Transitional dynamics and welfare effects of the public investment/output ratio*

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Abstract

In endogenous growth settings, long-run growth effects are important for welfare, but they should not be the only consideration for policy evaluation. In this paper, welfare effects along the first periods of the transition following a fiscal policy reform are found to be of opposite sign to long-run effects. Hence, fully characterizing the transitional dynamics is crucial when characterizing the effects of downsizing public investment, an important fiscal policy issue in industrialized economies.

Starting from a standard fiscal position and a benchmark parameter calibration, we show that downsizing public investment improves welfare under either capital or gross income taxes, provided public capital is not very productive. On the other hand, downsizing is found to improve welfare with independence of the tax system considered for high levels of the unproductive public expenditure/output ratio, or for low values of the elasticity of intertemporal substitution, the discount factor and/or the public capital elasticity in the aggregate technology.

Additionally, for high levels of the output elasticity of private capital, downsizing is shown to be optimal under the less distorting taxes, but not under gross income and capital income taxes.

Keywords: Transitional dynamics, endogenous growth, distorting taxes, public investment, simulation methods.

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1. Introduction

Reconsidering the fiscal role of governments is a central issue on economic policy in industrialized economies, and there is a wide consensus on the potential welfare gains from downsizing policies relative to the observed size of public sector in most industrialized economies. Public resources could be at a minimum classified as either expenditures that may *directly* affect productivity (infrastructure, public firms investment, etc.), or expenditures that do not have such effect (health, social security, law and order, defense, etc.) [Musgrave (1997)]. It is a standard characteristic in developed economies that governments face a level of mostly precommitted expenditures of the second kind because of previous social policies, and this paper characterizes the welfare-maximizing *productive* public expenditures/output ratio in a stylized endogenous growth model with private and non-congested public capital. We do not question how the level of unproductive public expenditure size is determined which,[1] as in Cassou and Lansing (1998) and Marrero and Novales (2005, 2007), we assume to be exogenous.[2] The analysis is performed under alternative tax systems, since welfare effects of a public investment policy could well be different, as suggested by Burgess and Stern (1993).

This paper relates to the vast literature on fiscal policy in dynamic settings. Starting with Lucas (1990), Jones and Manuelli (1990) and Barro (1990), endogenous growth models have become a standard environment to analyze the incidence of fiscal policies. Additionally, the link between public expenditures and the private production process was emphasized in Ratner (1983) and Aschauer (1989). Barro (1990), Futagami et al. (1993), Glomm and Ravikumar (1994, 1999), Turnovsky (1996, 2000), Aschauer (2000), Chen (2007), Marrero (2008), all discuss the optimality of productive public expenditures in growth models. Barro (1990) shows that the growth-maximizing public investment ratio coincides with the one maximizing welfare. In contrast, most other authors conclude that the welfare-maximizing ratio is strictly lower than the one maximizing growth. Futagami et al. (1993) point out that the reason for this discrepancy is the existence of transitional dynamics, which arises because public capital does not fully depreciate every period. Alternatively, in Glomm and Ravikumar (1994), public and private capital fully depreciate and thus the economy does not display transition. However, optimal predictions then differ from Barro (1990) because tax revenues are converted into infrastructure with some lag. Turnovsky (2000) extends the Barro (1990) model of productive government expenditure by introducing an elastic labor supply, and analyzes fiscal policy effects in the form of changes in public expenditures under diferent modes of tax financing. However,

[1]See Chen (2006) for a detailed analysis on this issue. This author stresses that certain economic factors (mostly *exogenous*) that alter the marginal utility of private consumption relative to the marginal utility of public consumption, would affect the size of public consumption.

[2]In fact, this is not an unrealistic assumption. A *current* government must often take as given some items included in public consumption, such as public wages, interest payments on public debt and bureaucratic or administrative disbursements, since they were approved before hand, possibly by previous governments. As a percentage of GDP, these public expenditure concepts are far from zero and have even increased over time in most developed countries. In addition, the political cost of cutting down these items could be high, even if their levels are above their *optimum* values.

the economy always lies on its balanced growth path, so there is no possibility of analyzing dynamic effects of such policies. Discrepancies from Barro then come from the presence of alternative tax systems and because leisure is a choice variable. Marrero (2008) reexamines the optimal choice of public investment in a more general framework, which allows for long-lasting capital stocks, a lower depreciation rate for public capital than for private capital, an elasticity of intertemporal substitution that differs from unity and the need to finance a non-trivial share of public, unproductive services in output. Under an income tax system, he showed that all this factors might be relevant in determining the optimal public investment policy.

In this paper we consider a framework with elastic labor supply which shows a non trivial transitional dynamics. We study, simultaneously, the sensitivity of the welfare-maximizing public investment/output ratio to alternative tax scenarios and the effect of public expenditure policies on welfare. In dynamic settings, an optimal policy can be described in terms of a trade-off between initial and future consumption. In general, the optimal policy induces a sacrifice in initial welfare (by reducing initial consumption and/or leisure), with a faster accumulation of capital along the former periods of the transition. This accumulation allows for an eventual faster growth in output, investment and consumption, compensating the initial loss of utility. Our contribution is twofold: *i*) to study in detail the relationship between this welfare trade-off and the tax system in an extended Barro-type framework with transitional dynamics, and *ii*) to characterize and interpret the magnitude of the welfare trade-off, which happens to be tax-dependent.

A complex framework like this easily becomes analytically intractable, so numerical techniques are needed to characterize the transitional dynamics. However, computing a numerical solution is not easy either, since the variables in the economy experience growth in steady-state. To avoid this problem, approximated equilibrium conditions obtained from the model written in stationary ratios are combined with the equilibrium equations for the level of the variables [similar to Novales et al. (1999)]. In computing welfare-maximizing policies, careful attention is given to the transitional dynamics associated with how the economy responds to such optimal policy. We compute the associated path of variables such as consumption, leisure, public and private investment and output and, finally, we characterize the time path for utility.

In addition to lump-sum taxation, four tax scenarios are alternatively considered: taxes on total income, gross capital income, labor income and private consumption. Under each tax system, a constant tax rate is chosen so that, in steady-state equilibrium, total public expenditures are entirely financed with the selected tax system, and lump-sum becomes zero. For simplicity, we assume that any possible deficit along the transition to steady-state will be financed with lump-sum transfers, while any surplus will be translated into positive transfers being assigned to the private sector.

We start by characterizing the welfare-maximizing public investment/output ratio, emphasizing the effect of such policy on the dynamic properties of welfare and the main macroeconomic variables. We relate the welfare-maximizing ratio to the initial level of 7% considered in the benchmark setting, and we discuss whether a downsizing in public investment is welfare improving, as well as the dependence of this analysis on the tax system considered. After that, we perform a *local* sensitivity analysis for the structural parameters in the economy to understand which are the main determinants of the welfare-maximizing policy. The robustness of conclusions obtained in the previous section to changes in parameter values is also discussed.

The rest of the paper is organized as follows: the model economy is presented in section 2; in section 3, the competitive equilibrium is described and a procedure to solve for the dynamics of the level of the variables is proposed; in section 4, we characterize the welfare-maximizing public investment/output ratio; in section 5, we carry out a local sensitivity analysis; finally, in section 6 the paper is closed by setting the main conclusions and possible extensions.

2. The environment

There are three different economic agents: a continuum of firms indexed by $i \in [0,1]$, households and a government, who cares only about fiscal policy.

2.1. Firms

A large number of *identical* firms, indexed by $i \in [0,1]$, produce the single consumption good in the economy. Each firm rents the same amount of private inputs from households (private capital, $\tilde{k}_t$, and labor, $\tilde{l}_t$) to produce $\tilde{y}_t$ units of output. The total amount of physical capital used by all the firms in the economy, $\tilde{K}_t$, is taken as a proxy for the index of knowledge available to each firm [as in Romer (1986)]. Additionally, public capital, $\tilde{K}_t^g$, affects the production process of all individual firms. Except for these externalities, the private production technology is a standard *Cobb-Douglas* function presenting constant returns to scale in the private inputs and increasing returns in the aggregate. For any firm,

$$\tilde{y}_t = f(\tilde{l}_t, \tilde{k}_t, \tilde{K}_t, \tilde{K}_t^g) = F\tilde{l}_t^{1-\alpha}\tilde{k}_t^{\alpha}\tilde{K}_t^{\theta_k}\left(\tilde{K}_t^g\right)^{\theta_g}, \ \theta_g, \ \alpha \in (0,1), \ \theta_k \geq 0, \tag{2.1}$$

where α is the economic share of capital, θ_g and θ_k are, respectively, the elasticities of output with respect to public capital and the knowledge index, and F is a technological scale factor, common to each firm.

Since firms are identical, from (2.1), aggregate output, $\tilde{Y}_t$, is produced according to,

$$\tilde{Y}_t = F\tilde{L}_t^{1-\alpha}\tilde{K}_t^{\alpha+\theta_k}\left(\tilde{K}_t^g\right)^{\theta_g}, \tag{2.2}$$

where $\tilde{L}_t$ is aggregate labor.

During period t, each firm pays the competitive-determined wage $\tilde{w}_t$ on the labor it hires and the rate r_t on the capital it rents. The dynamic problem faced by firms turns out to be static at each point in time,

$$\underset{\{l_t,k_t\}}{Max}\ f(\tilde{l}_t, \tilde{k}_t, \tilde{K}_t, \tilde{K}_t^g) - \tilde{w}_t\tilde{l}_t - r_t\tilde{k}_t,$$

and optimality leads to the usual marginal productivity conditions:

$$r_t = \alpha F \tilde{l}_t^{1-\alpha} \tilde{k}_t^{\alpha-1} \tilde{K}_t^{\theta_k} \left(\tilde{K}_t^g\right)^{\theta_g} = \alpha \frac{\tilde{y}_t}{\tilde{k}_t} = \alpha \frac{\tilde{Y}_t}{\tilde{K}_t}, \tag{2.3}$$

$$\tilde{W}_t = (1-\alpha) F \tilde{l}_t^{-\alpha} \tilde{k}_t^{\alpha} \tilde{K}_t^{\theta_k} \left(\tilde{K}_t^g\right)^{\theta_g} = (1-\alpha) \frac{\tilde{y}_t}{\tilde{l}_t} = (1-\alpha) \frac{\tilde{Y}_t}{\tilde{L}_t}, \tag{2.4}$$

where we have used the fact that each firm treats its own contribution to the aggregate capital stock as given, rents the same amounts of the private inputs and produces the same amount of output.

Aggregate private capital stock evolves according to,

$$\tilde{K}_{t+1} = (1-\delta^k)\tilde{K}_t + \tilde{I}_t^k, \tag{2.5}$$

where $\tilde{I}_t^k$ is gross private investment.

2.2. Households

The representative consumer chooses the fraction of time to spend as leisure, h_t. She is the owner of physical capital, and allocates her resources between consumption, $\tilde{C}_t$, and investment in physical capital, $\tilde{I}_t^k$. The price of the single consumption commodity and the time endowment of households are both normalized to one, and zero population growth is assumed every period.

Decisions are made each period to maximize the discounted aggregate value of the time separable utility function,

$$\max_{\left\{\tilde{C}_t, h_t, \tilde{K}_{t+1}\right\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t U(\tilde{C}_t, 1-h_t) \tag{2.6}$$

subject to her resource constraint in every period

$$(1+\tau^c)\tilde{C}_t + \tilde{K}_{t+1} + \tilde{T}_t \leq \tilde{W}_t h_t (1-\tau^w) + \tilde{K}_t \left[1 - \delta^k + r_t\left(1-\tau^k\right)\right],\ \tilde{C}_t \geq 0 \tilde{K}_{t+1} \geq 0 \text{ and } h_t \in [0,1] \tag{2.7}$$

and to the transversality condition, that places a limit on the accumulation of capital,

$$\lim_{t\to\infty} \beta^t \tilde{K}_{t+1} \frac{\partial u(\tilde{C}_t, h_t)}{\partial \tilde{C}_t} = 0. \tag{2.8}$$

where $\tilde{K}_{t+1}$ denotes the stock of physical capital at the end of time t, with $\tilde{K}_0 > 0$, τ^k, τ^w and τ^c are the flat tax rates on capital income, labor income and private consumption, respectively, and $\tilde{T}_t$ is a net transfer made by households to the public sector, which may be either positive or negative; $\beta \in (0,1)$ is the discount factor and $U(\tilde{C}_t, h_t)$ is a C^2 mapping, strictly concave, increasing in $\tilde{C}_t$ and $(1-h_t)$ and satisfying Inada conditions.[3] $U(\tilde{C}_t, h_t)$ is particularized to be

[3]Corner solutions in (2.6)-(2.8) are avoided, and restrictions (2.7) and (2.8) must hold with equality for utility to be maximized.

of the constant elasticity of substitution family [King et al. (1988)],

$$U(\tilde{C}_t, h_t) = \frac{\left[\tilde{C}_t^{\rho}(1-h_t)^{1-\rho}\right]^{1-\theta} - 1}{1-\theta}, \ \rho \in [0,1], \text{ if } \theta > 0 \text{ and } \theta \neq 1, \tag{2.9}$$

$$U(\tilde{C}_t, h_t) = \rho \ln \tilde{C}_t + (1-\rho)\ln(1-h_t), \ \rho \in [0,1], \ \theta = 1,$$

where $1/\theta$ is the constant elasticity of intertemporal substitution of private consumption and ρ denotes the importance of consumption relative to leisure in utility.

Optimality conditions are standard: the consumption-saving decision, (2.10), the consumption-leisure choice, (2.11),

$$\frac{\tilde{C}_{t+1}}{\tilde{C}_t} = \left\{ \beta \left(\frac{1-h_{t+1}}{1-h_t} \right)^{(1-\rho)(1-\theta)} \left[1 - \delta^k + r_{t+1}\left(1 - \tau_{t+1}^k\right)\right] \right\}^{\frac{1}{1-\rho(1-\theta)}}, \tag{2.10}$$

$$\frac{\rho}{1-\rho} = \frac{\tilde{C}_t}{(1-h_t)} \frac{(1+\tau_t^c)}{\tilde{W}_t}, \tag{2.11}$$

the budget constraint (2.7), $\tilde{C}_t > 0$, $\tilde{K}_{t+1} > 0$, $h_t \in (0,1)$ and the transversality condition (2.8).

2.3. The public sector

The public sector collects distorting and non-distorting taxes to finance its total current expenditures, divided into *productive* public expenditures, $\tilde{I}_t^g$, and *unproductive* public expenditures, $\tilde{C}_t^g$. For any tax system considered,

$$\tilde{I}_t^g = \varkappa_i \tilde{Y}_t, \tag{2.12}$$

$$\tilde{C}_t^g = \varkappa_c \tilde{Y}_t, \tag{2.13}$$

where $\varkappa_i$ is the policy instrument and $\varkappa_c$ is assumed to be exogenous and fixed. Once $\varkappa_i$ is chosen, we assume it remains constant forever. Public capital accumulates according to

$$\tilde{K}_{t+1}^g = \tilde{I}_t^g + (1-\delta^g)\tilde{K}_t^g, \tag{2.14}$$

where $\delta^g \in (0,1)$ is the public capital depreciation factor, which we consider to be different from that of private capital.

The government is not allowed to issue any debt, and only flat taxes are considered.[4] The budget constraint must balance every period,

$$\tilde{T}_t = \tilde{C}_t^g + \tilde{I}_t^g - \tau^c \tilde{C}_t - \tau^w h_t \tilde{W}_t - \tau^k \tilde{A}_t r_t. \tag{2.15}$$

In the simulation exercise, four tax scenarios are alternatively considered: taxing only total gross income, gross capital income, labor income or private consumption.

[4] Focusing on taxes as the public finance instrument has empirical justification: Taxation represents more than 80% of total public revenue in industrialized economies. Additionally, a state-owned industry could be treated in the similar way as taxes [see section 4 of Burgess and Stern (1993)] and printing money to finance public deficit is not permited in developed countries. Allowing for public debt would extend the model beyond the scope of the paper.

3. The equilibrium, the calibration, the simulation and the government problem

In this section, we define the competitive equilibrium and the balanced growth path and characterize the transitional dynamics of variables in levels.

3.1. The competitive equilibrium and the balanced growth path

Starting from an initial state, $\tilde{K}_0, \tilde{K}_0^g > 0$, the *competitive equilibrium* is a set of allocations $\left\{\tilde{C}_t, h_t, \tilde{L}_t, \tilde{K}_{t+1}, \tilde{I}_t^k, \tilde{Y}_t, \tilde{C}_t^g, \tilde{K}_{t+1}^g, \tilde{I}_t^g\right\}_{t=0}^{\infty}$, a set of prices $\tilde{p} = \{r_t, \tilde{w}_t\}_{t=0}^{\infty}$ and a fiscal policy $\tilde{\pi} = \left(\bar{\varkappa}_i, \eta, \tau^k, \tau^c, \left\{\tilde{T}_t\right\}_{t=0}^{\infty}\right)$, such that, given $\tilde{p}$ and $\tilde{\pi}$: (i) $\{\tilde{C}_t, h_t, \tilde{K}_{t+1}\}_{t=0}^{\infty}$ maximize households' welfare [satisfying (2.7), (2.8), (2.10) and (2.11)]; (ii) $\{\tilde{K}_{t+1}, \tilde{L}_t\}_{t=0}^{\infty}$ satisfy the profit-maximizing conditions [(2.2)-(2.3)], and $\tilde{K}_t$ accumulates according to (2.5); (iii) $\{\tilde{C}_t^g, \tilde{K}_{t+1}^g, \tilde{I}_t^g\}_{t=0}^{\infty}$ evolve according to (2.12)-(2.14); (iv) the budget constraint of the public sector, (2.15), and the technology constraint (2.2) to produce $\tilde{Y}_t$ holds; finally, (iv) markets clear every period,

$$\tilde{L}_t = h_t, \tag{3.1}$$

$$\tilde{Y}_t = \tilde{C}_t + \tilde{C}_t^g + \tilde{I}_t^k + \tilde{I}_t^g. \tag{3.2}$$

A *balanced growth path (bgp)* is defined as an equilibrium path along which aggregate variables either stay constant or grow at a constant rate. Hereinafter, variables with bar "–" denote values along the *bgp*. Jones and Manuelli (1997), among others, have shown that cumulative inputs must present constant returns to scale in the private production process for the existence of a steady-growth equilibrium (i.e., $\alpha + \theta_g + \theta_k = 1$). Additionally, r_t must be constant and high enough for the equilibrium to display positive steady-growth. From now on, we will focus on the special case: $\alpha + \theta_g + \theta_k = 1$. Under these conditions, it is easy to show from the equilibrium conditions, that $\tilde{Y}_t$, $\tilde{C}_t$, $\tilde{K}_t$, $\tilde{K}_t^g$, $\tilde{C}_t^g$ and $\tilde{T}_t$ must all grow at the same constant rate along the *bgp*, denoted by $\bar{\gamma}$ hereinafter, while bounded variables, such as tax rates, r_t and h_t, must be constant.

Particularizing condition (2.10) to a *bgp* equilibrium, a positive long-term growth rate is achieved whenever

$$\bar{\gamma} = \left\{\beta\left[1 - \delta^k + (1-\tau^k)\bar{r}\right]\right\}^{\frac{1}{1-\rho(1-\theta)}} - 1 > 0 \Leftrightarrow \bar{r} > \frac{1-\beta(1-\delta^k)}{(1-\tau^k)\beta}. \tag{3.3}$$

However, even though $\bar{\gamma}$ will then be positive, it cannot get so high that it allows households to follow a chain-letter action [(2.8) must hold on the *bgp*],

$$\lim_{t\to\infty} \frac{\rho(1-\bar{h})^{(1-\rho)(1-\theta)}\tilde{K}_0\,(1+\bar{\gamma})\,\beta^t\,(1+\bar{\gamma})^t}{\left[\tilde{C}_0(1+\bar{\gamma})^t\right]^{1-\rho(1-\theta)}} = 0 \Leftrightarrow \beta(1+\bar{\gamma})^{\rho(1-\theta)} < 1, \tag{3.4}$$

which ensures that time-aggregate utility (2.6) remains finite.

3.2. Alternative tax scenarios

Four alternative single-tax scenarios are considered, with taxes on either total gross income, gross capital income, labor income or private consumption. In each case, the tax rate is chosen so that total public expenditures, $\tilde{C}_t^g + \tilde{I}_t^g$, can be entirely financed along the *bgp* with the single tax being considered. Thus, $\tilde{T}_t = 0$ along this equilibrium path:

$$\begin{aligned}
\bar{I}_t^g + \bar{C}_t^g &= \tau\left(\bar{h}\bar{W}_t + \bar{r}\bar{K}_t\right) \Leftrightarrow \tau = \varkappa_i + \varkappa_c, && (3.5)\\
\bar{I}_t^g + \bar{C}_t^g &= \tau^k \bar{r}\bar{K}_t \Leftrightarrow \tau^k = \left(\varkappa_i + \varkappa_c\right)/\alpha, && (3.6)\\
\bar{I}_t^g + \bar{C}_t^g &= \tau^w \bar{h}\bar{W}_t \Leftrightarrow \tau^w = \left(\varkappa_i + \varkappa_c\right)/\left(1-\alpha\right), && (3.7)\\
\bar{I}_t^g + \bar{C}_t^g &= \tau^c \bar{C}_t \Leftrightarrow \tau^c = \left(\varkappa_i + \varkappa_c\right)\overline{\left(Y_t/C_t\right)}, && (3.8)
\end{aligned}$$

where we have imposed $\tau^k = \tau^w = \tau$ in (3.5), the tax rate applied to total income. By combining (2.15), (2.3) and (2.4), we obtain $\tilde{T}_t = 0$ every period under either output, capital or labor tax systems. Hence, it is only under consumption taxes that $\tilde{T}_t$ may be different from zero along the transition.

Regarding the alternative tax systems, we point out that (i) given $\varkappa_i$ and $\varkappa_c$, tax rates are time-invariant; (ii) any change in $\varkappa_i$ affects contemporaneously the tax rate being used as fiscal instrument; (iii) that change is more than proportional under either capital, labor or consumption taxes, while being proportional under income taxes; (iii) τ^k, τ^w and τ must be between zero and one, while τ^c cannot be so large that $\tilde{C}_t$ might ever become negative.

3.3. Benchmark calibration

The economy is calibrated following the standards in the literature. Wherever needed, we have chosen ratios of variables or parameter values so as to approximate industrialized economies during the eighties, considering the time unit to be one quarter. An annual after-tax net capital rate of return of 6%, an average labor share, $\bar{h}$, of 1/3, and an annual growth rate of 2.5%, are all replicated in steady-state under the benchmark calibration.

Private capital depreciates at a 10% annual rate, hence $\delta^k = .025$. We assume public capital depreciates at a lower rate of 5% , so that $\delta^g = .0125$.[5] For the instantaneous utility function, Mehra and Prescott (1985) suggest a relative risk aversion parameter between 1.0 and 2.0, and we choose $\theta = 1.20$. The elasticity of private capital, α, takes a standard value of .36, hence $\theta_g + \theta_k = .64$. The empirical literature discussing the productive nature of public capital shows controversial conclusions, different data sources and econometric techniques leading to rather different estimations of θ_g in a Cobb-Douglas production function.[6] In the benchmark

[5] Auerbach and Hines (1987) estimated a depreciation rate in the U.S. of 0.137 for equipment and 0.033 for structures. Since private capital includes a larger share of equipment than public capital, the estimated depreciation rate for private capital is expected to be larger. Ai and Cassou (1995) found support for this in the form of an estimated δ_g of just over half that of δ.

[6] Aschauer (1989) and Munnell (1990) estimate high values of θ_g, equal to 0.39 and 0.34, respectively. Accounting for non-stationarity in the data, Lynde and Richmond (1992) and Ai and Cassou (1995) obtain lower

calibration, we set $\theta_g = .20$, but we will perform a sensitivity analysis in section (5), allowing for θ_g to vary between .06 and .30.

Two different public expenditure concepts are considered: Schuknecht and Tanzi (1997) is used to set the public consumption-to-output ratio, $\varkappa_c$, equal to .17. Calibration of the initial level of the public investment ratio, $\varkappa_i$, is less evident. In international accounts of fiscal policy variables, public investment generally includes just central government activities, leaving aside local expenditures and public enterprises. Easterly and Rebelo (1993) try to correct for this deficiency. They estimate levels of $\varkappa_i$ in the eighties of .07 for New Zealand, .11 for Portugal, .07 for Australia, .08 in Japan, .02 in US, among others. Barro and Sala-i-Martin (1995) set total consolidated public investment to be about 30% of total investment. For industrialized economies, this leads to an average of $\varkappa_i$ about 7% in the eighties. We use .07 as benchmark value for $\varkappa_i$. In section 5, we will show that the welfare-maximizing public investment/output varies only slightly with the level of $\varkappa_i$, which will have important qualitative implications.

Finally, using steady-state equilibrium conditions for the model in ratios, β, ρ and F are chosen accordingly. In general, these values vary with θ_g and with the tax system considered. Table 1 shows calibrated values for all parameters, conditional on $\varkappa_i = .07$ and $\theta_g = .20$. Among the alternative tax systems, the main difference refers to the scale factor F. As expected, F must take a higher level to replicate the 2.5% annual growth rate under capital income taxes than under less distorting tax scenarios.

[INSERT TABLE 1 ABOUT HERE]

3.4. Simulating the competitive equilibrium

The competitive equilibrium cannot be solved analytically, so a numerical solution is required. Unfortunately, computing a numerical solution is far from trivial, since the variables in the economy experience growth in steady-state. In that setting, it is fairly simple to solve for stationary ratios, but level variables are needed to analyze welfare issues. The normalized level of a variable $\tilde{Z}_t$, $Z_t = \tilde{Z}_t/(1+\bar{\gamma})^t$, will grow at a zero rate along the *bgp*, but the steady-state value of Z_t is not well defined, so standard numerical methods applied directly to normalized variables cannot be used either. In the Appendix we describe a procedure that uses the dynamics of stationary ratios to recover the equilibrium path for normalized level variables, starting from a given initial state of the economy.[7]

but still significant estimates: the former get $\theta_g = 0.2$ using time series techniques, while the latter estimate φ between 0.15 and 0.2, using a GMM method. In a more recent paper, Shioji (2001) uses dynamic panel techniques to estimate the elasticity of output with respect to infrastructure to be somewhere around 0.1 and 0.15. On the other hand, papers by Holtz-Eaking (1994), Hulten and Schwab (1991) and Tatom (1991), among others, put that estimate very close to zero. Sturm *et al.* (1997) offer a selective review of these empirical studies.

[7]Novales et al. (1999) describes an alternative method to solve for the dynamics of level variables in endogenous growth models.

3.5. Solving the government's problem

The public sector maximizes welfare of the representative household along the competitive equilibrium. The policy instruments are $\varkappa_i$ and the associated tax rate, and the government commits itself to the announced policy. The economy is assumed to start on the *bgp* associated to the benchmark calibration, with an initial public investment/output ratio of 7% and a public capital stock, K_0^g, of 100.[8] A standard search method is used to numerically handle this control problem.

Given the tax system, the initial state (K_0, K_0^g) and a level of $\varkappa_i$: (*a*) (7.1)-(7.6) is solved for the *bgp* and the level of $\bar{\gamma}$ is obtained; (*b*) the process described in the previous subsection allows us to recover time series for C_t and h_t; (*c*) the utility of the representative consumer is evaluated[9]

$$\sum_{t=0}^{\infty}\left\{\frac{\left[\beta(1+\bar{\gamma})^{\rho(1-\theta)}\right]^t\left[C_t^{\rho}(1-h_t)^{1-\rho}\right]^{1-\theta}}{1-\theta}-\frac{\beta^t}{1-\theta}\right\}; \tag{3.9}$$

(*d*) the process is repeated for any feasible level of $\varkappa_i$, and the one maximizing (3.9) will be the welfare-maximizing choice.

To evaluate the infinite sum in (3.9), a truncated version with t^* periods is used, where t^* is chosen so that equilibrium time series are close enough to the *bgp*.[10] For each policy, time series $\{C_t, h_t\}_{t=0}^{t^*}$ are used to estimate welfare up to period t^*:

$$\sum_{t=0}^{t^*}\left\{\frac{\left[\beta(1+\bar{\gamma})^{\rho(1-\theta)}\right]^t\left[C_t^{\rho}(1-h_t)^{1-\rho}\right]^{1-\theta}}{1-\theta}\right\}-\frac{1}{(1-\beta)(1-\theta)}. \tag{3.10}$$

After period t^*, the economy is considered to be *close enough* to the *bgp* associated to the implemented policy. Therefore, according to (3.9), since $\beta(1+\bar{\gamma})^{\rho(1-\theta)} < 1$ [by (3.4)], the term

$$\begin{aligned} &\sum_{t=t^*+1}^{\infty}\frac{\left[\beta(1+\bar{\gamma})^{\rho(1-\theta)}\right]^t\left[C_{t^*}^{\rho}(1-\bar{h})^{1-\rho}\right]^{1-\theta}}{1-\theta} \\ &= \left[C_{t^*}^{\rho}(1-\bar{h})^{1-\rho}\right]^{1-\theta}\frac{\left[\beta(1+\bar{\gamma})^{\rho(1-\theta)}\right]^{t^*+1}}{(1-\theta)\left[1-\beta(1+\bar{\gamma})^{\rho(1-\theta)}\right]} \end{aligned} \tag{3.11}$$

approximates aggregate utility after period t^*, which is added-up to the numerical value obtained from (3.10).[11]

[8] The initial state is $K_0^g = 100$ and $K_0 = 100/\bar{k}^g$. The welfare-maximizing policy is shown to be invariant to this choice.

[9] The range of feasible lifetime utility values for (3.9) is bounded because: (i) the paths of C_t and h_t converge to bounded limits, since the numerical procedure imposes the competitive equilibrium to be on the stable manifold; (ii) $\bar{\gamma}$ is bounded from above by (**??**). In addition, the single period utility function is continuous and strictly concave, and the choice set is convex, conditions that ensure the existence of at most one interior solution to the problem of maximizing (3.9).

[10] T^* is chosen so that $\left|X_{T^*}-\bar{X}\right| < 10^{-3}$, with $T^* < 1500$, due to computational restrictions.

[11] Notice that (3.10) and (3.11) must be computed simultaneously, because t^* and C_{t^*} depend on the whole transitional dynamics up to period t^*.

4. The public investment policy under alternative tax scenarios

In this section we discuss the transitional dynamics and its relevance for correctly characterizing the welfare-maximizing public investment/output ratio. An optimal policy will generally imply a sacrifice in initial welfare (by reducing initial consumption and/or leisure), with a faster accumulation of capital along the former periods of the transition. This accumulation allows for an eventual faster growth in output, investment and consumption, compensating the initial loss of utility. We examine in detail the relationship between this welfare trade-off and the tax system characterizing and interpreting the magnitude of the welfare trade-off, which happens to be tax-dependent.

The theoretical analysis is supplemented by numerical results, focusing on the dependence of the welfare-maximizing policy with respect to the tax system and the main structural parameters. Moreover, to make the welfare analysis clear, we compare the welfare dynamics in the following situations: (i) when the $\varkappa_i$-ratio is chosen to maximize long-run growth, $\varkappa_i^*$, (ii) when the $\varkappa_i$-ratio is chosen to maximize welfare over 4 periods, $\varkappa_i^{sr}$, and finally (iii) when $\varkappa_i$ takes the value that maximizes welfare along the entire transition, $\varkappa_i^+$.

By comparing the obtained optimal ratio with the initial benchmark of 7%, we can discuss whether downsizing improves welfare, and its dependence on the tax system can be evaluated. Notice that the utility paths are not comparable among the alternative tax scenarios considered, since their calibrated scale factors, F, are different. However, ratios of variables are unaffected by scale factors and hence, the resulting welfare-maximizing public investment ratios under alternative tax scenarios can be compared.

4.1. Maximizing steady-state growth

As in a Barro-type setting, changes in $\varkappa_i$ may have two different effects on $\bar{\gamma}$: (i) a higher $\varkappa_i$ has a *positive* effect on output, since public capital is productive, and (ii) a *negative* effect, because an increment in $\varkappa_i$ leads to a parallel rise in the corresponding distorting tax rate, accordingly with (3.5)-(3.8). Hence, an inverted U-shaped relationship between $\bar{\gamma}$ and $\varkappa_i$ should be expected. The growth-maximizing public investment ratio, $\varkappa_i^*$, must equalize, in the margin, these two opposite effects. First, notice that the government also uses distorting taxes to finance unproductive expenses, which has an additional negative effect on growth. Secondly, the endogeneity of labor supply makes consumption and labor income taxes to also have adverse effects on growth, although they should be expected to be small.

Figure 4.1 shows the long-run savings rate,[12] $\bar{\gamma}$, $\overline{C/Y}$ and $\bar{h}$ as a function of $\varkappa_i$, for the benchmark economy with $\theta_g = .20$ and under the four alternative tax systems considered. The domain of $\varkappa_i$ is restricted to satisfy conditions (3.3) and (3.4) on growth, in addition to $\bar{C} > 0$ and $0 < \bar{h} < 1$. On the other hand, figure 4.2 shows the relationship between $\varkappa_i^*$ and θ_g, for θ_g between .06 and .30 under the alternative tax scenarios.

[12] The savings rate, s_t, is equal to $\frac{\tilde{K}_{t+1}-\tilde{K}_t}{\tilde{Y}_t}$, as in Barro (1990). Dividing this expression by $\tilde{K}_t$, $\bar{s} = \bar{\gamma}/\bar{M}$ in the *bgp*.

[INSERT FIGURE 4.1 AND 4.2 ABOUT HERE]

For the benchmark calibration, Figure 4.1 shows the inverted U-shaped relationship between $\bar{\gamma}$ and $\varkappa_i$ only under capital, labor and income taxes. Precisely, under these tax scenarios, the positive influence of public capital dominates when $\varkappa_i$ is low enough (the upward part of the curve). Thus, an increase in $\varkappa_i$, when $\varkappa_i$ is initially below $\varkappa_i^*$ (3.8%, 31.8% and 16.5%, under taxes on total income, labour or capital income, respectively) would produce faster growth. On the other hand, levels of $\varkappa_i$ above $\varkappa_i^*$ would lower the long-run saving rate, inducing a negative relationship between $\bar{\gamma}$ and $\varkappa_i$. Under consumption taxes, the disincentive effect on savings of an increase in the tax rate is not large enough to compensate the positive impact of $\varkappa_i$, and the relationship between $\bar{\gamma}$ and $\varkappa_i$ is positive and monotone, although strictly concave.

Several additional comments regarding the growth-maximizing public investment ratio are of interest.

First, under income taxes, the negative impact on $\bar{h}$ due to a tax raise is very small, since the tax burden is shared between labor and capital income. Hence, $\varkappa_i^*$ is in that case only slightly lower than $\theta_g(1-\varkappa_c)$, as obtained in section V in Barro (1990) and Marrero and Novales (2005, 2007) under a perfectly inelastic labour supply. Higher levels of θ_g and lower values of ρ in the calibration would surely magnify the impact on labor supply of an increase in $\varkappa_i$, making $\varkappa_i^*$ to depart from $\theta_g(1-\varkappa_c)$. For the benchmark calibration, as θ_g moves between .06 and .30, $\varkappa_i^*$ goes from 4.97% to 24.8% (see figure 4.2) while $\theta_g(1-\varkappa_c)$ is between 4.98% and 24.9%.

Second, as expected, the tax system on capital income is the one that better neutralizes the positive effect of public capital on growth. An increment in $\varkappa_i$ raises the associated tax rate more than proportionally [see (3.6)]. Hence, savings and growth are strongly discouraged above a level of $\varkappa_i$ well below $\theta_g(1-\varkappa_c)$. Consequently, $\varkappa_i^*$ goes from 1.2% to just 5.8% as θ_g changes between .06 and .30 (figure 4.2).

Third, an increment in $\varkappa_i$ has a significant (and negative) impact on $\bar{h}$ only under labor income taxes, and just for high levels of $\varkappa_i$[figure 4.1]. Consequently, for the mentioned range of θ_g, $\varkappa_i^*$ falls between 22.3% and 34.5%, far above the levels obtained under the previous tax scenarios (figure 4.2).

Finally, even though the effect on labor along the transition is significant under consumption taxes (as it will be seen latter), the steady-state effect is less relevant. Hence, $\varkappa_i^*$ is around 50.0%, the maximum feasible value, which drives $\overline{C/Y}$ down to a value close to zero. Clearly, this should be expected to be very different from the welfare-maximizing policy.

4.2. Short-run welfare effects

Downsizing public investment would reduce the ratio $\varkappa_i$ below its initial level of 7%, leaving a higher fraction of additional resources available to the private sector, which could be reassigned as additional consumption and savings.[13] Additionally, the single tax rate being used

[13] For illustrative purposes, only the short-run welfare impact of downsizing policies is discussed. A symmetric reasoning could be made for upsizing policies.

to generate revenues will be permanently reduced. Figure 4.3 shows the percent initial impact on working hours and public investment, for the benchmark economy with $\theta_g = .20$ and for alternative levels of $\varkappa_i$. The domain of $\varkappa_i$ is restricted so that conditions (3.3) and (3.4) hold, together with $C_t > 0$ and $0 < h_t < 1$ for all period t.

Figure 4.3 shows that, with independence of the tax system considered, the crowding-out of private consumption by public investment is shown to be less than perfect, since the initial impact on private investment is different from zero for changes in $\varkappa_i$. A cut in $\varkappa_i$ accompanied of a reduction in the tax rate on either labor income or capital income will initially produce an incentive to work or save, respectively, affecting consumption and saving decisions. Under consumption taxes, the cut in the associated tax rate is immediately transmitted as an incentive to work [by (2.11)],[14] which has a positive effect on the net return to capital and hence on short-term savings and private investment.

Under capital income taxes, saving becomes increasingly more attractive than consuming as the tax rate declines. Consequently, a *strong enough* public investment downsizing policy might take the substitution effect to the point of being more important than the direct income effect on private consumption, becoming optimal for households to initially reduce consumption and accumulate more assets (as is shown in figure 4.3 (a) under capital income taxes for $\varkappa_i < .04$). This will be more evident for low levels of the public capital elasticity, θ_g, for which the income effect of downsizing on consumption is lower. Under all other tax systems considered, private consumption initially increases with reductions in $\varkappa_i$, the higher impact being obtained under consumption and labor income taxes.

[INSERT FIGURE 4.3]

Figure 4.4 shows the public investment ratio achieving the maximum discounted utility over 4 periods following the policy intervention. This ratio is denoted by $\varkappa_i^{sr}$ hereinafter, and it is shown in the figure as a function of θ_g, $\theta_g \in [.06, .30]$, under the four alternative tax scenarios. Under capital income taxes, a strong reduction in $\varkappa_i$ has a negative short-run welfare effect, since private consumption, as well as leisure, initially declines (see figure 4.3). As a consequence, under capital income taxes, if the government only cares about the very short-run (i.e., 4 periods), the best strategy is to rise $\varkappa_i$ above the benchmark 7% when θ_g is higher than .16, the level of $\varkappa_i^{sr}$ being between 2.9% and 10.0% for the range of θ_g considered.

[INSERT FIGURE 4.4]

Under the remaining tax scenarios, downsizing initially increases private consumption but it also produces a decline in leisure, and the global impact on short-run welfare is unclear.

[14] Even though this incentive is lower than under labor income taxes.

Initial effects on main macroeconomic variables are very similar under labor income and private consumption taxes, and in figure 4.4, the two lines coincide. For the benchmark economy, $\varkappa_i^{sr}$ falls between .93% and 5.5% under these two tax systems, as θ_g takes values between .06 and .30. Hence, under these two tax systems, a certain amount of downsizing is always preferable, since it induces higher initial levels of private consumption without encouraging work excessively, which improves short-run welfare. Finally, under income taxes, $\varkappa_i^{sr}$ falls between 1.3% and 8.3% as θ_g changes in the range considered. Downsizing enhances short-run welfare in this case only for θ_g below .26.

4.3. The welfare-maximizing policy

In Barro (1990), the public investment ratio that maximizes welfare, denoted by $\varkappa_i^{+}$ hereinafter, is equal to $\varkappa_i^{*}$, the public investment ratio maximizing steady-state growth. Futagami et al. (1993) point out to the lack of transition among steady-states being a crucial feature behind this result, while Glomm and Ravikumar (1994) point out that if current public capital is considered as the productive factor instead of current public investment, there exists a one-period gap between tax revenues and its positive effect on output, inducing $\varkappa_i^{+}$ to be strictly lower than $\varkappa_i^{*}$. Precisely, these two forces contrary to the result of Barro (1990) are present in this model, implying $\varkappa_i^{+}$ to be strictly lower than $\varkappa_i^{*}$, with independence of the tax system considered [for this result, see also Marrero and Novales (2005, 2007) and Marrero (2008)].

Under each of the four tax scenarios considered, we have obtained contrary results when characterizing short-run effect on welfare and the long-run effect on growth of a public investment ratio. In general, downsizing public investment below the 7% benchmark improves steady-state growth under capital income taxes, but not under the alternative tax systems. However, the opposite happens when trying to maximize short-run welfare. Therefore, it is essential to analyze the entire transition to find out the time-invariant public investment ratio that equilibrates welfare loses and gains in the short- and the long-run.

Figure 4.5 shows $\varkappa_i^{+}$ as a function of θ_g under the four tax systems. From these figures and the results previously discussed in section 4.2, several characteristics of the welfare-maximizing public investment policy can be pointed out:

[INSERT FIGURE 4.5]

1. The direct relationship between $\varkappa_i^{+}$ and θ_g is independent of the tax system considered, since the positive effect of public capital on growth and welfare increases with θ_g. This relationship is linear under total income [as in Barro (1990)] and capital income taxes, while being concave under labor income and private consumption taxes (as it is also the case for the growth-maximizing ratio $\varkappa_i^{*}$).

2. Figure 4.5 shows that downsizing improves welfare under *capital income taxes* for any θ_g, with $\varkappa_i^{+}$ falling between 1.0% and 5.0%. The associated lower tax creates an important

incentive for *private capital* accumulation, with a negative initial impact on private consumption and leisure (figure 4.3 shows these initial impacts under capital income taxes and low enough levels of $\varkappa_i$). After a number of periods, the higher accumulation of *private capital* along the transition allows for private consumption and leisure to increase after a short number of periods. Besides, convergence to steady-state is now faster, and the steady-state rate of growth is higher. Hence, the welfare-maximizing policy favors higher future increases on private consumption and leisure in detriment of lower levels in the first periods after downsizing, and our result shows that the positive medium and long-term effects are more important for welfare than the short-run effects.

3. On the contrary, under *labor income taxes*, $\varkappa_i^+$ is always above 7% (between 10.8% and 28.2% for θ_g in $[.06, .30]$. Hence, it is now optimum to initially sacrifice private consumption and private investment in favor of leisure and public investment (see figure 4.3 for $\varkappa_i > .07$ under labor income taxes). Production and private consumption start growing faster after a short number of periods. Contrary to the case under capital income taxes, now the *engine* for the recovery of these variables is the accumulation of *public capital* in the short-run and along the whole transition, together with a small disincentive on the accumulation of private capital.

4. Under *total income taxes*, $\varkappa_i^+$ falls between those obtained under capital and labor income taxes. Consequently, the welfare effect of downsizing on public investment depends on θ_g. As it is shown in figure 4.5, downsizing raises welfare for $\theta_g \in [.06, .12]$, $\varkappa_i^+$ falling between 3.6% and the benchmark 7.0%, while $\varkappa_i^+$ goes from 7.0% to 19.4% for $\theta_g \in [.12, .36]$. When θ_g is below .12, public capital is not productive enough to compensate the negative impact on welfare of an increase in the tax rate, and the welfare gain due to a reduction in the tax rate on capital income prevails (recall that downsizing was optimal under capital income taxes).

5. Even though the short-run behavior of the main macroeconomic variables under consumption and labor income taxes is very similar when $\varkappa_i$ rises above the benchmark 7% , private consumption is greatly discouraged under consumption taxes as the economy approaches its new steady-state, while the opposite happens regarding savings and growth (see steady-state analysis from figure 4.1). Consequently, the difference between short- and long-run welfare effects of a particular public investment policy is more pronounced under consumption taxes. However, for the benchmark calibration and for the range of θ_g considered, $\varkappa_i^+$ is higher under consumption than under labor income taxes, falling between 13.9% and 34.9%, although this difference is lower than when comparing the growth-maximizing ratios. As it was the case under labor income taxes, the engine of the optimal behavior under consumption taxes is the strong *accumulation of public capital* along the transition.

6. The welfare-maximizing ratio is always lower than the one maximizing growth. For $\theta_g = .20$, the difference is 0.6, 3.9, 7.3 and 21.2 percentage points under taxes on capital

income, total income, labor income or consumption, respectively. As already mentioned, this difference is more pronounced under less distorting tax systems specially under consumption taxes, since, under that tax system, the growth-maximizing strategy initially reduces private consumption and leisure to a level close to zero.

This section has analyzed the welfare effects of a time-invariant public investment ratio policy, emphasizing the need to account for welfare along the transition as well as the importance of the tax system being implemented. Starting from a 7% public investment/output ratio, certain level of downsizing is welfare-improving under capital income taxes or under total income taxes, provided public capital is not very productive. On the other hand, welfare is lower when downsizing from the initial 7% ratio under labor income or consumption taxes (the less distorting tax systems). As we are about to see in the next section, this relationship between the welfare effect of downsizing and taxes reverses for some parameter values.

5. Sensitivity analysis

The main aim of this section is to clarify the determinants of the welfare-maximizing, time-invariant public investment ratio, $\varkappa_i^+$, and to discuss the robustness of the main findings in the previous section to changes in parameter values.[15]

In our setting, the transitional dynamics is affected by all economic fundamentals and, consequently, so is $\varkappa_i^+$. Even tough the competitive equilibrium cannot be analytically characterized, we use the numerical method described above to characterize a one-to-one mapping between $\varkappa_i^+$ and any structural parameter. We show that parameters of special importance in determining $\varkappa_i^+$ are: (i) the public capital share, θ_g, and the discount factor, β, with whom $\varkappa_i^+$ is positively related, a standard result in this literature; (ii) the unproductive public expenditure ratio, $\varkappa_c$, with whom $\varkappa_i^+$ is negatively related [as in Marrero (2008) and Marrero and Novales (2005, 2007)]; (iii) the inverse of the elasticity of substitution, θ, and the private capital elasticity, α, with whom the relationship of $\varkappa_i^+$ depends on the tax scenario. We also show that, at least in a neighborhood of the benchmark parameterization, changes in the values of the public and private capital depreciation factors or the initial public investment-to-output ratio just imply slight changes in $\varkappa_i^+$.

Figure 5.1 shows the relationship between $\varkappa_i^+$ and β, $\varkappa_c$, θ, α, displaying some interesting features. The direct relationship between $\varkappa_i^+$ and θ_g arises because the positive effect of public capital on growth and welfare increases with θ_g. The relationship is found to be linear under total income [as in Barro (1990)] and capital income taxes, being concave under labor income and private consumption taxes. On the other hand, a low β reduces the importance of growth on welfare, since β indicates the relative preference between present and future consumption. A higher slope between $\varkappa_i^+$ and β is found under labor income and consumption taxes than under

[15] The sensitivity analysis just considers the set of parameter values in a neighborhood around the benchmark levels, restricting initial growth to be non-negative, $C_t > 0$, $0 < h_t < 1$ for all t, and the NPG condition to hold.

capital income taxes. Consequently, $\varkappa_i^+$ approaches the initial level of 7% for low values of β independently of the tax system, although the ranking of $\varkappa_i^+$ stays constant under changes in β, the largest $\varkappa_i^+$ arising under private consumption taxes and the lowest under capital income taxes.

[INSERT FIGURE 5.1]

A high level of $\varkappa_c$ implies less resources for the private sector to save and consume. In addition, it amplifies the distortions on the strategies of the private sector because unproductive expenses are also financed with distortionary taxation. Consequently, the negative impact on consumption and leisure of a given public investment/output ratio becomes more harmful for welfare the higher is $\varkappa_c$, which explains the negative relationship between $\varkappa_i^+$ and $\varkappa_c$. In absolute value, the slope of this relationship is higher under less distorting taxes. Hence, for large enough values of $\varkappa_c$, even if unrealistic, $\varkappa_i^+$ will even get below the initial level of 7% independently of the tax system considered.

The elasticity of intertemporal substitution of private consumption is constant and equal to $1/\theta$. A smaller marginal rate of substitution between present and future consumption means that households have a higher preference for current consumption, relative to future consumption. Since $\beta < 1$, the long-run is less important than the short run for aggregate welfare under these circumstances. Consequently, a public investment ratio that strongly stimulates growth at the same time that significantly reducing private consumption initially, should not be expected to be optimal for high values of θ (low values of $1/\theta$). In the policy experiment, for $\theta > 13$, $\varkappa_i^+$ is systematically below 7% and far below the value of the investment ratio maximizing steady growth, with independence of the tax system considered.

Finally, the relationship between $\varkappa_i^+$ and α depends crucially on the tax system considered [see figure 5.1]. Since $\theta_k + \theta_g + \alpha = 1$, then, for a given value of θ_g, an increase in α amounts to a decrease in θ_k. But θ_k refers to a learning-by-doing externality in the production function, as in Romer (1986). It is well known that this externality leads in equilibrium to infra-accumulation of private capital, suggesting that welfare maximization might then require lower taxes on private capital. Effectively, we show that the relationship between $\varkappa_i^+$ and α is positive (i.e., a negative relationship between $\varkappa_i^+$ and θ_k) under capital as well as under total income taxes. However, when capital income is not being taxed, changes in the tax rate cannot reduce the learning-by-doing externality and hence, the relationship between $\varkappa_i^+$ and α is positive under labor and consumption taxes. As a consequence of all that, when the share of private capital is sufficiently high, $\varkappa_i^+$ may be lower under *less* distorting taxes, and downsizing might improve welfare under private consumption or under labor income taxes, but not under either capital or total income taxes, a very relevant result for fiscal policy making.

Summing-up, several important implications are found in this section:

- Under the alternative tax scenarios, the welfare-maximizing public investment/output ratio depends only slightly on its initial value. Consequently, downsizing public investment improves aggregate utility for economies starting with a public investment/output ratio slightly higher than the welfare-maximizing ratio characterized in section 4.3. For example, for the benchmark economy, if the initial level of $\varkappa_i$ is higher than 12.6%, 3.3%, 23.5% and 29.6%, respectively under total income, capital income, labor income and private consumption taxes, certain downsizing in public investment would improve welfare;

- For economies with high enough levels of $\varkappa_c$, or θ, and/or sufficiently low levels of β or θ_g, the welfare-maximizing public investment ratio is very similar, and generally lower than the benchmark 7%, with independence of the tax scenario considered;

- In economies with a high share of private capital in output, $\varkappa_i^+$ may be lower under labor income or consumption taxes than under capital or total income taxes.

6. Conclusions

We have characterized the welfare-maximizing, time-invariant, public investment/output ratio in an endogenous growth model with non-congested public capital and a constant ratio of unproductive public expenses.

In the first part of the chapter, the model was calibrated to be in line with industrialized economies in the 80s, under alternative distorting tax scenarios and different values of the share of public capital in output, a rather controversial parameter to evaluate. The economy was assumed to be initially along the balanced path associated to the benchmark calibration, the initial public investment-to-output ratio being 7% (a broad definition of public investment is used, as described in Easterly and Rebelo (1993)). Main findings in relation to this first part are:

(a1) Independently of the distorting tax scenario considered, short and long-run welfare effects are of opposite sign, and the welfare-maximizing public investment ratio is strictly lower than the growth-maximizing ratio;
(a2) These differences are more important under less distorting tax systems (labor income taxes and, specially, consumption taxes);
(a3) For the benchmark calibration, a downsizing in public investment improves welfare when taxing only capital income or total income, provided that public capital is not very productive.

Since the assumed tax scenarios are too extreme to be realistic, these results should not be interpreted as strict policy recommendations. However, they emphasize the importance of considering the transition between steady-states, as well as the relevance of the specific tax system in effect, when characterizing a welfare improving policy intervention.

In the second part of the paper, a local sensitivity analysis was performed. The simplicity of previous models implied a simple relationship between the welfare-maximizing time-invariant public investment/output ratio and economic fundamentals, which disappears in the more complex framework used in this paper. As in previous models, the discount factor and the public capital share of output positively affect the welfare-maximizing ratio. In addition, the *unproductive* public expenditure/output ratio, the private capital share and the elasticity of intertemporal substitution of private consumption also affect significantly the welfare-maximizing ratio. On the other hand, changes in the initial public investment ratio only barely affect the welfare-maximizing policy. In relation to this second part, the main findings are:

(b1) Qualitative findings (a1) and (a2) are robust, at least locally, to changes in structural parameters;
(b2) Regarding results in (a3), downsizing public investment becomes welfare improving independently of the tax system when the elasticity of intertemporal substitution of private consumption and the output elasticity of public capital are low enough or when the unproductive public expenditure-to-output ratio is sufficiently high;
(b3) Moreover, for a high value of the elasticity of intertemporal substitution, the *no-Ponzi game* condition limits the positive effect on growth of the public investment policy under labor income and private consumption taxes. As a consequence, the public investment ratio can be then only slightly higher than the 7% benchmark, so the welfare-maximizing ratio might be lower under less distorting taxes than under total income taxes;
(b4) Finally, when the output elasticity of private capital is sufficiently high, a downsizing in public investment is welfare improving under private consumption and labor income taxes, but not under capital income or under total income taxes.

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7. Appendix: Solving the dynamics

In this Appendix we show a procedure to solve for the dynamics of the competitive equilibrium. The outline of the procedure is as follows:

(i) Redefine competitive equilibrium conditions in terms of stationary ratios, $Z_t = \tilde{C}_t/\tilde{K}_t$, $V_t = \tilde{K}_t^g/\tilde{K}_t$, $M_t = \tilde{Y}_t/\tilde{K}_t$ and $\kappa_t = \tilde{K}_{t+1}/\tilde{K}_t$. After consolidating some equations, competitive equilibrium conditions reduce to a system of 6 equations in r_t, h_t and the Z_t, V_t, M_t, κ_t ratios:

$$\kappa_t \frac{Z_{t+1}}{Z_t} = \left\{ \beta \left(\frac{1-h_{t+1}}{1-h_t} \right)^{(1-\rho)(1-\theta)} \left[1-\delta^k + (1-\tau^k) r_{t+1} \right] \right\}^{\frac{1}{1-\rho(1-\theta)}}, \tag{7.1}$$

$$\frac{\rho}{1-\rho} = \frac{(1+\tau^c) Z_t}{(1-\alpha) M_t} \frac{h_t}{1-h_t}, \tag{7.2}$$

$$M_t = Z_t + \varkappa_c M_t + \varkappa_i M_t + \kappa_t - 1 + \delta^k, \tag{7.3}$$

$$r_t = \alpha M_t, \tag{7.4}$$

$$M_t = F h_t^{1-\alpha} W_t^{\theta_g}, \tag{7.5}$$

$$\kappa_t W_{t+1} = (1-\delta^g) W_t + \varkappa_i M_t. \tag{7.6}$$

Condition (7.1) comes directly from (2.10); (7.2) comes from (2.11) and $L_t = h_t$; (7.3) from combining (2.1), (2.7), (2.15), (2.12), (2.13); (7.4) and (7.5) come from (2.3) and (2.4), respectively; finally, (7.6) comes from dividing by $\tilde{K}_t$ in (2.12) and (2.14).

(ii) Pick structural parameters to solve (7.1)-(7.6) for the *bgp*.[16]

(iii) Log-linearize (7.1)-(7.6) around the *bgp* to solve for the dynamics of stationary ratios [Uhlig (1999)]. Let us denote by $W(t) = (k_t^g)$ the single beginning-of-period state variable and by $Q(t)$ the vector of real variables $Q(t) = (Z_t, M_t, r_t, h_t, \kappa_t)$,and by $w(t)$ and $\hat{q}(t)$ their log-deviations around their values along the *bgp,* $\bar{W}$ and $\bar{Q}$. The log-linear approximation to conditions (7.1)-(7.6) can then be rewritten more compactly as:

$$0 = Aw(t+1) + Bw(t) + C\hat{q}(t), \tag{7.7}$$

$$0 = Fw(t+2) + Gw(t+1) + Hw(t) + J\hat{q}(t+1) + K\hat{q}(t), \tag{7.8}$$

where those conditions showing dynamics of any variable in $Q(t)$ [(7.1) in our case] are included in (7.8), and matrices A, B, C,..., are functions of all structural and fiscal policy parameters. Details for the log-linearization of (7.1)-(7.6), together with matrices A, B, C,..., are shown below.

(iv) The following log-linear law of motion for $Q(t)$ and $W(t)$ is assumed:

$$w(t+1) = Pw(t), \tag{7.9}$$

$$\hat{q}(t) = S\hat{x}(t), \tag{7.10}$$

[16] In general, analytical expressions characterizing the *bgp* ara unavailable, so the existence and uniqueness of the *bgp* is checked through the numerical computation.

where P and S are free matrices of dimension (1x1) and (5x1). Next, conditions (7.7)-(7.8) are directly solved by the undetermined coefficients method, imposing the eigenvalues of P to be inside the unit circle, since otherwise $Q(t)$ and $W(t)$ would present explosive paths.

(v) Starting at (K_0, K_0^g), we have $k_0^g = K_0^g/K_0$, and values of C_0, Y_0, r_0, h_0 and k_1 are directly obtained from

$$\begin{pmatrix} C_0 \\ Y_0 \\ r_0 \\ h_0 \\ k_1 \end{pmatrix} = \begin{pmatrix} K_0 & 0 & 0 & 0 & 0 \\ 0 & K_0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} Q(0), \tag{7.11}$$

where, from (**??**), $Q(0) = \exp\left[S\left(\ln k_0^g - \ln \bar{k}^g\right) + \ln \bar{Q}\right] = \left(Z_0\ M_0\ r_0\ h_0\ \kappa_1\right)'$, and W_1 is obtained from (**??**).

(vi) K_1 and K_1^g (the state of the economy next period) are easily recovered from κ_0:

$$K_1 = \frac{K_0\kappa_0}{1+\bar{\gamma}} \text{ and } K_1^g = W_1 K_1. \tag{7.12}$$

(vii) Time series for normalized variables for successive periods are recovered by going recursively through steps (v) and (vi). Stability of the resulting time series is guaranteed since appropriate stability conditions were implemented when solving (7.9)-(7.10).

7.1. Log-linear optimal conditions

Uhlig (1999) proposes a procedure where optimal conditions are log-linearized without need of differentiating. Let $\hat{x}_t$ and $\hat{y}_t$ denote the variables in log-deviation to their steady-state, $\hat{x}_t = \ln\left(X_t/\bar{X}\right)$ and $\hat{y}_t = \ln\left(Y_t/\bar{Y}\right)$. X^a can be approximated:

$$\left(\frac{X}{\bar{X}}\right)^a = \exp\left(a \ln\left(\frac{X}{\bar{X}}\right)\right) = \exp(a\hat{x}) \simeq (1 + a\hat{x}) \Rightarrow X^a \simeq \bar{X}^a(1 + a\hat{x}). \tag{7.13}$$

In addition, $\hat{x}\hat{y} \simeq 0$ if variables are close enough to their steady-state values. Log-linearized versions of (7.1)-(7.6) are (all variables in log-deviations about steady-state):

$$\begin{aligned}
\hat{z}_{t+1} - \hat{z}_t + \hat{\kappa}_t + \frac{\tilde{\rho}\tilde{\theta}\bar{h}}{1-\bar{h}}(\hat{h}_{t+1} - \hat{h}_t) - \frac{\tilde{\theta}\bar{r}^k(1-\bar{\tau}^k)}{\bar{R}}\hat{r}_{t+1}^k &= 0, && (7.14)\\
\hat{z}_t - \hat{m}_t + \frac{1}{1-\bar{h}}\hat{h}_t &= 0, && (7.15)\\
\bar{M}\hat{m}_t - \bar{Z}\hat{z}_t - \varkappa_i\bar{M}\hat{m}_t - \varkappa_c\bar{M}\hat{m}_t - (1+\bar{\gamma})\hat{\kappa}_t &= 0, && (7.16)\\
\bar{r}^k\hat{r}_t^k - \alpha\bar{m}\hat{m}_t &= 0, && (7.17)\\
\hat{m}_t - (1-\alpha)\hat{h}_t - \theta_g\hat{w}_t &= 0 && (7.18)\\
\left(\hat{\kappa}_t + \hat{w}_{t+1}\right)(1+\bar{\gamma})\bar{W} - (1-\delta^g)\bar{W}\hat{w}_t - \varkappa_i\bar{M}\hat{m}_t &= 0, && (7.19)
\end{aligned}$$

where $\hat{\kappa}_t = \left[\ln\left(\frac{\tilde{K}_{t+1}}{\tilde{K}_t}\right) - \ln(1+\bar{\gamma})\right]$, $\tilde{\theta} = \frac{1}{1-\rho(1-\theta)}$, $\tilde{\rho} = (1-\rho)(1-\theta)$, $\bar{\gamma} = \varkappa_i F \bar{h}^{1-\alpha} \bar{W}^{\theta_g - 1} - \delta^g$, $\bar{M} = F\bar{h}^{1-\alpha\theta_g - 1\theta_g}$, $\bar{r}^k = \alpha\bar{M} - \delta^k$, and $\bar{R} = \left(1 + (1-\bar{\tau}^k)\bar{r}^k - \bar{\tau}^k\delta^k\right)$. Tax-rates are constant. Steady-state conditions have been used to simplify these expressions. The endogenous state variable is $\hat{v}_t = \ln W_t - \ln \bar{W}$, denoted in general terms by $\hat{x}(t)$. Endogenous control variables are included in $\hat{y}(t)' = \left(\hat{m}_t, \hat{z}_t, \hat{h}_t, \hat{r}_t^k, \hat{\kappa}_t\right)$. From (7.14)-(7.19), log-linear conditions can be rewritten:

$$A\hat{x}_{t+1} + B\hat{x}_t + C\hat{y}_t = 0, \tag{7.20}$$

$$F\hat{x}_{t+2} + G\hat{x}_{t+1} + H\hat{x}_t + J\hat{y}_{t+1} + K\hat{y}_t = 0, \tag{7.21}$$

where

$$A = \begin{pmatrix} 0 & 0 & 0 & 0 & (1+\bar{\gamma})\bar{W} \end{pmatrix}', \; B = \begin{pmatrix} 0 & 0 & 0 & -\theta_g & -(1-\delta^g)\bar{W} \end{pmatrix}',$$

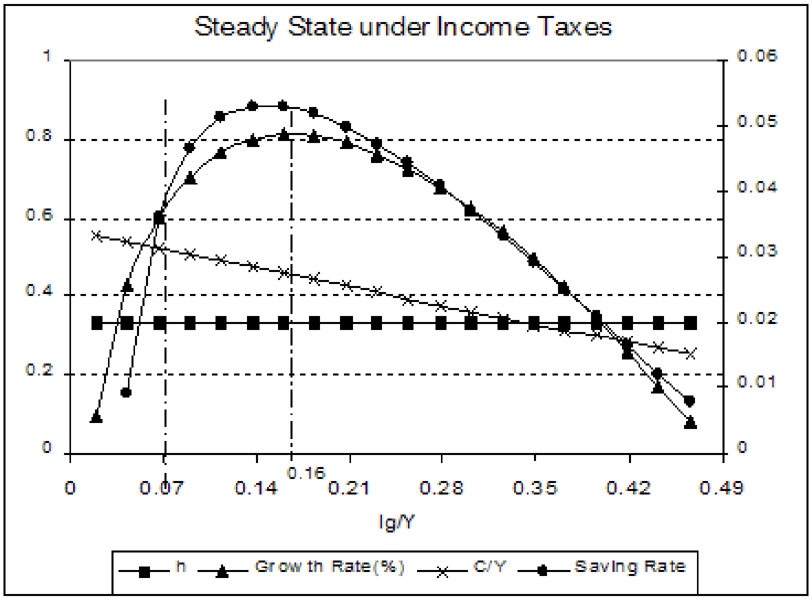
$$C = \begin{pmatrix} -1 & 1 & \frac{1}{1-\bar{h}} & 0 & 0 \\ \bar{M}(1-\varkappa_i-\varkappa_c) & -\bar{Z} & 0 & 0 & -(1+\bar{\gamma}) \\ -\alpha\bar{M} & 0 & 0 & \bar{r}^k & 0 \\ 1 & 0 & -(1-\alpha) & 0 & 0 \\ -\varkappa_i\bar{M} & 0 & 0 & 0 & (1+\bar{\gamma})\bar{W} \end{pmatrix},$$

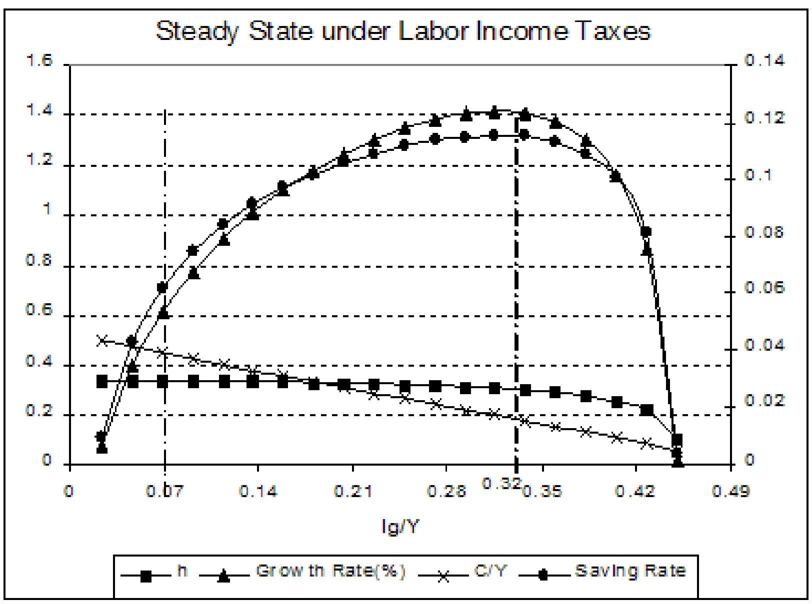
$$F = 0, \; G = 0, \; H = 0,$$

$$J = \begin{pmatrix} 0 & 1 & \frac{\tilde{\theta}\tilde{\rho}\bar{h}}{1-\bar{h}} & -\frac{\tilde{\theta}\bar{r}^k(1-\bar{\tau}^k)}{\bar{R}} & 0 \end{pmatrix}, \; K = \begin{pmatrix} 0 & -1 & -\frac{\tilde{\theta}\tilde{\rho}\bar{h}}{1-\bar{h}} & 0 & 1 \end{pmatrix}.$$

8. Appendix: List of figures

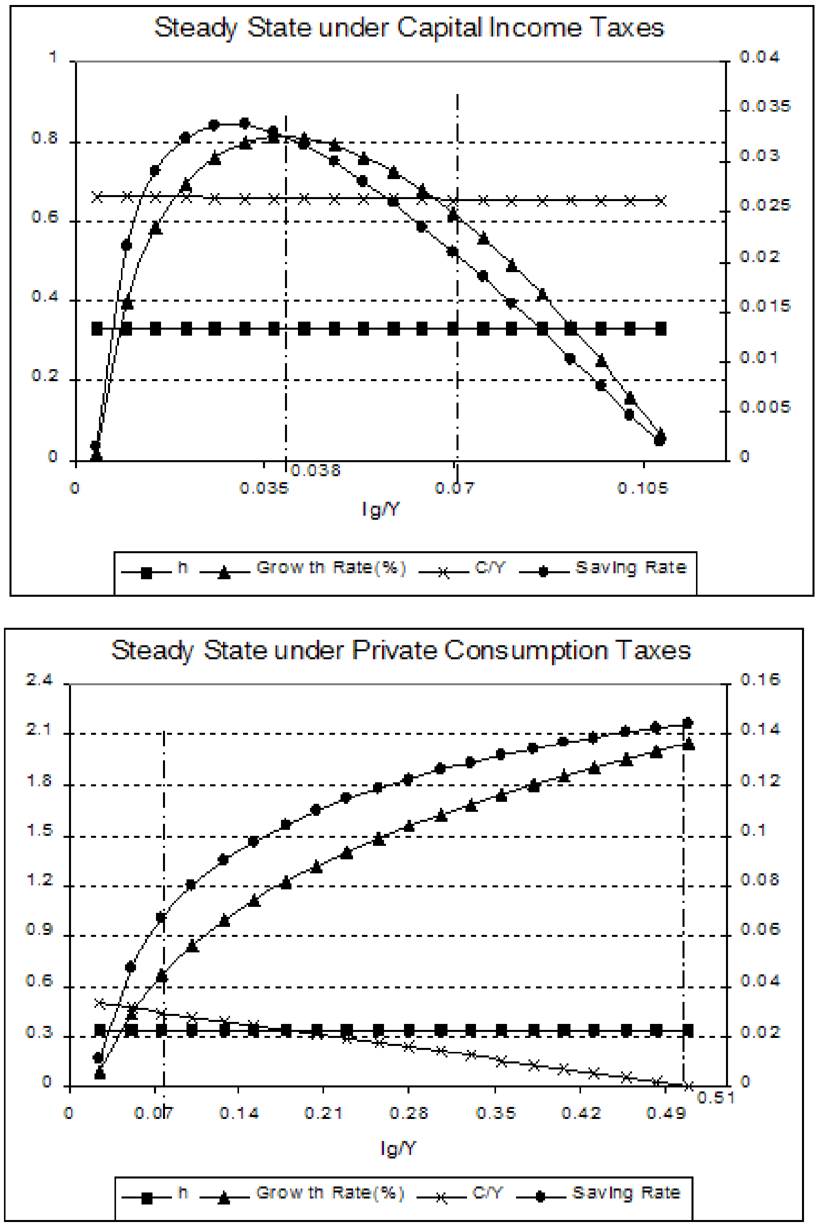
Figure 4.1 (a): Steady-state under alternative tax scenarios





Note: The left scale corresponds to $\bar{\mathrm{h}}$, $\bar{\mathrm{C}}/\bar{\mathrm{Y}}$ and $\bar{\gamma}$ in percent while the right one refers to the savings rate. The x-axis shows the steady-state public investment/output ratio, $I^g/Y = \varkappa_i$. Structural parameters are such that, for an initial $\varkappa_i$ of .07, the annual after-tax capital rate of return is 6%, $\bar{\mathrm{h}}$=1/3 and $\bar{\gamma}$=.62%. Public capital elasticity is .20. Growth-maximizing values of $\varkappa_i$ are .16, .04, .32 and .51 under total income, capital income, labor income and consumption taxes, respectively.

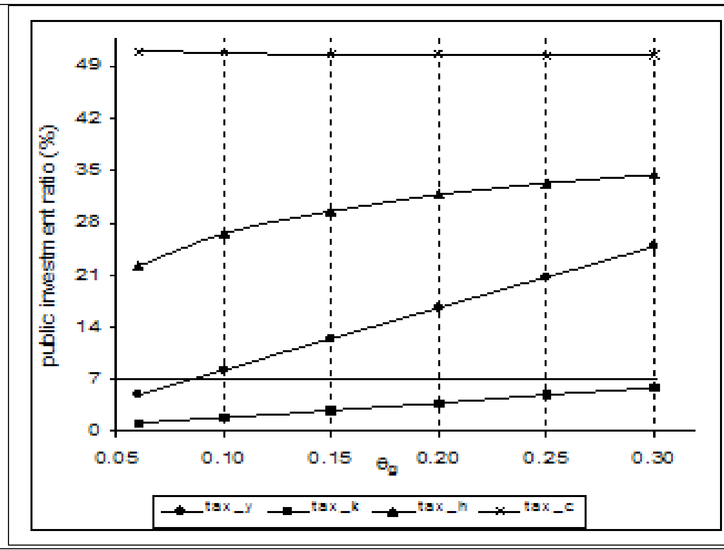
Figure 4.1 (b): Steady-state under alternative tax scenarios



Note: The left scale corresponds to $\bar{\mathrm{h}}$, $\bar{\mathrm{C}}/\bar{\mathrm{Y}}$ and $\bar{\gamma}$ in percent while the right one refers to the savings rate. The x-axis shows the steady-state public investment/output ratio, $I^g/Y = \varkappa_i$. Structural parameters are such that, for an initial $\varkappa_i$ of .07, the annual after-tax capital rate of return is 6%, $\bar{\mathrm{h}}$=1/3 and $\bar{\gamma}$=.62%. Public capital elasticity is

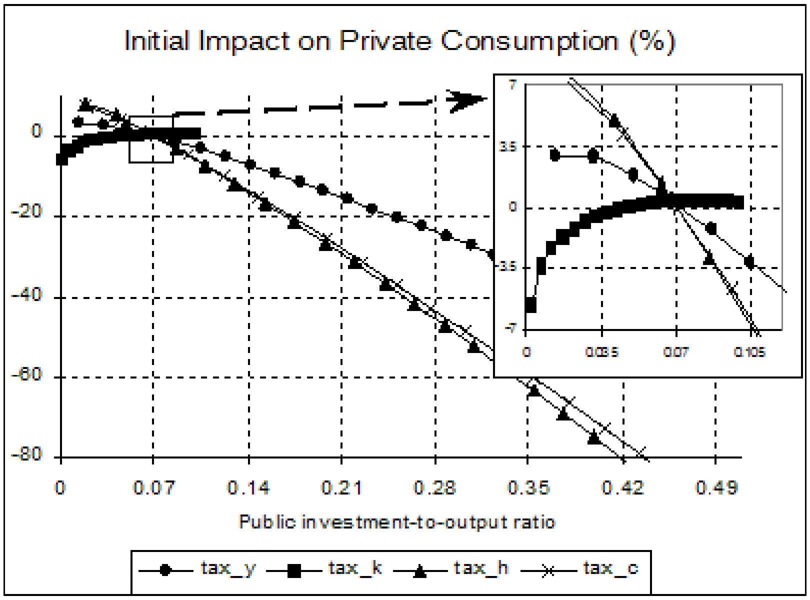
.20. Growth-maximizing values of $\varkappa_i$ are .16, .04, .32 and .51 under total income, capital income, labor income and consumption taxes, respectively.

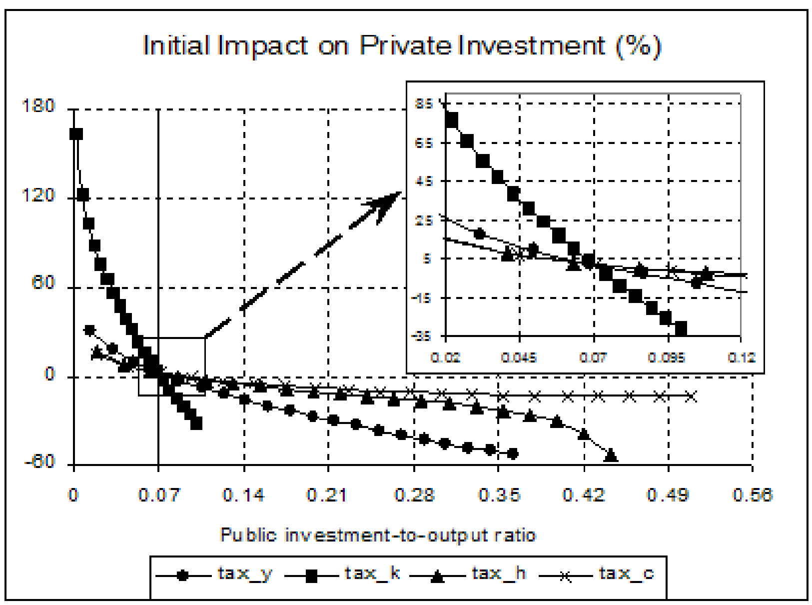
Figure 4.2: Public investment ratio maximizing steady-state growth



Note: Figures 4.2 shows the growth-maximizing public investment ratio, $\varkappa_i^*$, as function of the elasticity of public capital, under income taxes (tax_y), capital income taxes (tax_k), labor income taxes (tax_h) and private consumption taxes (tax_c). Structural parameters are such that, in all cases, the annual after-tax capital rate of return is 6%, $\bar{\text{h}}$=1/3 and $\bar{\gamma}$=.62%, with an initial public investment ratio of .07 (the solid line.)

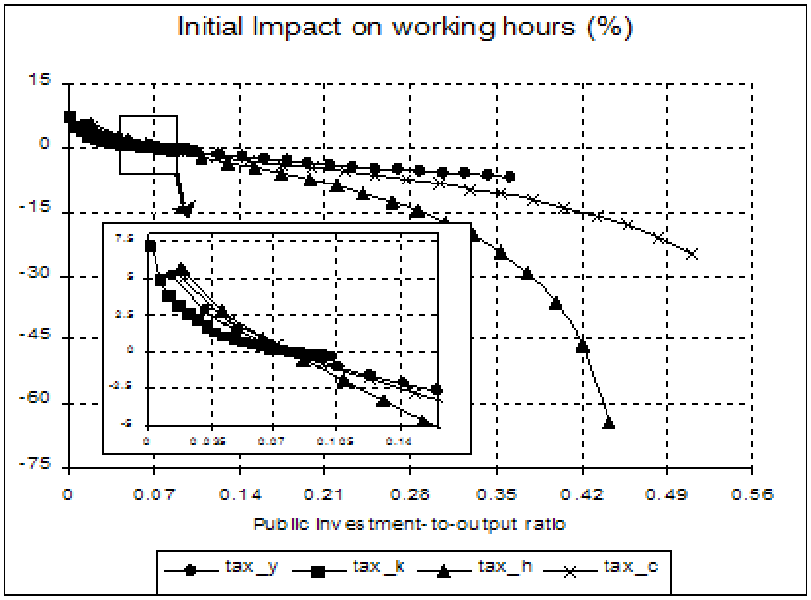
Figure 4.3 (a): Initial impact of main macroeconomic variables

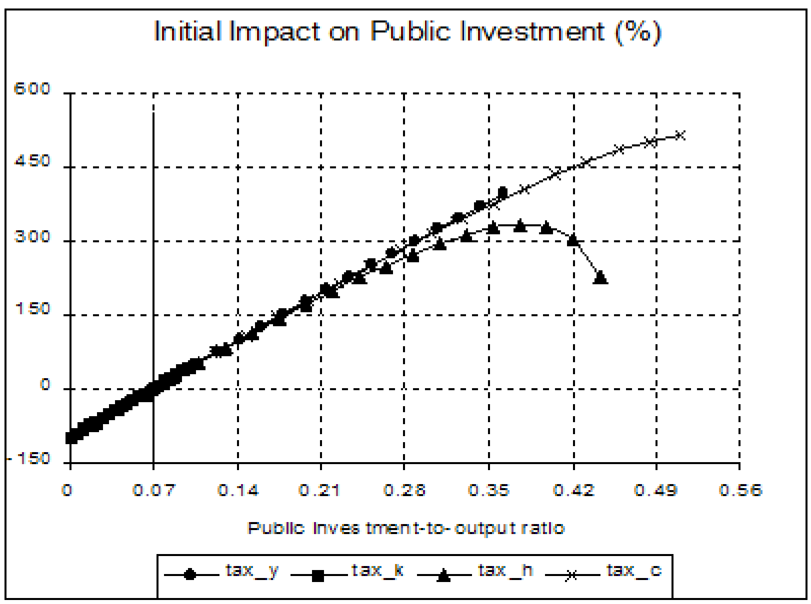




Note: The Y-axis shows the percent change of the variable between its initial level and the level attained one period after the policy intervention, under income (tax_y), capital income (tax_k), labor income (tax_h) and private consumption taxes (tax_c). The new level of $\varkappa_i$ is shown in the X-axis. Initial public investment ratio is .07 (where curves intersect) and elasticity of public capital is .20. Structural parameters are such that the annual after-tax capital rate of return is 6%, $\bar{h}$=1/3 and $\bar{\gamma}$=.62%.

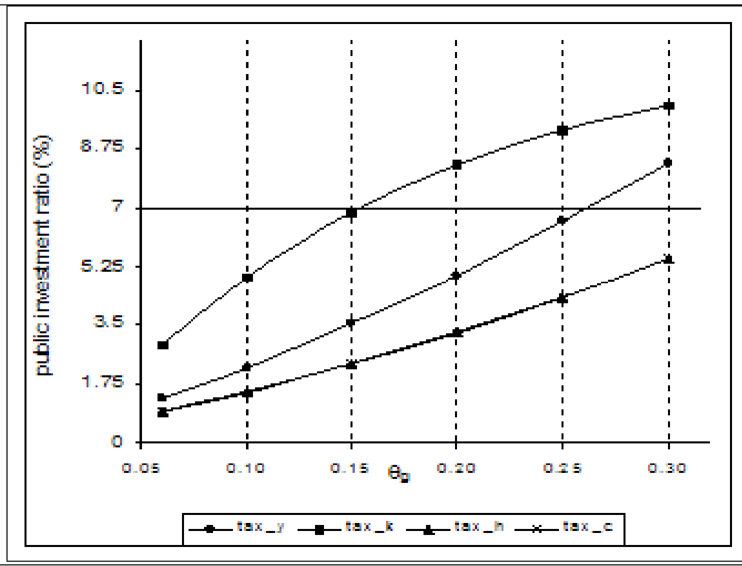
Figure 4.3 (b): Initial impact of main macroeconomic variables





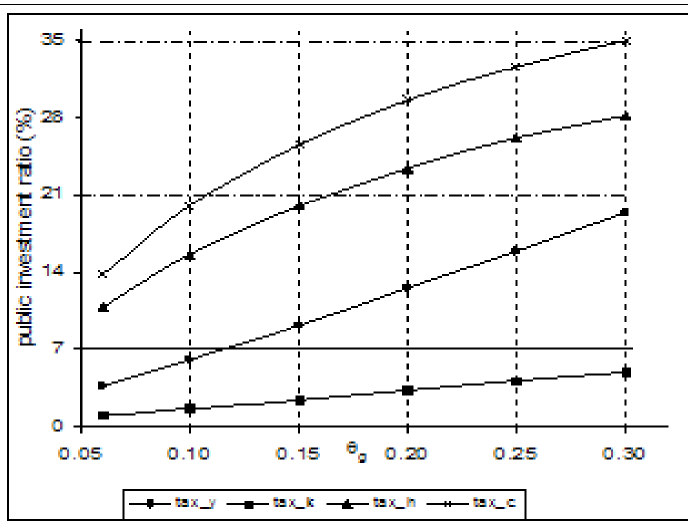
Note: The Y-axis shows the percent change of the variable between its initial level and the level attained one period after the policy intervention, under income (tax_y), capital income (tax_k), labor income (tax_h) and private consumption taxes (tax_c). The new level of $\varkappa_i$ is shown in the X-axis. Initial public investment ratio is .07 (where the lines intersect) and elasticity of public capital is .20. Structural parameters are such that the annual after-tax capital rate of return is 6%, $\bar{h}$=1/3 and $\bar{\gamma}$=.62%.

Figure 4.4: Public investment ratio maximizing welfare over 4 periods



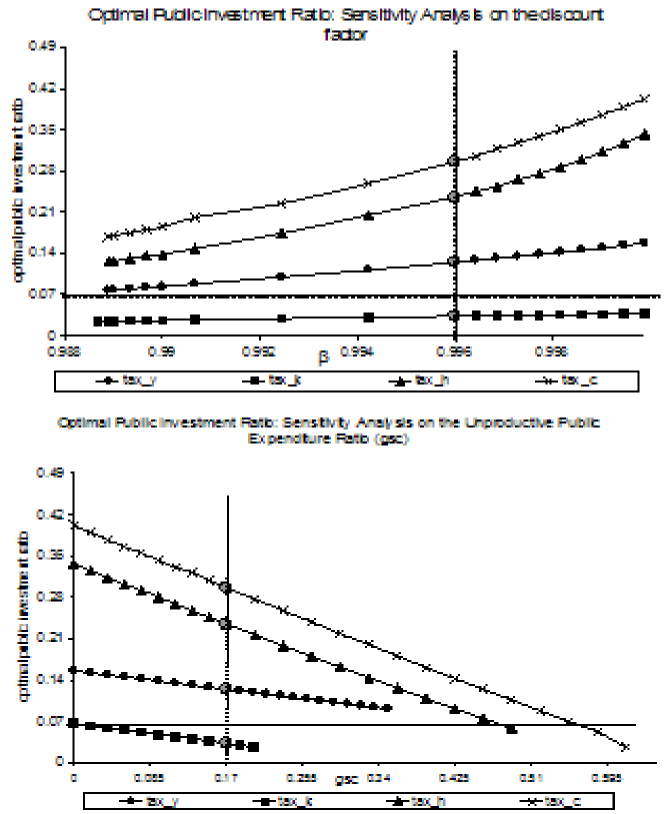
Note: Figure 4.4 shows the public investment ratio maximizing welfare over 4 periods, $\varkappa_i^{sr}$, as function of the elasticity of public capital, under income taxes (tax_y), capital income taxes (tax_k), labor income taxes (tax_h) and private consumption taxes (tax_c). Structural parameters are such that, in all cases, the annual after-tax capital rate of return is 6%, $\bar{\text{h}}$=1/3 and $\bar{\gamma}$=.62%, with an initial public investment ratio of .07 (the solid line).

Figure 4.5: Welfare-maximizing public investment ratio



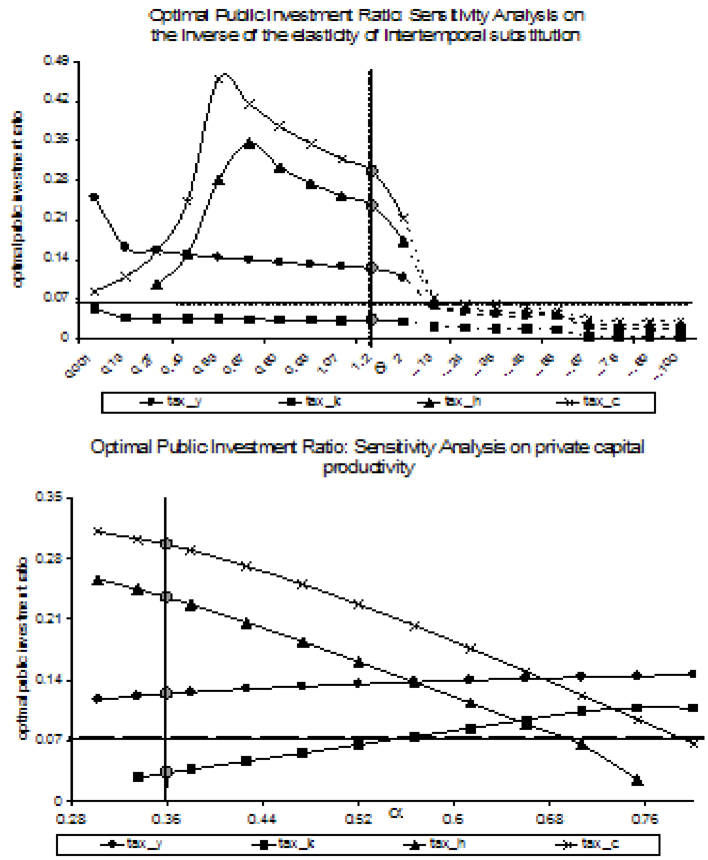
Note: Figure 4.5 shows the welfare-maximizing public investment ratio $\varkappa_i$, as a function of public capital elasticity, under total income (tax_y), capital income (tax_k), labor income (tax_h) and private consumption taxes (tax_c).

Figure 5.1 (a). Sensitivity analysis: the optimal public investment ratio as a function of β and $\varkappa_c$



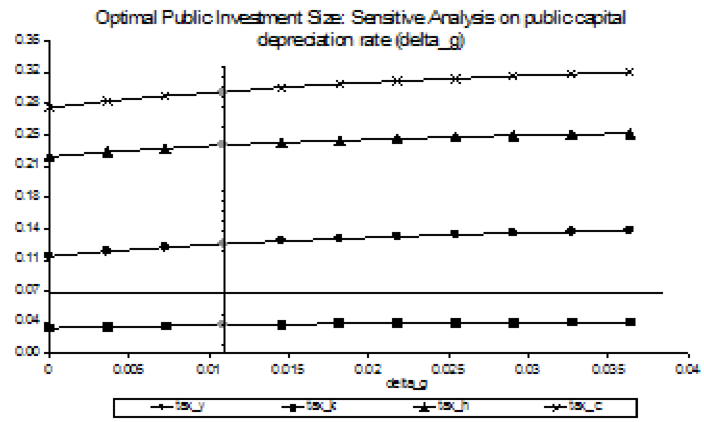
Note: The benchmark calibration sets α=.36, β=.996, δ^g=.0125, δ^k=.025, θ=1.20,$\varkappa_i$=.07 and $\varkappa_c$=.17 and θ_g=.20. For this benchmark parameterization, $\varkappa_i^+$ is shown as a grey circle for each alternative tax rule.

Figure 5.1 (b). Sensitivity analysis: the optimal public investment ratio as a function of θ and α



Note: The benchmark calibration sets α=.36, β=.996, δ^g=.0125, δ^k=.025, θ=1.20,$\varkappa_i$=.07, $\varkappa_c$=.17 and θ_g=.20. For this parameterization, $\varkappa_i^+$ is shown as a grey circle for each alternative tax rule.

Figure 5.1 (c). Sensitivity analysis: the optimal public investment ratio as a function of δ^g and δ^k



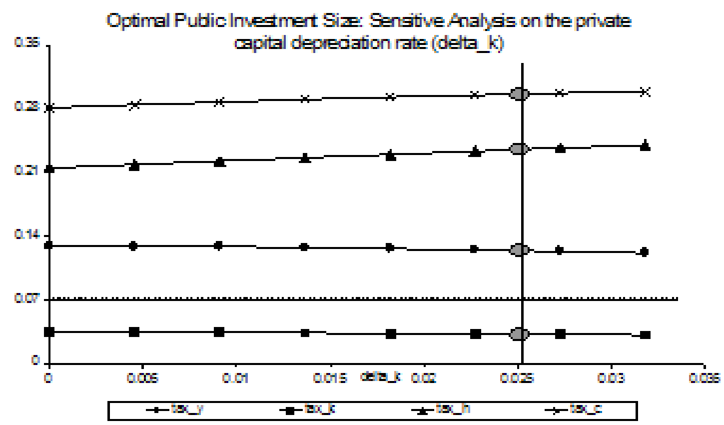


Table 1: Benchmark calibration

Taxes	θ_k	α	θ_g	F	δ^k	$\bar{h}$	β	θ	ρ	$\varkappa_c$	$\varkappa_i$	δ^g	τ^i
Income	.44	.36	.20	.30	.025	.33	.996	1.20	.35	.17	.07	.012	.24
Capital	.44	.36	.20	.59	.025	.33	.996	1.20	.34	.17	.07	.012	.67
Labor	.44	.36	.20	.24	.025	.33	.996	1.20	.36	.17	.07	.012	.38
Consumption	.44	.36	.20	.24	.025	.33	.996	1.20	.35	.17	.07	.012	.52

Note: Each row shows the benchmark calibration under each tax system for $\theta_g = .20$. They all reproduce a rate of growth $\bar{\gamma} = .62\%$, an after-tax capital rate of return of .015 and $\bar{h} = .33$. The column for τ^i shows the tax rate under each of the four tax rules.