A Component Delta Conditional Expected Shortfall ($Celta$ $CoES$) measure in the financial systemic risk framework *

Javier Ojea Ferreiro †

This version: April 15, 2018

Abstract

The early recognition of Systemically Important Financial Institutions ($SIFI$s) is a key step to monitor and track possible financial crisis. Recent literature has proposed several risk measures focusing on different characteristics of systemic events. Among the most important features for this identification is the size and the connection of the firms in the financial system, apart from their higher moments features (skewness and kurtosis). A systemic event might be imminent if institutions present extreme events more frequently than usually, and adverse scenarios arise more often than good ones. To date, no study has considered jointly these three aspects. I propose a systemic risk measure that merges these essential components of a systemic event by decomposing change in the expected losses of the financial system when a crisis arises. I employ a copula methodology comparing the results under different assumptions. The relevance of the new measure surfaces when higher moments and joint tail dependence are taking into account. The measure built on an accurate model can contribute to point out unforeseen systemic events where the unlikely extreme scenarios can be crucial, i.e., Black Swan events.

Keywords: *Systemic risk, Financial sector, Expected Shortfall, Conditional measures.*

1 Introduction

The 2008 financial crisis and the European sovereign debt crisis have implied new challenges for the ECB policy concerning macro-prudential supervision, where the identification and assessment of the Systemically Important Financial Institutions ($SIFI$s) plays a key role. The macro-prudential oversight should prevent financial sector from bringing about a breakdown of the economic system (ECB (2010b)). According to ECB (2010a), systemic risk can be defined as the risk of experiencing systemic events, which are financial failures likely to translate into adverse effects on welfare in the economy. The systemic risk sources can be divided in three types. The three ways in which this event can spread among the economy are contagion, financial imbalance and aggregate shocks. There are plenty of features of the financial system

* Any comment or suggestion is very welcome. I thank to my PhD supervisor, Prof. Alfonso Novales Cinca. This research was supported by the Ministry of Education of Spain (FPU grant FPU15/04241).

† PhD Candidate in Quantitative Finance and Economics, Complutense University of Madrid (UCM), Spain. Email: *jojea@ucm.es*.

that make financial sector susceptible to these systemic risk sources, e.g. externalities through transmition channel, asymmetric information due to agency problems and powerful feedback and amplification mechanism such as fire sales and herd behaviour.

Trichet (2009) points out the need of high-frequency systemic risk measures due to the speed of the crisis spillovers. The acceleration of the financial turmoil phases is consequence of the global financial integration, an increasing leverage in institutions, the accumulation of unsustainable global imbalance and technological innovations. Several high-frequency measures have been proposed based mainly on market data. Some market data based measures are the Marginal Expected Shortfall (MES) by Acharya et al. (2012), the Component Expected Shortfall (CES) by Banulescu and Dumitrescu (2015) the Delta Conditional Value-at-Risk ($\Delta CoVaR$) by Adrian and Brunnermeier (2016) and the Systemic RISK ($SRISK$) by Brownlees and Engle (2016). Each measure tries to put forward a certain feature of the systemic event. For instance MES is the conditional return of the financial firm when the market as a whole is on distress, whereas CES is the absolute contribution of each firm to the financial market crisis. Although from a time-series perspective MES and CES are almost identical, in the cross-sectional series there is a significant difference due to the inclusion off a size factor in the CES. Benoit et al. (2013) and Benoit et al. (2017) point out similarities between MES ranking and ranking based on market β under Gaussian assumptions. Because of that, Guntay and Kupiec (2014) concludes that MES is a measure where systemic and systematic risk are mixed, given an unreliable and noisy view of systemic risk. Löffler and Raupach (2017) and Kleinow et al. (2017) arrive to the same point. That is why these authors advocate from combining several systemic risk measures to identify $SIFI$s. The $\Delta CoVaR$ measures the change in the Conditional Value-at-Risk ($CoVaR$) of the financial market when the firm moves from normal to distress times. The original definition proposed by Adrian and Brunnermeier (2016)suffered from several drawbacks among which the most important were the impossibility to backtest $CoVaR$ and the counterintuitive fact that $CoVaR$ is not a monotonically increasing function of the dependence between the firms and the financial system (Mainik and Schaanning (2014)). Girardi and Ergün (2013) proposes a modification of $CoVaR$ definition that deals with these issues. However most of the articles concerning comparison between systemic risk measures as Benoit et al. (2017),Guntay and Kupiec (2014) or Löffler and Raupach (2017) use the original definition in spite of its problems. Moreover, $CoVaR$ has some limitation given its nature, i.e., it does not satisfy the sub-addictive property (see Artzner et al., 1999; Acerbi and Tasche, 2002). This issue can be solved if the Value-at-Risk dimension changes to a Expected Shortfall framework, i.e., building systemic risk measures based on Conditional Expected Shortfall ($CoES$). $\Delta CoES$ still can not be aggregated unlike CES or $SRISK$, i.e., $\Delta CoES$ does not allow to assess the effect in the $CoES$ of the financial system when a group of firms are in distress using the $\Delta CoES$ with each financial firm. Last but not least, $SRISK$ tries to assess the amount of capital needed by a firm in distress when the market is also in distress. For this propose, Brownlees and Engle (2016) mixes market and accountant data taking into account the common exposure of the firms to the financial market, the size and the leverage of institutions. The fact of using accountant data may introduce a discrepancy problem because of the differences between accounting systems. Moreover, the accountant data is scarce and only available at a low frequency. The sub-prime crisis has shown us that financial imbalance may arise from off-balance sheet activities, which adds an additional challenge to $SRISK$. Scott et al. (2016)

claims that SRISK can be appropriate for measuring systemic risk in the banking sector where the accountant data and the market leverage ratio may arise information about different business lines, but not for assessing systemic risk in other financial groups as the insurance sector. Salleo et al. (2016) finds $SRISK$ highly correlated with the leverage ratio doubting about the use of $SRISK$ as a benchmark for supervisory stress tests.

Constâncio (2017) takes concern about the two main systemic risks stemming from the non-bank financial sector that have not been captured properly by stress test and analytical tools for systemic risk, which have been built bearing the banking sector in mind.
First, the increasing size and growth of the non-bank financial sector which can potentially amplify financial stability risks. The size has played an important role during the recent crisis and it has increased during the last two decades (Laeven et al. (2014)).Actually, Bernanke (2010) highlighted the Too-Big-To-Fail ($TBTF$) problem as a key point concerning the 2008 financial crisis. Rose and Wieladek (2012) has found the bank size as a key determinant of public banking interventions in the UK. Size is important because there are evidences that large banks tend to engage more in risky business lines and be funded more with short-term debt making them vulnerable to liquidity constrains in case of crisis (Shleifer and Vishny (2009), Boot and Ratnovski (2012)). Large banks also use to incur in moral hazard behaviour taking excessive risk and having lower capital ratio due to the expectation of bailouts (Farhi and Tirole (2012)).
Second, the procyclical nature of margin and haircut-setting practices may lead to amplify liquidity and market risk via fire sales in a stress scenario, being the knowledge of interactions between agents a key element to prevent the propagation risk from individual institution to the financial system. Bernanke (2009) points out the relevance of mutual exposures of highly interconnected firms as a possible spillover source that can triggers out financial instabilities. Indeed, the interconnectedness of the firms with the financial system may be as determinant as the size feature to explain which are the $SIFI$s. International Monetary Fund et al. (2010) indicates that after the size factor, the interconnectedness is an essential determinant to identify $SIFI$s, as a consequence the Too-Connected-To-Fail ($TCTF$) problem can not be overlooked.

This article proposes a high-frequency market-based systemic risk measure that merges size and interconnectedness in a parsimonious way from the decomposition of the change in expected losses of the financial system when a crisis occurs. In addition to that, this measure takes also into account the possible effects of higher moments (skewness and kurtosis) from the firms to the financial system. Systemic event might be imminent if institutions present extreme events more frequently than usually, and adverse scenarios arise more often than good ones. The proposed measure named Component Delta Conditional Expected Shortfall ($Celta\ CoES$) can be also aggregated allowing the assessment of the jointly systemic risk of a set of countries. To the best of my knowledge, no article has combined these three overriding characteristics (size, dependence and tail-behaviour features) concerning systemic risk in a single measure which may help to monitor systemic risk. I perform an empirical application with European financial firms using weekly data during the period 2006-2016 through a copula methodology and under two different distribution assumptions. The benchmark assumption is the Gaussian case, i.e. financial firms are normally distributed for both marginal and joint distributions. The alternative assumption is a skewed-t distribution for marginal distribution and a Student t copula for the joint distribution. The aim of this alternative assumption is to point out the importance of higher moments and joint tail dependence when systemic risk measures are computed.

The remained of this paper is structured into four sections. The following section present the goal,the formula and the interpretation of the different systemic risk measure, including *CeltaCoES*. Section 3 shows the methodology employed to build these measures in the empirical exercise. Section 4 introduces the data of the exercise conducted in section 5. Finally, section 6 closes the article gathering the main findings and conclusions concerning the advantages that *Celta CoES* introduces in the literature.

2 The marked-based approach of systemic risk: tools and measures

The measures employed in this article focus on different feature of systemic risk although the ultimate goal is to assess systemic risk and to identify the $SIFI$s. This section provides the specific information that each measure tries to reach and the formula definition. The set of systemic risk analysed are MES, CES, $\Delta CoES$ and $Celta\ CoES$.
Lets define the financial system return as

$$r_{m,t} = \sum_{i=1}^{N} r_{i,t}\omega_{i,t-1}, \tag{1}$$

where $r_{i,t}$ stands for financial firm i's returns, N represents the number of financial institutions and $\omega_{i,t-1}$ indicates the market capitalization of firm i at $t-1$ over the total market capitalization. The mean losses in a distress scenario for the financial system is a good indicator to measure the scale of possible problems in the sectors if crisis occurs. The Expected Shortfall of the financial system is

$$ES_{m,t-1}(\alpha) = -E_{t-1}\left[r_{m,t}|r_{m,t} < -VaR(\alpha)\right]. \tag{2}$$

2.1 Marginal Expected Shortfall

The MES measure gives information about the mean losses of financial firm when a crisis arises to the financial system. This measure provides useful information concerning the average behaviour of financial institutions on a certain scenario and their conditional performance features.
MES measures the marginal contribution of an institution i to systemic risk

$$MES_{i,t}(\alpha) = \frac{\partial ES_{m,t}(\alpha)}{\partial w_{i,t}} = -E_{t-1}\left[r_{i,t}|r_{m,t} < VaR_{m,t}(\alpha)\right]. \tag{3}$$

Equation 3 can be rewritten as

$$MES_{i,t}(\alpha) = -\int_{-\infty}^{\infty} r_{i,t} f_{i,t}(r_i, t|r_{m,t} < VaR_{m,t}(\alpha))dr_{i,t}, \tag{4}$$

where $f_{i,t}(r_i, t|r_{m,t} < VaR_{m,t}(\alpha))$ is the probability density function for firm i conditioned to a scenario where the financial market is below its α100-th quantile.

2.2 Component Expected Shortfall

The CES measure indicates the contribution of each firm to the system mean losses in distress, giving a magnitude to the $MES(\alpha)$, i.e.,

$$CES_{i,t}(\alpha) = \omega_i MES_{i,t}(\alpha). \tag{5}$$

There can be established a relationship between CES and the ES of the financial system given Equation 3

$$ES_{m,t-1}(\alpha) = \sum_{i=1}^{N} \overbrace{\omega_{i,t-1} \underbrace{E_{t-1}\left(-r_{i,t} | r_{m,t} < VaR(\alpha)\right)}_{MES_{i,t}}}^{CES_{i,t}}. \tag{6}$$

Equation 6 points out two important advantages of CES over MES. First, CES introduces a size factor in the MES formula dealing with the $TBTF$ problem. Second, Equation 6 shows that the ES of the financial system can be expressed as a sum of CES. This means that CES can be aggregated, giving information about which would be the joint contribution the system losses in a financial crisis of a set of banks from the same country or sub-sector.

However, CES does not show the dependence with the market scenario nor give enough importance to tail features that could lead to a systemic event.

2.3 SRISK

$SRISK$ merges market and balance sheet information to provide the expected capital shortfall of a financial institution conditional on a prolonged market decline. The basic inputs for building the $SRISK$ are the market capitalization ($W_{i,t}$) and the total debt ($D_{i,t}$) for assessing the capital buffer and Long Run Marginal Expected Shortfall ($LRMES$) for estimating the future value of the financial institution, i.e.,

$$SRISK_{i,t} = [k(D_{i,t} + (1 - LRMES_{i,t+h})W_{i,t}) - (1 - LRMES_{i,t+h})W_{i,t}]^{+}, \tag{7}$$

where the superscript + indicates that only positive values are considered and the parameter k represent the capital requirements. For European banks, Engle et al. (2015) calibrates its value in 5.5%. $LRMES$ expresses the MES of the financial firm i over a six-month period where the market falls a 40%. An implementation of the market price-based measurement is the V-Lab (https://vlab.stern.nyu.edu/) where for assessing $LMRES$ without simulation suggest

$$LRMES_{i,t} = 1 - \exp\left(\left(1 - d\right)\beta_{i,t}\right) \tag{8}$$

where $\beta_{i,t}$ is the firm i's beta coefficient at time t and $d = 0.4$.

2.4 Δ CoES

$\Delta CoES_{m|i}$ measure indicates the change in the Expected Shortfall of the financial system when the financial firm i moves from normal times to a distress scenario, i.e.,

$$\Delta CoES_{m|i,t}(\beta) = CoES_{m|i,t}(\alpha_s, \beta) - CoES_{m|i,t}(\alpha_n, \beta) \tag{9}$$

where $CoES_{m|i}$ shows the ES of the market conditioned to a specific scenario for financial firm i. The $CoES$ is the coherent extension of $CoVaR$, i.e.,

$$CoES_{m|i,t}(\alpha, \beta) = \frac{1}{\beta} \int_0^\beta -CoVaR_{m|i,t}(\alpha, q) dq,$$

The $CoVaR_{m|i}(\alpha, \beta)$ definition improved by Girardi and Ergün (2013) expresses the minimum return in the financial system with a confidence level $(1 - \beta)100\%$ given that financial firm i is below its $\alpha 100$-th quantile, i.e.,

$$P_{t-1}[r_{m,t} \leq -CoVaR_{m|i,t}(\alpha, \beta) | r_{i,t} \leq VaR_{i,t}(\alpha)] = \beta. \tag{10}$$

The distress scenario for the conditioning firm, α_s in Equation 9, should be those where the returns are below its $\beta 100$-th quantile because the adverse scenario for the conditioned firm is defined as being below its $\beta 100\%$ worst case scenario, i.e., $\alpha_s = \beta$. While to define the normal times with the same precision as the distress ones, I consider a range of $\beta 100$ quantiles around the median for the normal scenario, i.e., $\alpha_n = (0.5 \pm \beta/2)$. Losses not considered in normal scenarios can trigger out a systemic event because of lack of liquidity, i.e. in a normal scenario capital needs can be fulfilled without spillover effect between sectors, but in a distress scenario capital needs could lead to bankrupt and bailout processes, triggering out a contagion event from the firm i to the financial market. Therefore, CoES is unsatisfactory measures for assessing the contagion because a benchmark for the losses is necessary to distinguish between the Expected Shortfall of the system and those arisen from the connection with a certain financial firms. Indeed, CoES and CoVaR may be enough to capture the losses in a given scenario but not the loss changes when the conditioning scenario moves. That is why Adrian and Brunnermeier (2016) conceives $\Delta CoES$ as a difference of $CoES$.

2.5 Celta CoES

With the exception of $\Delta CoES_{m|i}$, most of the systemic risk measures, such as MES or SRISK define systemic risk on the opposite way, i.e., measuring losses for financial institution i given a stress scenario for the financial system. The measure obtained by exchanging conditioned and conditioning variables in Equation 9 is a risk management tool similar to the stress test useful for tracking banks performance in terms of systemic risk. Whereas $\Delta CoES_{m|i}$ measures which financial institution contributes more to a financial crisis, $\Delta CoES_{i|m}$ measures which financial institution is more exposure to a contagion from the financial sector.

Regarding the relationship between $CoES_{i|m}$ and MES, MES would be divided in two sections depending on the own behaviour of institution i, so this division may arise sunken losses that may be ignored if the mean is assessed without considering the particular position of institution i. Figure 1 shows the MES as the area on the left of $VaR_{m,t}(\alpha)$, which can be split in two areas weighted by its occurrence probability provided that the financial market is below the threshold denoted by $VaR_{m,t}(\alpha)$. The threshold that divides MES in two areas is the Conditional Value-at-Risk ($CoVaR$). Losses higher than $-CoVaR$ would occur $\beta 100\%$ of the time whereas losses would be lower $(1 - \beta)100\%$ of the time.

[Insert Figure 1 here]

Expressing MES as a function of $CoES$

$$
\begin{aligned}
MES_{i,t}(\alpha) &= E_{t-1}\left(-r_{i,t}|r_{i,t} > CoVaR_{i|m,t}(\alpha,\beta)\right) \\
&\quad \underbrace{P_{t-1}[r_{i,t} > CoVaR_{i|m,t}(\alpha,\beta)|r_{m,t} \leq VaR_{m,t}(\alpha)]}_{1-\beta} + \\
&\quad \underbrace{E_{t-1}\left(-r_{i,t}|r_{i,t} \leq CoVaR_{i|m,t}(\alpha,\beta)\right)}_{CoES_{i|m,t}(\alpha,\beta)} \\
&\quad \underbrace{P_{t-1}[r_{i,t} \leq CoVaR_{i|m,t}(\alpha,\beta)|r_{m,t} \leq VaR_{m,t}(\alpha)]}_{\beta}, \qquad (11)
\end{aligned}
$$

where the interesting section of $MES_{i,t}$ is the one in which institution i is in distress while the other section, although more probable is less appealing. Actually, the ranking according MES could be quite different from the ranking following $CoES$ if there are losses on the tail of institution i that are hidden when all the distribution is considered.
The link between $CoES_{i|m}$ and the ES of the financial system arises combining Equations 6 and 11. There is an improvement of the accuracy in the systemic risk measure when we move from a marginal dimension, i.e. MES dimension, to a conditional marginal framework, i.e. $CoES$, due to the focus not only in the behaviour of the market but also in the performance of the financial firm.

To account for dependence, Equation (6) is expressed in differences from the benchmark in normal times, i.e. $ES(\alpha_s) - ES(\alpha_n)$, then Equation (6) would be

$$
\begin{aligned}
ES_{m,t-1}(\alpha_s) - ES_{m,t-1}(\alpha_n) &= \sum_{i=1}^{N} \omega_{i,t-1} \Bigg\{ \overbrace{E_{t-1}\left(-r_{i,t}|r_{m,t} < VaR(\alpha_s)\right)}^{MES_{i,t}(\alpha_s)} \\
&\quad - \underbrace{E_{t-1}\left(-r_{i,t}|VaR_{m,t}(\alpha_n^-) < r_{m,t} < VaR_{m,t}(\alpha_n^+)\right)}_{MES_{i,t}(\alpha_n)} \Bigg\},
\end{aligned}
$$

so in terms of conditional Expected Shortfall would be rewritten as

$$
\begin{aligned}
ES_{m,t-1}(\alpha_s) - ES_{m,t-1}(\alpha_n) &= \sum_{i=1}^{N} \overbrace{\omega_{i,t-1} \Delta CoES_{i|m,t}(\beta)}^{Celta \quad CoES_{i|m,t}} \beta + \\
&\quad \sum_{i=1}^{N} \omega_{i,t-1} \Delta CoReS_{i|m,t}(\beta)(1-\beta), \qquad (12)
\end{aligned}
$$

where $\Delta CoReS_{i|m,t}(\beta)$ would be the mean loss for firm i when the financial market moves from normal times to a distress period and firm i doesn't cross the threshold given by its $CoVaR$, i.e.

$$
\begin{aligned}
\Delta CoReS_{i|m,t}(\beta) &= E_{t-1}\left(-r_{i,t}|r_{i,t} > CoVaR(\alpha_s,\beta), r_{m,t} < VaR(\alpha_s)\right) - \qquad (13) \\
&\quad E_{t-1}\left(-r_{i,t}|r_{i,t} > CoVaR(\alpha_n,\beta), VaR(0.5-\beta/2) < r_{m,t} < VaR(0.5+\beta/2)\right).
\end{aligned}
$$

$\Delta CoReS_{i|m,t}(\beta)$ represents the change in the mean losses for firm i when it is able to keep itself above its $\beta 100\%$ worst case scenario when the financial sector moves from normal times to a distress scenario. **This ability of the institution to adapt and resist from an hazard change in the financial market expresses resilience.** Indeed, in biological terms the resilience of an ecosystem is defined as the measure of its ability to absorb changes and still exist (Holling 1973). Likewise, in economic terms resilience of the financial sector would be the ability of the financial firms to run properly, adapting themselves to a change in the financial market. Consequently, $CoReS$ stands for Conditional Resilience Shortfall. Figure 2 shows a diagram of the quantiles employed in the minuend and subtrahend of $\Delta CoES$.

[Insert Figure 2 here]

The appealing part of Equation (12) rests on how the change in the Expected Shortfall of financial firms when the financial market moves from a normal scenario to crisis times contributes to the change in the Expected Shortfall of the financial sector as a whole. *Celta CoES* is the $\Delta CoES_{i|m}$ of each firm weighted by its market capitalization, i.e.,

$$Celta\ CoES_{i,t}(\beta) \quad = \quad \omega_{i,t-1} \Delta CoES_{i|m,t}(\beta), \tag{14}$$

The loading factor of *Celta CoES* is directly related to the concept of **Too Big To Fall (TFTB)**. The second component of *Celta CoES*, $\Delta CoES_{i|m,t}$ gathers the exposure of the firm i to changes in the financial sector. By itself it is not a systemic risk measure, as Adrian and Brunnermeier (2016) stated, $\Delta CoES_{i|m,t}$ may be useful as a tool for monitoring the exposure of firm i to the financial system. The higher is the exposure, the stronger is the link with the financial system, as a consequence, this part of the *Celta CoES* is related to the **Too Connected To Fall (TCTF)** concept. It gathers the non linearities between the financial system and a certain firm because it is focused in extreme quantiles of the firm. As a matter of fact, the measure produces a similar result to the one obtained by a stress test representing the interconnectedness in high quantile of losses.

The economic interpretation of *Celta CoES* is direct assuming that $CoES_{i|m,t}(\alpha_n, \beta)$ represents the procyclical nature of haircuts and collateral of the financial institution i. $CoES_{i|m,t}(\alpha_n, \beta)$ acts as a kind of capital buffer obtained in normal times, when liquidity is high. Then $\Delta CoES_{i|m}(\beta)$ would reflect the uncovered capital needs in financial firm i when the financial sector is suffering a crisis, i.e., individual undercapitalization to a common shock. Finally, the contribution of each individual undercapitalization to the common undercapitalization of the financial sector is given by weighting $\Delta CoES_{i|m,t}(\beta)$ by $\omega_{i,t-1}$, i.e., *Celta CoES*. Note that the common undercapitalization of the system is higher that $ES(\alpha_s)$ when we focus on the *Celta CoES*.

A further advantage of *Celta CoES* is the fact that it can be aggregated, allowing to gather firms of the same country or of the same sub-sector in order to compute its joint contribution to systemic risk. This feature is shared with other systemic risk as $SRISK$ or CES but not with $\Delta CoES_{m|i,t}$. Besides, neither $\Delta CoES_{m|i,t}$ nor $SRISK$ nor CES give enough weight to tail features of the financial firm.

3 Methodology

I use a standard methodology for the empirical exercise, where the time-varying parameters for variance and correlation is given by a GJR-GARCH-DCC model, similar to Benoit et al. (2013). The conditional mean is given by an ARMA(1,1) model.
Lets us consider the following process for the return of the institution j

$$r_{j,t} \quad = \quad \underbrace{\psi_{j,0} + \psi_{j,1} r_{j,t-1} + \theta_{j,1}\epsilon_{j,t-1}}_{\mu_{j,t}} + \varepsilon_{j,t}, \quad j = i, m \tag{15}$$

where $\epsilon_{j,t} = r_{j,t} - \psi_{j,0} + \psi_{j,1} r_{j,t-1}$ and $\varepsilon_t = (\varepsilon_{m,t}, \varepsilon_{i,t})'$ is a vector with zero mean, no autocorrelation and heterocedasticity with time-varying correlation defined by a DCC model (Engle 2002).
The volatility model follows a GJR-GARCH(1,1) model.

$$\sigma^2_{j,t} \quad = \quad \omega_j + (\alpha_i + \theta_j \mathbb{1}_{(\varepsilon_{j,t-1}<0)})\varepsilon^2_{j,t-1} + \beta_j \sigma^2_{j,t-1}, \quad j = i, m \tag{16}$$

where, ω_j is the long term variance component, β_j is the persistence parameter of the past variance on the market and the financial firm, α_j accounts for the influence of news returns in the variance, θ_j gathers the leverage effect and $\mathbb{1}_{(\varepsilon_{j,t-1}<0)}$ is an indicator function that values one when $\varepsilon_{j,t-1} < 0$ and zero otherwise.
In order to model for time-varying serial correlation I employ the DCC model like Brownlees and Engle (2012). Let us define the conditional covariance matrix of ε_t as

$$E_{t-1}(\varepsilon_t \varepsilon_t') = \Sigma_t = \begin{pmatrix} \sigma^2_{m,t} & \sigma_{m,t}\sigma_{i,t}\rho_{im,t} \\ \sigma_{i,t}\sigma_{i,t}\rho_{im,t} & \sigma^2_{m,t} \end{pmatrix},$$

where $\varepsilon_t = (\varepsilon_{m,t}, \varepsilon_{i,t})'$ is the vector of zero mean returns from Equation (15), i.e. $\varepsilon_t = r_t - \mu_t$. The covariance matrix Σ_t can be rewritten as

$$\Sigma_t = D_t^{1/2} \Gamma_t D_t^{1/2}, \tag{17}$$

where $D_t = diag[\sigma^2_{m,t}, \sigma^2_{i,t}]$ is a diagonal matrix of returns variances and Γ_t is the conditional correlation matrix. Γ_t can directly be obtained from (17) as

$$E_{t-1}[\zeta_t \zeta_t'] = D_t^{-1/2} \Sigma_t D_t^{-1/2}, \tag{18}$$

where $\zeta_t = [\zeta_{m,t}, \zeta_{i,t}]'$ is the vector of standardized returns, i.e., $\zeta_t = D_t^{-1/2}(r_t - \mu_t)$. Let us define Q_t as the positive definite pseudo-covariance matrix with typical elements $q_{jj,t}$ such that the correlation estimation $\rho_{im,t} = \frac{q_{im,t}}{\sqrt{q_{ii,t} q_{mm,t}}}$. A parametrization of Q_t as a multivariate GARCH(p,q), which means as a function of information set at time $t-1$, allows each element of Γ_t to depend on q lagged of the squares and cross-product of ζ_t, as well as p lagged values of Q_t. Assuming a GARCH(1,1) model, i.e.,

$$Q_t = S(1 - \alpha - \beta) + \alpha \left(\zeta_{t-1}\zeta_{t-1}'\right) + \beta Q_{t-1}, \tag{19}$$

where S is the unconditional correlation matrix of the firm and the market return.
I employ the two-step method of Inference function for Margin (IFM) to estimate the parameters by maximum log-likelihood, where marginal distributions and copulas are estimated separately. The computational cost of finding the optimal set of parameters reduces significantly under this approach, Joe and Xu (1996) show that the estimated parameters using IFM method are consistent and asymptotically normal.

3.1 Benchmark framework: Gaussian assumption

The benchmark distribution framework is Gaussian. In this framework, the systemic risk measures have closed formulae. Proof of the following results can be checked in Appendices

Marginal Expected Shortfall

$$MES_{i,t}(\alpha) = \frac{\sigma_{i,t}\rho_t\phi\left(\Phi^{-1}(\alpha)\right)}{\alpha} - \mu_{i,t}.$$

Conditional Expected Shortfall and Δ CoES

$$CoES_{i,t}(\alpha,\beta) = \sigma_{i,t}\left(\sqrt{1-\rho_t^2}\frac{\phi\left(\Phi^{-1}(\beta)\right)}{\beta} + \rho_t\frac{\phi(\Phi(\alpha))}{\alpha}\right) - \mu_{i,t},$$

$$\Delta CoES_{i,t}(\beta) = \sigma_{i,t}\rho_t\left(\frac{\phi(\Phi(\alpha_s))}{\alpha_s} - \frac{\phi(\Phi(\alpha_n^+)) - \phi(\Phi(\alpha_n^-))}{\alpha_n^+ - \alpha_n^-}\right),$$

where $\alpha_s = \beta$, $\alpha_n^+ = 0.5 + \beta/2$ and $\alpha_n^- = 0.5 - \beta/2$.

3.2 Alternative framework: Student t copula and skewed-t marginal distributions

Higher moments, i.e. kurtosis and asymmetry, and joint tail dependence may arise important differences in systemic risk measures. Consequently, I employ an alternative distribution where the marginal distribution is a Hansen (1994)'s Skewed t marginal and a Student t copula is employed for building the joint distribution. It suits kurtosis, asymmetry and tail dependence.

The probability distribution function of the Student t is

$$f_\nu(x) = \frac{\Gamma\left(\frac{(\nu+1)}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)\sqrt{\nu\pi}}\left(1+\frac{x^2}{\nu}\right)^{-\frac{(\nu+1)}{2}},$$

and its copula density function is

$$c(s,q;\nu) = \frac{\Gamma\left(\frac{(\nu+1)}{2}\right)\Gamma\left(\frac{\nu}{2}\right)}{\Gamma\left(\frac{\nu+1}{2}\right)^2(1-\rho^2)^{1/2}}\left(1+\frac{x^2+y^2-2\rho xy}{(\nu)(1-\rho^2)}\right)^{-\frac{(\nu+2)}{2}}\left(1+\frac{x^2}{\nu}\right)^{\frac{\nu+1}{2}}\left(1+\frac{y^2}{\nu}\right)^{\frac{\nu+1}{2}},$$

where $x = t^{-1}(s;\nu)$ and $y = t^{-1}(q;\nu)$ where t^{-1} is the inverse cumulative t distribution function with ν number of degrees of freedom. Finally, according to Roncalli (2002) and (Cherubini et al., 2004, p. 117) the conditional t copula is

$$C_{2|1}(u_2,u_1;\rho,\nu) = t_{\nu+1}\left(\sqrt{\frac{\nu+1}{\nu+t_\nu^{-1}(u_1)^2}}\frac{t_\nu^{-1}(u_2)-\rho t_\nu^{-1}(u_1)}{\sqrt{1-\rho^2}}\right), \tag{20}$$

where t_ν stands for the cumulative t student distribution function and t_ν^{-1} is its inverse.

Regarding Hansen (1994)'s skewed t distribution, its density function is

$$h(\xi_t|\eta,\lambda) = \begin{cases} bc(1+\frac{1}{\eta-2}(\frac{b\xi_t+a}{1-\lambda})^2)^{-(\eta+1)/2} & \xi_t < -a/b \\ bc(1+\frac{1}{\eta-2}(\frac{b\xi_t+a}{1+\lambda})^2)^{-(\eta+1)/2} & \xi_t \geq -a/b \end{cases}, \tag{21}$$

where $2 < \eta < \infty$ and $-1 < \lambda < 1$. The constants a, b and c are given by

$$a = 4c\lambda\left(\frac{\eta-2}{\eta-1}\right), b = \sqrt{1+3\lambda^2-a^2}, c = \frac{\Gamma(\frac{\eta+1}{2})}{\sqrt{\pi(\eta-2)}\Gamma(\frac{\eta}{2})}.$$

Note that when $\lambda = 0$ Equation (21) reduces to the standard Gaussian distribution as $\eta \to \infty$. When $\lambda = 0$ and η finite, we obtain the standardized symmetric-t distribution.

Marginal Expected Shortfall

$$MES_{i,t}(\alpha) = \int_0^1 -(\mu_{i,t} + \sigma_{i,t}F_i^{-1}(s)) \frac{\overbrace{C_{m|i}(\alpha|s)}^{P(F_{i,t}(r_{i,t})=s|r_{m,t}<VaR_{m,t}(\alpha))}}{\alpha} ds$$

where $C_{m|i}$ is defined in Equation (20) and F_i^{-1} is the inverse cumulative function of i's marginal distribution.

Conditional Expected Shortfall The $CoES$ formula in the alternative framework would be

$$CoES_{i|m,t}(\alpha,\beta) = \frac{1}{\beta}\int_0^{s^*} \frac{C_{m|i}(\alpha|s)}{\alpha}(F_{i,t}^{-1}(s)\sigma_{i,t} + \mu_{i,t}) \quad ds,$$

where s^* is such that $P(F_{i,t}(r_{i,t}) < s^*|r_{m,t} < VaR_{m,t}(\alpha)) = \beta$ and $C_{m|i}$ is defined in Equation (20). The value of s^* can be found using copulas and the Bayes' Theorem as

$$\frac{1}{\alpha}\int_0^{s^*} C_{m|i,t}(\alpha|s)ds = \beta.$$

Note that $\frac{C_{m|i}(\alpha|s)}{\alpha} = P(F_{i,t}(r_{i,t}) = s|r_{m,t} < VaR_{m,t}(\alpha))$.

4 Data

I employ European financial market data from Datastream to perform an empirical exercise. I choose Euro zone financial institutions from September 2006 to the September 2016 on a weekly basis, filtering those institutions that are not enough liquid. There are a total of 201 financial institutions where 48 are banks, 75 are firms related to Real Estate, 19 are insurance firms and 59 are financial services oriented firms. Apart from the quotation of each institution I download also market capitalization for each institution from Datastream and annual total

debt from Thomson Reuters Worldscope.
Tables 1a, 1b and 1c show the considered banks gathered by country.

[Insert Table 1a here]
[Insert Table 1b here]
[Insert Table 1c here]

5 Results

Concerning cross-sectional sort, I assess at each moment t the ranking according to different measure ($SRISK$, MES, CES, $\Delta CoES_{i|m,t}$, $\Delta CoES_{m|i,t}$, capitalization share, β) and I compute the rank correlation of *Celta CoES* ranking at each moment t. Figure 3 presents on the left graph the rank correlation under Gaussian assumption and on the right graph under Student t copula and skewed-t marginals assumption. Risk measures are assessed using $\beta = 0.1$. *Celta CoES* has a rank correlation around 0.6-0.7 with the market risk measures (β, MES as was stated by Benoit et al. (2013) and $\Delta CoES_{i|m}$ as a exposure measure according to Adrian and Brunnermeier (2016)). The rank correlation is low with $SRISK$, probably because of the inclusion of accountant data in the later measure. The use of capitalization share (ω) in CES can explain the high rank correlation between both measures, which slightly decrease when we get away from Gaussian framework. The influence of the $TBTF$ firms may also explain the rank correlation between 0.85-0.7 with the measure capitalization. Finally, the rank correlation with $\Delta CoES_{m|i}$ is the highest correlation with a measure that does not use a size factor in its formula, which means that *Celta CoES* is giving a ranking of systemic risk that not only gathers size but also interconnectedness. To check if *Celta CoES* identifies properly the $SIFI$s, I check the common institutions that each measure shares with *Celta CoES* in the highest decile. The percentage of common institutions with β and MES criteria is practically the same, around a 60%. The evolution of common institutions with $\Delta CoES_{i|m}$ seems to present two peaks around 2009 and 2012. Those peaks are clearly identify when the common institutions are compared to $SRISK$ top 20. Finally, *Celta CoES* presents a high similarity on the top 20 of $\Delta CoES_{m|i}$ and the market capitalization, good proxies of $TCTF$ and $TBTF$.

[Insert Figure 3 here]
[Insert Figure 4 here]

To identify how the decomposition of the change in the ES of the system following Equation 12 is split between *Celta CoES* and the $\Delta CoReS$ weighted by its size, Figures 5a (pre-crisis), 5b (after Lehman Brother's failure), 5c (during the European sovereign debt crisis) and 5d (after Mario Draghi's speech on July 26^{th}, 2012) show this information in different time periods for a set of nine financial institutions (Allianz, Commerzbank, BBVA, Sabadell, Mapfre, BNP Paribas, Natixis, Intesa Sanpaolo, Mediobanca).
Top bar figure presents the decomposition of the change in the Expected Shortfall of the financial systems when it moves from normal to distress scenario showing at the top the value of both components. The evolution of some institutions as Allianz, Commerzbank, BBVA, BNP Paribas and Intesa Sanpaolo is less steady than for instance Sabadell, Mapfre or Mediobanca. Bottom left bar graph isolate each component and assess the importance of each component

taking as a percentage of the sum of the total, i.e. blue bars represent $Celta\ CoES_i$ as $\frac{Celta\ CoES_i}{\sum_{i=1}^{N} Celta\ CoES_i}100\%$ and red bars represent $\omega_i \Delta CoRes_{i|m}$ as $\frac{\omega_i \Delta CoRes_{i|m}}{\sum_{i=1}^{N} \omega_i \Delta CoRes_{i|m}}100\%$, while on the top of each measure is the ranking according to $Celta\ CoES_i$ or $\omega_i \Delta CoRes_{i|m}$, the higher is the discrepancy, the more information is given $Celta\ CoES$. Finally, bottom right bar graph presents the $\omega_i \Delta CoES_{i|m,t}(\alpha_s, \beta)$ and the blue section shows the subtrahend of $CeltaCoES$, i.e., $\omega_i \Delta CoES_{i|m,t}(\alpha_n, \beta)$, for some institutions as Mediobanca or Sabadell represents around the half of $\omega_i \Delta CoES_{i|m,t}(\alpha_s, \beta)$.

[Insert Figure 5a here]
[Insert Figure 5b here]
[Insert Figure 5c here]
[Insert Figure 5d here]

The top 5 $SIFI$s according different indicators at those moments are presented in tables 2a and 2b, highlighting the presence of some common institutions as Unicredit, Santander or BNP Paribas across the diverse risk measures.

[Insert Table 2a here]
[Insert Table 2b here]

This decomposition of the change in the Expected Shortfall of the market is presented in a time-varying perspective in the set of figures 6b, 7b, 8b, 9b, 10b, 11b, 12b, 13b and 14b. The top figures point out the fact that the times-series evolution of each component of the change in the ES may be quite diverse depending if we are measuring in absolute value or as a percentage of the cross-section, see for instance Figure 6b concerning Allianz and Figure 7b regarding Commerzbank. Bottom figures show the decomposition of $\Delta CoES_{i|m,t}$ weighted (right) or non-weighted (left) by firm size.

[Insert Figure 6b here]
[Insert Figure 7b here]
[Insert Figure 8b here]
[Insert Figure 9b here]
[Insert Figure 10b here]
[Insert Figure 11b here]
[Insert Figure 12b here]
[Insert Figure 13b here]
[Insert Figure 14b here]

The last set of figures point out differences in the time-series of $Celta\ CoES$ and CES that may not be appreciated in the cross section due to the influence of size factor. Besides the different time-series of $Celta$ and CES, it should be highlighted the change that is produced in the $CeltaCoES$ when we move from the benchmark framework to the alternative one while CES is not modified, as can be seen for instance in the bottom left and centre plots of figure 9a for Sabadell.

[Insert Figure 6a here]
[Insert Figure 7a here]
[Insert Figure 8a here]
[Insert Figure 9a here]
[Insert Figure 10a here]
[Insert Figure 11a here]
[Insert Figure 12a here]
[Insert Figure 13a here]
[Insert Figure 14a here]

Figure 15 and 16 aggregate systemic risk measures depending on the country or subsector. *Celta CoES* presents a slightly higher presence of Irish and Greek financial firms during 2012-2015 compared to CES while $SRISK$ gives quite different weights to each country. $SRISK$ shows clearly its impossibility to give a reasonable weight to non-bank sectors as it is shown in figure 16.

[Insert Figure 15 here]
[Insert Figure 16 here]

6 Conclusion

The size and the interconnectedness factor are the two main elements that determine the Systemically Importance Financial Institutions ($SIFI$s). These two factors are linked with the Too-Big-To-Fail ($TBFT$) and the Too-Connected-To-Fail ($TCTF$) concepts respectively.

I propose a systemic risk measure named *Celta CoES* that merges these two components highlighting the role of higher moments, i.e. skewness and kurtosis, to explain systemic risk. A systemic event might be imminent if institutions present extreme events more frequently than usually, and adverse scenarios arise more often than good ones. Further advantages of *Celta CoES* is the fact of being aggregated across institutions as a results of its intitutive link with the decomposition of the change in market losses due to the economic cycle.

Results from an empirical exercise using European financial data during the last 10 years address some considerations about *Celta CoES* and its relationship with other systemic risk measures.
First, *Celta CoES* presents a high rank correlation with size and interconnectedness proxies apart of similarities around 80% in the common institutions on the highest decile of $SIFI$. This result points out the accuracy of *Celta CoES* to combine the $TBTB$ and the $TCTF$ concepts in one measure.
Second, concerning the connection with other systemic risk measures, *Celta CoES* presents a high rank correlation with CES which is explained by the common factor size in their formulae. However, once the size factor is taking into account, i.e., considering MES, correlation with *Celta CoES* decreases sharply, having then a similar rank correlation as the one if institutions were sorted by β. This is not surprising given the relationship between MES and β pointed out by Benoit et al. (2013) and Benoit et al. (2017) in a Gaussian context. The differences between *Celta CoES* and CES increases when the framework is changed to a one

where there are tail dependence and non-Gaussian higher moments for financial institutions, specially in the time-series. Conceptually, CES and *Celta* $CoES$ express different features of systemic risk. More precisely, CES is focused on the contribution of each firm to the Expected Shortfall of the financial system while *Celta* $CoES$ gathers also dependence and the own tail behaviour of firms, so it is not unexpected that the trend is different when we settle down a more sophisticated framework, i.e. non-Gaussian. Regarding the relationship *Celta* $CoES$- $SRISK$, it presents two peaks on the top 20 common $SIFI$ that correspond to the 2008 crisis and the European sovereign debt crisis. Furthermore, *Celta* $CoES$ does not undervalue the role of non-banking sectors to trigger a potential financial crisis, whereas $SRISK$ gives an excessive weight to the role of banking system due to leverage component in its formula.

The usefulness of *Celta* $CoES$ lies on a proper model that considers the differential features where this measure gives relevant information, i.e. tail dependence, kurtosis, skewness. A methodology where the joint and marginal tail behaviour is correctly described is a *conditio sine qua non* for the significance of *Celta* $CoES$ in the set of systemic risk measures. Recently, Zhou (2009) and Hartmann et al. (2007) have employed an Extreme Value Theory (EVT) framework to estimate systemic risk measures. The challenge associated with a state-of-the-art methodology is the model risk, which should not be underestimated. *Celta* $CoES$ tracks market sentiments in real-time allowing to act as measure to identify $SIFI$s because of being a high-frequency measure. However, *Celta* $CoES$ may be also too procyclical to be employed for establishing countercyclical capital requirements. A forward looking systemic risk measure may be obtain by doing a Long Run *Celta* $CoES$ via simulation as Brownlees and Engle (2016) does with the $LRMES$ or using accountant data to forecast future values as Adrian and Brunnermeier (2016) does with the *forward* $\Delta CoVaR$ alleviating the procyclical behaviour.

Policy makers need indicators for identify SIFIs, in order to manage their behaviour and prevent from harmful effects in the real economy. *Celta* $CoES$ is a measure that combines $TBFT$ and $TCTF$ features focusing in the events in the upper tail of losses, those more pernicious that can potentially leads to a systemic event. The application of this measure has practical implications on macroprudential policy and stress testing allowing to identify the importance of the financial firms based on its exposure at high quantile of losses to changes in the financial system and its feedback effect to the system.

References

Acerbi, C. and Tasche, D. On the coherence of Expected Shortfall. *Journal of Banking and Finance*, 26:1487–1503, 2002.

Acharya, V., Engle, R., and Richardson, M. Capital shortfall: A new Approach to Ranking and Regulating Systematic Risks. *American Economic Review*, 102(3):59–64, 2012.

Adrian, T. and Brunnermeier, M. K. CoVaR. *The American Economic Review*, 106(7): 1705–1741, 2016.

Artzner, P., Delbaen, F., Eber, J.-M., and Heath, D. Coherent measures of risk. *Mathematical finance*, 9(3):203–228, 1999.

Banulescu, G.-D. and Dumitrescu, E.-I. Which are the sifis? a component expected shortfall approach to systemic risk. *Journal of Banking & Finance*, 50:575–588, 2015.

Benoit, S., Colletaz, G., Hurlin, C., and Prignon, C. A Theoretical and Empirical Comparison of Systemic Risk Measures. Working Papers halshs-00746272, HAL, June 2013.

Benoit, S., Colliard, J.-E., Hurlin, C., and Pérignon, C. Where the risks lie: A survey on systemic risk. *Review of Finance*, 21(1):109–152, 2017.

Bernanke, B. Financial Reform to Address Systemic Risk, 2009. Speech delivered at the Council on Foreign Relations.

Bernanke, B. Causes of the Recent Financial and Economic Crisis, 2010. Statement of the FED Chairman before the Financial Crisis Inquiry Commission.

Boot, A. W. A. and Ratnovski, L. Banking and Trading. CEPR Discussion Papers 9148, C.E.P.R. Discussion Papers, September 2012.

Brownlees, C. and Engle, R. F. Srisk: A conditional capital shortfall measure of systemic risk. *The Review of Financial Studies*, 30(1):48–79, 2016.

Brownlees, C. T. and Engle, R. Volatility, correlation and tails for systemic risk measurement. *Available at SSRN*, 1611229, 2012.

Cherubini, U., Luciano, E., and Vecchiato, W. *Copula methods in finance*. John Wiley & Sons, 2004.

Constâncio, V. Macroprudential stress-tests and tools for the non-bank sector, 2017. ESRB Annual Conference.

ECB. New quantitative measures of systemic risk. Technical report, Financial Stability Review, 12 2010a.

ECB. Analytical models and tools for the identification and assessment of systemic risks. Technical report, Financial Stability Review, 06 2010b.

Engle, R. Dynamic conditional correlation: a simple class of multivariate generilized autoregressive conditional heteroskedasticity models. *American Statistical Association Journal of Business and Economic stastistics*, 20(3):339–350, 2002.

Engle, R., Jondeau, E., and Rockinger, M. Systemic risk in Europe. *Review of Finance*, 19 (1):145–190, 2015.

Farhi, E. and Tirole, J. Collective Moral Hazard, Maturity Mismatch, and Systemic Bailouts. *American Economic Review*, 102(1):60–93, February 2012.

Girardi, G. and Ergün, A. T. Systemic risk measurement: Multivariate GARCH estimation of CoVaR. *Journal of Banking and Finance*, 37(8):3169–3180, 2013.

Guntay, L. and Kupiec, P. Taking the risk out of systemic risk measurement. Technical report, American Enterprise Institute (AEI) Research Paper, 01 2014.

Hansen, B. E. Autoregressive conditional density estimation. *International Economic Review*, pages 705–730, 1994.

Hartmann, P., Straetmans, S., and de Vries, C. Banking System Stability. A Cross-Atlantic Perspective. In *The Risks of Financial Institutions*, pages 133–192. National Bureau of Economic Research, 2007.

Holling, C. S. Resilience and stability of ecological systems. *Annual review of ecology and systematics*, 4(1):1–23, 1973.

International Monetary Fund, Bank for International Settlements, and Financial Stability Board. Guidance to assess the systemic importance of financial institutions, markets and instruments: Initial considerations: Report to the g-20 finance ministers and central bank governors. *Staff of the International Monetary Fund and the Bank for International Settlements, and the Secretariat of the Financial Stability Board*, 2010.

Joe, H. and Xu, J. J. The estimation method of inference functions for margins for multivariate models. Technical report, Department of Statistics, University of British Columbia, 1996.

Kleinow, J., Moreira, F., Strobl, S., and Vhmaa, S. Measuring systemic risk: A comparison of alternative market-based approaches. *Finance Research Letters*, 21:40 – 46, 2017.

Laeven, L., Ratnovski, L., and Tong, H. Bank Size and Systemic Risk. IMF Staff Discussion Notes 14/4, International Monetary Fund, May 2014.

Löffler, G. and Raupach, P. Pitfalls in the use of systemic risk measures. *Journal of Financial and Quantitative Analysis*, 2017.

Mainik, G. and Schaanning, E. On dependence consistency of covar and some other systemic risk measures. *Statistics and Risk Modeling*, 31(1):49–77, 2014.

Roncalli, T. Gestion des risques multiples ou copules et aspects multidimensionnels du risque. Technical report, Groupe de Recherche Opérationnelle-Crédit Lyonnais, 2002.

Rose, A. and Wieladek, T. Too big to fail: some empirical evidence on the causes and consequences of public banking interventions in the United Kingdom. Bank of England working papers 460, Bank of England, August 2012.

Salleo, C., Homar, T., and Kick, H. Making sense of the EU wide stress test: a comparison with the SRISK approach. Working Paper Series 1920, European Central Bank, June 2016.

Scott, H. S., Ricci, K., and Sarfatti, A. SRISK as a Measure of Systemic Risk for Insurers: Oversimplified and Inappropriate. Technical report, Harvard Law School Working Paper, 2016.

Shleifer, A. and Vishny, R. W. Unstable Banking. NBER Working Papers 14943, National Bureau of Economic Research, Inc, May 2009.

Trichet, J.-C. Clare Distinguished Lecture in Economics and Public Policy, 2009. Speech at University of Cambridge organised by the Clare College.

Zhou, C. Are banks too big to fail? DNB Working Papers 232, Netherlands Central Bank, Research Department, December 2009.

Appendices

A Tables

Table 1a: Banks of the empirical exercise

AUSTRIA	14	CYPRUS	11
ATRIUM EUROPEAN RLST.	Real Estate	APOLLO INVESTMENT FUND PUBLIC	Financial Services
BK.FUR TIROL UND VBG.	Bank	ATLANTIC INSURANCES	Real Estate
BKS BANK	Bank	CPI HOLDINGS PUBLIC	Financial Services
CA IM.ANLAGEN	Real Estate	DEMETRA INVESTMENT	Financial Services
ERSTE GROUP BANK	Bank	ELLINAS FINANCE	Financial Services
IMMOFINANZ	Real Estate	HELLENIC BANK	Bank
OBERBANK	Bank	JUPITER PORTFOLIO INVS.	Financial Services
OBERBANK PREF.	Bank	K & G COMPLEX	Real Estate
RAIFFEISEN BANK INTL.	Bank	PANDORA INVS.	Real Estate
S IMMO	Real Estate	TRIENA INV.(CAPITAL)	Financial Services
UBM DEVELOPMENT	Real Estate	WOOLWORTH(CYPRUS) PROPS.	Real Estate
UNIQA INSU GR AG	Insurance	**GERMANY**	**30**
UNTERNEHMENS INVEST	Financial Services	AAREAL BANK	Real Estate
VIENNA INSURANCE GROUP A	Insurance	ADLER REAL ESTATE	Real Estate
BELGIUM	**27**	ALLIANZ	Insurance
ACKERMANS & VAN HAAREN	Financial Services	AURELIUS SE & CO.KGAA	Financial Services
AGEAS (EX-FORTIS)	Insurance	CAPITAL STAGE	Financial Services
ATENOR	Real Estate	COMDIRECT BANK	Financial Services
BANQUE NALE.DE BELGIQUE	Bank	COMMERZBANK	Bank
BEFIMMO	Real Estate	DEUTSCHE BANK	Bank
BREDERODE	Financial Services	DEUTSCHE BETEILIGUNGS	Financial Services
CARE PROPERTY INV	Real Estate	DEUTSCHE BOERSE	Financial Services
COFINIMMO	Real Estate	DEUTSCHE EUROSHOP	Real Estate
CIE.DU BOIS SAUVAGE	Financial Services	DEUTSCHE WOHNEN BR.SHS.	Real Estate
GBL NEW	Financial Services	DIC ASSET	Real Estate
GIMV	Financial Services	EUWAX	Financial Services
HOME INVEST BELGIUM	Real Estate	HAMBORNER REIT	Real Estate
IMMOBEL	Real Estate	HANNOVER RUCK.	Insurance
INTERVEST OFFICES & WAREHOUSES REIT	Real Estate	HSBC TRINKAUS & BURKHD.	Financial Services
KBC ANCORA	Bank	MBB	Financial Services
KBC GROUP	Bank	MLP	Financial Services
LEASINVEST	Real Estate	MUENCHENER RUCK.	Insurance
QUEST FOR GROWTH	Financial Services	NUERNBERGER BETS.	Insurance
RETAIL ESTATES	Real Estate	OLDENBURGISCHE LB.	Bank
SOFINA	Financial Services	PATRIZIA IMMOBILIEN	Real Estate
TEXAF	Financial Services	SEDLMAYR GRUND & IM.	Real Estate
VASTNED RETAIL BEL REIT	Real Estate	STINAG STUTTGART INVEST	Financial Services
WAREHOUSES REITS	Real Estate	TAG IMMOBILIEN	Real Estate
WDP	Real Estate	VIB VERMOEGEN	Real Estate
WERELDHAVE BELGIUM CVA REIT	Real Estate	WCM BETS.-UND GRUNBSZ.	Real Estate
WOLUWE EXTENS	Real Estate	WESTGRUND	Real Estate
WOLUWE SHOPPING	Real Estate	WUESTENROT & WUERTT.	Financial Services

Table 1b: Banks of the empirical exercise

SPAIN	**13**
ALANTRA PARTNERS	Financial Services
BBV.ARGENTARIA	Bank
BANCO DE SABADELL	Bank
BANCO SANTANDER	Bank
BANKINTER R	Bank
BOLSAS Y MERCADOS ESPANOLES	Financial Services
CEVASA	Real Estate
CORPORACION FINCA.ALBA	Financial Services
GRUPO CATALANA OCCIDENTE	Real Estate
INMOBILIARIA COLONIAL	Real Estate
INMOBILIARIA DEL SUR LIMITED DATA	Real Estate
MAPFRE	Insurance
QUABIT INMOBILIARIA	Real Estate
FINLAND	**3**
CITYCON	Real Estate
SAMPO A	Real Estate
TECHNOPOLIS	Real Estate
GREECE	**11**
ALPHA ASTIKA AKINITA	Real Estate
ALPHA BANK	Bank
BANK OF GREECE	Bank
BANK OF PIRAEUS	Bank
EUROBANK ERGASIAS	Bank
EUROPEAN REL.GEN.INS.CR	Real Estate
GRIVALIA PROPERTIES REIC	Real Estate
HELLENIC EXCHANGES HDG.	Financial Services
LAMDA DEVELOPMENT	Real Estate
MARFIN INV.GP.HDG.	Financial Services
NATIONAL BK.OF GREECE	Bank
IRELAND	**5**
AIB GROUP	Bank
BANK OF IRELAND GROUP	Bank
FBD HOLDINGS	Insurance
IFG GROUP	Financial Services
PERMANENT TSB GHG.	Bank

FRANCE	**39**
ABC ARBITRAGE	Financial Services
ALTAMIR	Financial Services
ALTAREA	Real Estate
ALTAREIT	Real Estate
ANF IMMOBILIER	Real Estate
APRIL	Insurance
ARTOIS INDFIN.DE LARTO.	Financial Services
AXA	Insurance
BNP PARIBAS	Bank
CAMBODGE (CIE DU)	Financial Services
CARMILA	Real Estate
CEGEREAL REIT	Real Estate
CNP ASSURANCES	Insurance
CRCAM NORD DE FRANCE CCI	Bank
CREDIT AGRICOLE	Bank
CREDIT AGR.ILE DE FRANCE	Bank
CREDIT FONCIER DE MONACO	Bank
EULER HERMES GROUP	Real Estate
FIDUCIAL REAL ESTATE	Real Estate
FONCIERE DES MURS	Real Estate
FONCIERE DES REGIONS	Real Estate
GECINA REIT	Real Estate
ICADE REIT	Real Estate
IDI	Financial Services
KLEPIERRE	Real Estate
MERCIALYS REIT	Real Estate
MONCEY FINANCIERE	Financial Services
NATIXIS	Bank
NEXITY	Real Estate
ROTHSCHILD & CO	Financial Services
SCOR SE	Insurance
SOCIETE FONC.LYONNAISE	Real Estate
SC.FONFNC.ET DE PARTS.	Financial Services
SOCIETE GENERALE	Bank
TOUR EIFFEL	Real Estate
UNION FINC.FRANC.	Financial Services
VIEL ET CIE	Financial Services
WENDEL	Financial Services
UNIBAIL-RODAMCO SE REIT	Real Estate
UNITED KINGDOM	**2**
AP ALTERNAT ASSETS	Financial Services
EUROCASTLE INV.	Financial Services

Table 1c: Banks of the empirical exercise

ITALY	24	NETHERLANDS	19
ASSICURAZIONI GENERALI	Insurance	AEGON	Insurance
AZIMUT HOLDING	Financial Services	BEVER HOLDING	Real Estate
BANCA CARIGE	Bank	BINCKBANK	Financial Services
BANCA IFIS	Financial Services	EUROCOMMERCIAL	Real Estate
BANCA MEDIOLANUM	Insurance	EUROPEAN ASSET TRUST	Financial Services
BANCA MONTE DEI PASCHI	Bank	GROOTHANDELSGEB.	Real Estate
BANCA PPO.DI SONDRIO	Bank	HAL TRUST	Financial Services
BANCO BPM	Bank	ING GROEP	Bank
BNC.DI DESIO E DELB.	Bank	KARDAN N V	Real Estate
BENI STABILI	Real Estate	KAS BANK	Financial Services
BPER BANCA	Bank	NSI	Real Estate
CATTOLICA ASSICURAZIONI	Insurance	ROBECO DH EUR ICVC	Financial Services
CREDITO EMILIANO	Bank	ROBECO GLOBAL STAR	Financial Services
DEA CAPITAL	Financial Services	ROLINCO	Financial Services
IMMOBILIARE GRDE. DTBZ. SO.DI INVM.IMMB.	Real Estate	ROLINCO 6.5% CUM.PF.	Financial Services
INTESA SANPAOLO	Bank	V LANSCHOT KEMPEN	Bank
INTESA SANPAOLO RSP	Bank	VALUE8	Financial Services
MEDIOBANCA BC.FIN	Bank	VASTNED RETAIL	Real Estate
TAMBURI INV.PARTNERS	Financial Services	WERELDHAVE	Real Estate
UNICREDIT	Bank	**PORTUGAL**	**3**
UNIONE DI BANCHE ITALIAN	Bank	BANCO BPI	Bank
UNIPOL GRUPPO FINANZIARI	Insurance	BANCO COMR.PORTUGUES R	Bank
UNIPOLSAI	Real Estate	OREY ANTUNES	Financial Services
VITTORIA ASSICURAZIONI	Insurance		

Table 2a: Top 5 systemic risk institutions according to each risk measure

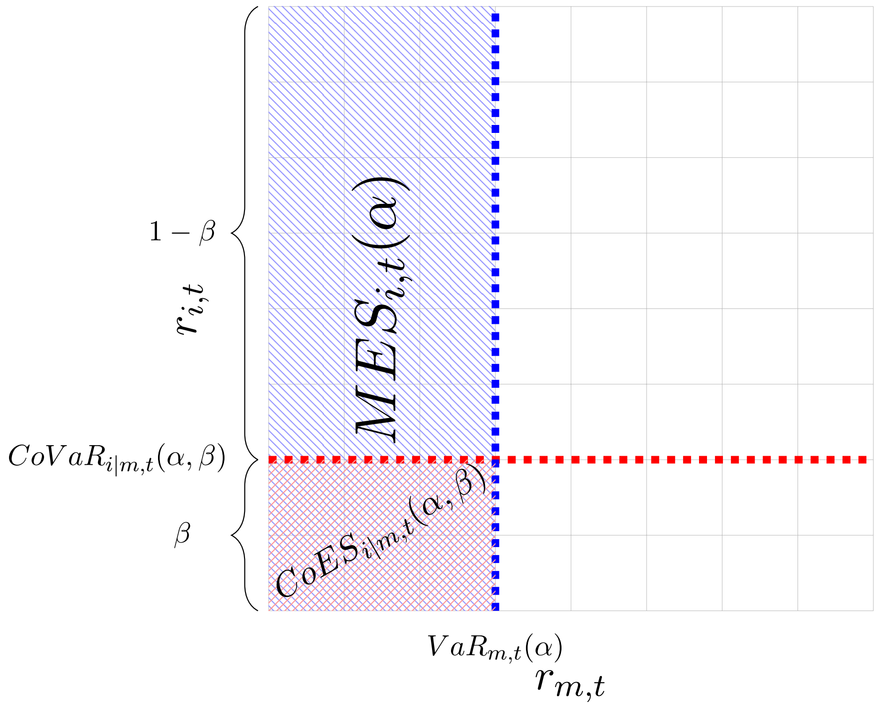
	SRISK	*MES*	*CES*	$\Delta CoES_{i\|m,t}$
			February 6^{th}*, 2008*	
#1	COMMERZBANK	QUABIT INMOBILIARIA	BANCO SANTANDER	QUABIT INMOBILIARIA
#2	DEUTSCHE BANK	WENDEL	ALLIANZ	WENDEL
#3	INTESA SANPAOLO RSP	AGEAS (EX-FORTIS)	AXA	AGEAS (EX-FORTIS)
#4	AGEAS (EX-FORTIS)	COMMERZBANK	UNICREDIT	INMOBILIARIA COLONIAL
#5	BANQUE NALE.DE BELGIQUE	AXA	ING GROEP	ATRIUM EUROPEAN RLST.
			November 12^{th}*, 2008*	
#1	DEUTSCHE BANK	AGEAS (EX-FORTIS)	BANCO SANTANDER	AGEAS (EX-FORTIS)
#2	AGEAS (EX-FORTIS)	IMMOFINANZ	AXA	IMMOFINANZ
#3	COMMERZBANK	BANK OF IRELAND GROUP	BBV.ARGENTARIA	EUROCASTLE INV.
#4	INTESA SANPAOLO RSP	KBC ANCORA	SOCIETE GENERALE	PERMANENT TSB GHG.
#5	UNICREDIT	ING GROEP	UNICREDIT	BANK OF IRELAND GROUP
			April 6^{th}*, 2011*	
#1	INTESA SANPAOLO RSP	PERMANENT TSB GHG.	BNP PARIBAS	PERMANENT TSB GHG.
#2	COMMERZBANK	AIB GROUP	BANCO SANTANDER	AIB GROUP
#3	NATIXIS	BANK OF IRELAND GROUP	ALLIANZ	BANK OF IRELAND GROUP
#4	BANK OF IRELAND GROUP	QUABIT INMOBILIARIA	AXA	QUABIT INMOBILIARIA
#5	AIB GROUP	AXA	DEUTSCHE BANK	WENDEL
			May 8^{th}*, 2013*	
#1	INTESA SANPAOLO RSP	ALPHA BANK	BNP PARIBAS	ALPHA BANK
#2	COMMERZBANK	NATIONAL BK.OF GREECE	BANCO SANTANDER	BANK OF PIRAEUS
#3	UNICREDIT	BANK OF PIRAEUS	DEUTSCHE BANK	EUROBANK ERGASIAS
#4	CREDIT AGRICOLE	EUROBANK ERGASIAS	BBV.ARGENTARIA	NATIONAL BK.OF GREECE
#5	NATIXIS	BANCO BPM	ALLIANZ	BANCO COMR.PORTUGUES R

Table 2b: Top 5 systemic risk institutions according to each risk measure

	$\Delta CoES_{m\|i,t}$	*Celta CoES*	Capitalization	β
			February 6^{th}*, 2008*	
#1	ING GROEP	ALLIANZ	BANCO SANTANDER	CAMBODGE (CIE DU)
#2	BBV.ARGENTARIA	BANCO SANTANDER	UNICREDIT	QUABIT INMOBILIARIA
#3	AXA	ING GROEP	BNP PARIBAS	AXA
#4	SOCIETE GENERALE	UNICREDIT	INTESA SANPAOLO	COMMERZBANK
#5	BANCO SANTANDER	AGEAS (EX-FORTIS)	ALLIANZ	WENDEL
			November 12^{th}*, 2008*	
#1	ALLIANZ	BANCO SANTANDER	BANCO SANTANDER	CAMBODGE (CIE DU)
#2	BBV.ARGENTARIA	BNP PARIBAS	BNP PARIBAS	AGEAS (EX-FORTIS)
#3	SOCIETE GENERALE	SOCIETE GENERALE	BBV.ARGENTARIA	IMMOFINANZ
#4	KBC ANCORA	BBV.ARGENTARIA	AXA	BANK OF IRELAND GROUP
#5	DEUTSCHE BANK	UNICREDIT	INTESA SANPAOLO	KBC ANCORA
			April 6^{th}*, 2011*	
#1	INTESA SANPAOLO	ALLIANZ	BANCO SANTANDER	CAMBODGE (CIE DU)
#2	BNP PARIBAS	BNP PARIBAS	BNP PARIBAS	PERMANENT TSB GHG.
#3	INTESA SANPAOLO RSP	BANCO SANTANDER	ALLIANZ	AIB GROUP
#4	AXA	ING GROEP	BBV.ARGENTARIA	BANK OF IRELAND GROUP
#5	UNICREDIT	AXA	DEUTSCHE BANK	QUABIT INMOBILIARIA
			May 8^{th}*, 2013*	
#1	BBV.ARGENTARIA	BNP PARIBAS	BANCO SANTANDER	CAMBODGE (CIE DU)
#2	BNP PARIBAS	AIB GROUP	BNP PARIBAS	ALPHA BANK
#3	SOCIETE GENERALE	DEUTSCHE BANK	ALLIANZ	BANK OF PIRAEUS
#4	ING GROEP	BANCO SANTANDER	BBV.ARGENTARIA	NATIONAL BK.OF GREECE
#5	AXA	ALLIANZ	DEUTSCHE BANK	EUROBANK ERGASIAS

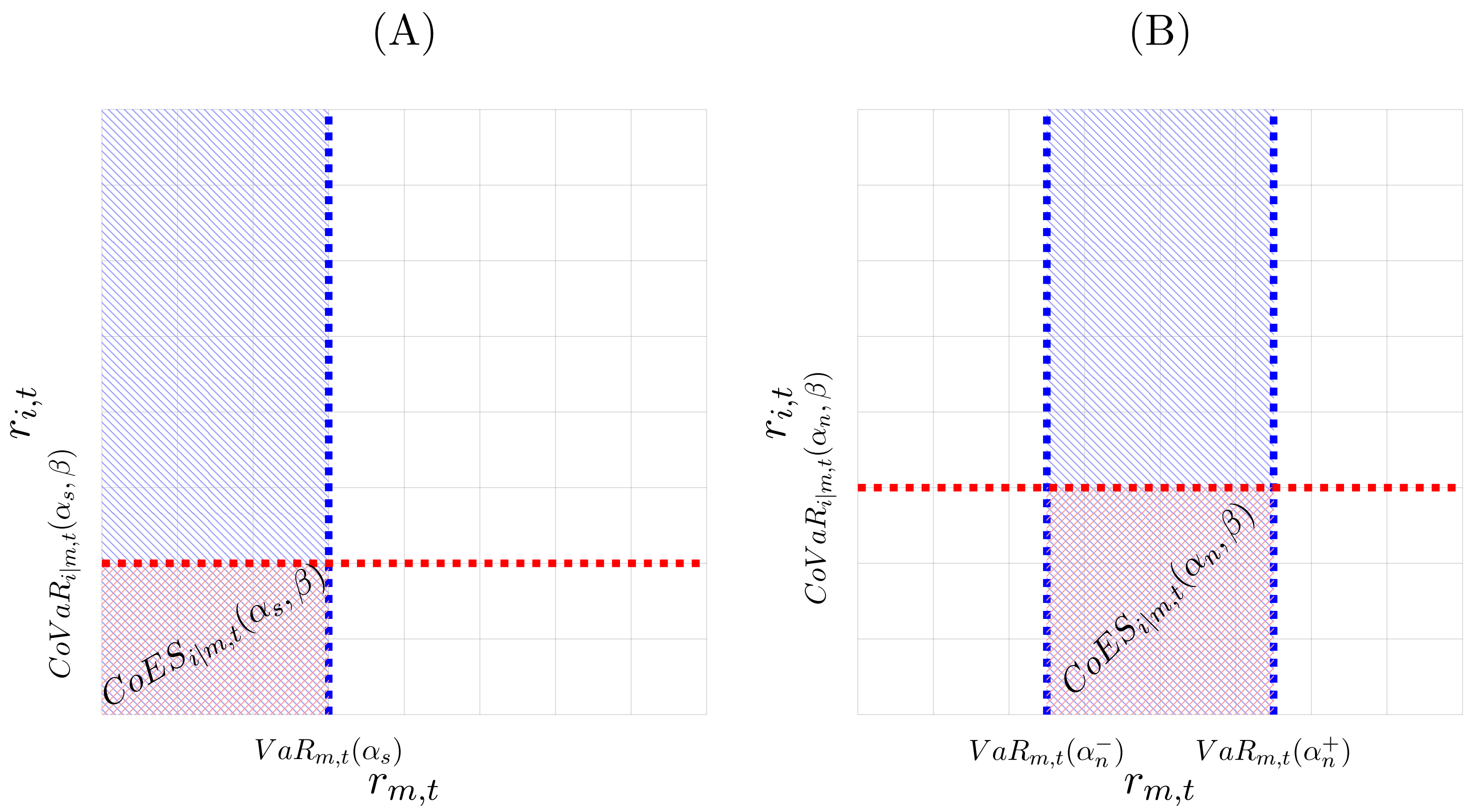
B Figures

Figure 1: Relationship between MES and $CoES$



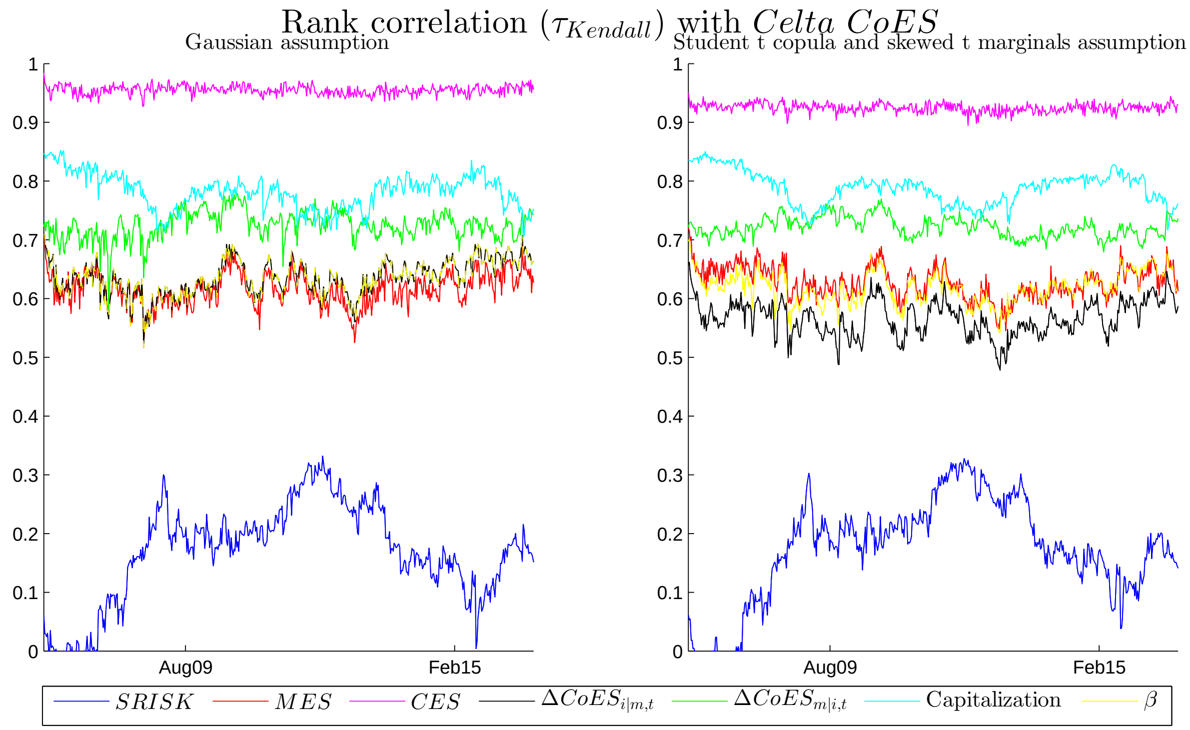
Striped blue rectangle shows $MES_{i,t}(\alpha)$ while the striped red rectangle shows $CoES_{i|m,t}(\alpha,\beta)$. Note that all the measures for institution i are built on the left of the threshold $VaR_{m,t}(\alpha)$ of the financial market. The Marginal Expected Shortfall can be divided in two means weighted by its probability. The threshold that divides both areas is the $CoVaR_{i|m,t}(\alpha,\beta)$ and the area weighted by a probability β would be the $CoES_{i|m,t}(\alpha,\beta)$.

Figure 2: $\Delta CoES$ minuend and subtrahend



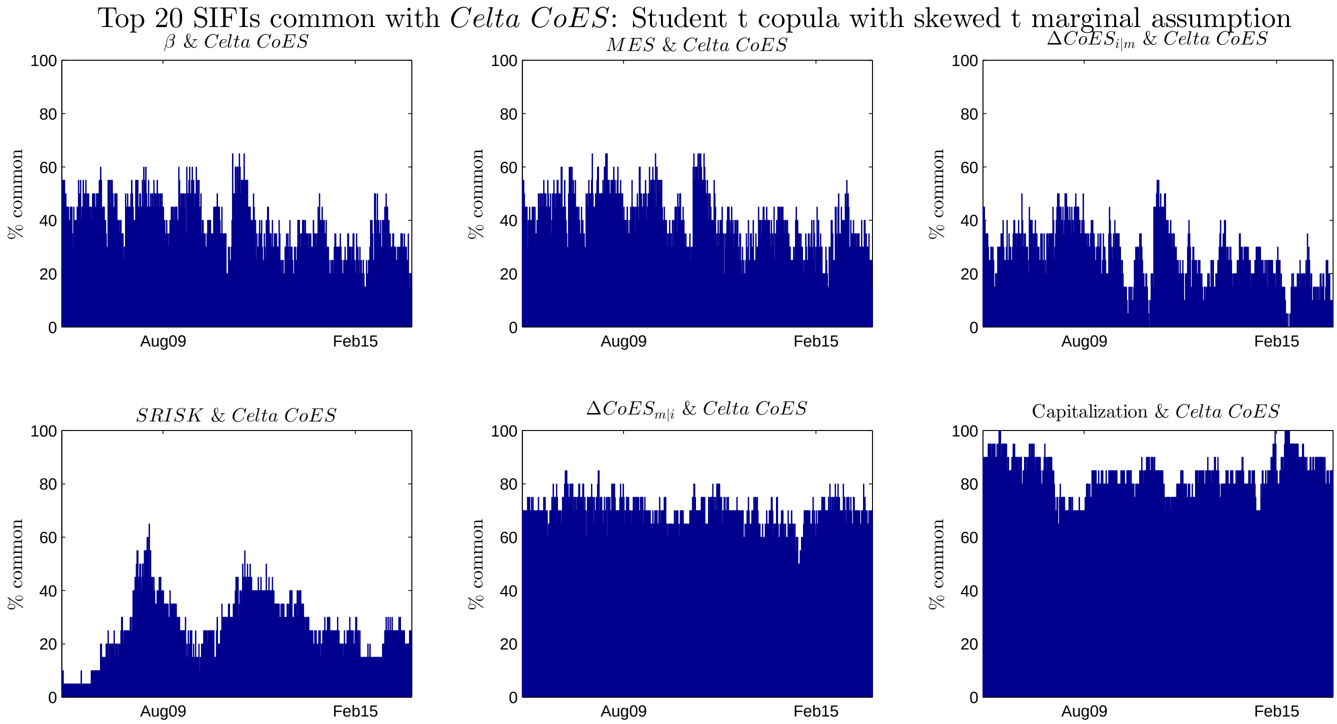
Striped blue rectangle shows the scenario considered for the financial market. This scenario is a distress one in figure **(A)** where returns are lower than $VaR_{m,t}(\alpha_s)$ with $\alpha_s = \beta$ while in figure **(B)** the chosen scenario is around the median, i.e. normal times, where financial market's returns are lower than $VaR(\alpha_n^+)$ but higher than $VaR(\alpha_n^-)$ with $\alpha_n^+ = 0.5 + \beta/2$ and $\alpha_n^- = 0.5 - \beta/2$. Red lines show $CoVaR_{i|m,t}(\alpha,\beta)$ that, given a certain scenario for the financial market, leave below the $\beta 100\%$ worst case scenarios for institution i. The mean loss in these set of scenarios gives $CoES_{i|m,t}(\alpha,\beta)$. The difference of this measure when is assessed in case **(A)** or in case **(B)** is $\Delta CoES(\beta)$, i.e. $\Delta CoES(\beta) = CoES_{i|m,t}(\alpha_s,\beta) - CoES_{i|m,t}(\alpha_n,\beta)$.

Figure 3: Time-series evolution of rank correlation of *Celta CoES* with different systemic risk measures



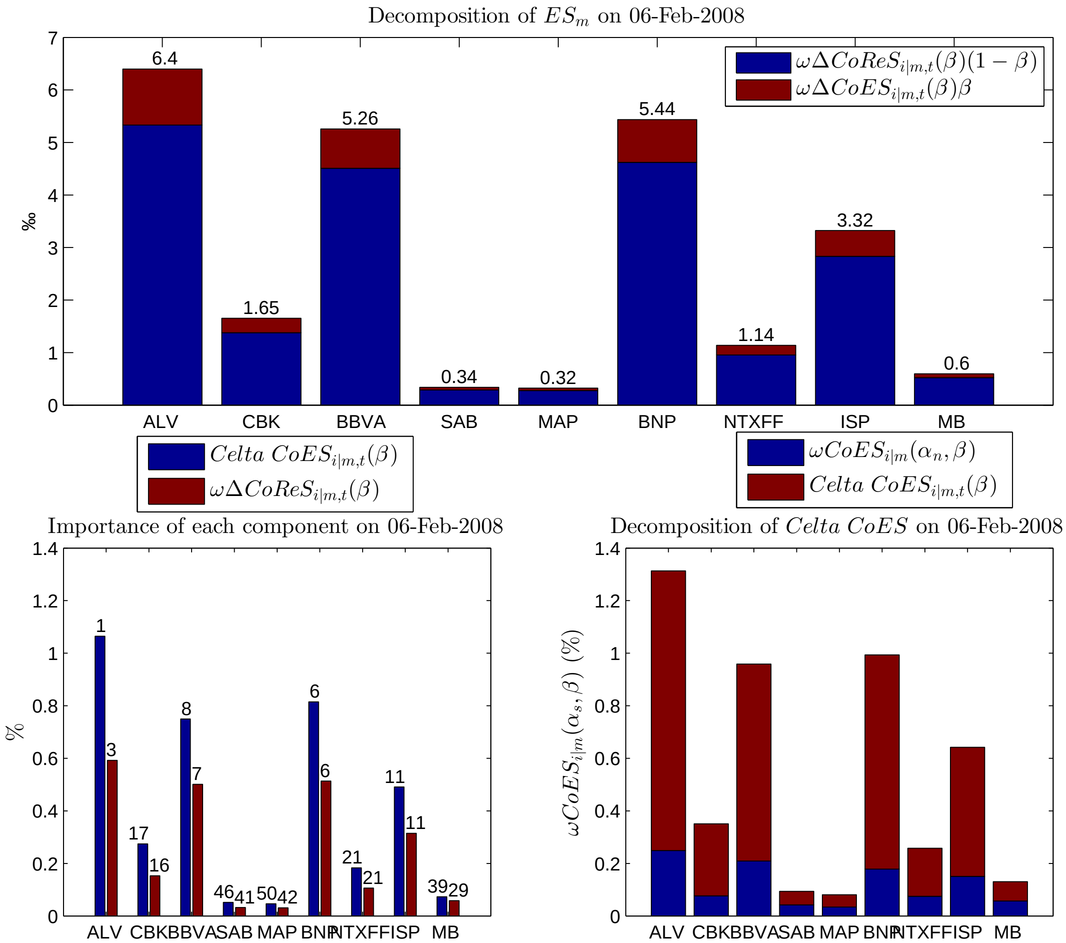
Each subplot assumes different distribution for returns innovation. At each time t the ranking according to *Celta CoES* and other systemic risk measure are compared using Kendall's τ.

Figure 4: Time-series evolution of common SIFIs in the top20 under Student t copula and skewed t marginal distributions.



Each subplot shows the evolution of the common SIFIs percentage in the Top 20 with *Celta CoES*. Comparing *MES* Top 20 with $\Delta CoES_{i|m,t}$ Top 20 is equivalent to compare *Celta CoES* with *CES* if the size factor is overlooked. *MES*, $\Delta CoES_{i|m}$, $\Delta CoES_{m|i}$, *CES* and *CeltaCoES* are measured with a 90% confidence level.

Figure 5a: Decomposition of the Expected Shortfall of the market system for a set of financial firms on February 6^{th}, 2008

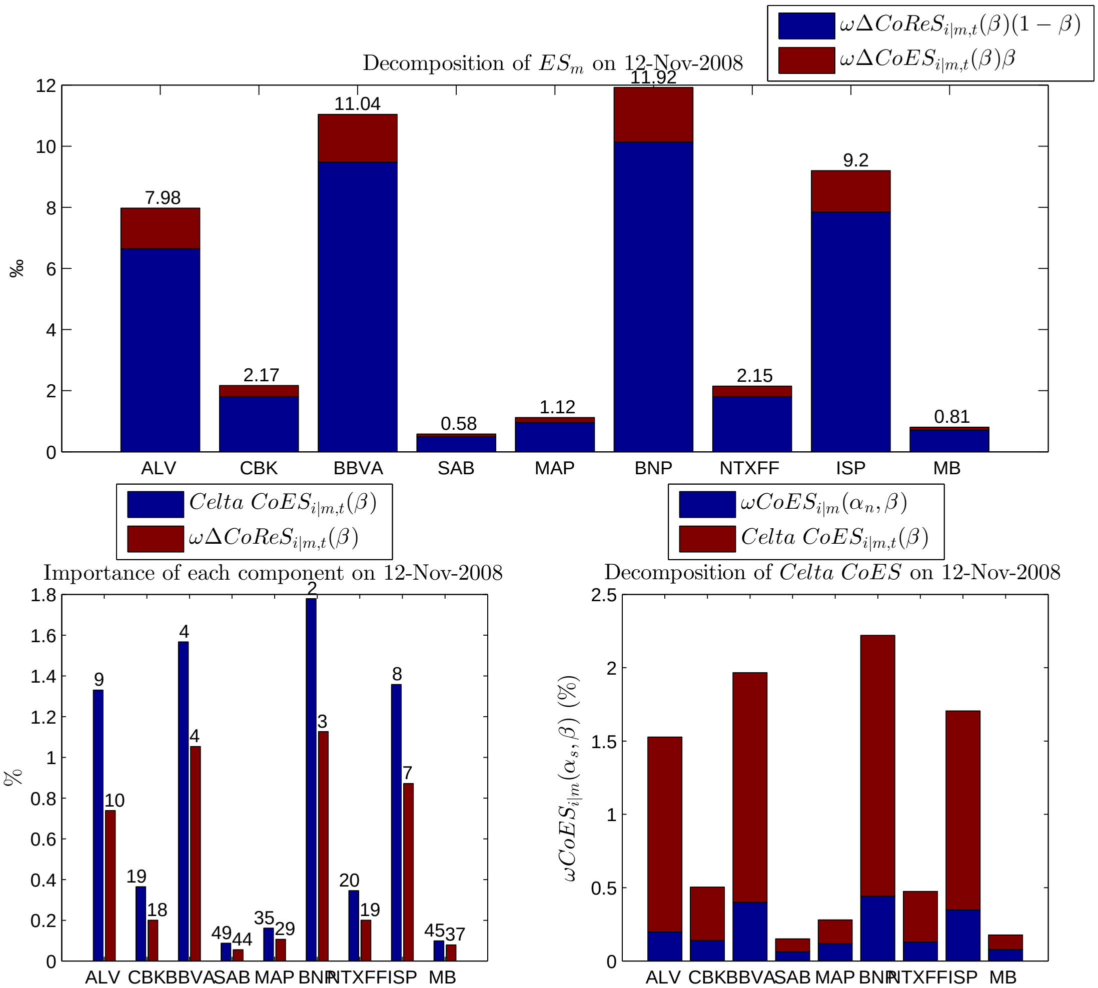


Top figure shows the change in the Expected Shortfall contribution of each financial firm when a crisis occurs. The change in rates per thousands of the expected shortfall contribution of each institution is on the top of each bar. Bottom left figure shows in the blue bars the $Celta\ CoES$ for each firm as a percentage of the $Celta\ CoES$ of the system and in the red bars the $\omega_i \Delta CoReS_{i|m,t}(\beta)$ for each firm as a percentage of the sum of the $\omega_i \Delta CoReS_{i|m,t}(\beta)$ for all the considered firms. On the top of the bars is the ranking according to each component, the higher is the different in ranking the more important are the features in the tail of the distribution. Bottom left figure shows the $\omega_i CoES_{i|m,t}(\alpha_s, \beta)$ of the firms in percentage, where the blue section is the subtrahend of the $Celta\ CoES$, $\omega_i CoES_{i|m,t}(\alpha_n, \beta)$, and the red section is the $Celta\ CoES$. It allows to compare the minuend and the subtrahend that compose the $Celta\ CoES$, i.e. $\omega_i CoES_{i|m,t}(\alpha_s, \beta)$ and $\omega_i CoES_{i|m,t}(\alpha_n, \beta)$.

ES and $CoES$ are measured with a 90% confidence level, i.e., $\beta = 0.1$. It is assumed a Student t copula and skewed t marginal distributions.

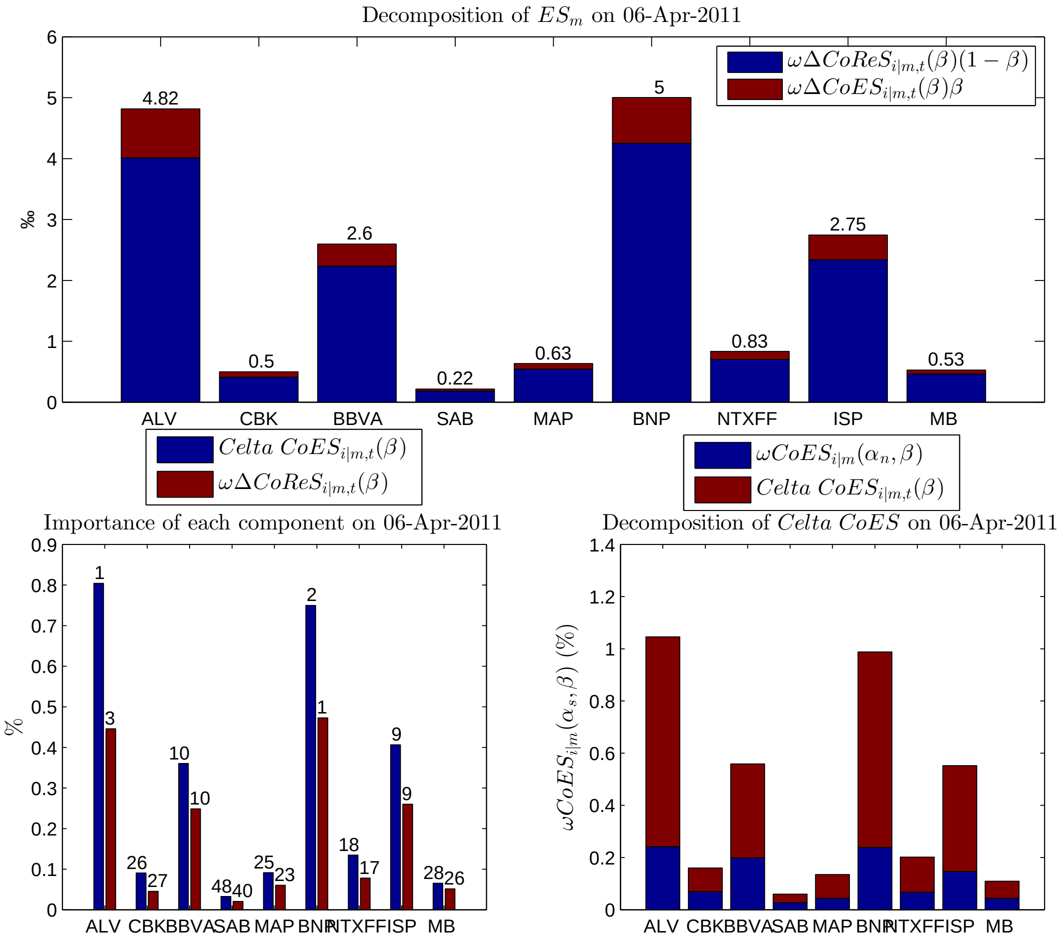
ALV: ALLIANZ, CBK: COMMERZBANK, BBVA: BBV.ARGENTARIA, SAB: BANCO DE SABADELL, MAP: MAPFRE, BNP: BNP PARIBAS, NTXFF: NATIXIS, ISP: INTESA SANPAOLO, MB: MEDIOBANCA BC.FIN.

Figure 5b: Decomposition of the Expected Shortfall of the market system for a set of financial firms on November 12^{th}, 2008



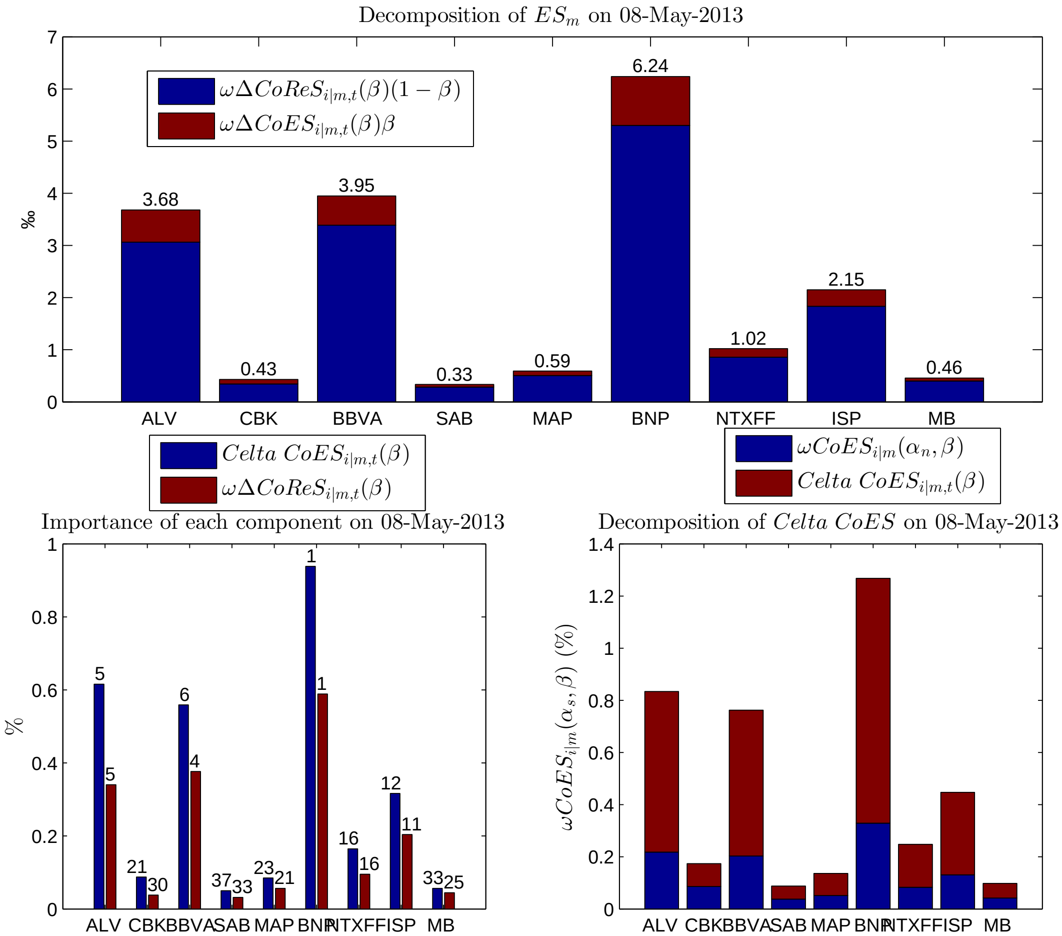
Top figure shows the change in the Expected Shortfall contribution of each financial firm when a crisis occurs. The change in rates per thousands of the expected shortfall contribution of each institution is on the top of each bar. Bottom left figure shows in the blue bars the $Celta\ CoES$ for each firm as a percentage of the $Celta\ CoES$ of the system and in the red bars the $\omega_i \Delta CoReS_{i|m,t}(\beta)$ for each firm as a percentage of the sum of the $\omega_i \Delta CoReS_{i|m,t}(\beta)$ for all the considered firms. On the top of the bars is the ranking according to each component, the higher is the different in ranking the more important are the features in the tail of the distribution. Bottom left figure shows the $\omega_i CoES_{i|m,t}(\alpha_s, \beta)$ of the firms in percentage, where the blue section is the subtrahend of the $Celta\ CoES$, $\omega_i CoES_{i|m,t}(\alpha_n, \beta)$, and the red section is the $Celta\ CoES$. It allows to compare the minuend and the subtrahend that compose the $Celta\ CoES$, i.e. $\omega_i CoES_{i|m,t}(\alpha_s, \beta)$ and $\omega_i CoES_{i|m,t}(\alpha_n, \beta)$.
ES and $CoES$ are measured with a 90% confidence level, i.e., $\beta = 0.1$. It is assumed a Student t copula and skewed t marginal distributions.
ALV: ALLIANZ, CBK: COMMERZBANK, BBVA: BBV.ARGENTARIA, SAB: BANCO DE SABADELL, MAP: MAPFRE, BNP: BNP PARIBAS, NTXFF: NATIXIS, ISP: INTESA SANPAOLO, MB: MEDIOBANCA BC.FIN.

Figure 5c: Decomposition of the Expected Shortfall of the market system for a set of financial firms on April 6^{th}, 2011



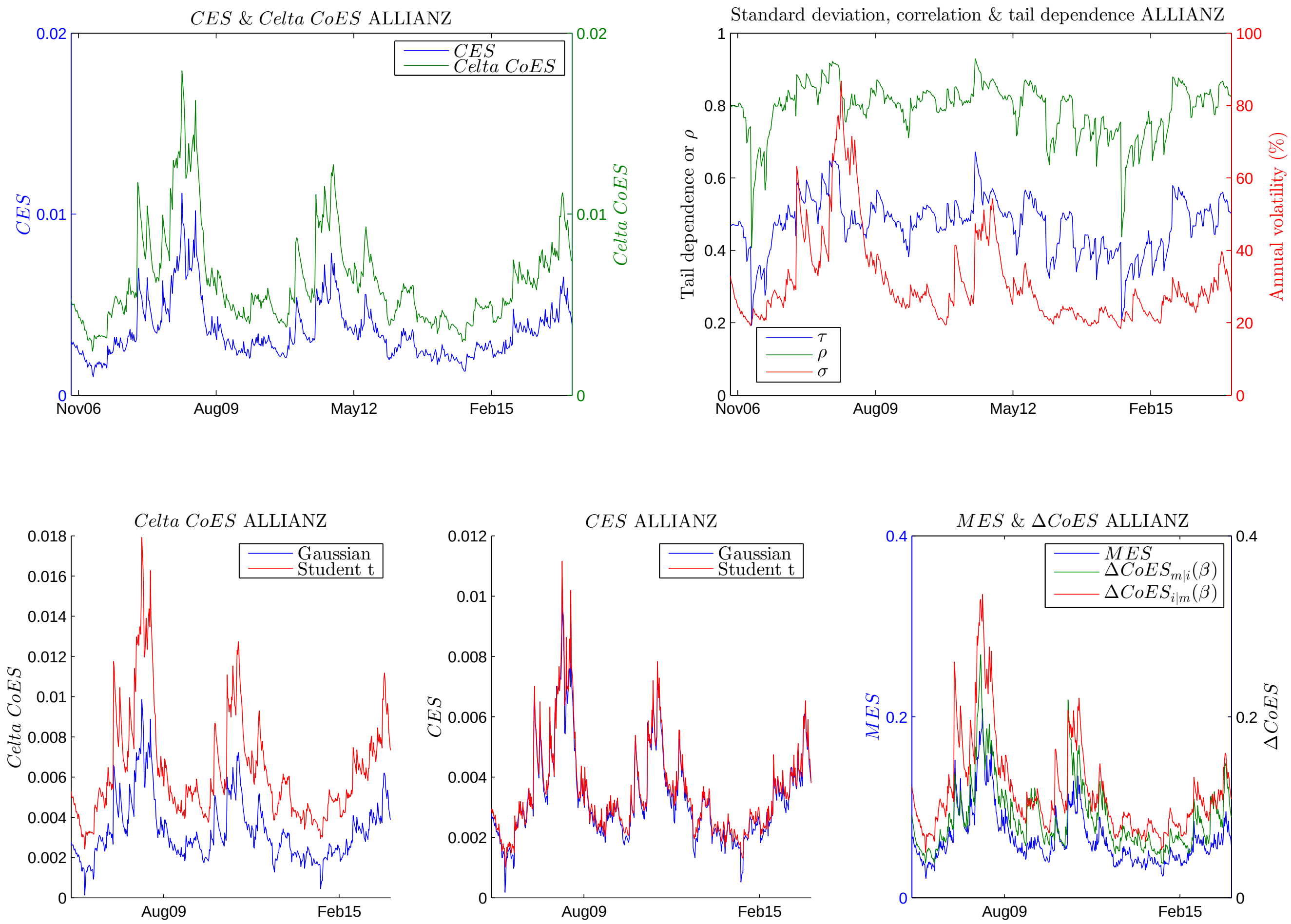
Top figure shows the change in the Expected Shortfall contribution of each financial firm when a crisis occurs. The change in rates per thousands of the expected shortfall contribution of each institution is on the top of each bar. Bottom left figure shows in the blue bars the $Celta\ CoES$ for each firm as a percentage of the $Celta\ CoES$ of the system and in the red bars the $\omega_i \Delta CoReS_{i|m,t}(\beta)$ for each firm as a percentage of the sum of the $\omega_i \Delta CoReS_{i|m,t}(\beta)$ for all the considered firms. On the top of the bars is the ranking according to each component, the higher is the different in ranking the more important are the features in the tail of the distribution. Bottom left figure shows the $\omega_i CoES_{i|m,t}(\alpha_s, \beta)$ of the firms in percentage, where the blue section is the subtrahend of the $Celta\ CoES$, $\omega_i CoES_{i|m,t}(\alpha_n, \beta)$, and the red section is the $Celta\ CoES$. It allows to compare the minuend and the subtrahend that compose the $Celta\ CoES$, i.e. $\omega_i CoES_{i|m,t}(\alpha_s, \beta)$ and $\omega_i CoES_{i|m,t}(\alpha_n, \beta)$.
ES and $CoES$ are measured with a 90% confidence level, i.e., $\beta = 0.1$. It is assumed a Student t copula and skewed t marginal distributions.
ALV: ALLIANZ, CBK: COMMERZBANK, BBVA: BBV.ARGENTARIA, SAB: BANCO DE SABADELL, MAP: MAPFRE, BNP: BNP PARIBAS, NTXFF: NATIXIS, ISP: INTESA SANPAOLO, MB: MEDIOBANCA BC.FIN.

Figure 5d: Decomposition of the Expected Shortfall of the market system for a set of financial firms on May 8^{th}, 2013



Top figure shows the change in the Expected Shortfall contribution of each financial firm when a crisis occurs. The change in rates per thousands of the expected shortfall contribution of each institution is on the top of each bar. Bottom left figure shows in the blue bars the *Celta CoES* for each firm as a percentage of the *Celta CoES* of the system and in the red bars the $\omega_i \Delta CoReS_{i|m,t}(\beta)$ for each firm as a percentage of the sum of the $\omega_i \Delta CoReS_{i|m,t}(\beta)$ for all the considered firms. On the top of the bars is the ranking according to each component, the higher is the different in ranking the more important are the features in the tail of the distribution. Bottom left figure shows the $\omega_i CoES_{i|m,t}(\alpha_s, \beta)$ of the firms in percentage, where the blue section is the subtrahend of the *Celta CoES*, $\omega_i CoES_{i|m,t}(\alpha_n, \beta)$, and the red section is the *Celta CoES*. It allows to compare the minuend and the subtrahend that compose the *Celta CoES*, i.e. $\omega_i CoES_{i|m,t}(\alpha_s, \beta)$ and $\omega_i CoES_{i|m,t}(\alpha_n, \beta)$.
ES and $CoES$ are measured with a 90% confidence level, i.e., $\beta = 0.1$. It is assumed a Student t copula and skewed t marginal distributions.
ALV: ALLIANZ, CBK: COMMERZBANK, BBVA: BBV.ARGENTARIA, SAB: BANCO DE SABADELL, MAP: MAPFRE, BNP: BNP PARIBAS, NTXFF: NATIXIS, ISP: INTESA SANPAOLO, MB: MEDIOBANCA BC.FIN.

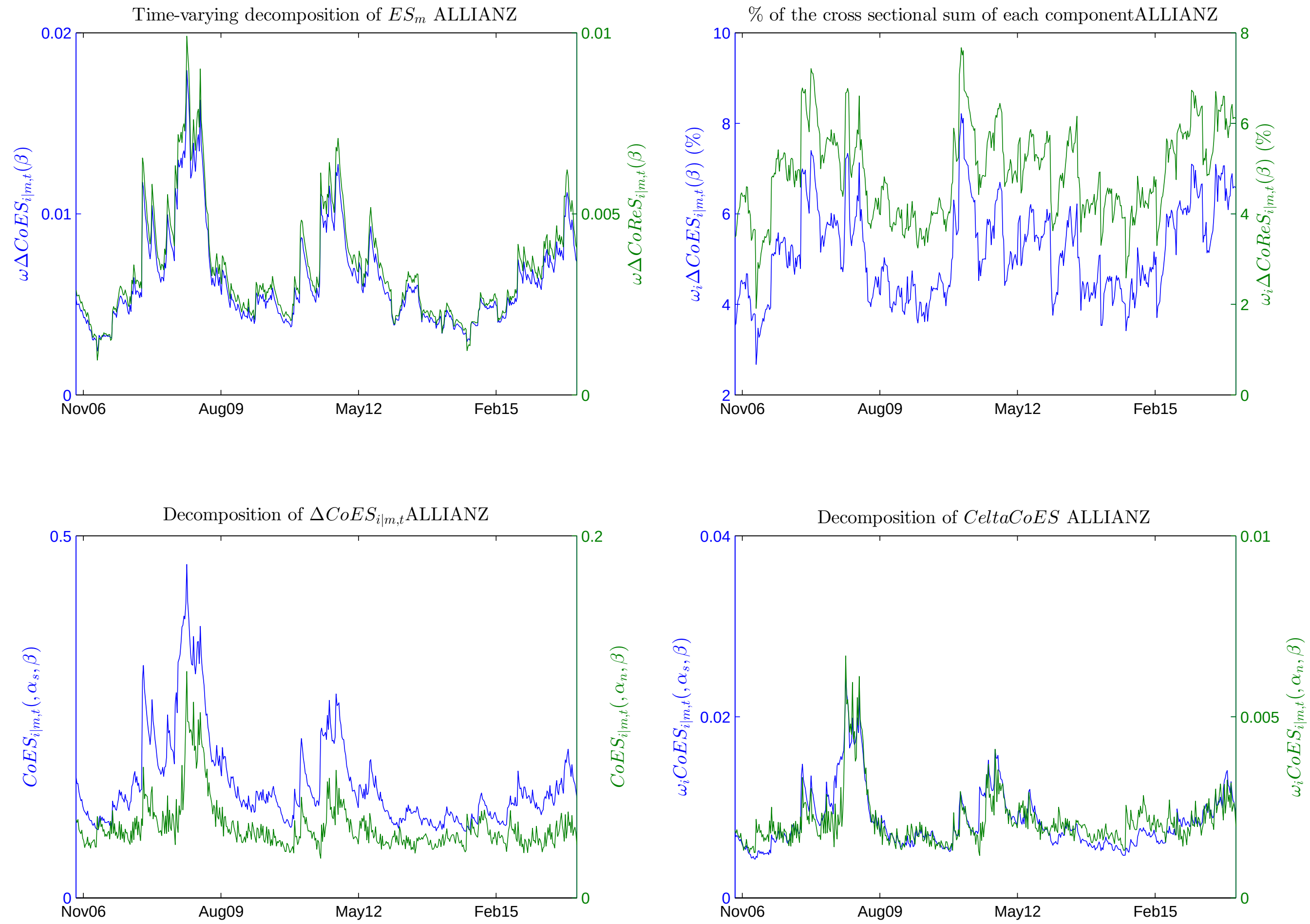
Figure 6a: Time-series comparison across risk measures for ALLIANZ



Top left figure shows *CES* and *Celta CoES* measures for ALLIANZ during the period 2006-2016. Left y-axis corresponds to *CES* measure and right y-axis to *Celta CoES*. Right top figure represents correlation (ρ) and tail dependence (τ) between ALLIANZ and the financial market on the left y-axis and annual volatility in percentage on the right y-axis. Bottom left and centre figures show the change in *Celta CoES* and *CES* when the assumptions changes from Gaussian to Student t copula and skewed-t marginal distributions. Bottom right figure points out the evolution of MES on the left y-axis and the evolution of $\Delta CoES$ on the right y-axis.

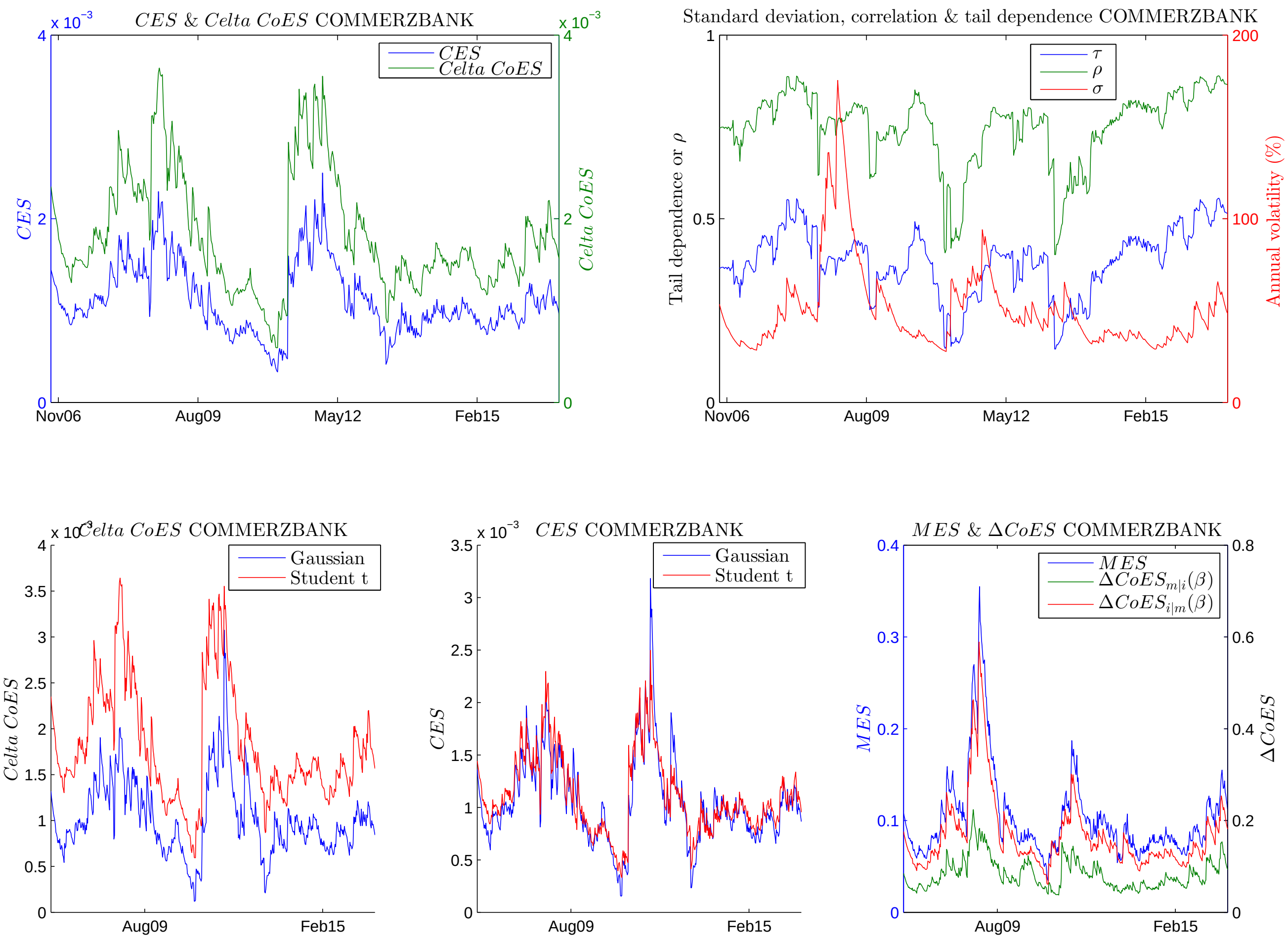
MES, $\Delta CoES_{i|m}$, $\Delta CoES_{m|i}$, CES and $CeltaCoES$ are measured with a 90% confidence level, i.e., $\beta = 0.1$. It is assumed a Student t copula and skewed t marginal distributions.

Figure 6b: Time-series decomposition of the contribution of ALLIANZ to the Expected Shortfall of the financial market



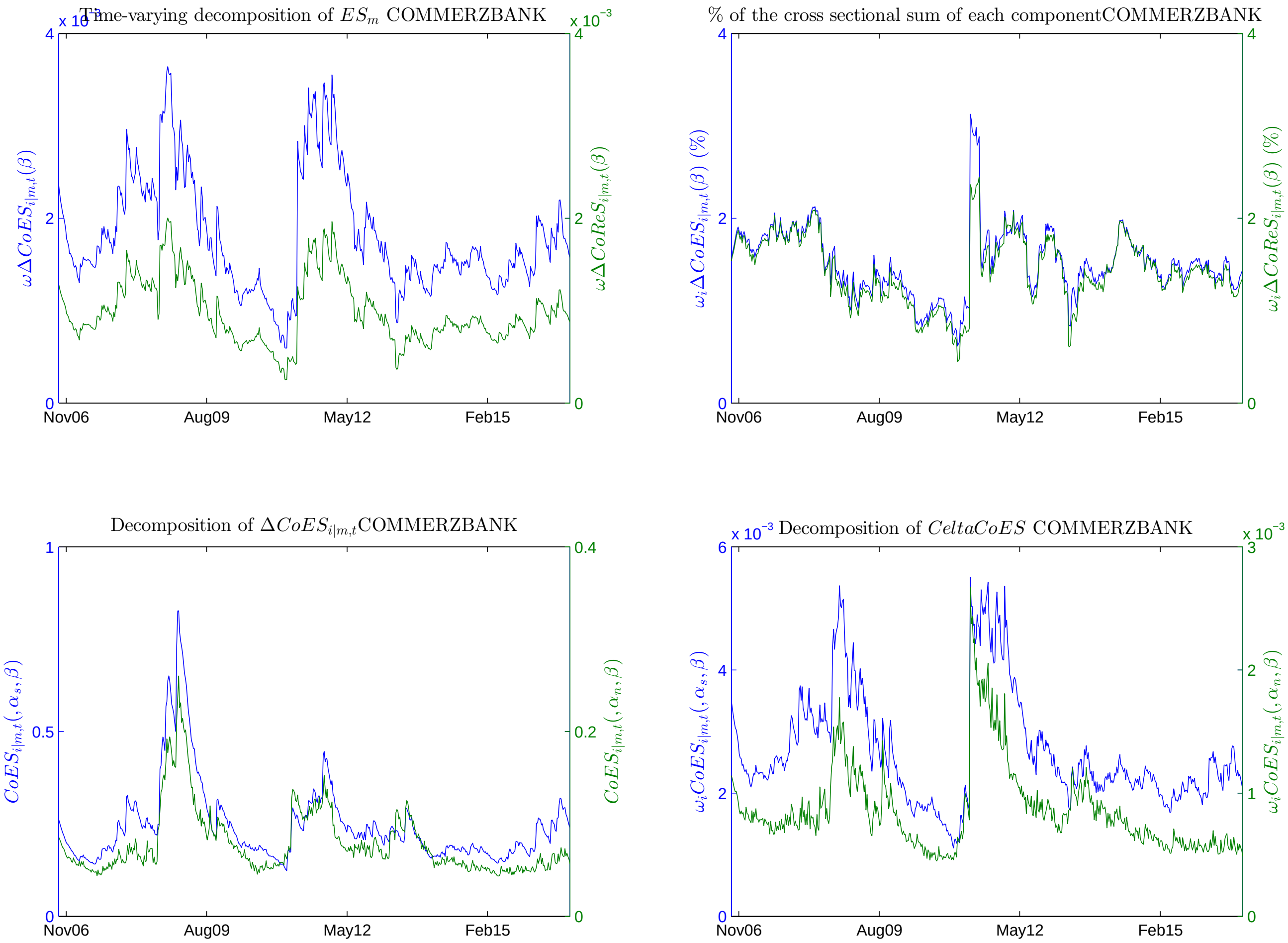
Top left figure shows the time-series evolution of $\omega_i \Delta CoES_{i|m,t}$ on the left y-axis and the time-series evolution of $\omega_i \Delta CoReS_{i|m,t}$ on the right y-axis for the analysed institution. Top right figure shows the same time-series that top left figure but in percentage of the cross-section sum of $\omega_i \Delta CoES_{i|m,t}/\omega_i \Delta CoReS_{i|m,t}$. Bottom left figure decomposes the $\Delta CoES_{i|m,t}$ for the chosen institution. The minuend is shown on the left y-axis and the substrahend on the right y-axis. Bottom right figure is the bottom left figure weighted by the size of the selected firm in the financial system. MES and $\Delta CoES$ are measured with a 90% confidence level, i.e., $\beta = 0.1$. It is assumed a Student t copula and skewed t marginal distributions.

Figure 7a: Time-series comparison across risk measures for COMMERZBANK



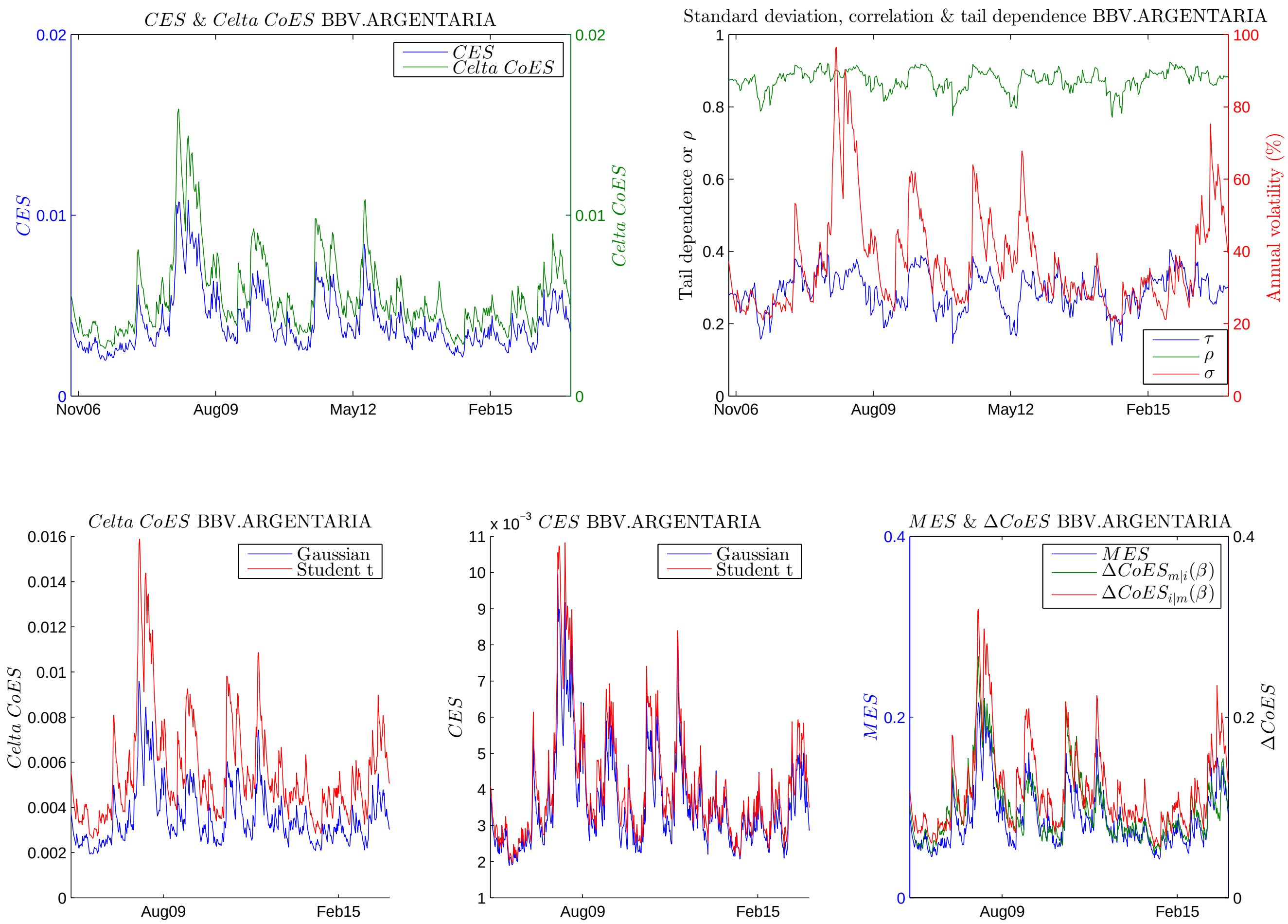
Top left figure shows CES and $Celta\ CoES$ measures for COMMERZBANK during the period 2006-2016. Left y-axis corresponds to CES measure and right y-axis to $Celta\ CoES$. Right top figure represents correlation (ρ) and tail dependence (τ) between COMMERZBANK and the financial market on the left y-axis and annual volatility in percentage on the right y-axis. Bottom left and centre figures show the change in $Celta\ CoES$ and CES when the assumptions changes from Gaussian to Student t copula and skewed-t marginal distributions. Bottom right figure points out the evolution of MES on the left y-axis and the evolution of $\Delta CoES$ on the right y-axis. MES, $\Delta CoES_{i|m}$, $\Delta CoES_{m|i}$, CES and $CeltaCoES$ are measured with a 90% confidence level, i.e., $\beta = 0.1$. It is assumed a Student t copula and skewed t marginal distributions.

Figure 7b: Time-series decomposition of the contribution of COMMERZBANK to the Expected Shortfall of the financial market



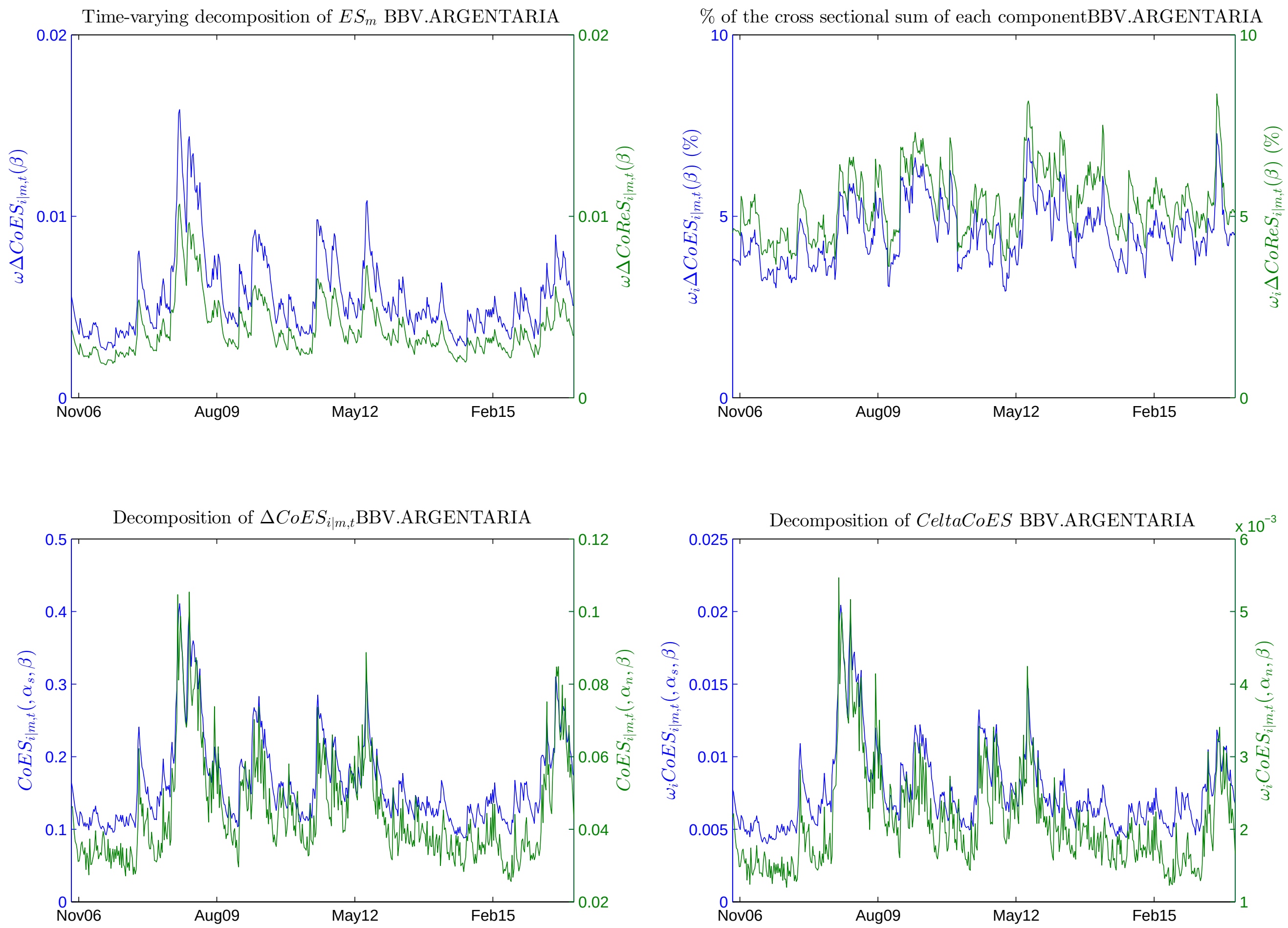
Top left figure shows the time-series evolution of $\omega_i \Delta CoES_{i|m,t}$ on the left y-axis and the time-series evolution of $\omega_i \Delta CoReS_{i|m,t}$ on the right y-axis for the analysed institution. Top right figure shows the same time-series that top left figure but in percentage of the cross-section sum of $\omega_i \Delta CoES_{i|m,t}/\omega_i \Delta CoReS_{i|m,t}$. Bottom left figure decomposes the $\Delta CoES_{i|m,t}$ for the chosen institution. The minuend is shown on the left y-axis and the substrahend on the right y-axis. Bottom right figure is the bottom left figure weighted by the size of the selected firm in the financial system. MES and $\Delta CoES$ are measured with a 90% confidence level, i.e., $\beta = 0.1$. It is assumed a Student t copula and skewed t marginal distributions.

Figure 8a: Time-series comparison across risk measures for BBV.ARGENTARIA



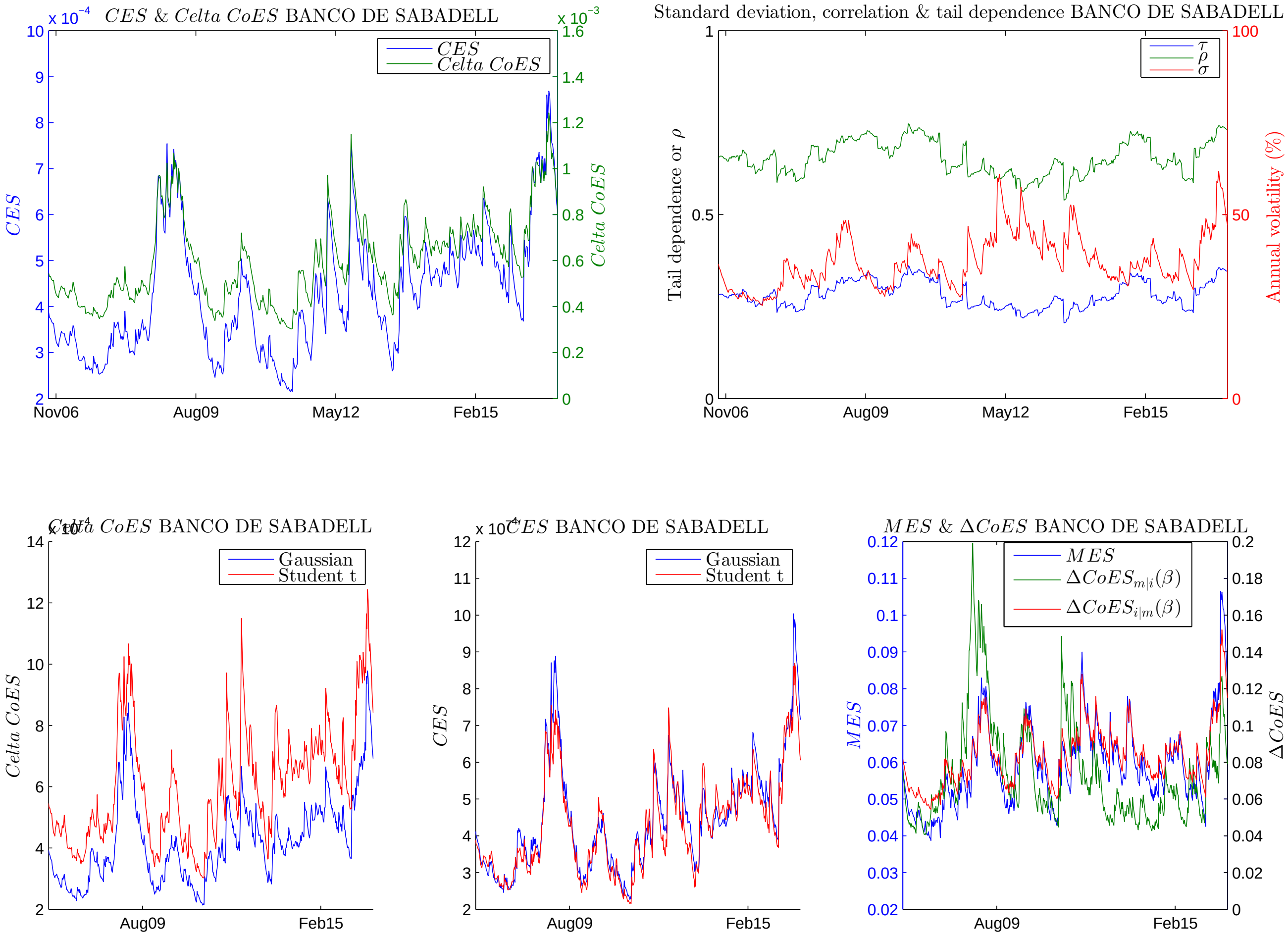
Top left figure shows CES and $Celta\ CoES$ measures for BBV.ARGENTARIA during the period 2006-2016. Left y-axis corresponds to CES measure and right y-axis to $Celta\ CoES$. Right top figure represents correlation (ρ) and tail dependence (τ) between BBV.ARGENTARIA and the financial market on the left y-axis and annual volatility in percentage on the right y-axis. Bottom left and centre figures show the change in $Celta\ CoES$ and CES when the assumptions changes from Gaussian to Student t copula and skewed-t marginal distributions. Bottom right figure points out the evolution of MES on the left y-axis and the evolution of $\Delta CoES$ on the right y-axis. MES, $\Delta CoES_{i|m}$, $\Delta CoES_{m|i}$, CES and $CeltaCoES$ are measured with a 90% confidence level, i.e., $\beta = 0.1$. It is assumed a Student t copula and skewed t marginal distributions.

Figure 8b: Time-series decomposition of the contribution of BBV.ARGENTARIA to the Expected Shortfall of the financial market



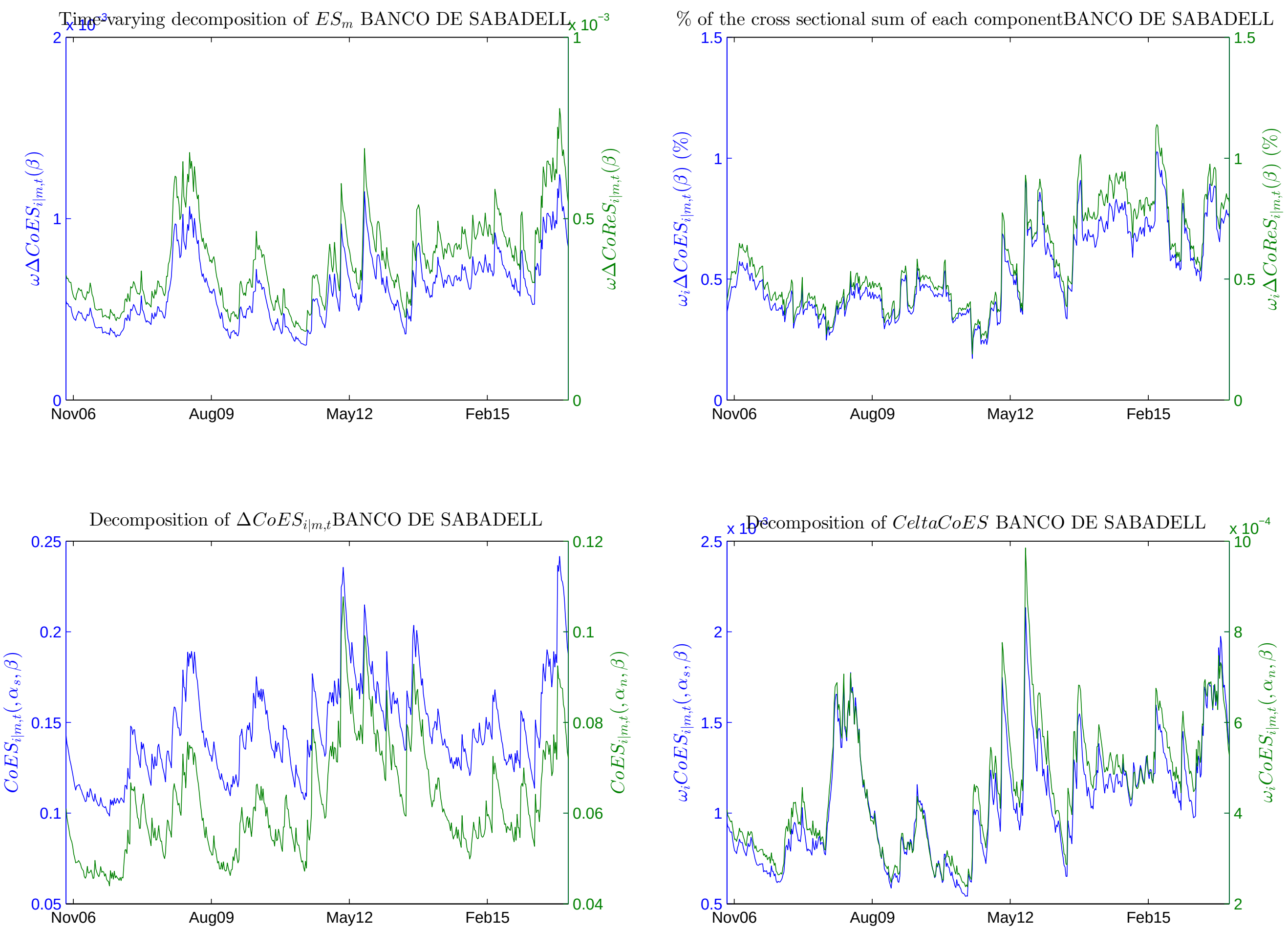
Top left figure shows the time-series evolution of $\omega_i\Delta CoES_{i|m,t}$ on the left y-axis and the time-series evolution of $\omega_i\Delta CoReS_{i|m,t}$ on the right y-axis for the analysed institution. Top right figure shows the same time-series that top left figure but in percentage of the cross-section sum of $\omega_i\Delta CoES_{i|m,t}/\omega_i\Delta CoReS_{i|m,t}$. Bottom left figure decomposes the $\Delta CoES_{i|m,t}$ for the chosen institution. The minuend is shown on the left y-axis and the substrahend on the right y-axis. Bottom right figure is the bottom left figure weighted by the size of the selected firm in the financial system. MES and $\Delta CoES$ are measured with a 90% confidence level, i.e., $\beta = 0.1$. It is assumed a Student t copula and skewed t marginal distributions.

Figure 9a: Time-series comparison across risk measures for BANCO DE SABADELL



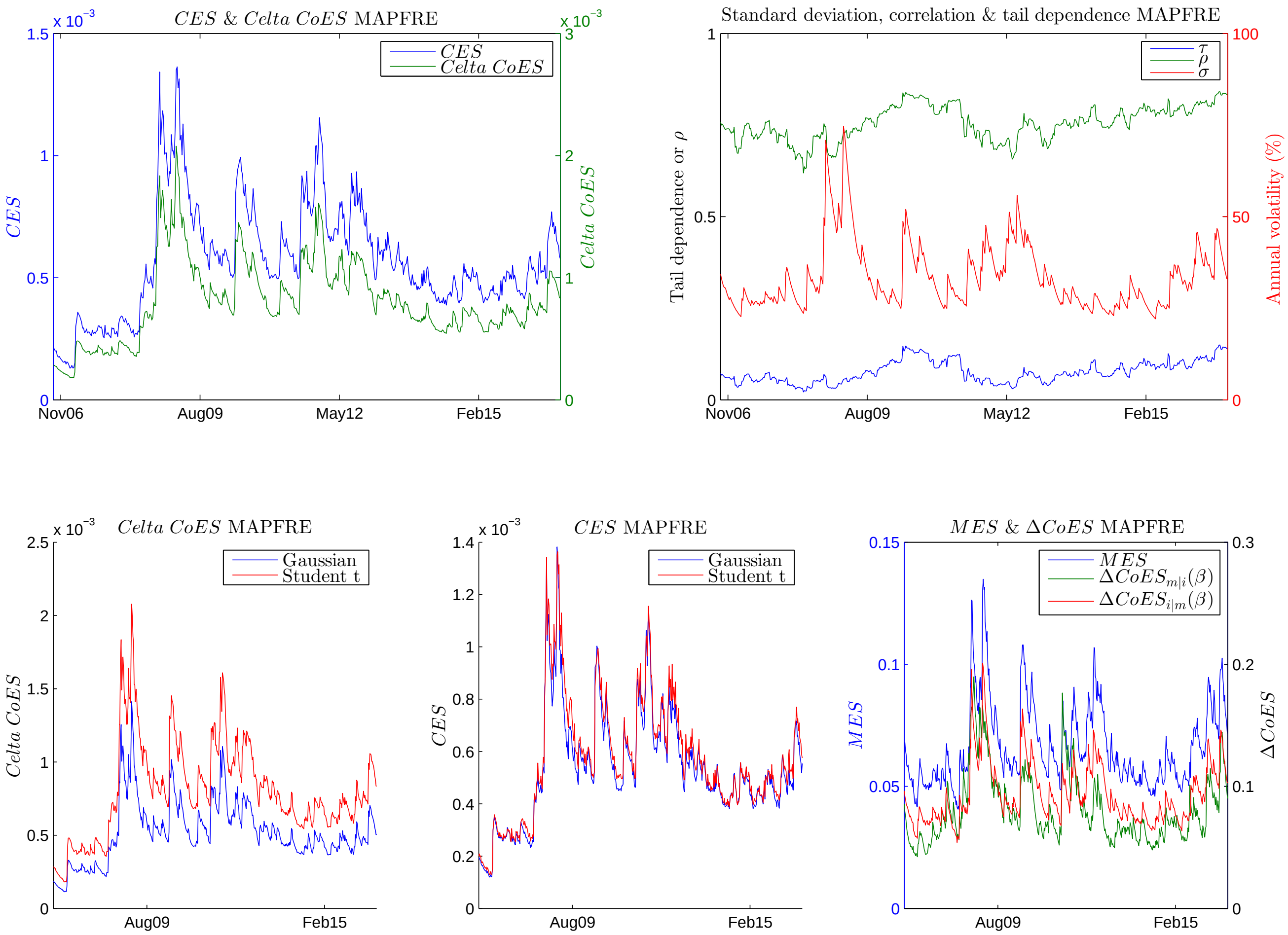
Top left figure shows CES and $Celta\ CoES$ measures for BANCO DE SABADELL during the period 2006-2016. Left y-axis corresponds to CES measure and right y-axis to $Celta\ CoES$. Right top figure represents correlation (ρ) and tail dependence (τ) between BANCO DE SABADELL and the financial market on the left y-axis and annual volatility in percentage on the right y-axis. Bottom left and centre figures show the change in $Celta\ CoES$ and CES when the assumptions changes from Gaussian to Student t copula and skewed-t marginal distributions. Bottom right figure points out the evolution of MES on the left y-axis and the evolution of $\Delta CoES$ on the right y-axis. MES, $\Delta CoES_{i|m}$, $\Delta CoES_{m|i}$, CES and $CeltaCoES$ are measured with a 90% confidence level, i.e., $\beta = 0.1$. It is assumed a Student t copula and skewed t marginal distributions.

Figure 9b: Time-series decomposition of the contribution of BANCO DE SABADELL to the Expected Shortfall of the financial market



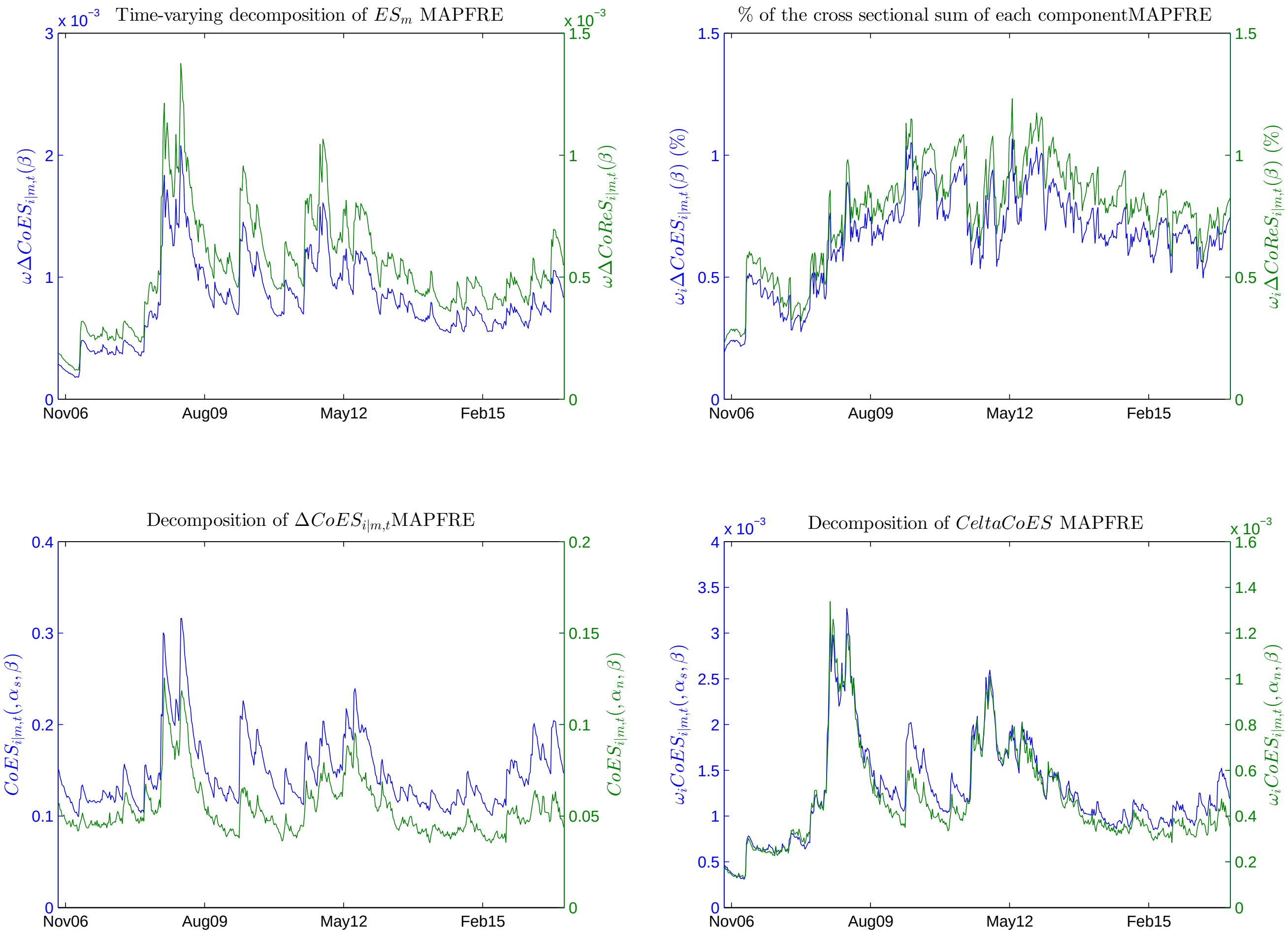
Top left figure shows the time-series evolution of $\omega_i\Delta CoES_{i|m,t}$ on the left y-axis and the time-series evolution of $\omega_i\Delta CoReS_{i|m,t}$ on the right y-axis for the analysed institution. Top right figure shows the same time-series that top left figure but in percentage of the cross-section sum of $\omega_i\Delta CoES_{i|m,t}/\omega_i\Delta CoReS_{i|m,t}$. Bottom left figure decomposes the $\Delta CoES_{i|m,t}$ for the chosen institution. The minuend is shown on the left y-axis and the substrahend on the right y-axis. Bottom right figure is the bottom left figure weighted by the size of the selected firm in the financial system. MES and $\Delta CoES$ are measured with a 90% confidence level, i.e., $\beta = 0.1$. It is assumed a Student t copula and skewed t marginal distributions.

Figure 10a: Time-series comparison across risk measures for MAPFRE



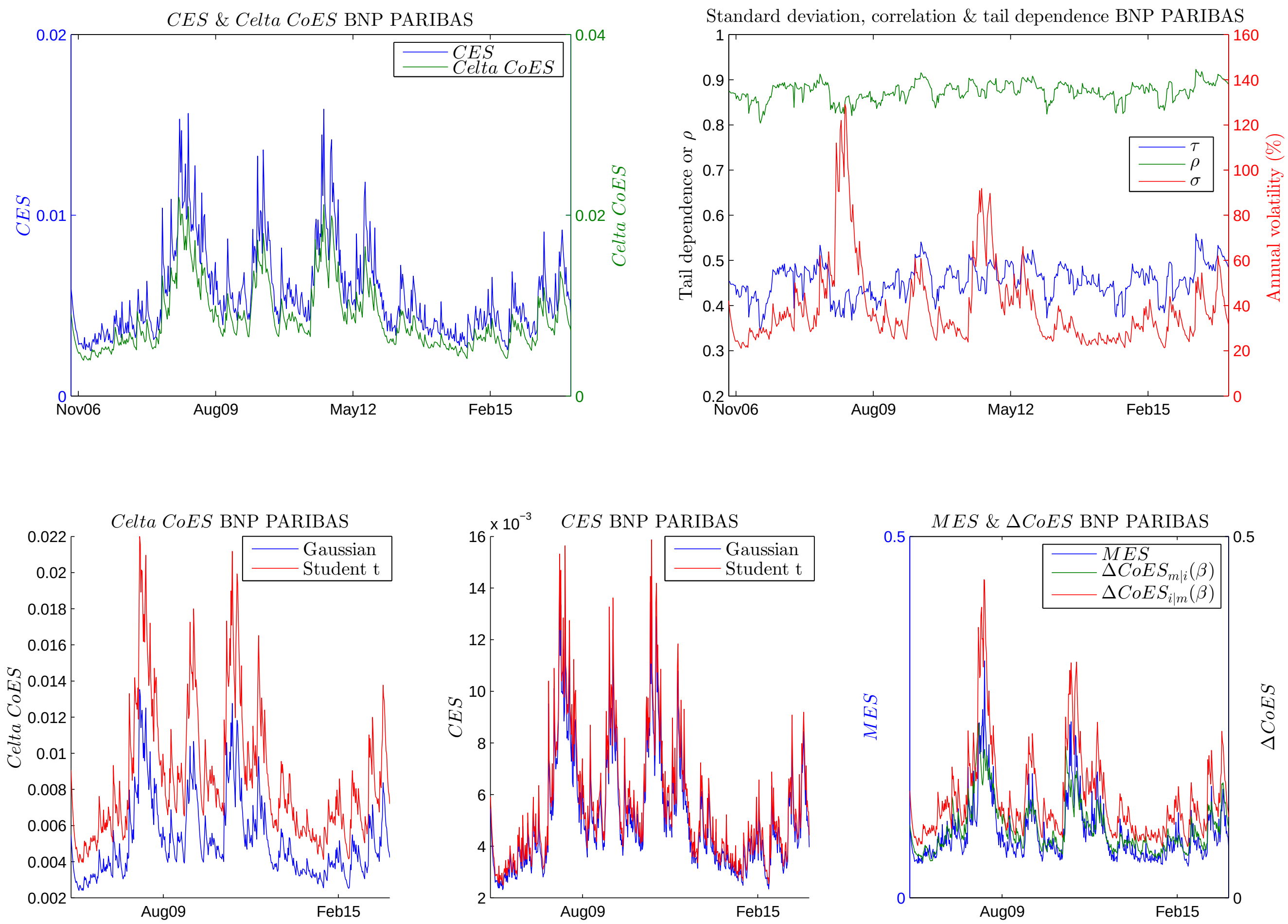
Top left figure shows CES and $Celta\ CoES$ measures for MAPFRE during the period 2006-2016. Left y-axis corresponds to CES measure and right y-axis to $Celta\ CoES$. Right top figure represents correlation (ρ) and tail dependence (τ) between MAPFRE and the financial market on the left y-axis and annual volatility in percentage on the right y-axis. Bottom left and centre figures show the change in $Celta\ CoES$ and CES when the assumptions changes from Gaussian to Student t copula and skewed-t marginal distributions. Bottom right figure points out the evolution of MES on the left y-axis and the evolution of $\Delta CoES$ on the right y-axis.
MES, $\Delta CoES_{i|m}$, $\Delta CoES_{m|i}$, CES and $CeltaCoES$ are measured with a 90% confidence level, i.e., $\beta = 0.1$. It is assumed a Student t copula and skewed t marginal distributions.

Figure 10b: Time-series decomposition of the contribution of MAPFRE to the Expected Shortfall of the financial market



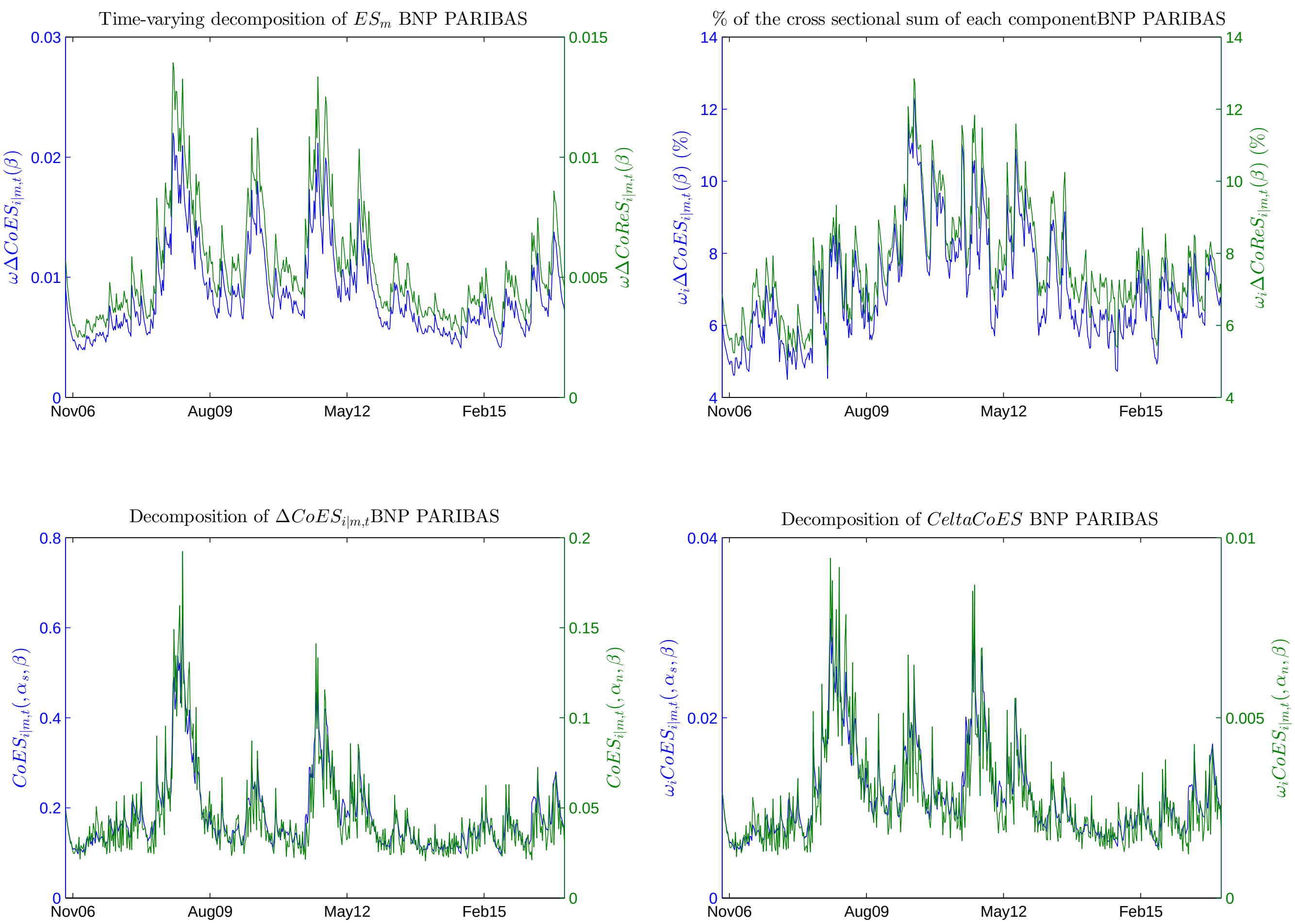
Top left figure shows the time-series evolution of $\omega_i\Delta CoES_{i|m,t}$ on the left y-axis and the time-series evolution of $\omega_i\Delta CoReS_{i|m,t}$ on the right y-axis for the analysed institution. Top right figure shows the same time-series that top left figure but in percentage of the cross-section sum of $\omega_i\Delta CoES_{i|m,t}/\omega_i\Delta CoReS_{i|m,t}$. Bottom left figure decomposes the $\Delta CoES_{i|m,t}$ for the chosen institution. The minuend is shown on the left y-axis and the subtrahend on the right y-axis. Bottom right figure is the bottom left figure weighted by the size of the selected firm in the financial system. MES and $\Delta CoES$ are measured with a 90% confidence level, i.e., $\beta = 0.1$. It is assumed a Student t copula and skewed t marginal distributions.

Figure 11a: Time-series comparison across risk measures for BNP PARIBAS



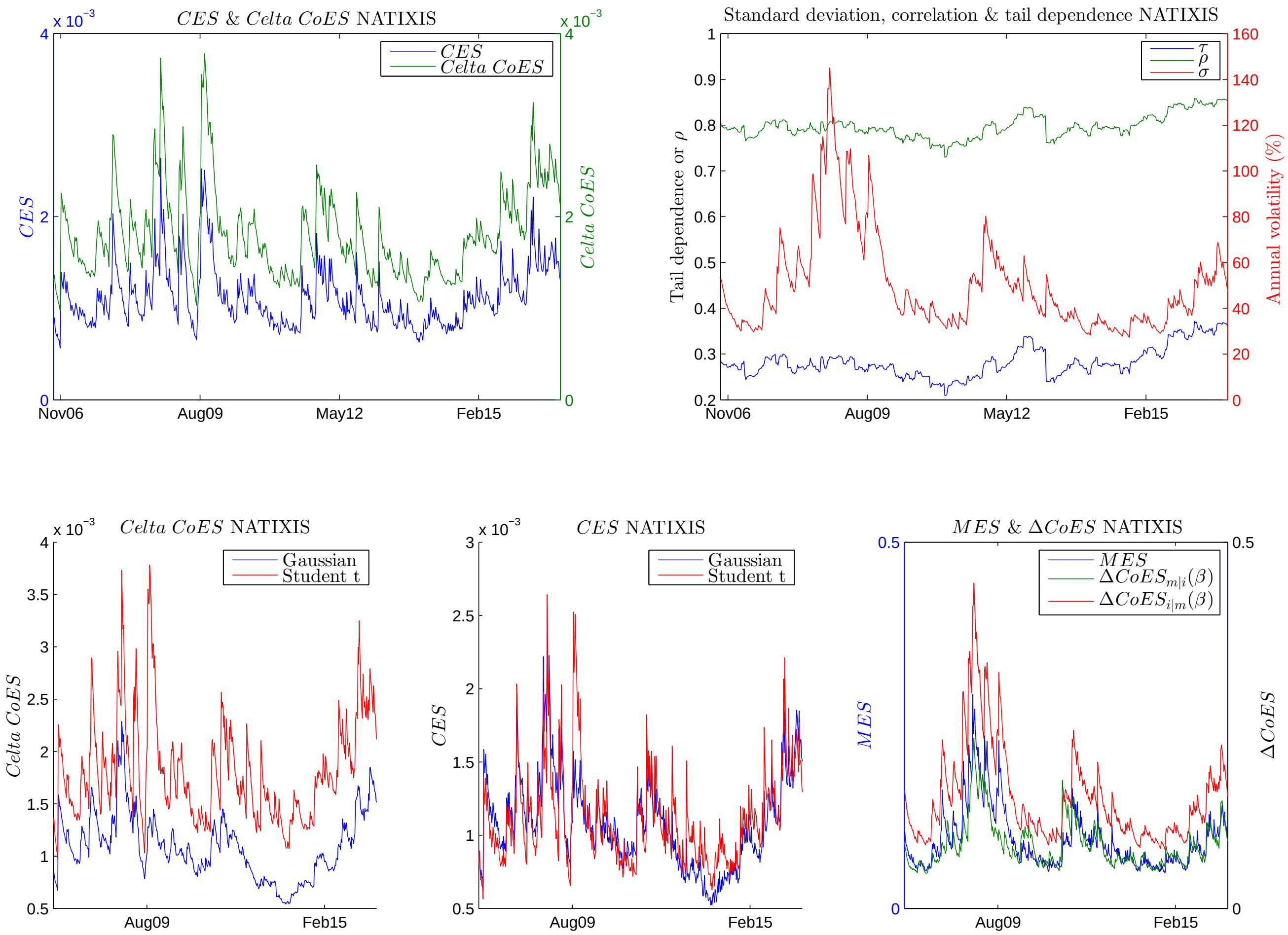
Top left figure shows CES and $Celta\ CoES$ measures for BNP PARIBAS during the period 2006-2016. Left y-axis corresponds to CES measure and right y-axis to $Celta\ CoES$. Right top figure represents correlation (ρ) and tail dependence (τ) between BNP PARIBAS and the financial market on the left y-axis and annual volatility in percentage on the right y-axis. Bottom left and centre figures show the change in $Celta\ CoES$ and CES when the assumptions changes from Gaussian to Student t copula and skewed-t marginal distributions. Bottom right figure points out the evolution of MES on the left y-axis and the evolution of $\Delta CoES$ on the right y-axis.
MES, $\Delta CoES_{i|m}$, $\Delta CoES_{m|i}$, CES and $CeltaCoES$ are measured with a 90% confidence level, i.e., $\beta = 0.1$. It is assumed a Student t copula and skewed t marginal distributions.

Figure 11b: Time-series decomposition of the contribution of BNP PARIBAS to the Expected Shortfall of the financial market



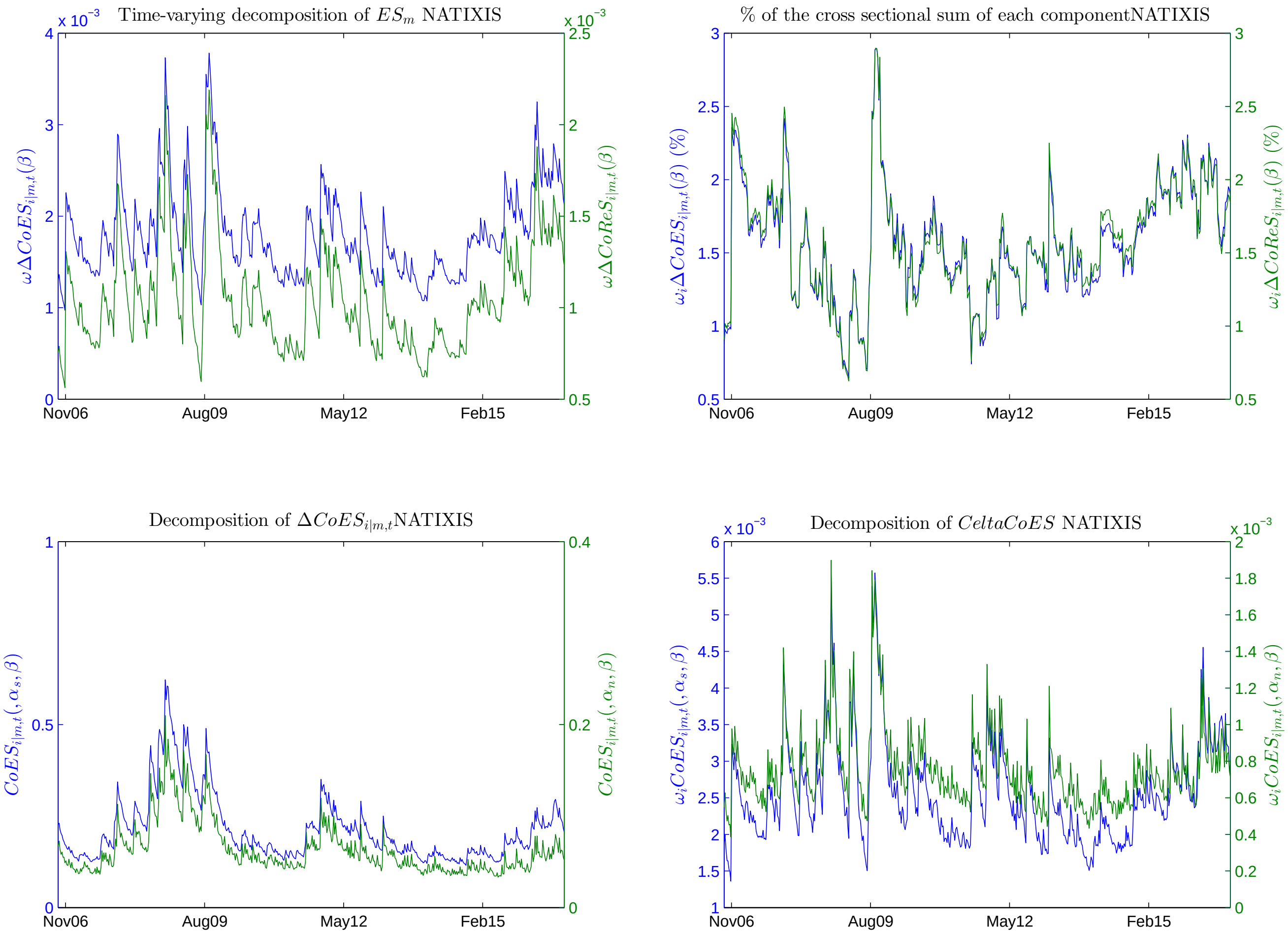
Top left figure shows the time-series evolution of $\omega_i \Delta CoES_{i|m,t}$ on the left y-axis and the time-series evolution of $\omega_i \Delta CoReS_{i|m,t}$ on the right y-axis for the analysed institution. Top right figure shows the same time-series that top left figure but in percentage of the cross-section sum of $\omega_i \Delta CoES_{i|m,t}/\omega_i \Delta CoReS_{i|m,t}$. Bottom left figure decomposes the $\Delta CoES_{i|m,t}$ for the chosen institution. The minuend is shown on the left y-axis and the subtrahend on the right y-axis. Bottom right figure is the bottom left figure weighted by the size of the selected firm in the financial system. MES and $\Delta CoES$ are measured with a 90% confidence level, i.e., $\beta = 0.1$. It is assumed a Student t copula and skewed t marginal distributions.

Figure 12a: Time-series comparison across risk measures NATIXIS



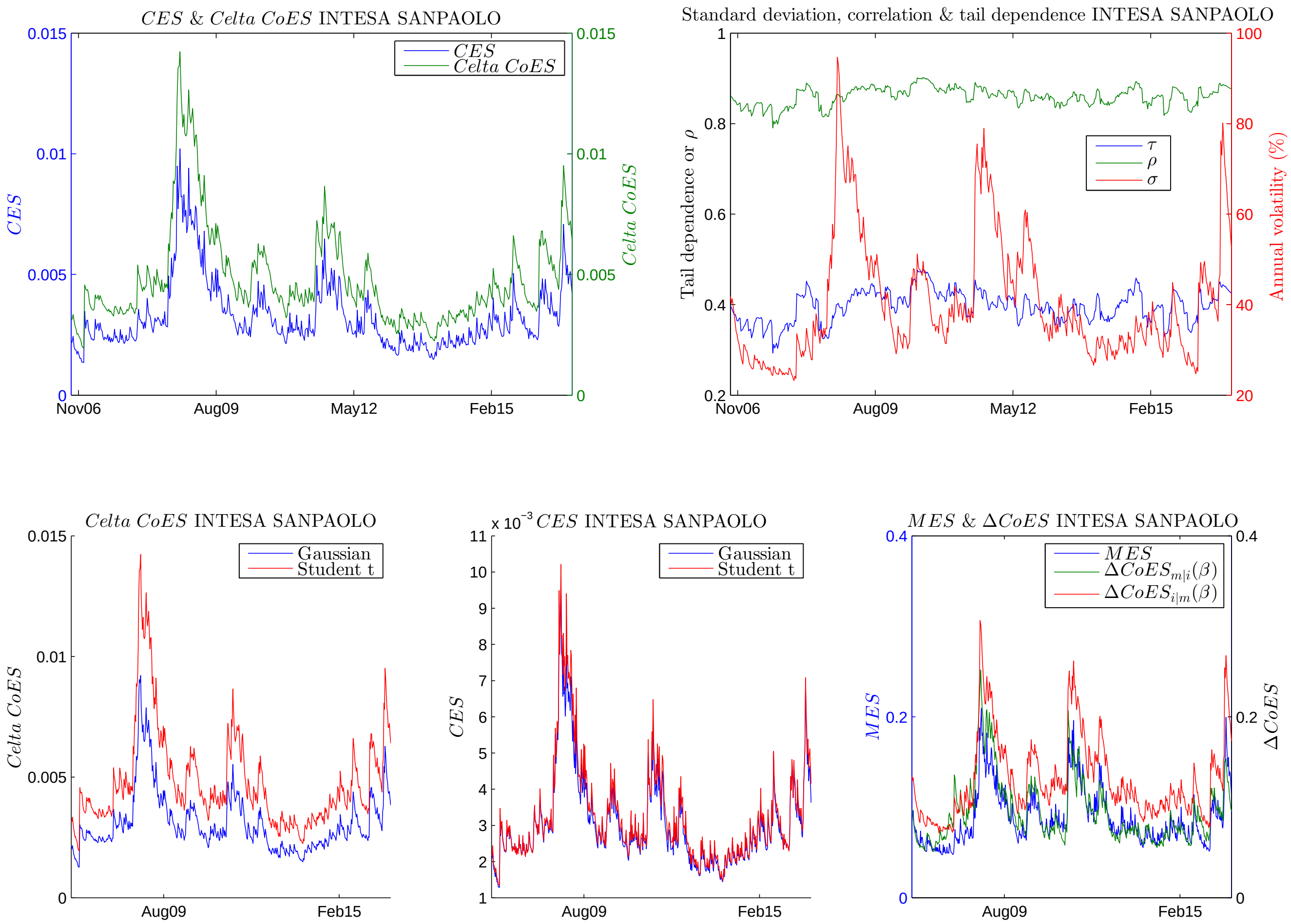
Top left figure shows CES and $Celta\ CoES$ measures for NATIXIS during the period 2006-2016. Left y-axis corresponds to CES measure and right y-axis to $Celta\ CoES$. Right top figure represents correlation (ρ) and tail dependence (τ) between NATIXIS and the financial market on the left y-axis and annual volatility in percentage on the right y-axis. Bottom left and centre figures show the change in $Celta\ CoES$ and CES when the assumptions changes from Gaussian to Student t copula and skewed-t marginal distributions. Bottom right figure points out the evolution of MES on the left y-axis and the evolution of $\Delta CoES$ on the right y-axis.
MES, $\Delta CoES_{i|m}$, $\Delta CoES_{m|i}$, CES and $CeltaCoES$ are measured with a 90% confidence level, i.e., $\beta = 0.1$. It is assumed a Student t copula and skewed t marginal distributions.

Figure 12b: Time-series decomposition of the contribution of NATIXIS to the Expected Shortfall of the financial market



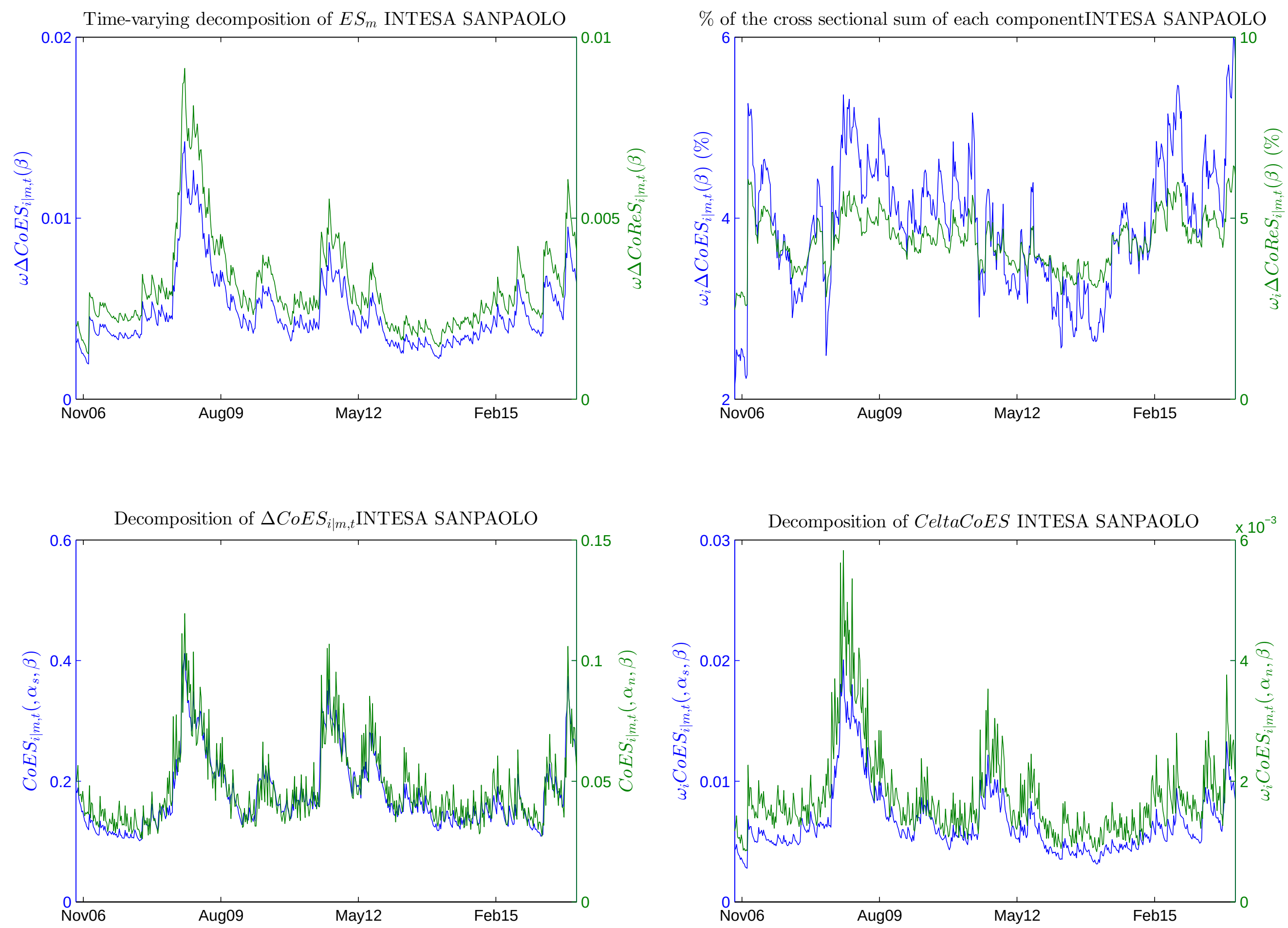
Top left figure shows the time-series evolution of $\omega_i\Delta CoES_{i|m,t}$ on the left y-axis and the time-series evolution of $\omega_i\Delta CoReS_{i|m,t}$ on the right y-axis for the analysed institution. Top right figure shows the same time-series that top left figure but in percentage of the cross-section sum of $\omega_i\Delta CoES_{i|m,t}/\omega_i\Delta CoReS_{i|m,t}$. Bottom left figure decomposes the $\Delta CoES_{i|m,t}$ for the chosen institution. The minuend is shown on the left y-axis and the subtrahend on the right y-axis. Bottom right figure is the bottom left figure weighted by the size of the selected firm in the financial system. MES and $\Delta CoES$ are measured with a 90% confidence level, i.e., $\beta = 0.1$. It is assumed a Student t copula and skewed t marginal distributions.

Figure 13a: Time-series comparison across risk measures INTESA SANPAOLO



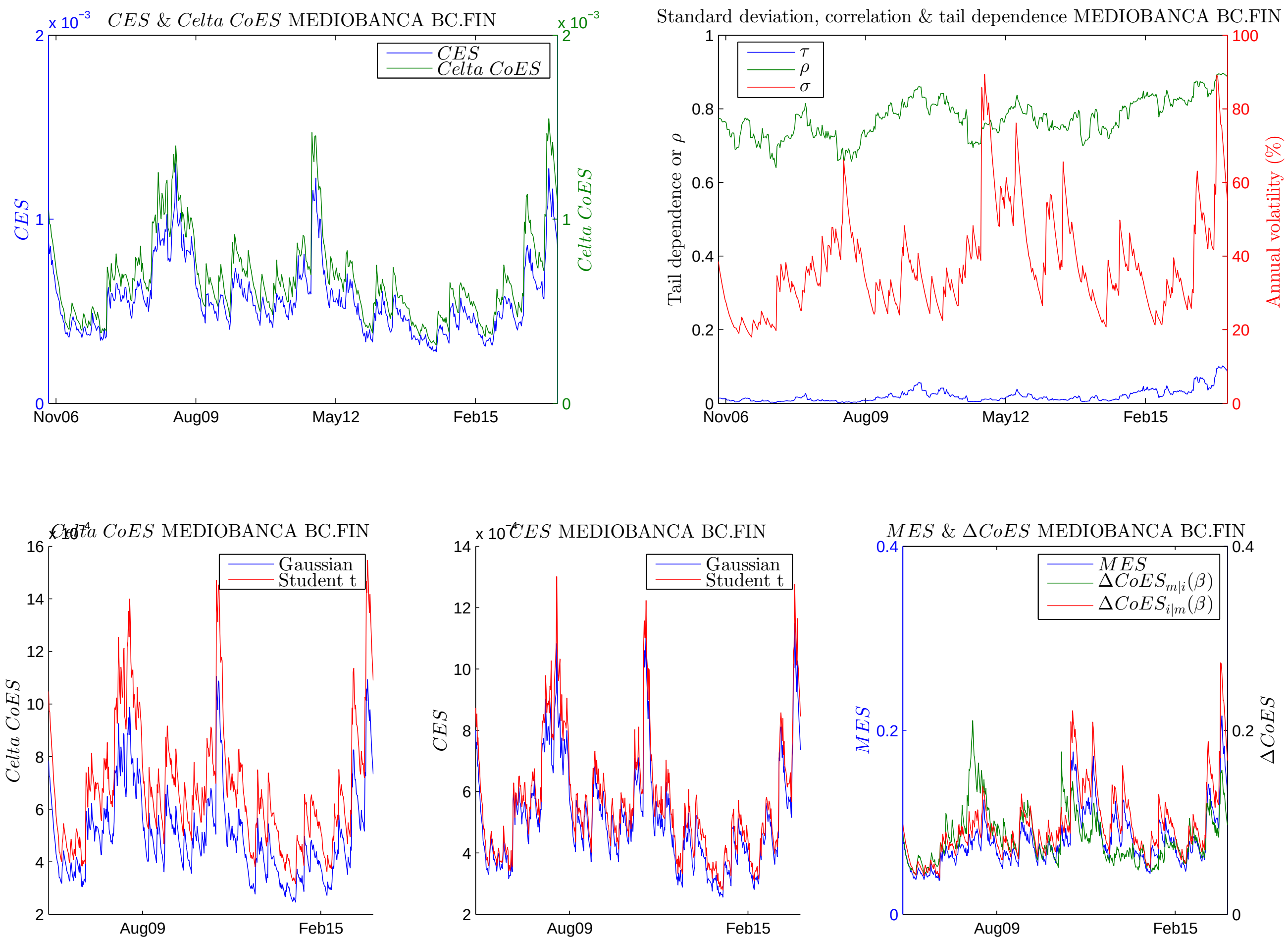
Top left figure shows *CES* and *Celta CoES* measures for INTESA SANPAOLO during the period 2006-2016. Left y-axis corresponds to *CES* measure and right y-axis to *Celta CoES*. Right top figure represents correlation (ρ) and tail dependence (τ) between INTESA SANPAOLO and the financial market on the left y-axis and annual volatility in percentage on the right y-axis. Bottom left and centre figures show the change in *Celta CoES* and *CES* when the assumptions changes from Gaussian to Student t copula and skewed-t marginal distributions. Bottom right figure points out the evolution of *MES* on the left y-axis and the evolution of $\Delta CoES$ on the right y-axis. MES, $\Delta CoES_{i|m}$, $\Delta CoES_{m|i}$, CES and $CeltaCoES$ are measured with a 90% confidence level, i.e., $\beta = 0.1$. It is assumed a Student t copula and skewed t marginal distributions.

Figure 13b: Time-series decomposition of the contribution of INTESA SANPAOLO to the Expected Shortfall of the financial market



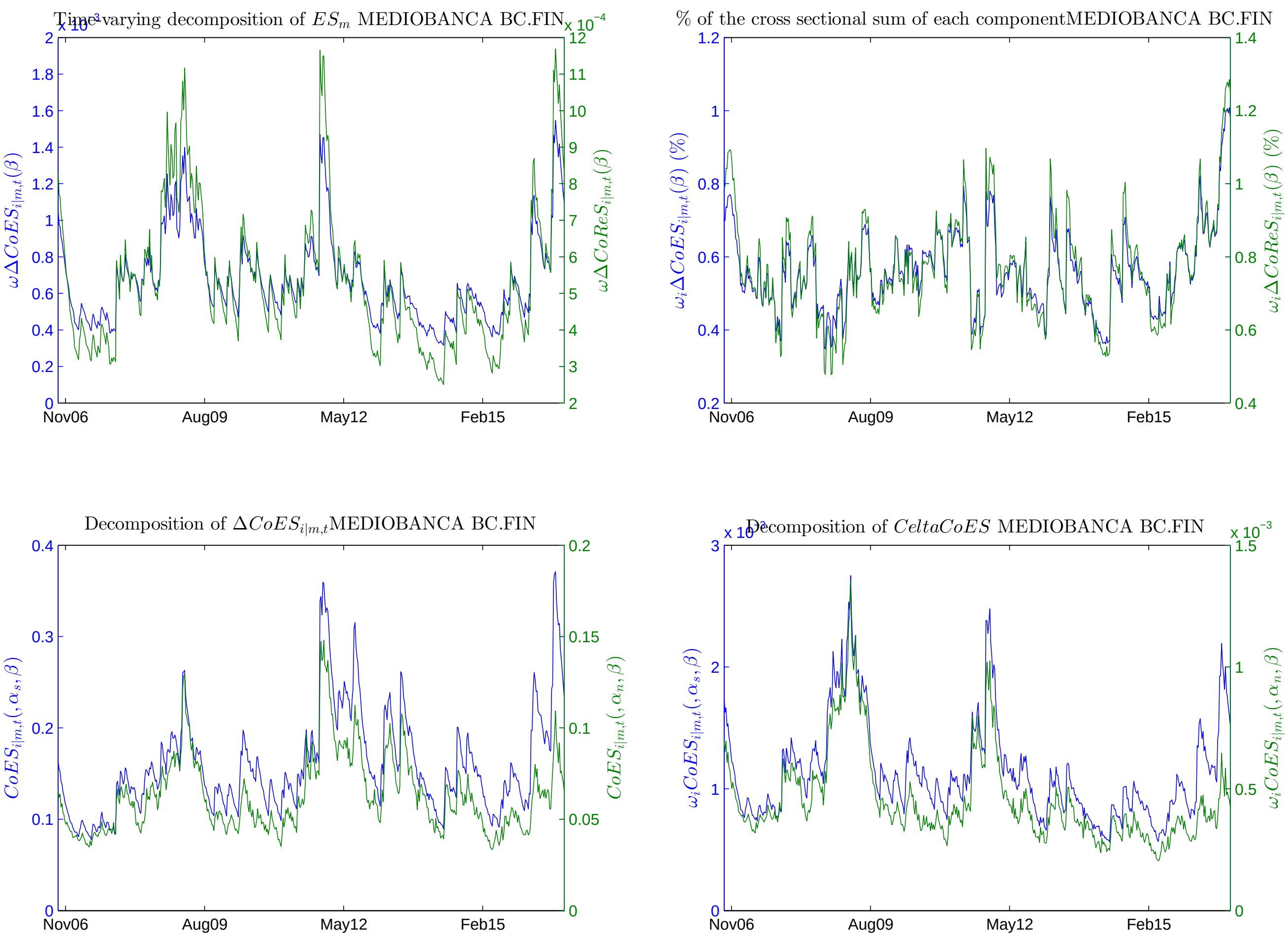
Top left figure shows the time-series evolution of $\omega_i\Delta CoES_{i|m,t}$ on the left y-axis and the time-series evolution of $\omega_i\Delta CoReS_{i|m,t}$ on the right y-axis for the analysed institution. Top right figure shows the same time-series that top left figure but in percentage of the cross-section sum of $\omega_i\Delta CoES_{i|m,t}/\omega_i\Delta CoReS_{i|m,t}$. Bottom left figure decomposes the $\Delta CoES_{i|m,t}$ for the chosen institution. The minuend is shown on the left y-axis and the subtrahend on the right y-axis. Bottom right figure is the bottom left figure weighted by the size of the selected firm in the financial system. MES and $\Delta CoES$ are measured with a 90% confidence level, i.e., $\beta = 0.1$. It is assumed a Student t copula and skewed t marginal distributions.

Figure 14a: Time-series comparison across risk measures MEDIOBANCA BC.FIN



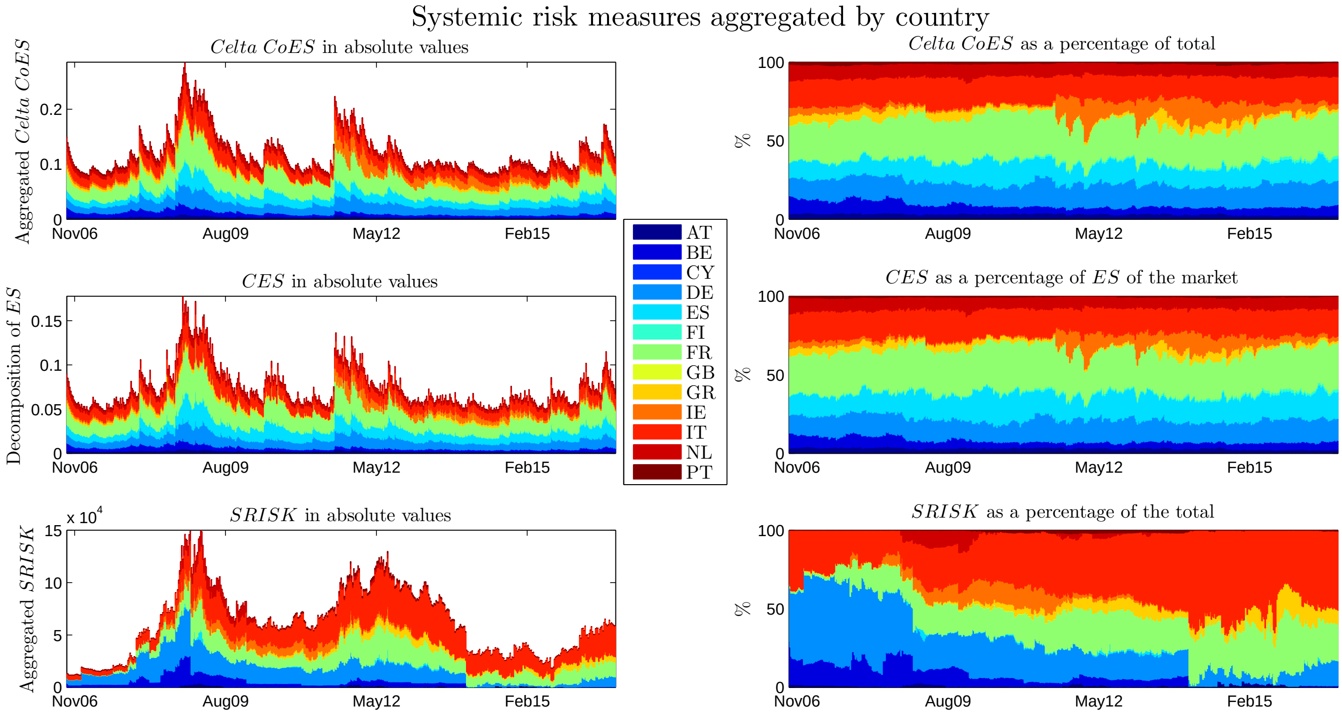
Top left figure shows CES and $Celta\ CoES$ measures for MEDIOBANCA BC.FIN during the period 2006-2016. Left y-axis corresponds to CES measure and right y-axis to $Celta\ CoES$. Right top figure represents correlation (ρ) and tail dependence (τ) between MEDIOBANCA BC.FIN and the financial market on the left y-axis and annual volatility in percentage on the right y-axis. Bottom left and centre figures show the change in $Celta\ CoES$ and CES when the assumptions changes from Gaussian to Student t copula and skewed-t marginal distributions. Bottom right figure points out the evolution of MES on the left y-axis and the evolution of $\Delta CoES$ on the right y-axis. MES, $\Delta CoES_{i|m}$, $\Delta CoES_{m|i}$, CES and $CeltaCoES$ are measured with a 90% confidence level, i.e., $\beta = 0.1$. It is assumed a Student t copula and skewed t marginal distributions.

Figure 14b: Time-series decomposition of the contribution of MEDIOBANCA BC.FIN to the Expected Shortfall of the financial market



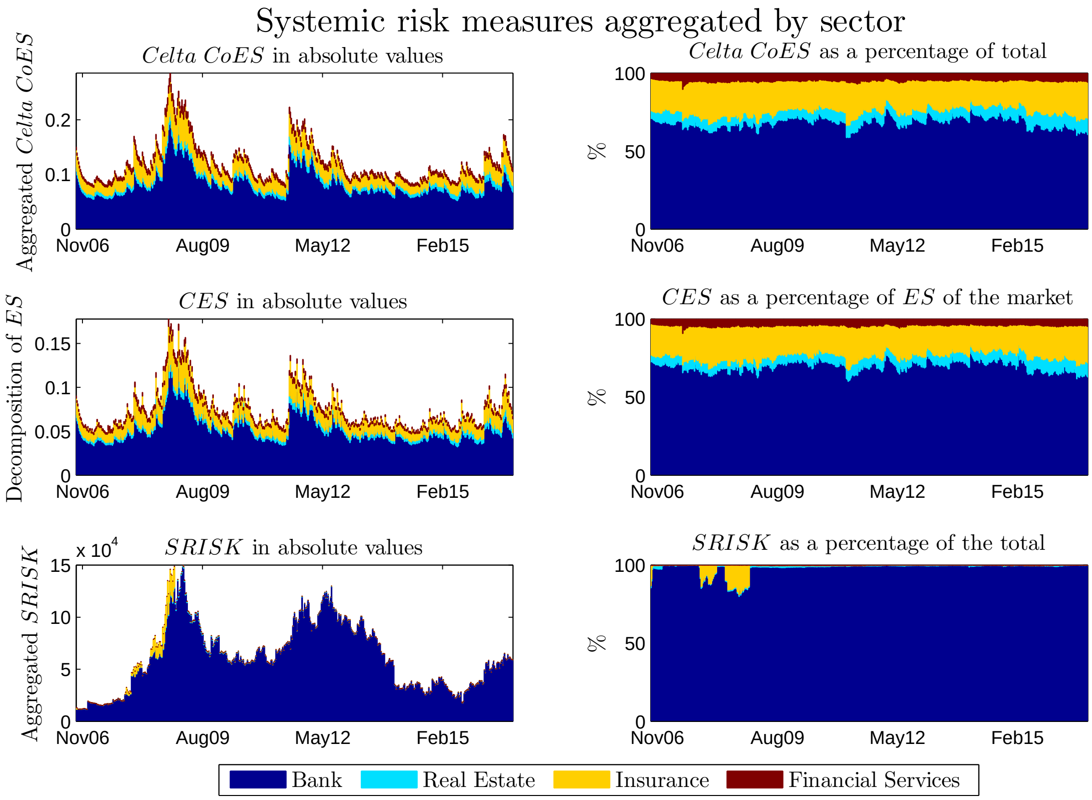
Top left figure shows the time-series evolution of $\omega_i\Delta CoES_{i|m,t}$ on the left y-axis and the time-series evolution of $\omega_i\Delta CoReS_{i|m,t}$ on the right y-axis for the analysed institution. Top right figure shows the same time-series that top left figure but in percentage of the cross-section sum of $\omega_i\Delta CoES_{i|m,t}/\omega_i\Delta CoReS_{i|m,t}$. Bottom left figure decomposes the $\Delta CoES_{i|m,t}$ for the chosen institution. The minuend is shown on the left y-axis and the subtrahend on the right y-axis. Bottom right figure is the bottom left figure weighted by the size of the selected firm in the financial system. MES and $\Delta CoES$ are measured with a 90% confidence level, i.e., $\beta = 0.1$. It is assumed a Student t copula and skewed t marginal distributions.

Figure 15: Time-series comparison between CES and $Celta\ CoES$ and $SRISK$ on a country level



I assume Student t copula and skewed-t marginals.
AT: AUSTRIA, BE: BELGIUM, CY: CYPRUS, DE: GERMANY, ES: SPAIN, FI: FINLAND, FR:FRANCE, GB: GREAT BRITAIN, IE: IRELAND, IT: ITALY, NT: NETHERLANDS, PT: PORTUGAL.

Figure 16: Time-series comparison between CES and $Celta\ CoES$ and $SRISK$ on a sector level



I assume Student t copula and skewed-t marginals.

C Building systemic risk measures

The expression for each measure is provided first in a general formula and then the particular one under the chosen methodology.

C.1 Expected Shortfall (ES) of the financial market

The $VaR_{m,t}(\alpha)$ gives information about how large is the minimum return for the financial market m with $(1-\alpha)100\%$ confidence level. It is obtained by solving the implicit equation

$$P_{t-1}\left[r_{m,t} \leq VaR_{m,t}(\alpha)\right] = \alpha. \tag{22}$$

Expressing Equation (22) following the proposed model under Gaussian assumptions

$$VaR_{m,t}(\alpha) = \mu_{m,t} + \sigma_{m,t}\Phi^{-1}(\alpha) \tag{23}$$

where Φ^{-1} is the inverse standardized cumulative Gaussian distribution function. For the skewed t distribution would be

$$VaR_{m,t}(\alpha) = \mu_{m,t} + \sigma_{m,t}F^{-1}(\alpha;\lambda,\nu),$$

where F^{-1} is the Hansen (1994)'s skewed t inverse cumulative distribution function with asymmetry parameter λ and ν degrees of freedom.

The Value-at-Risk only looks at a certain quantile, consequently it isn't a subadditive measure. The properties of this risk measure can be enhanced if we look further than the quantile of interest for the VaR. The Expected Shortfall tells us how large are the average losses in the financial market if these losses are higher than $-VaR(\alpha)$, i.e.,

$$\begin{aligned} ES_{m,t-1}(\alpha) &= \mathbb{E}_{m,t-1}\left[-r_{m,t}|r_{m,t} < VaR_{i,t}(\alpha)\right] \\ &= \frac{1}{\alpha}\int_0^{\alpha} -VaR_{m,t}(s) \quad ds \end{aligned} \tag{24}$$

where for the Gaussian case is a closed form without computing numerically the integral

$$ES_{m,t-1}(\alpha) \;=\; \sigma_{m,t}\alpha^{-1}\phi(\Phi^{-1}(\alpha)) - \mu_{m,t}. \tag{25}$$

where ϕ is the probability standardized Gaussian distribution function.

Expected Shortfall (ES) under Gaussian framework Equation (25) can be rewritten in a Gaussian framework using VaR definition provided in (23), i.e.

$$\begin{aligned} ES_{m,t-1}(\alpha) &= \frac{1}{\alpha}\int_0^{\alpha} -\mu_{m,t} - \sigma_{m,t}\Phi^{-1}(s) \quad ds \\ &= -\mu_{m,t} - \frac{\sigma_{m,t}}{\alpha}\int_0^{\alpha} \Phi^{-1}(s) \quad ds. \end{aligned}$$

Consequently, the problem is reduced to the integration of the inverse cumulative Gaussian distribution function from 0 to α. Define a change of variable $s = \Phi(r)$, then $ds = \phi(r)dr$ so $\int_0^{\alpha} \Phi^{-1}(s) \quad ds = \int_{-\infty}^{\Phi^{-1}(\alpha)} r\phi(r) \quad dr$ where ϕ is the probability Gaussian distribution function. Subsequently,

$$\begin{aligned} \int_{-\infty}^{\Phi^{-1}(\alpha)} r\phi(r) \quad dr &= \int_{-\infty}^{\Phi^{-1}(\alpha)} \frac{r}{\sqrt{2\pi}}\exp(-r^2/2) \quad dr \\ &= \frac{1}{\sqrt{2\pi}}\left[-\exp(-r^2/2)\right]_{-\infty}^{\Phi^{-1}(\alpha)} \\ &= -\phi(\Phi^{-1}(\alpha)). \end{aligned}$$

As a result the ES is

$$ES_{m,t-1}(\alpha) \;=\; -\mu_{m,t} + \frac{\sigma_{m,t}}{\alpha}\phi(\Phi^{-1}(\alpha)).$$

When the ES has an upper (α^+) and a lower bound, the expression is slightly modified

$$ES_{m,t-1}(\alpha) \;=\; -\mu_{m,t} + \frac{\sigma_{m,t}}{\alpha^+ - \alpha^-}\left\{\phi(\Phi^{-1}(\alpha^+)) - \phi(\Phi^{-1}(\alpha^-))\right\}.$$

C.2 Marginal Expected Shortfall (MES)

The Marginal Expected Shortfall of financial institution i is the mean loss of firm i when financial market's returns are below its $VaR_{m,t}(\alpha)$, i.e.

$$\begin{aligned} MES_{i,t}(\alpha) &= E_{t-1}\left(-r_{i,t}|r_{m,t} < VaR_{m,t}(\alpha)\right) \\ &= \int_0^1 P(F_{i,t}(r_{i,t}) = s|r_{m,t} < VaR_{m,t}(\alpha))F_{i,t}^{-1}(s) \quad ds, \end{aligned} \tag{26}$$

where $F_{i,t}$ is the cumulative distribution function of firm i's returns at time t and $F_{i,t}^{-1}$ is its inverse. For the Gaussian case, the MES expression is

$$MES_{i,t}(\alpha) = \frac{\sigma_{i,t}\rho_t\phi\left(\Phi^{-1}(\alpha)\right)}{\alpha} - \mu_{i,t}. \tag{27}$$

Concerning Student t joint distribution, the solution would be

$$MES_{i,t}(\alpha) = \int_0^1 -(\mu_{i,t} + \sigma_{i,t}F_i^{-1}(s)) \qquad \frac{\overbrace{C_{m|i}(\alpha|s)}^{P(F_{i,t}(r_{i,t})=s|r_{m,t}<VaR_{m,t}(\alpha))}}{\alpha} \qquad ds$$

where $C_{m|i}$ is defined in Equation (20) and F_i^{-1} is the inverse cumulative function of i's marginal distribution. Note that if we wanted to do the integral over the returns instead of its cumulative distribution function, we would have an additional component because $s = F_{i,t}(r_{i,t})$ so $ds = \overbrace{f_{i,t}(r_{i,t})dr_{i,t}}^{dF_{i,t}(r_{i,t})}$. Component Expected Shortfall (CES) is directly obtained weighting MES by the market capitalization for each firm.

Marginal Expected Shortfall (MES) in a Gaussian framework $r_t = (r_{m,t}, r_{i,t})'$ can be expressed as

$$\begin{pmatrix} r_{m,t} \\ r_{i,t} \end{pmatrix} = \underbrace{\begin{pmatrix} \mu_{m,t} \\ \mu_{i,t} \end{pmatrix}}_{\mu_t} + \underbrace{\begin{pmatrix} \sigma_{m,t} & 0 \\ 0 & \sigma_{i,t} \end{pmatrix}}_{D^{1/2}} \underbrace{\begin{pmatrix} 1 & 0 \\ \rho_t & \sqrt{1-\rho_t^2} \end{pmatrix}}_{L_t} \begin{pmatrix} \Phi^{-1}(U_m) \\ \Phi^{-1}(U_i) \end{pmatrix} \tag{28}$$

where L_t matrix represents Choleski decomposition and ρ_t is the correlation parameter obtained from the DCC model. U_m and U_i are uniform independent distributed variables while Φ^{-} is the inverse cumulative Gaussian distribution function.

The vector r_t is normally distributed with mean μ_t and covariance matrix $D^{1/2}L_tL_t'D^{1/2}$. Given a value for financial market returns $r_{m,t}$, the returns distribution of firm i becomes $r_{i,t}|r_{m,t}\ N\left(\mu_{i,t} + \frac{\sigma_{i,t}\rho_t}{\sigma_{m,t}}\left(r_{m,t} - \mu_{m,t}\right), \sqrt{1-\rho_t^2}\sigma_{i,t}\right)$, where N refers to the Gaussian distribution where the first input is the mean ($\mu_{i|m,t}$) and the second one is the standard deviation ($\sigma_{i|m,t}$).

If the realization of $r_{m,t}$ is expressed in terms of quantiles, i.e. $r_{m,t} = \Phi^{-1}(q)\sigma_{m,t} + \mu_{m,t}$, the mean value of $r_{i,t}$ given that $r_{m,t}$ is in its q quantile is $\mu_{i,t} + \sigma_{i,t}\rho_t\Phi^{-1}(q)$, i.e. $E_{t-1}\left(r_{i,t}|r_{m,t} = VaR_{m,t}(q)\right)$. Then, the mean value of $r_{i,t}$ given that $r_{m,t}$ is at most in its α quantile would be

$$E_{t-1}\left(r_{i,t}|r_{m,t} < VaR_{m,t}(\alpha)\right) = \mu_{i,t} + \sigma_{i,t}\rho_t \qquad \underbrace{\frac{\int_0^\alpha \Phi^{-1}(q)dq}{\alpha}}_{E\left(\frac{r_{m,t}-\mu_{m,t}}{\sigma_{m,t}}|r_{m,t}<VaR_{m,t}(\alpha)\right)} \qquad .$$

Because of the solution of previous integral, the MES expression is

$$MES_{i,t}(\alpha) = \frac{\sigma_{i,t}\rho_t\phi\left(\Phi^{-1}(\alpha)\right)}{\alpha} - \mu_{i,t}.$$

C.3 Conditional Expected Shortfall ($CoES$) and Delta Conditional Expected Shortfall ($\Delta CoES$)

The Conditional Expected Shortfall of financial institution i given that the financial market m is below its quantile α is expressed as

$$\begin{aligned} CoES_{i,t}(\alpha,\beta) &= E_{t-1}\left(-r_{i,t}|r_{i,t} < CoVaR_{m,t}(\alpha,\beta)\right) \\ &= \frac{1}{\beta}\int_0^{s^*} P(F_{i,t}(r_{i,t}) = s|r_{m,t} < VaR_{m,t}(\alpha))F_{i,t}^{-1}(s) \quad ds, \end{aligned} \tag{29}$$

where s^* is such that $P(F_{i,t}(r_{i,t}) < s^*|r_{m,t} < VaR_{m,t}(\alpha)) = \beta$. In a Gaussian framework this expression can be rewritten as

$$CoES_{i,t}(\alpha,\beta) = \sigma_{i,t}\left(\sqrt{1-\rho_t^2}\frac{\phi\left(\Phi^{-1}(\beta)\right)}{\beta} + \rho_t\frac{\phi(\Phi(\alpha))}{\alpha}\right) - \mu_{i,t}, \tag{30}$$

while for $\Delta CoES$ is

$$\Delta CoES_{i,t}(\beta) = \sigma_{i,t}\rho_t\left(\frac{\phi(\Phi(\alpha_s))}{\alpha_s} - \frac{\phi(\Phi(\alpha_n^+)) - \phi(\Phi(\alpha_n^-))}{\alpha_n^+ - \alpha_n^-}\right), \tag{31}$$

where $\alpha_s = \beta$, $\alpha_n^+ = 0.5 + \beta/2$ and $\alpha_n^- = 0.5 - \beta/2$.
The $CoES$ formula in the non Gaussian framework would be

$$CoES_{i|m,t}(\alpha,\beta) = \frac{1}{\beta}\int_0^{s^*} P(F_{i,t}(r_{i,t}) = s|r_{m,t} < VaR_{m,t}(\alpha))(F_{i,t}^{-1}(s)\sigma_{i,t} + \mu_{i,t}) \quad ds,$$

where s^* is such that $P(F_{i,t}(r_{i,t}) < s^*|r_{m,t} < VaR_{m,t}(\alpha)) = \beta$ and $P(F_{i,t}(r_{i,t}) = s|r_{m,t} < VaR_{m,t}(\alpha)) = \frac{C_{m|i}(\alpha|s)}{\alpha}$. The value of s^* can be found using copulas and the Bayes' Theorem as

$$\frac{1}{\alpha}\int_0^{s^*} C_{m|i,t}(\alpha|s)ds = \beta$$

.

Component Delta Expected Shortfall ($Celta\ CoES$) is straightforward obtained weighting $\Delta CoES_{i|m,t}(\beta)$ by the market capitalization for each firm.

Conditional Expected Shortfall (CoES) in a Gaussian framework From Equation (27) and taking under consideration the representation of r_t in Equation (28), Equation (11) can be rewritten as

$$
\begin{aligned}
MES_{i,t}(\alpha) &= \sigma_{i,t}\left\{\frac{\rho_t\phi\left(\Phi^{-1}(\alpha)\right)}{\alpha} - \sqrt{1-\rho_t^2}\left(\int_0^\beta \Phi^{-1}(q)dq + \int_\beta^1 \Phi^{-1}(q)dq\right)\right\} - \mu_{i,t} \\
&= \underbrace{\sigma_{i,t}\frac{\rho_t\phi\left(\Phi^{-1}(\alpha)\right)}{\alpha} - \mu_{i,t}}_{-\mu_{i|m,t}} - \\
&\quad \overbrace{\sigma_{i,t}\sqrt{1-\rho_t^2}}^{\sigma_{i|m,t}}\underbrace{\left(\frac{1}{\beta}\int_0^\beta \Phi^{-1}(q)dq\right)}_{E_{t-1}(A)}\overbrace{\beta}^{P_{t-1}(A)} - \\
&\quad \overbrace{\sigma_{i,t}\sqrt{1-\rho_t^2}}^{\sigma_{i|m,t}}\underbrace{\left(\frac{1}{1-\beta}\int_\beta^1 \Phi^{-1}(q)dq\right)}_{E_{t-1}(A^C)}\overbrace{(1-\beta)}^{P_{t-1}(A^C)}
\end{aligned}
$$

where

$$
\begin{aligned}
E_{t-1}(A) &= E_{t-1}\left(\frac{(r_{i,t}-\mu_{i|m,t})}{\sigma_{i|m,t}}|r_{i,t} < CoVaR_{i|m,t}(\alpha,\beta), r_{m,t} < VaR_{m,t}(\alpha)\right), \\
P_{t-1}(A) &= P(r_{i,t} < CoVaR_{i|m,t}|r_{m,t} < VaR_{m,t}(\alpha)), \\
E_{t-1}(A^C) &= E_{t-1}\left(\frac{(r_{i,t}-\mu_{i|m,t})}{\sigma_{i|m,t}}|r_{i,t} > CoVaR_{i|m,t}(\alpha,\beta), r_{m,t} < VaR_{m,t}(\alpha)\right) \text{ and} \\
P_{t-1}(A^C) &= P(r_{i,t} > CoVaR_{i|m,t}|r_{m,t} < VaR_{m,t}(\alpha)).
\end{aligned}
$$

From the solution of these integrals,

$$
\begin{aligned}
E_{t-1}(A) &= \frac{-1}{\beta}\phi\left(\Phi^{-1}(\beta)\right) \\
E_{t-1}(A^C) &= \frac{1}{1-\beta}\phi\left(\Phi^{-1}(\beta)\right).
\end{aligned}
$$

Consequently

$$
CoES_{i|m,t}(\alpha,\beta) = \sigma_{i,t}\left(\frac{\sqrt{1-\rho_t^2}\phi\left(\Phi^{-1}(\beta)\right)}{\beta} + \frac{\rho_t\phi(\Phi(\alpha))}{\alpha}\right) - \mu_{i,t},
$$

and in case the scenario for the financial market were defined between two bounds

$$
CoES_{i|m,t}(\alpha,\beta) = \sigma_{i,t}\left(\frac{\sqrt{1-\rho_t^2}\phi\left(\Phi^{-1}(\beta)\right)}{\beta} + \frac{\rho_t\left\{\phi(\Phi(\alpha^+)) - \phi(\Phi(\alpha^-))\right\}}{\alpha^+ - \alpha^-}\right) - \mu_{i,t},
$$

where α^+ defines the upper bound and α^- is the quantile that defines the lower bound. Then, $\Delta CoES_{i|m,t}(\beta)$ formula in Gaussian framework is straightforward deduced.