment ratio from \varkappa_i^+ will always reduce welfare, but we show that the welfare loss is always smaller when that change is compensated with income taxes. Even though we focus on deviations in the public investment ratio, a similar argument can be used to establish that the optimality of income tax financing also applies to the case when it is the public consumption ratio, \varkappa_c , that deviates from its optimal level. The result would apply again to the case of a deviation from the optimal tax mix, *i.e.*, when compensating changes are simultaneously introduced in distortionary and non-distortionary taxes, while keeping the total public expenditures ratio to output unchanged. The effects of alternative financing methods for these two deviations from optimal policy in some calibrated economies are illustrated in the next section.

Expression (33) decomposes welfare into a term emerging from initial private and public consumption, and a second term which depends on the rate of growth. The analysis in the previous sections showed that a change in public investment would affect initial private consumption and the rate of growth, but not initial public consumption. Hence, if we denote by $V^{(\tau)}(\varepsilon) = V(\varkappa_i^+ + \varepsilon, \varkappa_c^+, \varkappa_w, \tau^+ + \varepsilon, v^+)$ and $V^{(v)}(\varepsilon) = V(\varkappa_i^+ + \varepsilon, \varkappa_c^+, \varkappa_w, \tau^+, v^+ + \varepsilon)$, the levels of welfare under each financing alternative, expression (69) in part 1.b of the Appendix

can be used to write the difference $D(\varepsilon)$ between them as,

$$\begin{aligned}
(1 - \beta) D(\varepsilon) &= V^{(\tau)}(\varepsilon) - V^{(v)}(\varepsilon) = D^c(\varepsilon) + D^\gamma(\varepsilon), \text{ with} \\
D^c(\varepsilon) &= \ln \left(1 + \frac{\beta\alpha(1 + \varphi)}{(1 - \varkappa_w)(1 - \beta) - \varepsilon(1 + \varphi)} \varepsilon \right), \\
D^\gamma(\varepsilon) &= \frac{\beta(1 + \varphi)}{(1 - \beta)} (1 - \theta) \ln \left(1 - \frac{\alpha}{(1 - \varkappa_w)(1 - \theta)} \varepsilon \right).
\end{aligned} \tag{35}$$

where $D^c(\varepsilon)$ and $D^\gamma(\varepsilon)$ capture the relative welfare effects of a deviation from \varkappa_i^+ on *initial private consumption* and *growth*, respectively. Both effects are zero for $\varepsilon = 0$. $D^c(\varepsilon)$ shares the same sign than ε : an increase in the public investment ratio above \varkappa_i^+ , financed by higher lump-sum taxes, will lower initial consumption by more than if income taxes are raised, so $D^c(\varepsilon)$ will be positive, and the opposite effect would apply if the public investment ratio falls below \varkappa_i^+ . On the other hand, $D^\gamma(\varepsilon)$ has the opposite sign to ε : an increase in income taxation to finance a higher public investment ratio will lower the rate of growth more than if lump-sum taxes are used. As a consequence, $D^\gamma(\varepsilon)$ will be negative for $\varepsilon > 0$, and the opposite would happen when $\varepsilon < 0$, the observed investment ratio falling below \varkappa_i^+ . That the effects on initial consumption and growth have opposite sign illustrates the trade-off embedded in each tax policy. Hence, in principle, the sign of the net effect on $D(\varepsilon)$ of a change on ε is unclear. The following proposition shows that the net effect is always favorable to compensating a deviation from \varkappa_i^+ using income taxes.

Proposition 9 *From the point of view of welfare maximization, it is always better to compensate deviations from the optimal expenditure policy using factor income taxes than using lump-sum taxes.*

Proof. See part 2 in the appendix ■

This is an important result, that complements the characterization of optimal policy we made above. It states that if the government is forced to run into a sub-optimal public expenditure policy, the deviation in expenditures should be compensated with movements in the tax rate on factor incomes, while leaving non-distortionary taxation unchanged.

6 A numerical illustration

Further insights into the optimal policy can be obtained by illustrating our fiscal policy results with numerical examples in alternative calibrated economies. We keep $\varphi = 0.40$ and $\beta = 0.90$ constant in all calibrations, and change other more relevant parameters to find stationary equilibrium values and optimal policy variables. Aggregate variables are shown as a percentage of output.

6.1 Optimal policy

Table 2 shows numerical results for various optimal policies under alternative calibrations, all consistent with our discussion in Sections 5.1 and 5.2.¹⁶ The first four columns show the parameterization used, with the ratio of exogenous public expenditures to output in the first column. Columns five and six show private consumption and investment decisions, followed by optimal decisions on public consumption and investment. Together with exogenous public spending, in the last column in this panel, these four decisions exhaust total output. Public and private investment are high in our economy, and consumption levels are relatively small, because of the full depreciation assumption on both types of capital. The next columns show lump-sum taxes and distortionary taxes, both as a proportion of total output, followed by total revenues and by the relevance of each tax concept relative to exogenous public expenditures.

[INSERT TABLE 2 ABOUT HERE]

The first panel considers an elasticity of private capital $\alpha = 0.75$, which suggests a broad interpretation of this factor as incorporating physical as well as human capital.¹⁷ Under the constraint $\alpha + \phi + \theta = 1$, that leaves little room for a spillover for private capital, and taxing factor incomes becomes optimal. Distortionary taxation revenues amount to 15.0% of output in the absence of spillover, reducing their significance to 3.7% of output when $\phi = 0.10$. For a higher spillover $\phi = 0.25$, it is optimal to subsidize factor incomes.

A comparison between the first and second panels shows that a higher level of exogenous public spending in the economy leads to taxing factor incomes more intensively, while lump-sum tax revenues decrease, as a percentage of output. Total taxes increase, although less than proportionally, while the public investment and consumption ratios both decrease. All this goes as mentioned in Sections 5.1 and 5.2,¹⁸ and it reflects the crowding-out effect of the fiscal constraint on public consumption and investment characterized in Table 1 and discussed in the previous section. This is a rather broad crowding-out effect, with private and public decisions on consumption and investment all being affected.

The third and fourth panels show that, for a given spillover parameter, an increase in the elasticity of public capital, together with a corresponding decrease in α to maintain

¹⁶The full depreciation assumption restrains us from any strict comparison between the optimal ratios implied by the calibrated economy and the ones observed in actual economies. To make a sound comparison, other economic fundamentals not included in the model, such as the presence of heterogeneous agents, capital markets imperfections, and the presence of congestion effects in public infrastructures, should be taken into account when characterizing optimal fiscal policy.

¹⁷Being a one sector AK-version model, private capital may be interpreted broadly as an amalgam of physical and human capital.

¹⁸The same result arises in the comparisons between panels 3 and 4.

$\alpha + \theta + \phi = 1$, tends to induce higher public investment, as expected. They also increase lump-sum revenues while reducing revenues from distortionary taxation. For the parameterization considered in the panel, the higher elasticity of public capital leads to increased factor subsidies, and the comparison between rows 10 and 11 in the table shows how a shift may arise in optimal policy from taxing to subsidizing factor rents. Total revenues increase, since the only change in expenditure components is the increase in public investment. The last panel presents two AK economies, in which an increment in the public externality again produces a crowding-out effect on consumption and investment decisions, increasing distortionary taxation as well as total revenues. This is again in agreement with Table 1.

The more relevant factor determining optimality of factor income taxes seems to be that the spillover of aggregate private capital not be too important. However, as shown in row 10, even with a large spillover externality, taxing factor income will be optimal if the previously committed public expenditures claim a sizeable proportion of output. Finally, as shown in Proposition 7, when taxes on income factors are levied, tax revenues are never above exogenous public expenditures, showing that even though this expenditure component is the only justification for factor income taxes, it is itself financed through a combination of lump-sum and income taxes. On the other hand, as shown in Proposition 8, optimal non-distortionary taxes are positive in all cases, reflecting the fact that it is never optimal to give positive net lump-sum transfers to the private sector.

6.2 Uncompensated changes

Table 3 summarizes the effects of deviations from optimal policy. For different parameterizations, we examine the welfare effects of *uncompensated changes* of 0.01 in either the public consumption or the public investment ratios, in the centralized as well as in the decentralized economy. Results for the decentralized economy depend on the way deviations from optimal policy are financed, and we alternatively consider a raise in lump-sum taxes, income taxes, or a simultaneous increase of equal size in both types of revenues. Effects are shown in the table as the percentage increase in private consumption that would be needed each period to reach the same level of welfare as before the deviation from the optimal policy.

[INSERT TABLE 3 ABOUT HERE]

We take again $\varphi = 0.4$ and $\beta = 0.9$ in all cases, and the technological constant A is chosen so that growth is 4.0% under the optimal policy in all simulated economies. We consider \varkappa_w of either 15% of output, in economies 1, 2, and 5, or 30% of output, in economies 3 and 4. In economy 1 there is no spillover from aggregate private capital. The spillover effect, measured by ϕ , is strong in economies 2 and 3, and weaker in economies 4 and 5. The elasticity of public capital, θ , is higher in economies 4 and 5.

The first two economies share exogenous public spending of 15% of output, and optimal public investment and consumption turn out to be 19.1% and 2.4% of output, respectively, for both of them. In the first case, however, optimal policy leads to financing total expenditures by a combination of lump-sum and income taxes, whereas in the second economy, optimal financing is in the form of strong subsidies to factor incomes, together with high lump-sum taxes.

An increase of 0.01 in the public consumption ratio above its optimal value in the centralized economy would lead to a welfare loss similar to a decrease of 2.77% in consumption every period. In the decentralized economy, the loss would be of 4.12% of consumption if the change is financed with lump-sum taxes, or 2.85% of consumption if it is financed through income taxes. Any combination of taxes would produce intermediate results, and the table shows results for an equal split between lump-sum and income taxes. The nature of effects of a 0.01 increase in the public investment ratio is similar: the welfare loss amounts to 0.55% of consumption in the planner's economy, being of either 1.92% or 0.63% of consumption in the decentralized economy, depending on whether the deviation is financed with lump-sum or income taxes.

Effects are more important in the economies with a higher ratio of public waste spending. This numerical exercise illustrates two facts: *i*) as expected, welfare losses are always smaller in the centralized than in the decentralized economy, and *ii*) in line with the discussion in the previous section (Proposition 9), welfare losses are always smaller if the deviation from optimal policy is financed with income taxes than with lump-sum taxes.

6.3 Compensated changes

The first two rows in the *Compensated changes* panel refer to simultaneous changes in both public expenditure ratios so that total expenditures do not change, as a proportion of output. Alternatively, in the last two rows, the label refers to simultaneous changes in lump-sum and income taxes, maintaining total revenues constant as a percentage of total output.

An increase of 0.01 in the public investment ratio, compensated with an equal decrease in the public consumption ratio, produces a welfare loss comparable to a decrease in consumption of 5.04% every period. The same numerical effects apply to the centralized and the decentralized economies. An increase of 0.01 in the public consumption ratio, compensated with an equal decrease in the public investment ratio, has always a smaller welfare effect than the reverse deviation. The welfare effects of deviations from optimal policy are clearly not symmetric. Effects are again higher in the two economies with the higher public externality.

An increase of 0.01 in the income tax rate, compensated with an equal decrease in lump-sum taxes to maintain total revenues unchanged as a percentage of output, leads in the first economy to a welfare loss comparable to a fall of 0.64% in consumption every period,

whereas the reverse deviation has the same effect as a 0.73% reduction in consumption every period. Again, welfare effects are not symmetric. Effects are smaller in the other parametric cases considered, most likely because tax rates are highest in the first economy.

7 Robustness of results

In Section ??, several assumptions were needed to solve the Ramsey problem analytically: log utility, Cobb-Douglas technology, uniform income tax rates and fully depreciation of private and public capital. The main result of the paper shows that a positive income tax rate may be optimal, in spite of the allowance of lump-sum taxation and for positive spillover and public capital externalities. For a general CES utility function, we study the robustness of this result when utility moves away from the logarithmic specification and when public and private capital are long-lived. We show that these generalizations do not affect the optimal income tax rate, although they may have important implications on the optimal public investment and lump-sum tax policies. Under these circumstances, a fully analytical analysis is unfeasible. Since C_g does not affect the optimal tax structure, for simplicity, we do not consider C^g in utility and we set $\varkappa_c = 0$ throughout this section. Thus, public expenditure is either productive or wasteful.

Conditions (9), (8), (4) and (7) are now:

$$U = \sum_{t=0}^{\infty} \beta^t \frac{C^{1-1/\sigma} - 1}{(1 - 1/\sigma)}, \quad \sigma > 0, \quad (36)$$

$$K_{t+1} = (1 - \delta)K_t + I_t, \quad (37)$$

$$G_{t+1} = (1 - \delta^g)G_t + I_t^g, \quad (38)$$

$$I_t^g + C_t^w = \tau_t Y_t + X_t \Leftrightarrow \varkappa_i + \varkappa_w = \tau_t + v_t. \quad (39)$$

where σ is the constant elasticity of intertemporal substitution and δ and δ^g are the linear depreciation rates of private and public capital, respectively, both between zero and one and not necessarily equal. The consumption-saving condition (23) for the representative household becomes,

$$C_{t+1}/C_t = \beta^\sigma [1 - \delta + r_{t+1} (1 - \tau_{t+1})]^\sigma, \quad (40)$$

The Ramsey problem turns out to be,

$$\max_{\{C_t, K_{t+1}, G_{t+1}, \tau_t, v_t\}_0^\infty} \mathcal{L} = \sum \beta^t \frac{C_t^{1-1/\sigma} - 1}{(1 - 1/\sigma)}, \quad \text{subject to} \quad (41)$$

$$C_t^{-1/\sigma} = \beta C_{t+1}^{-1/\sigma} \left(1 - \delta + \alpha A (G_{t+1}/K_{t+1})^\theta (1 - \tau_{t+1}) \right), \quad (42)$$

$$G_{t+1} = (v_t + \tau_t - \varkappa_c) A (G_t/K_t)^\theta K_t - (1 - \delta_g)G_t, \quad (43)$$

$$C_t + K_{t+1} - (1 - \delta)K_t = (1 - \tau_t - v_t)A (G_t/K_t)^\theta K_t, \quad (44)$$

where the first constraint corresponds to the household consumption-saving condition, the second represents the accumulation of public capital and the third is the global resource constraint of the economy.¹⁹

Fiscal policy must be stationary along the BGP and all aggregate variables grow at the same constant rate γ . We define the following stationary ratios: $\tilde{G} = G_t/K_t$ and $\tilde{u}_c = (C_{t+1}/C_t)^{1/\sigma} = (1 + \bar{\gamma})^{1/\sigma}$, which is the marginal rate of substitution between next-period and current consumption. For the BGP, the solution of the above Ramsey problem leads to the following conditions on public-to-private capital:²⁰

$$\beta \left[1 + (1 - \varkappa_w)\theta A\tilde{G}^{\theta-1} - \delta_g \right] = (1 + \bar{\gamma})^{1/\sigma}, \quad (45)$$

$$\varkappa_i (\alpha + \phi) A\tilde{G}^\theta + \left[1 - \delta + (1 - \tau - \nu)(\alpha + \phi) A\tilde{G}^\theta \right] = (1 + \bar{\gamma})^{1/\sigma} / \beta. \quad (46)$$

In addition, from the public capital accumulation rule (43) and the Euler condition (42),

$$\theta A\tilde{G}^{\theta-1} = \theta \frac{\delta^g + \gamma}{\varkappa_i}, \quad (47)$$

$$1 - \delta + \alpha A\tilde{G}^\theta (1 - \tau) = (1 + \gamma)^{1/\sigma} / \beta. \quad (48)$$

Substituting (47) into (45), we obtain an equation for the optimal public investment ratio,

$$\varkappa_i^+ = \beta\theta(1 - \varkappa_w) \frac{\delta^g + \gamma}{(1 + \gamma)^{1/\sigma} - \beta(1 - \delta^g)}. \quad (49)$$

This expression is equivalent to one found in Marrero (2006a). In general, since γ depends on policy variables, an explicit expression for \varkappa_i^+ cannot be obtained. Only under full depreciation ($\delta_g = 1$) and a logarithmic utility function ($\sigma = 1$) do we get an explicit expression for the optimal public investment ratio, $\beta\theta(1 - \varkappa_w)$, the same than that shown in Proposition 3.²¹

Combining (47) with (45) yields the optimal income tax rate,

$$\tau^+ = 1 - (1 - \varkappa_w) \frac{1 - \theta}{\alpha}, \quad (50)$$

¹⁹As under $\sigma, \delta, \delta^g = 1$, it is easy to show that the resource allocation emerging from the fiscal policy that solves the Ramsey problem reproduces that of the planner's problem [see Marrero (2006b) for more details about this point].

²⁰See Marrero (2006b) for a detailed description of the solution of the Ramsey problem.

²¹It is easy to show that $\varkappa_i^+ < \beta\theta(1 - \varkappa_w)$ if $\sigma < 1$ or if $\sigma = 1$ and $\delta^g < 1$. Moreover, \varkappa_i^+ can never be higher than the ratio maximizing the long-run growth rate, $\theta(1 - \varkappa_w)$. See Marrero (2006a) for more details about this point.

which is the same as that shown in Proposition 3, so the result in Proposition (5). Finally, using (49), we obtain the optimal stationary level of lump-sum transfers as a fraction of output,

$$v^+ = (1 - \varkappa_w) \left(\varphi \frac{\gamma + \delta^g}{(1 + \gamma)^{1/\sigma} / \beta - 1 + \delta^g} + \frac{\phi}{\alpha} \right). \quad (51)$$

In Marrero (2006a), it is shown that, for $\sigma \neq 1$, \varkappa_i^+ is positive and strongly related with δ^g and σ , while the relationship is also positive but less intense with respect to δ . Hence, since τ^+ is not affected by these parameters, the relationship between v^+ and these parameters is negative and of the same sign than with respect to \varkappa_i^+ .

8 Final Remarks

We have characterized the optimal tax and expenditure policies in an economy where public capital is productive and there is a spillover effect from aggregate private capital. We assume debt cannot be issued, so that the government has three decisions to make every period: *i*) investment on public capital, *ii*) public consumption and *iii*) the tax mixture between income and lump-sum taxation to finance public expenditures, even allowing for the possibility of subsidizing factors' rents. The presence of public capital as an input in the production function and the spillover externality, both tend to favor subsidizing factor rents. In addition, we incorporate into our model economy the fact that the government needs to finance an exogenous component of public expenditures, due to commitments made in the past. This is assumed to be a constant proportion of output. The crowding-out effect on private consumption of raising tax revenues is larger when it is lump-sum taxes that are increased, rather than income taxes. This difference turns out to be more important the larger the requirement of the exogenous component of public expenditures. Hence, this constraint on public expenditures will tend to reduce subsidies on productive factors and may even lead to income taxation being optimal.

The main result in this paper is that taxing factor incomes may be optimal for reasonable parameterizations in the presence of the public capital externality and the spillover effect from aggregate private capital, in spite of the fact that both externalities in the economy tend to favor subsidizing production factors. Moreover, welfare losses from deviations from the optimal policy are always smaller when compensated with changes in income taxes than when adjusting lump-sum taxes. Factor taxes are always optimal in the absence of the spillover externality, but they can also be optimal in cases when that externality is sizeable, if combined with the constraint to pay for a relatively high level of the exogenous component of public expenditures. The optimality of factor income taxes is more likely in economies with a low elasticity of public capital, a weak spillover from private capital, and also when exogenous public expenditures claim a large proportion of output.

The presence of public consumption as an argument in the utility function of the representative agent turns out not to be necessary for the optimality of income taxes in our setup. An extension of this paper should go in the direction of avoiding the dichotomy between factors affecting the structure of revenues and expenditures by considering nonseparability of private and public consumption in consumers' preferences. A second consideration along this line would lead to explicitly including leisure in the utility function. That would allow us to characterize the optimal combination of lump-sum taxes with labor and capital income taxes to finance public investment and consumption. A final extension should look at an economy with a non-trivial transition to steady-state, as it would be the case under long-lived public and private stocks of capital.

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Appendix

Part 1:

1.a. Solving the planner's problem

Optimality conditions to maximize the Lagrangian,

$$\underset{\{C_t, G_{t+1}, K_{t+1}\}_{t=0}^{\infty}}{Max} \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{l} \ln C_t + \varphi \ln(\varkappa_c A K_t^{1-\theta} G_t^\theta) - \lambda_t [G_{t+1} - \varkappa_i A K_t^{1-\theta} G_t^\theta] - \\ \mu_t [K_{t+1} + G_{t+1} + C_t - (1 - \varkappa_c - \varkappa_w) A K_t^{1-\theta} G_t^\theta] \end{array} \right\}$$

are,

$$\beta^t \left(\frac{1}{C_t} - \mu_t \right) = 0, \quad (52)$$

$$-\beta^t (\lambda_t + \mu_t) + \beta^{t+1} \left[\frac{\theta \varphi}{G_{t+1}} + A \left(\frac{K_{t+1}}{G_{t+1}} \right)^{1-\theta} \theta [\lambda_{t+1} \varkappa_i + \mu_{t+1} (1 - \varkappa_c - \varkappa_w)] \right] = 0, \quad (53)$$

$$-\beta^t \mu_t + \beta^{t+1} \left[\frac{\varphi(1-\theta)}{K_{t+1}} + A \left(\frac{K_{t+1}}{G_{t+1}} \right)^{-\theta} (1-\theta) [\lambda_{t+1} \varkappa_i + \mu_{t+1} (1 - \varkappa_c - \varkappa_w)] \right] = 0, \quad (54)$$

$$K_{t+1} + G_{t+1} + C_t - (1 - \varkappa_c - \varkappa_w) A K_t^{1-\theta} G_t^\theta = 0, \quad (55)$$

$$G_{t+1} - \varkappa_i A K_t^{1-\theta} G_t^\theta = 0, \quad (56)$$

together with sign conditions, $C_t, K_{t+1}, Y_t, \mu_t, \lambda_t \geq 0$, and the standard transversality condition regarding the accumulation of private and public capital.

Combining (55) and (56) and dividing by K_t we get,

$$\frac{K_{t+1}}{K_t} + \frac{C_t}{K_t} - (1 - \varkappa_c - \varkappa_i - \varkappa_w) \left(\frac{G_t}{K_t} \right)^\theta = 0. \quad (57)$$

On the other hand, plugging (52) into (53) and (54):

$$-\lambda_t - \frac{1}{C_t} + \beta \left[\frac{\theta \varphi}{G_{t+1}} + \lambda_{t+1} \varkappa_i \theta \left(\frac{G_{t+1}}{K_{t+1}} \right)^{\theta-1} + \frac{(1 - \varkappa_c - \varkappa_w) \theta}{C_{t+1}} \left(\frac{G_{t+1}}{K_{t+1}} \right)^{\theta-1} \right] = 0, \quad (58)$$

$$-\frac{1}{C_t} + \beta (1 - \theta) \left[\frac{\varphi}{K_{t+1}} + \lambda_{t+1} \varkappa_i \left(\frac{G_{t+1}}{K_{t+1}} \right)^\theta + \frac{(1 - \varkappa_c - \varkappa_w)}{C_{t+1}} \left(\frac{G_{t+1}}{K_{t+1}} \right)^\theta \right] = 0, \quad (59)$$

which imply that along the balanced growth path, $1/\lambda_t$ must growth at the same constant rate than the rest of the variables in the economy.

To solve the dynamics of control and state variables, we make a linear guess: $C_t = aY_t$, $K_{t+1} = bY_t$, $1/\lambda_t = dY_t$ and show that such a solution actually exists. If we are able to find expressions for a , b and d so that first order and sign conditions hold, we will have shown that the centralized problem has at least a linear solution. On the other hand, uniqueness of such an equilibrium comes from the theorem of the maximum, since the choice set is compact and the objective function is continuous.²² Plugging the previous guess together with $G_{t+1} = \varkappa_i Y_t$ into (57)-(59),

$$[b + a - (1 - \varkappa_c - \varkappa_w - \varkappa_i)] = 0, \quad (60)$$

$$-1/dY_t - \frac{1}{aY_t} + \beta\theta \left[\frac{\varphi}{\varkappa_i Y_t} + \frac{\varkappa_i}{db^{1-\theta} \varkappa_i^\theta Y_t} \left(\frac{\varkappa_i}{b}\right)^{\theta-1} + \frac{(1 - \varkappa_c - \varkappa_w)}{ab^{1-\theta} \varkappa_i^\theta Y_t} \left(\frac{\varkappa_i}{b}\right)^{\theta-1} \right] = 0, \quad (61)$$

$$-\frac{1}{aY_t} + \beta(1 - \theta) \left[\frac{\varphi}{bY_t} + \frac{\varkappa_i}{db^{1-\theta} \varkappa_i^\theta Y_t} \left(\frac{\varkappa_i}{b}\right)^\theta + \frac{(1 - \varkappa_c - \varkappa_w)}{ab^{1-\theta} \varkappa_i^\theta Y_t} \left(\frac{\varkappa_i}{b}\right)^\theta \right] = 0, \quad (62)$$

where we have also used $Y_{t+1} = AK_{t+1}^{1-\theta} G_{t+1}^\theta = Ab^{1-\theta} \varkappa_i^\theta Y_t$ and $C_{t+1} = Aab^{1-\theta} \varkappa_i^\theta Y_t$. After some manipulations we get a system of three equations in a , b , d :

$$[b + a - (1 - \varkappa_c - \varkappa_w - \varkappa_i)] = 0, \quad (63)$$

$$-\frac{1}{d} - \frac{1}{a} + \beta \frac{\theta\varphi}{\varkappa_i} + \frac{\beta\theta}{d} + \frac{\beta(1 - \varkappa_c - \varkappa_w)\theta}{a\varkappa_i} = 0, \quad (64)$$

$$-\frac{1}{a} + \beta(1 - \theta) \frac{1}{b} \left[\varphi + \frac{\varkappa_i}{d} + \frac{(1 - \varkappa_c - \varkappa_w)}{a} \right] = 0, \quad (65)$$

from which we get expressions for a and b as functions of structural and policy parameters,

$$a = (1 - \beta) \frac{1 - \varkappa_i - \varkappa_c - \varkappa_w}{1 - \beta(\theta - \varphi + \varphi\theta)}, \quad (66)$$

$$b = \beta(1 + \varphi)(1 - \theta) \frac{1 - \varkappa_i - \varkappa_c - \varkappa_w}{1 - \beta(\theta - \varphi + \varphi\theta)}. \quad (67)$$

Along the balanced growth path, the constant output growth rate is: $Y_{t+1}/Y_t = (1 + \gamma^p) = Ab^{1-\theta} \varkappa_i^\theta$, so that,

$$1 + \gamma^p = A \left[\frac{\beta(1 + \varphi)(1 - \theta)(1 - \varkappa_i - \varkappa_c - \varkappa_w)}{1 - \beta(\theta - \varphi + \varphi\theta)} \right]^{1-\theta} \varkappa_i^\theta. \quad (68)$$

²²An explicit argument on uniqueness of the competitive equilibrium allocation is made in Marrero and Novales (2005) and Glomm and Ravikumar (1994,1999).

1.b. Optimal government expenditures under the planner's solution

After using expressions for C_0 and γ^p from (15) and (18) together with $C_0^g = \varkappa_c Y_0$, (19) amounts to,

$$\max_{\zeta^p} V(\zeta^p) = \frac{1}{1-\beta} \left\{ \begin{aligned} & \ln \left[(1-\beta) \frac{1-\varkappa_i-\varkappa_c-\varkappa_w}{1-\beta(\theta-\varphi+\varphi\theta)} \right] + \varphi \ln \varkappa_c + (1+\varphi) \ln Y_0 + \\ & + \frac{\beta(1+\varphi)}{1-\beta} \ln \left[\left(\frac{\beta(1+\varphi)(1-\theta)(1-\varkappa_i-\varkappa_c-\varkappa_w)}{1-\beta(\theta-\varphi+\varphi\theta)} \right)^{1-\theta} \varkappa_i^\theta \right] \end{aligned} \right\}, \quad (69)$$

where we search for the public expenditures stationary policy $(\varkappa_i, \varkappa_c)$ maximizing utility on the efficient frontier. Optimality conditions for (69) are,

$$\begin{aligned} \frac{\partial V(\zeta)}{\partial \varkappa_c} &= \frac{-\varkappa_c + \varphi - \beta\varphi - \varphi\varkappa_i + \varphi\varkappa_i\beta - \varphi\varkappa_c - \varphi\varkappa_w + \varphi\varkappa_w\beta + \varkappa_c\beta\theta + \varkappa_c\beta\varphi\theta}{(1-\varkappa_i-\varkappa_c-\varkappa_w)\varkappa_c(1-\beta)} = 0, \\ \frac{\partial V(\zeta)}{\partial \varkappa_i} &= -\frac{\beta\theta\varkappa_w + \beta\theta\varphi\varkappa_w + -\varkappa_c\beta\varphi\theta + \varphi\varkappa_i\beta - \beta\varphi\theta - \beta\theta + \varkappa_c\beta\theta + \varkappa_i}{(1-\varkappa_i-\varkappa_c-\varkappa_w)\varkappa_i(1-\beta)} = 0, \end{aligned}$$

with solution,

$$\begin{aligned} \varkappa_c^{p+} &= \varphi(1-\varkappa_w) \frac{1-\beta}{1+\varphi}, \\ \varkappa_i^{p+} &= \theta\beta(1-\varkappa_w). \end{aligned}$$

Finally, the optimal paths for consumption and private capital are,

$$\begin{aligned} \frac{C_t}{Y_t} &= (1-\beta) \frac{1-\varkappa_i^{p+} - \varkappa_c^{p+} - \varkappa_w}{1-\beta(\theta-\varphi+\varphi\theta)} = (1-\varkappa_w) \frac{1-\beta}{1+\varphi}, \\ \frac{K_t}{Y_t} &= \beta(1+\varphi)(1-\theta) \frac{1-\varkappa_i^{p+} - \varkappa_c^{p+} - \varkappa_w}{1-\beta(\theta-\varphi+\varphi\theta)} = \beta(1-\varkappa_w)(1-\theta), \\ \frac{G_t}{Y_t} &= \varkappa_i^{p+} = \theta\beta(1-\varkappa_w). \end{aligned}$$

1.c. The Ramsey solution

Plugging expressions (29) for C_0 , (32) for γ and $C_0^g = \varkappa_c Y_t$ into (33) we get,

$$V(\varsigma) = \frac{1}{1-\beta} \left\{ \begin{aligned} & \ln [(1-\varkappa_i-\varkappa_c-\varkappa_w) - \beta\alpha(1-\varkappa_i-\varkappa_c-\varkappa_w+v)] + \varphi \ln \varkappa_c + \\ & (1+\varphi) \ln Y_0 + \frac{\beta(1+\varphi)}{1-\beta} \ln \left\{ A\varkappa_i^\theta [\alpha\beta(1-\varkappa_i-\varkappa_c-\varkappa_w+v)]^{1-\theta} \right\} \end{aligned} \right\}. \quad (70)$$

It is easy to show that $V(\cdot)$ is non monotonic in either \varkappa_i, \varkappa_c and v . Moreover, for any given v , we have $\lim_{\varkappa_i, \varkappa_c \rightarrow 0^+} V = \lim_{\varkappa_i, \varkappa_c \rightarrow 1^-} V = -\infty$, so the welfare-maximizing levels of \varkappa_i, \varkappa_c fall in the interior of $[0, 1] \times [0, 1]$. This is not the case for v , whose optimal value may fall above 1 or below 0. $V(\zeta)$ defined in (70) is strictly concave in ζ , so condition $\nabla V(\zeta) = 0$ is necessary and sufficient to characterize a global maximum,

$$\frac{\partial V}{\partial \varkappa_i} = \frac{-(1 - \beta\alpha)}{1 - \varkappa - \beta\alpha(1 - \varkappa + v)} + \frac{\beta(1 + \varphi)\theta}{(1 - \beta)\varkappa_i} - \frac{\beta(1 + \varphi)(1 - \theta)}{(1 - \beta)(1 - \varkappa + v)} = 0, \quad (71)$$

$$\frac{\partial V}{\partial v} = -\frac{\beta\alpha}{1 - \varkappa - \beta\alpha(1 - \varkappa + v)} + \frac{\beta(1 + \varphi)(1 - \theta)}{(1 - \beta)(1 - \varkappa + v)} = 0, \quad (72)$$

$$\frac{\partial V}{\partial \varkappa_g} = -\frac{1 - \beta\alpha}{1 - \varkappa - \beta\alpha(1 - \varkappa + v)} + \frac{\varphi}{\varkappa_c} - \frac{\beta(1 + \varphi)(1 - \theta)}{(1 - \beta)(1 - \varkappa + v)} = 0, \quad (73)$$

where $\varkappa = \varkappa_i + \varkappa_c + \varkappa_w$. Adding up (72) and (73), we get,

$$v = (1 - \varkappa)\left(\frac{1}{\alpha\beta} - 1\right) - \frac{\varkappa_c}{\varphi} \frac{1}{\alpha\beta}, \text{ and thus} \quad (74)$$

$$1 - \varkappa + v = 1 - \tau = \left(1 - \varkappa - \frac{\varkappa_c}{\varphi}\right) \frac{1}{\alpha\beta}. \quad (75)$$

Plugging (75) into (72) we get,

$$-\frac{\beta\alpha}{1 - \varkappa - \left(1 - \varkappa - \frac{\varkappa_c}{\varphi}\right)} + \frac{\beta(1 + \varphi)(1 - \theta)}{(1 - \beta)\left(1 - \varkappa - \frac{\varkappa_c}{\varphi}\right) \frac{1}{\beta\alpha}} = 0,$$

from where we get the relationship between the welfare maximizing levels of \varkappa_i and \varkappa_c ,

$$\varkappa_c^+ = \frac{\varphi}{1 + \varphi} \frac{1 - \beta}{1 - \beta + \beta(1 - \theta)} (1 - \varkappa_i^+ - \varkappa_w). \quad (76)$$

On the other hand, combining (71) and (73), we get an alternative relationship between \varkappa_i^+ and \varkappa_c^+ ,

$$\varkappa_i^+ = \beta \frac{1 + \varphi}{1 - \beta} \frac{\theta}{\varphi} \varkappa_c^+. \quad (77)$$

Combining (76) with (77), we get,

$$\begin{aligned} \varkappa_c^+ &= \frac{\varphi}{1 + \varphi} (1 - \beta) (1 - \varkappa_w), \\ \varkappa_i^+ &= \beta\theta (1 - \varkappa_w). \end{aligned}$$

Then,

$$\begin{aligned}
v^+ &= (1 - \varkappa^+) \left(\frac{1}{\alpha\beta} - 1 \right) - \frac{\varkappa_c^+}{\varphi} \frac{1}{\alpha\beta} \\
&= \left(\frac{(1 - \theta)(1 - \alpha\beta)}{\alpha} - \frac{1 - \beta}{1 + \varphi} \right) (1 - \varkappa_w), \\
\tau^+ &= \varkappa^+ - v^+ = 1 - (1 - \varkappa_w) \frac{1 - \theta}{\alpha}.
\end{aligned}$$

Part 2: Proof of Proposition 9

Lemma 10 $\frac{\partial |D^C(\varepsilon)|}{\partial \varepsilon} > \frac{\partial |D^\gamma(\varepsilon)|}{\partial \varepsilon}$ for $0 < \varepsilon < \frac{(1 - \varkappa_w)(1 - \beta)}{1 + \varphi}$, while $\frac{\partial |D^C(\varepsilon)|}{\partial \varepsilon} < \frac{\partial |D^\gamma(\varepsilon)|}{\partial \varepsilon}$ for $-\varkappa_i^+ < \varepsilon < 0$.

Proof: First order derivatives are: $\frac{\partial D^C(\varepsilon)}{\partial \varepsilon} = \frac{\beta\alpha(1+\varphi)(1-\varkappa_w)(1-\beta)}{[(1-\varkappa_w)(1-\beta)-\varepsilon(1+\varphi)]^2} \frac{1}{1 + \frac{\beta\alpha(1+\varphi)}{(1-\varkappa_w)(1-\beta)-\varepsilon(1+\varphi)}\varepsilon}$, $\frac{\partial D^\gamma(\varepsilon)}{\partial \varepsilon} = -\frac{\beta(1+\varphi)}{(1-\beta)} \frac{\alpha(1-\theta)}{(1-\varkappa_w)(1-\theta)-\alpha\varepsilon}$. Since $1 - \beta < 1 + \varphi$ and $\alpha \leq 1 - \theta$, we have that $\frac{\partial D^C(\varepsilon)}{\partial \varepsilon}$ is positive, and $\frac{\partial D^\gamma(\varepsilon)}{\partial \varepsilon}$ is negative for any ε in the range $-\varkappa_i^+ < \varepsilon < \frac{(1-\varkappa_w)(1-\beta)}{1+\varphi}$.

i) For $0 < \varepsilon < \frac{(1-\varkappa_w)(1-\beta)}{1+\varphi}$, we have: $\frac{\partial |D^C(\varepsilon)|}{\partial \varepsilon} > \frac{\partial |D^\gamma(\varepsilon)|}{\partial \varepsilon} \Leftrightarrow \frac{\partial D^C(\varepsilon)}{\partial \varepsilon} > -\frac{\partial D^\gamma(\varepsilon)}{\partial \varepsilon} \Leftrightarrow \frac{(1-\beta)(1-\varkappa_w)}{[(1-\beta)(1-\varkappa_w)-\varepsilon(1+\varphi)]^2} > \frac{1}{(1-\beta)} \frac{1-\theta}{(1-\varkappa_w)(1-\theta)-\alpha\varepsilon}$. Since $\varphi \geq 0$ and $\alpha \leq 1$, we have $\frac{(1-\varkappa_w)(1-\beta)}{1+\varphi} < \frac{(1-\varkappa_w)(1-\beta)}{\alpha}$, so that $\varepsilon < \frac{(1-\varkappa_w)(1-\beta)}{1+\varphi}$ implies $\alpha\varepsilon < (1 - \varkappa_w)(1 - \theta)$, and the previous inequality will be equivalent to,

$$(1 - \beta)^2 (1 - \varkappa_w) \alpha < (1 - \theta)(1 - \beta)(1 - \varkappa_w)(1 + \varphi) + \Phi,$$

with

$$\Phi = (1 - \theta)(1 + \varphi)(1 - \beta\alpha)(1 - \beta)(1 - \varkappa_w) - (1 - \theta)(1 + \varphi)^2(1 - \beta\alpha)\varepsilon.$$

Since Φ is positive for the range of ε considered, and $(1 - \beta)\alpha < (1 + \varphi)(1 - \theta)$, the inequality above holds.

ii) For $-\varkappa_i^+ < \varepsilon < 0$ we have $\frac{\partial |D^C(\varepsilon)|}{\partial \varepsilon} < \frac{\partial |D^\gamma(\varepsilon)|}{\partial \varepsilon} \Leftrightarrow -\frac{\partial D^C(\varepsilon)}{\partial \varepsilon} < \frac{\partial D^\gamma(\varepsilon)}{\partial \varepsilon} \Leftrightarrow \frac{\partial D^C(\varepsilon)}{\partial \varepsilon} > -\frac{\partial D^\gamma(\varepsilon)}{\partial \varepsilon}$ which, by the same previous argument, it is seen to hold for any negative ε .

Proposition 9 follows directly from the previous lemma.

Tables

Table 1: Sign effects on main variables

	\varkappa_i^+	\varkappa_c^+	\varkappa^+	v^+	τ^+	$v^+ + \tau^+$	a^+	b^+
$\varkappa_w \uparrow$	\downarrow	\downarrow	\uparrow	\downarrow	\uparrow	\uparrow	\downarrow	\downarrow
$\theta \uparrow (\Delta\alpha = -\Delta\theta)$	\uparrow	$=$	\uparrow	\uparrow	\downarrow	\uparrow	$=$	\downarrow
$\phi \uparrow (\Delta\alpha = -\Delta\phi)$	$=$	$=$	$=$	\uparrow	\downarrow	$=$	$=$	$=$

Table 2: Optimal fiscal policy and stationary equilibrium values

Parameterizations				% of total output					Taxes, %			ratio to \varkappa_w , %	
$\beta = 0.90; \varphi = 0.40$				C	I	I^g	C^g	C^w	v	τ	$v + \tau$	v	τ
\varkappa_w	α	ϕ	θ										
0.15	0.75	0.00	0.25	6.1	57.4	19.1	2.4	15.0	21.6	15.0	36.6	143.7	100
0.15	0.75	0.10	0.15	6.1	65.0	11.5	2.4	15.0	25.2	3.7	28.9	168.2	24.4
0.15	0.75	0.25	0.00	6.1	76.5	0.0	2.4	15.0	30.8	-13.3	17.4	205.1	-88.9
0.30	0.75	0.00	0.25	5.0	47.3	15.8	2.0	30.0	17.8	30.0	47.8	59.2	100.0
0.30	0.75	0.10	0.15	5.0	53.6	9.5	2.0	30.0	20.8	20.7	41.5	69.3	68.9
0.30	0.75	0.25	0.00	5.0	63.0	0.0	2.0	30.0	25.3	6.7	32.0	84.4	22.2
0.15	0.50	0.20	0.30	6.1	53.6	23.0	2.4	15.0	59.4	-19.0	40.4	395.9	-126.7
0.15	0.40	0.20	0.40	6.1	45.9	30.6	2.4	15.0	75.5	-27.5	48.0	503.5	-183.3
0.15	0.30	0.20	0.50	6.1	38.3	38.3	2.4	15.0	97.3	-41.7	55.7	649.0	-277.8
0.30	0.50	0.20	0.30	5.0	44.1	18.9	2.0	30.0	48.9	2.0	50.9	163.0	6.7
0.30	0.40	0.20	0.40	5.0	37.8	25.2	2.0	30.0	62.2	-5.0	57.2	207.3	-16.7
0.30	0.30	0.20	0.50	5.0	31.5	31.5	2.0	30.0	80.2	-16.7	63.5	267.2	-55.6
0.15	1.00	0.00	0.00	6.1	76.5	0.0	2.4	15.0	2.4	15.0	17.4	16.2	100.0
0.30	1.00	0.00	0.00	5.0	63.0	0.0	2.0	30.0	2.0	30.0	32.0	6.7	100.0

Note: bold letters shows those cases where taxing factor rents is optimal

Table 3: Welfare effects of deviations from the optimal policy

ECONOMIES	1	2	3	4	5
Parameterizations					
α	0.75	0.40	0.40	0.40	0.40
ϕ	0.00	0.35	0.35	0.15	0.15
θ	0.25	0.25	0.25	0.45	0.45
\varkappa_w	0.15	0.15	0.30	0.30	0.15
Optimal Policies (d)					
\varkappa_i	0.191	0.191	0.158	0.284	0.344
\varkappa_c	0.024	0.024	0.020	0.020	0.024
τ	0.150	-0.594	-0.313	0.038	-0.169
v	0.216	0.959	0.790	0.566	0.687
$1 + \gamma$	1.040	1.040	1.040	1.040	1.040
Uncompensated Changes (a)					
Higher \varkappa_c , Centralized ec.	-2.769	-2.769	-3.897	-3.957	-2.810
Higher \varkappa_c , Higher v	-4.116	-4.116	-5.914	-5.914	-4.116
Higher \varkappa_c , Higher τ	-2.850	-3.240	-4.597	-4.606	-3.247
Higher \varkappa_c , Higher τ and v	-3.281	-3.618	-5.163	-5.165	-3.620
Higher \varkappa_i , Centralized ec.	-0.546	-0.546	-0.800	-0.599	-0.407
Higher \varkappa_i , Higher v	-1.924	-1.924	-2.881	-2.624	-1.746
Higher \varkappa_i , Higher τ	-0.629	-1.028	-1.522	-1.271	-0.855
Higher \varkappa_i , Higher τ and v	-1.069	-1.414	-2.106	-1.849	-1.237
Compensated Changes (a)					
Higher \varkappa_i , Lower \varkappa_c (b)	-5.039	-5.039	-7.997	-7.753	-4.867
Higher \varkappa_c , Lower \varkappa_i (b)	-3.075	-3.075	-4.347	-4.058	-2.879
Higher τ , Lower v (c)	-0.640	-0.188	-0.275	-0.285	-0.194
Higher v , Lower τ (c)	-0.730	-0.201	-0.299	-0.309	-0.208

(a) % increase in private consumption needed to match the level of welfare attained under the optimal policy;
(b) Results for the centralized and the decentralized economies are the same; (c) It applies to the decentralized economy; (d) Bold figures refer to economies where optimal policy implies taxing factor incomes