OPTIMAL TIME-CONSISTENT FISCAL POLICY UNDER ENDOGENOUS GROWTH WITH ELASTIC LABOUR SUPPLY

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ABSTRACT

In an endogenous growth model with public consumption and investment and an elastic labour supply, we explore the time-consistent optimal choice for two policy instruments: an income tax rate and the split of government spending between consumption and investment. We compare the Markovian optimal policy with the Ramsey policy, extending previous work that characterized optimal fiscal policy either in an exogenous growth framework, assuming an exogenously given split of income between consumption and investment, or an inelastic supply of labour. The Markov-perfect policy implies a higher income tax rate. To compensate for the lower disposable income, a larger proportion of government spending is allocated to consumption than those chosen under a commitment constraint on the part of the government. As a result, economic growth is slightly lower under the Markov-perfect policy than under the Ramsey policy. The welfare loss relative to the benevolent planner's solution is mainly due to the difference in growth rates.

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1. Introduction

Considerable effort is being devoted to characterizing optimal policy in the absence of commitment by the current government with respect to policies taken by future governments. This is to recognize a peculiarity of actual economies that, if ignored, might lead to significant biases in policy design. An additional motivation is to avoid proposing supposedly optimal policies from which we know beforehand that the government will have an incentive to deviate.

Klein, Krusell and Rios-Rull (2008) characterize the optimal time-consistent tax policy in an exogenous growth model with leisure and public consumption in the utility function. They propose a repeated game structure with successive governments, each one of them governing only for one period. Each government is supposed to be a dominant player that takes the optimal reaction of private agents as given when deciding the optimal policy. Ortigueira (2006) compares the results obtained under the structure in Klein, Krusell and Rios-Rull with those from an alternative design of the game in which the government and private agents make their respective decisions simultaneously, characterizing the behavior of the economy along the transition to the optimal steady-state. These two papers consider alternative fiscal structures, always with a single policy instrument: either a single tax levied on total income, a single tax on capital income or a single tax on labor income. Martin (2010) follows the same game structure as Klein, Krusell and Rios-Rull (2008), extending the analysis to the simultaneous consideration of different tax rates for capital and labor income, solving for the optimal time consistent choice for both fiscal instruments. A further exogenous growth analysis is done by Azzimonti et al. (2009), who characterize the Markovian tax rate raised on total income when used to finance public investment.

However, endogenous growth considerations should be important when searching for the design of an optimal policy. They not only allow for a more plausible representation of actual economies, but also for explicitly taking into account the effect of fiscal policy on the rate of growth. That extension has been done by Malley et al. (2002), who use an endogenous growth model and solve for the time-consistent path of distorting income tax rates to finance both productive and consumption government expenditures, under an exogenous split of total spending, logarithmic preferences and full capital depreciation. Novales, Perez and Ruiz (2013) develop a further extension of the Malley et al. analysis to the consideration of an endogenously determined split of government spending between consumption and investment, assuming that private capital has incomplete depreciation each period and that private agents have preferences represented by a CRRA utility function in private and public consumption, as well as on leisure time. These extensions are very relevant, since in Malley et al. (2002) setup, the Ramsey policy is not subject to a time consistency problem and it coincides with the Markov perfect solution, a result shown by Novales, Perez, Ruiz (2013).[1]

In this paper we extend the analysis of Novales, Pérez and Ruiz, to include an elastic labor supply. We consider an endogenous growth economy with elastic labor supply in which the government takes decisions on public consumption and investment each period. The government raises revenues through income taxes, and decides which percentage to devote to public consumption or to investment.

We first compute the analytical solution in a relatively simple environment similar to the one used by Malley et al. (2002), where consumers have logarithmic preferences that are separable across commodities and over time, and private capital fully depreciates each period. The novelty with respect to those authors is that we solve not only for the Markov-perfect tax rate but also for the time-consistent composition of total government expenditures, further considering an elastic labor supply. The optimal time-consistent policy and the resulting allocation of resources are then compared to the Ramsey solution.

For the more general case with incomplete depreciation of private capital and CRRA preferences in private and public consumption, as well as leisure, the optimal fiscal policies cannot be characterized analytically. There, we numerically compute the three solutions. For the Markovian time-consistent policy we assume that our model lacks transitional dynamics, as is typical in AK-type economies like the one in this paper. Besides, absence of transitional dynamics has been analytically shown by Novales, Pérez and Ruiz (2013) in a similar economy, under an inelastic labor supply.

Our qualitative results are similar to those in Novales, Pérez and Ruiz (2013), with the Markov policy placing a higher income tax and devoting a higher proportion of revenues to public consumption than the Ramsey optimal policy. Hence, the normative results suggest that the loss of welfare due to the larger fiscal pressure suffered by the private sector is partially mitigated by the Markov government through a direct welfare

[1] Azzimonti et al. (2009) also show this result for an exogenous growth economy.

compensation by means of a larger supply of the public consumption good. The consequences of implementing this time consistent policy are a slightly lower working time and also a lower rate of growth relative to the Ramsey solution.

The paper is organized as follows: we describe the model economy in section 2, characterizing section 3 the solutions to the Markov, Ramsey and Planner´s optimization programs. In section 4, an analytical solution is obtained under the more restricted assumptions on preferences and capital depreciation. Section 5 characterizes the optimal policies and allocations of resources under the three solution concepts, under a CRRA utility function and incomplete depreciation of private capital. Welfare analysis is developed in section 6, and section 7 concludes.

2. The model economy

We consider an economy with a private agent and a firm that maximizes profits subject to a technology that produces the single consumption commodity. We assume that the private agent has each period a unit of time to allocate between leisure and working time. The stocks of private and public capital, k_t and $k_{p,t}$, are used together with labour time, l_t, as inputs in a production technology: $y_t = Bk_t^{\alpha}(l_t k_{p,t})^{1-\alpha}$, where B is a scale parameter. The firm pays a rent $r_t k_t + w_t l_t$ to the private agent for the use of private capital and labor, solving each period the static profit optimization problem:

$$\underset{\{k_t,l_t\}}{Max}\,\Pi_t = Bk_t^{\alpha}(l_t k_{p,t})^{1-\alpha} - r_t k_t - w_t l_t .$$

Markets for production inputs are competitive. At each point in time, input prices are equal to their marginal product:

$$r_t = r\left(k_t, k_{p,t}, l_t\right) = \alpha B\left(k_t/(l_t k_{p,t})\right)^{\alpha-1}, \tag{1}$$

$$w_t = w\left(k_t, k_{p,t}, l_t\right) = (1-\alpha)B\left(k_t/(l_t k_{p,t})\right)^{\alpha} k_{p,t} . \tag{2}$$

The government uses the proceeds from income taxes to finance public consumption and to accumulate public capital. We denote by η_t the proportion of revenues used at time t to purchase public consumption, the remaining public resources being used to pay for public investment. The government budget constraint is *, where

$$g_t = \eta_t \tau_t\left(r_t k_t + w_t l_t\right), \tag{3}$$

$$k_{p,t} = (1-\eta_t)\tau_t\left(r_t k_t + w_t l_t\right). \tag{4}$$

In line with Barro (1990), and Cazzavillan (1996), it is public investment that is productive, since the same $k_{p,t}$ variable enters as an argument in the production function and as an expenditure component in the government budget constraint. Alternatively, we could think of public capital as fully depreciating each period.

The consumer maximizes her life-time discounted aggregate utility, $\sum_{t=0}^{\infty} \rho^t U(c_t, g_t, l_t)$, defined over private and public consumption, c_t, g_t, and leisure, $1\text{-}l_t$, subject to a flat tax τ_t on total income. She knows the current values of τ_t and η_t, and expect future governments to follow policies $\tau_{t+1} = \mathcal{T}(k_{t+1}, k_{p,t+1})$ and $\eta_{t+1} = \mathcal{H}(k_{t+1}, k_{p,t+1})$, solving the problem:

$$\upsilon\left(k_t, k_{p,t}; \tau_t; \eta_t; \mathcal{T}; \mathcal{H}\right) = \underset{\{c_t, k_{t+1}, l_t\}}{Max} \left[U\left(c_t, g_t, l_t\right) + \rho \tilde{\upsilon}(k_{t+1}, k_{p,t+1}; \mathcal{T}; \mathcal{H})\right] \tag{5}$$

given k_0, taking as given all policy variables: $\left\{\tau_t, \eta_t, k_{p,t}, g_t\right\}_{t=0}^{\infty}$ and subject to the budget constraint,

$$c_t + k_{t+1} - (1-\delta)k_t = (1-\tau_t)\left[w_t \cdot l_t + r_t \cdot k_t\right], \tag{6}$$

leading to the following optimality equations

$$\frac{U_l\left(c_t, g_t, l_t\right)}{U_c\left(c_t, g_t, l_t\right)} = \left(1-\tau_t\right)(1-\alpha)\left[(1-\eta_t)\tau_t\right]^{\frac{1-\alpha}{\alpha}} B^{1/\alpha} l_t^{(1-2\alpha)/\alpha} k_t \tag{7}$$

$$U_c\left(c_t, g_t, l_t\right) = \rho U_c\left(c_{t+1}, g_{t+1}, l_{t+1}\right)\left[1-\delta + \left(1-\tau_{t+1}\right)\alpha B\left(k_{t+1} / (l_{t+1} k_{p,t+1})\right)^{\alpha-1}\right]. \tag{8}$$

The first equation is the equality between the marginal rate of substitution between consumption and leisure and the after tax real wage, while the second equation is the Euler equation,[2] in which we have already taken (1) into account.

From the government budget expenditure rules and the optimality conditions for the competitive firms we get,

$$g_t = \mathcal{G}(k_t, k_{p,t}; \tau_t, \eta_t) = \eta_t \tau_t B k_t^{\alpha} l_t^{1-\alpha} k_{p,t}^{1-\alpha}, \tag{9}$$

$$k_{p,t} = (1-\eta_t)\tau_t B k_t^{\alpha} l_t^{1-\alpha} k_{p,t}^{1-\alpha}, \tag{10}$$

as well as the global constraint of resources:

$$k_{t+1} = (1-\delta)k_t + (1-\tau_t)B k_t^{\alpha} k_{p,t}^{1-\alpha} \ell\left(k_t, k_{p,t}; \tau_t, \eta_t\right)^{1-\alpha} - \mathcal{C}\left(k_t, k_{p,t}; \tau_t, \eta_t\right), \tag{11}$$

[2] Along the paper we denote partial derivatives by $F_v \equiv \frac{\partial F}{\partial v}$.

where $\mathcal{C}\left(k_t,k_{p,t};\tau_t,\eta_t\right)$ and $\ell\left(k_t,k_{p,t};\tau_t,\eta_t\right)$ are the consumption and working time decision functions that solve the Euler equation (7) and (8).

From (9) and (10), we have:

$$k_{p,t}=\left[(1-\eta_t)\tau_t\right]^{1/\alpha}B^{1/\alpha}l_t^{(1-\alpha)/\alpha}k_t \tag{12}$$

$$g_t=\mathcal{G}(k_t,k_{p,t};\tau_t,\eta_t)=\eta_t\tau_t Bk_t^{\alpha}l_t^{1-\alpha}k_{p,t}^{1-\alpha}=\eta_t\left(1-\eta_t\right)^{(1-\alpha)/\alpha}\tau_t^{1/\alpha}B^{1/\alpha}l_t^{(1-\alpha)/\alpha}k_t \tag{13}$$

so that the conditions for competitive equilibrium are:

$$\frac{U_l(c_t,l_t,g_t)}{U_c(c_t,l_t,g_t)}=(1-\alpha)(1-\tau_t)\left[(1-\eta_t)\tau_t\right]^{\frac{1-\alpha}{\alpha}}B^{1/\alpha}l_t^{\frac{1-\alpha}{\alpha}}k_t, \tag{14}$$

$$U_c(c_t,l_t,g_t)=\rho U_c\left(c_{t+1},l_{t+1},g_{t+1}\right)\left[1-\delta+\alpha(1-\tau_{t+1})[(1-\eta_{t+1})\tau_{t+1}]^{\frac{1-\alpha}{\alpha}}B^{1/\alpha}l_t^{\frac{1-\alpha}{\alpha}}\right] \tag{15}$$

Substituting (12) in the technology function, output is given by

$$y_t=B^{1/\alpha}\left[(1-\eta_t)\tau_t\right]^{(1-\alpha)/\alpha}l_t^{(1-\alpha)/\alpha}k_t\ . \tag{16}$$

As a consequence of (16) and (12), in the competitive equilibrium allocation, the ratio of public capital to output is equal to $(1-\eta_t)\tau_t$, an extension of the result in Barro (1990). As is typical in the Barro family of *AK* models, the constant returns to scale in the cumulative factors is the source of endogenous growth in our model economy.

Finally, in competitive equilibrium we will have

$$\upsilon\left(k_t,k_{p,t};\mathcal{T}\left(k_t,k_{p,t}\right);\mathcal{H}\left(k_t,k_{p,t}\right);\mathcal{T};\mathcal{H}\right)=\tilde{\upsilon}\left(k_t,k_{p,t};\mathcal{T};\mathcal{H}\right).$$

3. Optimal fiscal policy under an elastic labor supply

In this section we characterize the allocations of resources that would be achieved in our model economy under three policy arrangements: the Markov equilibrium, the Ramsey equilibrium and the solution to the benevolent planner's problem.

3.1 The Markov, time-consistent optimal policy

To characterize the Markov solution, we use the same game structure as in Klein et al. (2008), with successive governments that take decisions on policy variables only for the single period they are in office. Accordingly, the Markov government recognizes its inability to commit on future policies taken by the following governments.

In contrast, the Ramsey allocation of resources would be presumably achieved by a planning government that would ignore such limitation. This solution would be unrealistic, since a future government would have an incentive to deviate from the previously designed policy so that the initially conceived Ramsey allocation would never be achieved.

We maintain the assumption on the availability of a production technology: $y_t = Bk_t^{\alpha}\left(k_{p,t}l_t\right)^{1-\alpha}$. Consumers solve the time aggregate utility maximization problem subject to the constraint: $k_{t+1} - (1-\delta)k_t + c_t = (1-\tau_t)\left(r_t k_t + w_t l_t\right)$, given k_0 and taking into account policy announcements. The government raises revenues imposing a global tax rate on total income and distributes the proceeds between public consumption and investment. The government budget constraint is: $\tau_t\left(r_t k_t + w_t l_t\right) = g_t + k_{p,t}$.

The government knows that the consumption and working time decision rules of the household are the solutions to (7) and (8). Acting as a leader, it chooses the current tax rate and the split of public resources between consumption and investment, taking as given the policies followed by future governments and taking into account the reaction of the household to the policy choices, to solve the problem:

$$V\left(k_t, k_{p,t}\right) = \underset{\{\tau_t,\eta_t\}}{Max}\Big[U\Big(\mathcal{C}\left(k_t, k_{p,t};\tau_t,\eta_t\right), \ell(k_t, k_{p,t};\tau_t,\eta_t), \mathcal{G}\left(k_t, k_{p,t};\tau_t,\eta_t\right)\Big) + \rho V(k_{t+1}, k_{p,t+1})\Big] \qquad \text{[P1]}$$

where $\mathcal{G}\left(k_t, k_{p,t};\tau_t,\eta_t\right)$, $k_{p,t}$ and k_{t+1} are given by (13), (12) and (11), respectively.

Proposition 1. *The time consistent policy corresponding to the Markov equilibrium is the solution to the set of Generalized Euler Equations (GEE):*

$$\frac{U_{c_t}\mathcal{C}_{\tau_t} + U_{g_t}\mathcal{G}_{\tau_t} + U_{l_t}\ell_{\tau_t}}{y_t\left(1-(1-\tau_t)\dfrac{1-\alpha}{\alpha}\left(\dfrac{\ell_{\tau_t}}{\ell_t}+\dfrac{1}{\tau_t}\right)\right)+\mathcal{C}_{\tau_t}} = \frac{U_{c_t}\mathcal{C}_{\eta_t} + U_{g_t}\mathcal{G}_{\eta_t} + U_{l_t}\ell_{\eta_t}}{\dfrac{1-\alpha}{\alpha}(1-\tau_t)y_t\left(\dfrac{1}{1-\eta_t}-\dfrac{\ell_{\eta_t}}{\ell_t}\right)+\mathcal{C}_{\eta_t}} \tag{17}$$

and

$$\frac{U_{c_t}\mathcal{C}_{\tau_t} + U_{g_t}\mathcal{G}_{\tau_t} + U_{l_t}\ell_{\tau_t}}{y_t\left(1-(1-\tau_t)\dfrac{1-\alpha}{\alpha}\left(\dfrac{\ell_{\tau_t}}{\ell_t}+\dfrac{1}{\tau_t}\right)\right)+\mathcal{C}_{\tau_t}} = \rho\Bigg\{U_{c_{t+1}}\mathcal{C}_{k_{t+1}} + U_{g_{t+1}}\mathcal{G}_{k_{t+1}} + U_{l_{t+1}}\ell_{k_{t+1}} +$$

$$\frac{U_{c_{t+1}}\mathcal{C}_{\tau_{t+1}} + U_{g_{t+1}}\mathcal{G}_{\tau_{t+1}} + U_{l_{t+1}}\ell_{\tau_{t+1}}}{y_{t+1}\left(1-(1-\tau_{t+1})\dfrac{1-\alpha}{\alpha}\left(\dfrac{\ell_{\tau_{t+1}}}{\ell_{t+1}}+\dfrac{1}{\tau_{t+1}}\right)\right)+\mathcal{C}_{\tau_{t+1}}}\left[1-\delta+(1-\tau_{t+1})\left(\frac{y_{t+1}}{k_{t+1}}+\frac{1-\alpha}{\alpha}\frac{y_{t+1}}{\ell_{t+1}}\ell_{k_{t+1}}\right)-\mathcal{C}_{k_{t+1}}\right]\Bigg\} \tag{18}$$

where $y_t = \left[(1-\eta_t)\tau_t\right]^{\frac{1-\alpha}{\alpha}} B^{1/\alpha} \ell_t^{\frac{1-\alpha}{\alpha}} k_t$, $\mathcal{G}_{\tau_t} = \mathcal{G}_t\left(\frac{1}{\alpha\tau_t} + \frac{1-\alpha}{\alpha}\frac{\ell_{\tau_t}}{\ell_t}\right)$,

$\mathcal{G}_{\eta_t} = \mathcal{G}_t\left[\frac{1}{\eta_t} - \frac{1-\alpha}{\alpha}\frac{1}{1-\eta_t} + \frac{1-\alpha}{\alpha}\frac{\ell_{\eta_t}}{\ell_t}\right]$, $\mathcal{G}_{k_t} = \mathcal{G}_t / k_t$, $\mathcal{C}_t \equiv \mathcal{C}\left(k_t, k_{p,t}; \tau_t, \eta_t\right)$ *and*

$\ell_t \equiv \ell\left(k_t, k_{p,t}; \tau_t, \eta_t\right), \mathcal{G}_t \equiv \mathcal{G}\left(k_t, k_{p,t}; \tau_t, \eta_t\right)$.

Proof: *See Appendix 1.* □

In what follows, we assume that preferences can be represented by the utility function, $U(c_t, l_t, g_t) = \frac{\left(c_t(1-l_t)^{\varepsilon} g_t^{\theta}\right)^{1-\sigma} - 1}{1-\sigma}$. Under this structure of preferences, the allocation of resources under the Markov equilibrium is obtained as the solution to the system made up by the competitive equilibrium conditions:

$$\varepsilon\mathcal{C}_t \frac{\ell_t}{1-\ell_t} = (1-\alpha)(1-\tau_t)\left[(1-\eta_t)\tau_t\right]^{\frac{1-\alpha}{\alpha}} B^{1/\alpha} \ell_t^{\frac{1-\alpha}{\alpha}} k_t,$$

$$\mathcal{C}_t^{-\sigma}\left(1-\ell_t\right)^{\varepsilon(1-\sigma)} \mathcal{G}_t^{\theta(1-\sigma)} = \rho\mathcal{C}_{t+1}^{-\sigma}\left(1-\ell_{t+1}\right)^{\varepsilon(1-\sigma)} \mathcal{G}_{t+1}^{\theta(1-\sigma)} \times \left[1-\delta+\alpha(1-\tau_{t+1})[(1-\eta_{t+1})\tau_{t+1}]^{\frac{1-\alpha}{\alpha}} B^{1/\alpha} \ell_{t+1}^{\frac{1-\alpha}{\alpha}}\right],$$

together with the global constraint of resources and the two Generalized Euler equations:

$$k_{t+1} = (1-\delta)k_t + (1-\tau_t)\left[(1-\eta_t)\tau_t\ell_t\right]^{\frac{1-\alpha}{\alpha}} B^{1/\alpha} k_t - \mathcal{C}_t,$$

$$\frac{\frac{\mathcal{C}_{\tau_t}}{\mathcal{C}_t} + \theta\frac{\mathcal{G}_{\tau_t}}{\mathcal{G}_t} - \varepsilon\frac{\ell_{\tau_t}}{\ell_t}\frac{\ell_t}{1-\ell_t}}{y_t\left(1-(1-\tau_t)\frac{1-\alpha}{\alpha}\left(\frac{\ell_{\tau_t}}{\ell_t}+\frac{1}{\tau_t}\right)\right)+\mathcal{C}_{\tau_t}} = \frac{\frac{\mathcal{C}_{\eta_t}}{\mathcal{C}_t} + \theta\frac{\mathcal{G}_{\eta_t}}{\mathcal{G}_t} - \varepsilon\frac{\ell_{\eta_t}}{\ell_t}\frac{\ell_t}{1-\ell_t}}{\frac{1-\alpha}{\alpha}(1-\tau_t)y_t\left(\frac{1}{1-\eta_t}-\frac{\ell_{\eta_t}}{\ell_t}\right)+\mathcal{C}_{\eta_t}}$$

$$\frac{\frac{\mathcal{C}_{\tau_t}}{\mathcal{C}_t} + \theta\frac{\mathcal{G}_{\tau_t}}{\mathcal{G}_t} - \varepsilon\frac{\ell_{\tau_t}}{\ell_t}\frac{\ell_t}{1-\ell_t}}{y_t\left(1-(1-\tau_t)\frac{1-\alpha}{\alpha}\left(\frac{\ell_{\tau_t}}{\ell_t}+\frac{1}{\tau_t}\right)\right)+\mathcal{C}_{\tau_t}} = \rho\left\{\frac{\mathcal{C}_{k_{t+1}}}{\mathcal{C}_{t+1}} + \theta\frac{\mathcal{G}_{k_{t+1}}}{\mathcal{G}_{t+1}} - \varepsilon\frac{\ell_{k_{t+1}}}{\ell_{t+1}}\frac{\ell_{t+1}}{1-\ell_{t+1}} + \right.$$

$$\left.\frac{\frac{\mathcal{C}_{\tau_{t+1}}}{\mathcal{C}_{t+1}} + \theta\frac{\mathcal{G}_{\tau_{t+1}}}{\mathcal{G}_{t+1}} - \varepsilon\frac{\ell_{\tau_{t+1}}}{\ell_{t+1}}\frac{\ell_{t+1}}{1-\ell_{t+1}}}{y_{t+1}\left(1-(1-\tau_{t+1})\frac{1-\alpha}{\alpha}\left(\frac{\ell_{\tau_{t+1}}}{\ell_{t+1}}+\frac{1}{\tau_{t+1}}\right)\right)+\mathcal{C}_{\tau_{t+1}}}\left[1-\delta+(1-\tau_{t+1})\left(\frac{y_{t+1}}{k_{t+1}}+\frac{1-\alpha}{\alpha}\frac{y_{t+1}}{\ell_{t+1}}\ell_{k_{t+1}}\right)-\mathcal{C}_{k_{t+1}}\right]\right\}.$$

This system can be written in terms of ratios of variables with respect to private capital, as:

$$\frac{\varepsilon\,\chi_t\ell_t}{1-\ell_t}=(1-\alpha)(1-\tau_t)\left[(1-\eta_t)\tau_t\right]^{\frac{1-\alpha}{\alpha}}B^{1/\alpha}\ell_t^{\frac{1-\alpha}{\alpha}}, \tag{19}$$

$$\chi_t^{-\sigma}\left(1-\ell_t\right)^{\varepsilon(1-\sigma)}\phi_t^{\theta(1-\sigma)}\gamma_{t+1}^{-\sigma+\theta(1-\sigma)}=\rho\chi_{t+1}^{-\sigma}\left(1-\ell_{t+1}\right)^{\varepsilon(1-\sigma)}\phi_{t+1}^{\theta(1-\sigma)}\times \left[1-\delta+\alpha(1-\tau_{t+1})[(1-\eta_{t+1})\tau_{t+1}]^{\frac{1-\alpha}{\alpha}}B^{1/\alpha}\ell_{t+1}^{\frac{1-\alpha}{\alpha}}\right] \tag{20}$$

$$\gamma_{t+1}\equiv\frac{k_{t+1}}{k_t}=(1-\delta)+(1-\tau_t)\left[(1-\eta_t)\tau_t\ell_t\right]^{\frac{1-\alpha}{\alpha}}B^{1/\alpha}-\chi_t \tag{21}$$

$$\frac{\dfrac{\chi_{\tau_t}}{\chi_t}+\theta\dfrac{\phi_{\tau_t}}{\phi_t}-\varepsilon\dfrac{\ell_{\tau_t}}{\ell_t}\dfrac{\ell_t}{1-\ell_t}}{\tilde{y}_t\left(1-(1-\tau_t)\dfrac{1-\alpha}{\alpha}\left(\dfrac{\ell_{\tau_t}}{\ell_t}+\dfrac{1}{\tau_t}\right)\right)+\chi_{\tau_t}}=\frac{\dfrac{\chi_{\eta_t}}{\chi_t}+\theta\dfrac{\phi_{\eta_t}}{\phi_t}-\varepsilon\dfrac{\ell_{\eta_t}}{\ell_t}\dfrac{\ell_t}{1-\ell_t}}{\dfrac{1-\alpha}{\alpha}(1-\tau_t)\tilde{y}_t\left(\dfrac{1}{1-\eta_t}-\dfrac{\ell_{\eta_t}}{\ell_t}\right)+\chi_{\eta_t}} \tag{22}$$

$$\tilde{U}_t\gamma_{t+1}^{-\sigma+\theta(1-\sigma)}\frac{\dfrac{\chi_{\tau_t}}{\chi_t}-\varepsilon\dfrac{\ell_{\tau_t}}{\ell_t}\dfrac{\ell_t}{1-\ell_t}+\theta\dfrac{\phi_{\tau_t}}{\phi_t}}{\tilde{y}_t\left(1-(1-\tau_t)\dfrac{1-\alpha}{\alpha}\left(\dfrac{\ell_{\tau_t}}{\ell_t}+\dfrac{1}{\tau_t}\right)\right)+\chi_{\tau_t}}=\rho\tilde{U}_{t+1}\left\{1+\frac{k_{t+1}\chi_{k_{t+1}}}{\chi_{t+1}}-\varepsilon\frac{\ell_{k_{t+1}}}{1-\ell_{t+1}}+\theta\left(1+\frac{k_{t+1}\phi_{k_{t+1}}}{\phi_{t+1}}\right)+\right.$$
$$\left.\frac{\dfrac{\chi_{\tau_{t+1}}}{\chi_{t+1}}-\varepsilon\dfrac{\ell_{\tau_{t+1}}}{\ell_{t+1}}\dfrac{\ell_{t+1}}{1-\ell_{t+1}}+\theta\dfrac{\phi_{\tau_{t+1}}}{\phi_{t+1}}}{\tilde{y}_{t+1}\left(1-(1-\tau_{t+1})\dfrac{1-\alpha}{\alpha}\left(\dfrac{\ell_{\tau_{t+1}}}{\ell_{t+1}}+\dfrac{1}{\tau_{t+1}}\right)\right)+\chi_{\tau_{t+1}}}\left[1-\delta+(1-\tau_{t+1})\left(\tilde{y}_{t+1}+\frac{1-\alpha}{\alpha}\frac{\tilde{y}_{t+1}}{\ell_{t+1}}\ell_{k_{t+1}}\right)-\chi_{k_{t+1}}\right]\right\} \tag{23}$$

where $\chi_t\equiv\mathcal{C}_t/k_t$, $\chi_{\tau_t}=\mathcal{C}_{\tau_t}/k_t$, $\chi_{\eta_t}=\mathcal{C}_{\eta_t}/k_t$, $\phi_t\equiv\mathcal{G}_t/k_t=\eta_t(1-\eta_t)^{\frac{1-\alpha}{\alpha}}\tau_t^{1/\alpha}B^{1/\alpha}\ell_t^{\frac{1-\alpha}{\alpha}}$,

$\phi_{\tau_t}=\mathcal{G}_{\tau_t}/k_t$, $\phi_{\eta_t}=\mathcal{G}_{\eta_t}/k_t$, $\tilde{U}_t=\chi_t^{1-\sigma}\phi_t^{\theta(1-\sigma)}\ell_t^{\varepsilon(1-\sigma)}$, $\tilde{y}_t\equiv\dfrac{y_t}{k_t}=\left[(1-\eta_t)\tau_t\ell_t\right]^{\frac{1-\alpha}{\alpha}}B^{1/\alpha}$.

3.2 The Ramsey policy

As usual, we define the benchmark “Ramsey equilibrium” as the solution to an optimal-policy problem where the government can commit to future policies. The Ramsey optimal policy is then the solution to the problem of maximizing the time aggregate utility of the household, subject to the equilibrium conditions (7), (8), (11), and (13) as constraints:

$$
\begin{aligned}
&\underset{\{\tau_t,\eta_t\}}{Max}\sum_{t=0}^{\infty}\rho^t U\left(c_t,l_t,g_t\right)\\
&\text{subject to:}\\
&k_{t+1}=(1-\delta)k_t+B^{1/\alpha}(1-\tau_t)\left[(1-\eta_t)\tau_t\right]^{\frac{1-\alpha}{\alpha}}l_t^{\frac{1-\alpha}{\alpha}}k_t-c_t\\
&U_{c_t}=\rho U_{c_{t+1}}\left[1-\delta+\alpha B^{1/\alpha}(1-\tau_{t+1})\left[(1-\eta_{t+1})\tau_{t+1}\right]^{\frac{1-\alpha}{\alpha}}l_{t+1}^{\frac{1-\alpha}{\alpha}}\right] \qquad \text{[P2]}\\
&\frac{U_l\left(c_t,g_t,l_t\right)}{U_c\left(c_t,g_t,l_t\right)}=\left(1-\tau_t\right)(1-\alpha)B^{1/\alpha}\left[\left(1-\eta_t\right)\tau_t\right]^{(1-\alpha)/\alpha}l_t^{(1-\alpha)/\alpha}k_t\\
&g_t=\eta_t\tau_t^{1/\alpha}(1-\eta_t)^{\frac{1-\alpha}{\alpha}}B^{1/\alpha}l_t^{\frac{1-\alpha}{\alpha}}k_t\ .
\end{aligned}
$$

where we have used (12) to eliminate public capital from the system.

The Ramsey policy takes into account the optimal reactions of private agents. However, it is time inconsistent, since once private agents adjust their decisions to the announced economic policy it will be optimal for the government to change policy. Full analytical details for computing the Ramsey policy and the implied allocation of resources are provided in Appendix 2.

Given the complexity involved in characterizing optimal policy under lack of commitment, attention has often been restricted to Ramsey policies, in spite of their well-known limitation of assuming commitment on the part of the government. It is therefore important to evaluate to what extent the Markov-perfect fiscal policy differs from the Ramsey policy in our setup. We will perform such analysis in Section 4.

3.3 The planner's problem

A benevolent planner who can impose lump-sum taxes would allocate resources so as to maximize time aggregate utility with the global constraint of resources as its sole restriction, thereby solving the problem,

$$
\begin{aligned}
&\underset{\{c_t,l_t,k_{t+1},k_{pt},g_t\}}{Max}\sum_{t=0}^{\infty}\rho^t\frac{c_t^{1-\sigma}(1-l_t)^{\varepsilon(1-\sigma)}g_t^{\theta(1-\sigma)}-1}{1-\sigma}\\
&\text{subject to:} \qquad \text{[P3]}\\
&k_{t+1}-(1-\delta)k_t+c_t+g_t+k_{p,t}=Bk_t^{\alpha}k_{p,t}^{1-\alpha}l_t^{1-\alpha}.
\end{aligned}
$$

Appendix 3 contains the analytical details of the optimal policy and the implied allocation of resources under the benevolent planner´s rule. We use $\tau_t^P=\dfrac{g_t+k_{p,t}}{y_t}$ as a measure of the size of the public sector, and $\eta_t^P=\dfrac{g_t}{g_t+k_{p,t}}$ for the composition of public expenditures under the planner solution. Both of them will be used in the graphs and tables we present below.

4 An analytical solution under elastic labor supply: logarithmic utility and full depreciation of private capital.

Let us assume a logarithmic utility and full depreciation, $\sigma = \delta = 1$. The competitive equilibrium conditions become:

$$\frac{\varepsilon c_t l_t}{1 - l_t} = (1-\alpha)(1-\tau_t) B^{1/\alpha} \left[(1-\eta_t)\tau_t l_t\right]^{\frac{1-\alpha}{\alpha}} k_t \tag{24}$$

$$\frac{c_{t+1}}{c_t} = \rho \left[(1-\tau_{t+1}) \alpha B^{1/\alpha} \left[(1-\eta_{t+1})\tau_{t+1} l_{t+1}\right]^{\frac{1-\alpha}{\alpha}} \right] \tag{25}$$

$$k_{t+1} + c_t = (1-\tau_t) B^{1/\alpha} \left[(1-\eta_t)\tau_t l_t\right]^{\frac{1-\alpha}{\alpha}} k_t ,. \tag{26}$$

Proposition 2. *The competitive equilibrium allocations are given by,*

$$k_{t+1} = \rho\alpha \left[\frac{1-\alpha}{\varepsilon(1-\rho\alpha) + 1 - \alpha} \right]^{\frac{1-\alpha}{\alpha}} \Omega(\tau_t, \eta_t) k_t ,$$

$$c_t = (1-\rho\alpha) \left[\frac{1-\alpha}{\varepsilon(1-\rho\alpha) + 1 - \alpha} \right]^{\frac{1-\alpha}{\alpha}} \Omega(\tau_t, \eta_t) k_t ,$$

$$l_t = \frac{1-\alpha}{\varepsilon(1-\rho\alpha) + 1 - \alpha},$$

$$\textit{where} \quad \Omega(\tau_t, \eta_t) \equiv (1-\tau_t) B^{1/\alpha} \left[(1-\eta_t)\tau_t\right]^{\frac{1-\alpha}{\alpha}} .$$

Proof.- *Plugging in system (24)-(26) a guess:* $k_{t+1} = A\,\Omega(\tau_t, \eta_t) k_t$, $c_t = B\,\Omega(\tau_t, \eta_t) k_t$, *$l_t$=D, for the functional form of the competitive equilibrium allocation, with A, B, and D being unknown constants, it is easy to show that,*

$$A = \rho\alpha \left[\frac{1-\alpha}{\varepsilon(1-\rho\alpha) + 1 - \alpha} \right]^{\frac{1-\alpha}{\alpha}}, B = (1-\rho\alpha) \left[\frac{1-\alpha}{\varepsilon(1-\rho\alpha) + 1 - \alpha} \right]^{\frac{1-\alpha}{\alpha}}, D = \frac{1-\alpha}{\varepsilon(1-\rho\alpha) + 1 - \alpha}. \square$$

The conditions in Proposition 2 allow us to characterize the equilibrium paths for $\{k_{t+1}, c_t, l_t\}_{t=0}^{\infty}$ under a given fiscal policy $\{\tau_t, \eta_t\}_{t=0}^{\infty}$, starting from an initial stock of physical capital.

The next propositions characterize analytically the optimal Markov and Ramsey policies for this economy.

Proposition 3. *i) Under full depreciation of private capital and a logarithmic utility function, the optimal time-consistent fiscal policy is:*[3]

$$\tau_t^M = \tau^M = 1 - \frac{\alpha(1+\rho\theta)}{1+\theta}, \quad \forall t$$

$$\eta_t^M = \eta^M = \frac{\alpha\theta(1-\rho)}{1-\alpha+\theta(1-\alpha\rho)}, \quad \forall t.$$

ii) The constant values of the two policy variables under the optimal Markov policy are related by: $\tau^M = \dfrac{1-\alpha}{1-\eta^M}$

Proof. *See Appendix 4 for the proof of i). The proof of ii) is straightforward.* □

As we can see, under full depreciation of physical capital and logarithmic preferences, separable in public and private consumption, the optimal policy is the same as that obtained by Novales, Pérez and Ruiz (2013) for a version of this economy with inelastic labour supply.

Proposition 4. *Under full depreciation of private capital and a logarithmic utility function, the Ramsey fiscal policy is the same as the optimal Markov policy.*

Proof. *Particularizing the system of equations in Appendix 4 for the case of complete depreciation of private capital and logarithmic preferences, the result is obtained after some tedious algebra.* □

5. Comparing the Ramsey and Markov solutions

For the more general case with incomplete depreciation of private capital and CRRA preferences in private and public consumption, as well as leisure, the optimal fiscal policies cannot be characterized analytically.

It is well known that in the family of AK models, the economy lacks any transitional dynamics under the competitive equilibrium mechanism, as well as under

[3] Malley et al. (2002) obtain a similar expression for the Markov perfect tax rate. Our result generalizes Malley et al. (2002) in two directions: by considering an endogenous labor supply and by characterizing the optimal value of the split of government spending in addition to the income tax rate as policy instruments.

the benevolent planner's rule or the Ramsey solution. That is also the case in the economy we consider when labor is supplied inelastically [Novales, Pérez and Ruiz (2013)].[4] Taking all this into account, to numerically compute the allocation of resources under this policy we assume that the economy lacks transitional dynamics. As a consequence, control variables for the private agent and the government cannot depend on state variables, so that partial derivatives: $\chi_{k_t} = \phi_{k_t} = \ell_{k_t} = 0, \forall t,$ at the second Generalized Euler condition (23).

Let us now compare the Markov and Ramsey solutions between themselves, as well as with the allocation of resources that would be achieved under the rule of a benevolent planner who can impose lump-sum taxes, as characterized in Appendix 3.

The Markov equilibrium is obtained solving the system (19)-(23). As shown in Appendix 2, the solution to the Ramsey problem [P2] is characterized by a system of 10 dynamic equations in $\{\gamma, \chi, \phi, l, \eta, \tau, \tilde{\mu}_1, \tilde{\mu}_2, \tilde{\mu}_3, \tilde{\mu}_4\}$ that allows us to compute the balanced growth path for the Ramsey policy (τ^R, η^R) as well as the implied allocation of resources, characterized by $(\gamma^R, \chi^R, \phi^R, l^R)$ and four multipliers $\{\tilde{\mu}_1, \tilde{\mu}_2, \tilde{\mu}_3, \tilde{\mu}_4\}$. That system is made up only by control variables, with no participation of any state variable. Hence, in the absence of local indeterminacy of equilibrium, the only possible solution is that control variables stay on the balanced growth path (BGP) from the initial period, with no transition.

Under incomplete depreciation of private capital, the choice of parameter values: $\theta = 0.4,\ 1-\alpha = 0.20,\ \rho = 0.99,\ \delta = 0.10,\ B = 0.573, \varepsilon = 1.1,$ when generating annual data lead to sensible properties of the Markov solution. Parameter values are standard in the literature for annual data except for θ, which is chosen so that the ratio of public consumption to private consumption for the Markov solution is in line with data for the postwar US economy (g/c=0.25). For instance, for $\sigma = 2$, we get a ratio of public to private consumption of 0.26, an annual growth rate γ=1.5%, and a gross real interest rate: $1/\left(\rho\gamma^{(1-\sigma)(1+\theta)}\right) \approx 1.0314$. As it is standard in the endogenous growth literature, the value chosen for α is consistent with a broad concept of capital that includes both physical and human components (see Cazzavillian, 1996). The elasticity of output with

[4] As shown in Section 4, a simpler version of our model economy, with full depreciation of capital and logarithmic preferences also lacks transition under the Markov time-consistent policy.

respect to public capital, 1-α, is in line with previous literature, although the range of values varies significantly across authors between the 0.03 estimated by Eberts (1986), and the 0.39 estimated by Aschauer (1989).

Figure 1 shows values for the main variables in the economy under the three equilibrium concepts as a function of the risk aversion parameter, σ. Over the whole range of values considered, the optimal income tax increases with risk aversion. It always falls between 20% and 31%, being higher under the Markov-perfect policy than under the time-inconsistent Ramsey policy. The proportion of public resources devoted to consumption, relative to investment, is also increasing in σ, staying between 5% and 35%. It is higher under the Markov-perfect solution than under the Ramsey policy, so the first mechanism assigns a lower proportion of public resources to investment. In the absence of commitment, the government partially compensates the higher fiscal pressure on the private sector by increasing the direct welfare benefit obtained from public consumption.

Steady state growth is slightly higher under the Ramsey policy. Growth rates are large for low values of the risk aversion parameter, but they become quite realistic for values of σ above 1.5. As a proportion of output, private consumption is higher under the Ramsey policy, while public consumption is higher under the Markov policy. In terms of specific values, private consumption never exceeds 35% of output under either policy, while public consumption remains below 11% of output, both observations below the levels observed in actual economies. However, the public to private consumption ratio is around 25%, as in observed data.[5] For the Markov and Ramsey solutions we could obtain ratios of public and private consumption to output similar to those in actual data, at the expense of getting income tax rates implausibly high.

A benevolent planner raising lump-sum taxes would devote an even higher proportion of public resources to consumption than the Markov and Ramsey solutions, and the growth rate would be considerably higher than under the alternative solutions.

The working time choice is very similar for the Ramsey and Markov solutions for low risk aversion parameters, while for $\sigma > 2$ is slightly larger under the time-consistent optimal policy.

[5] The ratio of public to private consumption increases from 20% to 35% under the Markov rule, as sigma increases between 1.5 and 5.0. Under the Ramsey policy, that ratio increases from 15% to 20%.

That the income tax is higher under the Markov-perfect policy than under the Ramsey solution is consistent with the result obtained by Ortigueira (2006) in an exogenous growth economy under inelastic labor supply.[6] This result arises because the Markovian government cannot internalize the distortionary effects of current taxation on past investment, while in the Ramsey solution, the government takes fully into account the negative effect of the income tax on future investment. A similar argument explains that the Markov government devotes a higher proportion of public resources to consumption, which has a direct impact on current utility, to the expense of public investment, which would have a positive effect mainly on future utility. A lower income tax rate and a higher share of investment in public expenditures make the growth rate to be higher under the Ramsey than under the Markov solution.

Figure 2 presents results for $\sigma = 2$, and values of the relative weight of public consumption in the utility function, θ, between 0.2 and 1.5, the remaining parameters being as in Figure 1. As expected, public consumption as a share of total public spending increases with θ. Qualitative results stay the same, with the Markov-perfect policy imposing a higher income tax than the Ramsey policy and devoting a higher proportion of public resources to consumption. The growth rate is again higher under the Ramsey than under the Markov policy.

Table 1 summarizes the results by displaying a single point from Figure 1 and Figure 2 (a vertical section of both Figures at σ=2.0).

Table 2 analyzes the effects of a change in α. The value of B has been chosen to guarantee positive growth rates under the Markov and Ramsey solutions.

A comparison of the two panels in Table 1 shows that as public consumption becomes more appreciated the optimal tax rate increases, as it does the proportion of public resources devoted to consumption. These two changes, that are quantitatively important, lead to a lower rate of growth. The ratios of both types of capital to output remain unchanged.

Table 2 shows that an increase in the productivity of public capital (lower α) leads to higher tax rates. The government detracts more aggregate resources from the economy and devotes a larger proportion of them to investment. Because of the increase

[6] Even though the two results are not strictly comparable, since one of them refers to an exogenous growth economy and the other to an endogenous growth economy.

in the tax rate generated by a lower α parameter, the productivity of private capital and hence, the rate of growth, both decrease.

A comparison of the left panels from Tables 1 and 2 allows us to also analyze the effects of an increase in ε, the weight of leisure in the utility function. A larger appreciation for leisure would reduce working time while reducing the proportion of public resources devoted to consumption. The optimal income tax rate slightly decreases, but the negative effect from the lower working time is more intense, so that the growth rate is reduced.

Since the ratio of public capital to output is the same for the three solutions, and the resource allocations obtained under the three solution concepts satisfy the conditions for competitive equilibrium, then the product $(1-\eta)\tau$ is also the same for the three solution concepts. This property implies that the ratio of private capital to output is also the same for the three solutions under any parameterization. The common value of $(1-\eta)\tau$ turns out to be equal to the elasticity of output with respect to public capital, again an extension of the result obtained by Barro (1990) in a model with just public capital ($\eta = 0$). That also explains the relationship $\tau^M(1-\eta^M) = (1-\alpha)$ that we showed for the optimal Markov policy in the case of logarithmic preferences and full depreciation of private capital. Now we see that such a relationship holds for the three equilibrium concepts in any parameterization of our model economy.

The solution under lump-sum taxes leads to the largest public sector and devotes a lowest share of public resources to investment. Since taxes are not distortionary under the planner's solution, a larger proportion of resources extracted by the public sector can be made compatible with a higher rate of growth.

Table 1. Values for the main variables under the three solution concepts. Effects of a change in θ						
	$B = 0.573$, $\sigma = 2.00$, $\theta = 0.40$, $\alpha = 0.80$, $\delta = 0.10$, $\rho = 0.99$ ε=1.1			$B = 0.573$, $\sigma = 2.00$, $\theta = 1.00$, $\alpha = 0.80$, $\delta = 0.10$, $\rho = 0.99$ ε=1.1		
	Planner	*Markov*	*Ramsey*	*Planner*	*Markov*	*Ramsey*
η (%)	28.4	26.3	20.5	44.3	41.0	31.0
τ (%)	27.9	27.1	25.2	35.9	33.9	29.0
l (%)	47.8	32.7	32.3	53.4	33.7	32.4
γ *(%)*	4.47	1.51	1.65	3.75	0.79	1.07
c/y(%)	19.9	27.2	28.5	15.9	23.6	27.0
g/y(%)	7.95	7.12	5.17	15.9	13.9	9.00
kp/y	0.2	0.2	0.2	0.2	0.2	0.2
k/y	3.61	3.97	3.98	3.51	3.94	3.98

Note to the table: for the planner solution $\tau_t^P = \frac{g_t + k_{p,t}}{y_t}$ and $\eta_t^P = \frac{g_t}{g_t + k_{p,t}}$.

Table 2. Values for the main variables under the three solution concepts. Effects of a change in ε and α						
	$B = 0.573$, $\sigma = 2.00$, $\theta = 0.40$, $\alpha = 0.80$, $\delta = 0.10$, $\rho = 0.99$ ε=1.4			$B = 0.573$, $\sigma = 2.00$, $\theta = 0.40$, $\alpha = 0.85$, $\delta = 0.10$, $\rho = 0.99$ ε=1.4		
	Planner	*Markov*	*Ramsey*	*Planner*	*Markov*	*Ramsey*
η (%)	28.0	25.6	19.6	38.4	37.3	32.6
τ (%)	27.8	26.9	24.9	24.3	23.9	22.2
l (%)	42.4	28.2	27.8	31.4	21.2	21.0
γ *(%)*	4.21	1.31	1.45	5.84	2.94	3.08
c/y(%)	19.4	26.6	27.8	23.4	30.3	31.4
g/y(%)	7.76	6.87	4.88	9.35	8.93	7.24
kp/y	0.2	0.2	0.2	0.15	0.15	0.15
k/y	3.72	4.12	4.13	3.30	3.54	3.54

Note to the table: for the planner solution $\tau_t^P = \frac{g_t + k_{p,t}}{y_t}$ and $\eta_t^P = \frac{g_t}{g_t + k_{p,t}}$.

Qualitative results are similar to the version of this model with inelastic labor supply analyzed in Novales, Perez and Ruiz (2013). In both economies, the Markov solution leads to a higher income tax rate and a lower proportion of public resources devoted to investment than the Ramsey solution. Additionally, we find that the amount of time devoted to work is distinctly higher in the Markov than in the Ramsey solution, which compensates for a slightly lower *k/y* ratio, making production to be a more labor intensive activity.

6. Welfare

In this section we compare the levels of welfare that would arise along the balanced growth path under the time consistent Markov policy and under a benevolent planner.[7] We compute the consumption compensation as a percentage of output [as in Lucas (1987)] that would be needed under the Markov rule to achieve the same level of welfare than under the resource allocation of the planner with non-distortionary taxation.

Under a CRRA utility, welfare can be written,

$$W_i = \sum_{t=0}^{\infty} \rho^t \frac{c_{t,i}^{1-\sigma}(1-l_{t,i})^{\varepsilon(1-\sigma)} g_{t,i}^{\theta(1-\sigma)} - 1}{1-\sigma} = \frac{1}{1-\sigma}\left[\frac{\chi_i^{1-\sigma}\phi_i^{\theta(1-\sigma)}(1-l_{t,i})^{\varepsilon(1-\sigma)}}{1-\rho\,\gamma_i^{(1-\sigma)(1+\theta)}} - \frac{1}{1-\rho}\right], i = Planner, Markov$$

.

Let $\{c_{t,i}, g_{t,i}\}$, $i=P,M$, be the optimal path for private and public consumption for the planner's solution and the Markov solution, respectively, that is: $c_{t,i} = \chi_i k_{t,i} = \chi_i k_0 \gamma_i^t \underbrace{=}_{k_0=1} \chi_i \gamma_i^t$, $g_{t,i} = \phi_i k_{t,i} = \phi_i k_0 \gamma_i^t \underbrace{=}_{k_0=1} \phi_i \gamma_i^t$, $\quad i = P, M$, where we have used the normalization $k_0 = 1$.

The consumption compensation λ needed for the Markov solution to achieve the same level of welfare as under the planner's allocation can be obtained by solving the following equation:

$$W_P = \sum_{t=0}^{\infty} \rho^t \frac{(1+\lambda)^{1-\sigma} c_{t,M}^{1-\sigma}(1-l_{t,M})^{\varepsilon(1-\sigma)} g_{t,M}^{\theta(1-\sigma)} - 1}{1-\sigma},$$

that is,

$$\frac{1}{1-\sigma}\left[\frac{\chi_P^{1-\sigma}(1-l_{t,P})^{\varepsilon(1-\sigma)}\phi_P^{\theta(1-\sigma)}}{1-\rho\,\gamma_P^{(1-\sigma)(1+\theta)}} - \frac{1}{1-\rho}\right] = \frac{1}{1-\sigma}\left[\frac{(1+\lambda)^{1-\sigma}\chi_M^{1-\sigma}(1-l_{t,M})^{\varepsilon(1-\sigma)}\phi_M^{\theta(1-\sigma)}}{1-\rho\,\gamma_M^{(1-\sigma)(1+\theta)}} - \frac{1}{1-\rho}\right],$$

and finally,

$$1+\lambda = \left[\frac{1-\rho\,\gamma_M^{(1-\sigma)(1+\theta)}}{1-\rho\,\gamma_P^{(1-\sigma)(1+\theta)}}\right]^{\frac{1}{1-\sigma}} \frac{\chi_P}{\chi_M}\left(\frac{\phi_P}{\phi_M}\right)^{\theta} \frac{(1-l_{t,P})^{\varepsilon}}{(1-l_{t,M})^{\varepsilon}}. \tag{27}$$

This compensation is translated into output units through the ratio $100\lambda \frac{c_{t,M}}{y_{t,M}}$, which is shown in Figure 3.

[7] We do not consider the level of welfare under the Ramsey solution because of its time-inconsistent nature.

Welfare compensations in Figure 3 have been computed under the same parameterizations as in Tables 1 and 2. As the risk aversion parameter changes between 1.5 and 5.0, the Markov consumption compensation falls from 35% to 4% of output. In particular, for $\sigma = 2$, the compensation that would be necessary to achieve the planner's welfare is 14.5% of output. The decrease in consumption compensation is due to the decline in the value of the first factor in (27). That factor, which depends on growth rates, falls by 70%, from 4.15 for σ=1.5, to 1.28 when σ=5. The second factor increases by 51%, from 0.66 to 1.00; the third factor decreases by 5%, while the last factor increases by 25%.

The consumption compensation increases with θ. For σ=2, the Markov consumption compensation increases from 15% to 37% of output. Again, this increase in the consumption compensation is mainly due to the first factor in (35), that increases by 66% as θ increases between θ=0.5 and θ=1.5. The factor in the ratio of public consumption to capital, that increases by 35%, is second in importance. The ratio of private consumption to capital decreases by 8%, while the leisure factor falls by 15%.

The consumption compensation needed to achieve the level of welfare produced by the planner's allocation of resources is also slightly increasing in the weight of leisure in the utility function. A compensation of about 11% of consumption is needed when $\varepsilon = 0.5$, increasing to 14.3% when $\varepsilon = 1.5$. In this case, the leisure factor in (27), that falls by 12%, it is almost as important as the first factor which increases by 18%. The other two factors increase by about 3% and are less important to explain the change in consumption compensation as leisure becomes more appreciated.

By and large, the difference in growth rates is the main determinant of the welfare loss of the Markov solution relative to the planner's solution, over and above the effects of differences in the ratios of private or public consumption to output.

7. Conclusions

We have described analytical conditions for the characterization of the optimal fiscal policy and the implied allocation of resources in an endogenous growth economy of the AK-type, in which the government can raise income taxes and split tax revenues between public consumption and investment. The representative consumer has preferences over private and public consumption as well as on leisure time, so the labor supply is elastic. This work generalizes Novales, Perez and Ruiz (2013) in a version of this economy with inelastic labor supply. It also extends existing work on optimal time-consistent fiscal policy by allowing for potential effects of policy on growth in an endogenous growth economy, as well as by endogenously determining the split of public resources between consumption and investment.

We can analytically characterize the optimal time consistent fiscal policy, as well as the Ramsey policy in the special case of logarithmic preferences and full depreciation of private capital. For the more general case, with incomplete depreciation of private capital and CRRA preferences, there is no analytical solution. We have then numerically computed the optimal policy and the allocation of resources under the time-consistent (Markov) and the Ramsey (time-inconsistent) policy regimes, assuming absence of transitional dynamics under the Markov rule. As a benchmark for comparison, we have also obtained the optimal policy and the allocation of resources that would arise under a benevolent planner's rule.

Results are qualitatively similar to those obtained by Novales, Perez and Ruiz (2013) under an inelastic labour supply, with the time-consistent Markov policy imposing a higher income tax rate and devoting a higher proportion of public resources to consumption, rather than to investment in public capital.

The consumption compensation that would be needed to achieve under the time consistent policy the same level of welfare as under the planner's rule is mainly due to the difference between the growth rates under both policy regimes.

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Appendix 1: *Proof of Proposition* 1

The problem solved by the government is:

$$V\left(k_t,k_{p,t}\right)=\underset{\{\tau_t,\eta_t\}}{Max}\left[U\left(\mathcal{C}\left(k_t,k_{p,t};\tau_t,\eta_t\right),\ell(k_t,k_{p,t};\tau_t,\eta_t),\mathcal{G}\left(k_t,k_{p,t};\tau_t,\eta_t\right)\right)+\rho V(k_{t+1},k_{p,t+1})\right]$$

where $\mathcal{G}\left(k_t,k_{p,t};\tau_t,\eta_t\right)$, $k_{p,t}$ and k_{t+1} are given by (13), (12) and (11), respectively.

First order optimality conditions for the government's problem

- with respect to τ:

$$U_{c_t}\mathcal{C}_{\tau_t}+U_{g_t}\mathcal{G}_{\tau_t}+U_{l_t}\ell_{\tau_t}+\rho\,V_{k_{t+1}}\left[y_t\left((1-\tau_t)\frac{1-\alpha}{\alpha}\left(\frac{\ell_{\tau_t}}{\ell_t}+\frac{1}{\tau_t}\right)-1\right)-\mathcal{C}_{\tau_t}\right]=0,$$

where: $y_t=\left[(1-\eta_t)\tau_t\right]^{\frac{1-\alpha}{\alpha}}B^{1/\alpha}\ell_t^{\frac{1-\alpha}{\alpha}}k_t$, $\mathcal{G}_{\tau_t}=\mathcal{G}_t\left(\frac{1}{\alpha\tau_t}+\frac{1-\alpha}{\alpha}\frac{\ell_{\tau_t}}{\ell_t}\right)$.

- with respect to η:

$$U_{c_t}\mathcal{C}_{\eta_t}+U_{g_t}\mathcal{G}_{\eta_t}+U_{l_t}\ell_{\eta_t}+\rho\,V_{k_{t+1}}\left[\frac{1-\alpha}{\alpha}(1-\tau_t)y_t\left(\frac{\ell_{\eta_t}}{\ell_t}-\frac{1}{1-\eta_t}\right)-\mathcal{C}_{\eta_t}\right]=0,$$

where: $\mathcal{G}_{\eta_t}=\mathcal{G}_t\left(\frac{1}{\eta_t}-\frac{1-\alpha}{\alpha}\frac{1}{1-\eta_t}+\frac{1-\alpha}{\alpha}\frac{\ell_{\eta_t}}{\ell_t}\right)$.

The envelope condition is:

$$\begin{aligned}V_{k_t}=&\,U_{c_t}\mathcal{C}_{k_t}+U_{g_t}\mathcal{G}_{k_t}+U_{l_t}\ell_{k_t}+\frac{\partial\tau_t}{\partial k_t}\left(U_{c_t}\mathcal{C}_{\tau_t}+U_{g_t}\mathcal{G}_{\tau_t}+U_{l_t}\ell_{\tau_t}\right)+\frac{\partial\eta_t}{\partial k_t}\left(U_{c_t}\mathcal{C}_{\eta_t}+U_{g_t}\mathcal{G}_{\eta_t}+U_{l_t}\ell_{\eta_t}\right)+\\&\rho\,V_{k_{t+1}}\left\{1-\delta+(1-\tau_t)\left(\frac{y_t}{k_t}+\frac{1-\alpha}{\alpha}\frac{y_t}{\ell_t}\ell_{k_t}\right)-\mathcal{C}_{k_t}+\frac{\partial\tau_t}{\partial k_t}\left[y_t\left((1-\tau_t)\frac{1-\alpha}{\alpha}\left(\frac{\ell_{\tau_t}}{\ell_t}+\frac{1}{\tau_t}\right)-1\right)-\mathcal{C}_{\tau_t}\right]+\right.\\&\left.\frac{\partial\eta_t}{\partial k_t}\left[\frac{1-\alpha}{\alpha}(1-\tau_t)y_t\left(\frac{\ell_{\eta_t}}{\ell_t}-\frac{1}{1-\eta_t}\right)-\mathcal{C}_{\eta_t}\right]\right\}\end{aligned}$$

where $\mathcal{G}_{k_t}=\mathcal{G}_t/k_t$. After using the first order conditions derived above, this condition can be written as

$$V_{k_t}=U_{c_t}\mathcal{C}_{k_t}+U_{g_t}\mathcal{G}_{k_t}+U_{l_t}\ell_{k_t}+\rho\,V_{k_{t+1}}\left[1-\delta+(1-\tau_t)\left(\frac{y_t}{k_t}+\frac{1-\alpha}{\alpha}\frac{y_t}{\ell_t}\ell_{k_t}\right)-\mathcal{C}_{k_t}\right].$$

From the optimality conditions above we get,

$$\rho V_{k_{t+1}} = \frac{U_{c_t}\mathcal{C}_{\tau_t} + U_{g_t}\mathcal{G}_{\tau_t} + U_{l_t}\ell_{\tau_t}}{y_t\left(1-(1-\tau_t)\frac{1-\alpha}{\alpha}\left(\frac{\ell_{\tau_t}}{\ell_t}+\frac{1}{\tau_t}\right)\right)+\mathcal{C}_{\tau_t}},$$

$$\rho V_{k_{t+1}} = \frac{U_{c_t}\mathcal{C}_{\eta_t} + U_{g_t}\mathcal{G}_{\eta_t} + U_{l_t}\ell_{\eta_t}}{\frac{1-\alpha}{\alpha}(1-\tau_t)y_t\left(\frac{1}{1-\eta_t}-\frac{\ell_{\eta_t}}{\ell_t}\right)+\mathcal{C}_{\eta_t}},$$

which leads to condition (16).

Plugging the first equation into the envelope condition we get,

$$V_{k_t} = U_{c_t}\mathcal{C}_{k_t} + U_{g_t}\mathcal{G}_{k_t} + U_{l_t}\ell_{k_t} + \frac{U_{c_t}\mathcal{C}_{\tau_t} + U_{g_t}\mathcal{G}_{\tau_t} + U_{l_t}\ell_{\tau_t}}{y_t\left(1-(1-\tau_t)\frac{1-\alpha}{\alpha}\left(\frac{\ell_{\tau_t}}{\ell_t}+\frac{1}{\tau_t}\right)\right)+\mathcal{C}_{\tau_t}} \times \left[1-\delta+(1-\tau_t)\left(\frac{y_t}{k_t}+\frac{1-\alpha}{\alpha}\frac{y_t}{\ell_t}\ell_{k_t}\right)-\mathcal{C}_{k_t}\right],$$

To finally get equation (17):

$$\frac{U_{c_t}\mathcal{C}_{\tau_t} + U_{g_t}\mathcal{G}_{\tau_t} + U_{l_t}\ell_{\tau_t}}{y_t\left(1-(1-\tau_t)\frac{1-\alpha}{\alpha}\left(\frac{\ell_{\tau_t}}{\ell_t}+\frac{1}{\tau_t}\right)\right)+\mathcal{C}_{\tau_t}} =$$

$$\rho\left[\begin{array}{l} U_{c_{t+1}}\mathcal{C}_{k_{t+1}} + U_{g_{t+1}}\mathcal{G}_{k_{t+1}} + U_{l_{t+1}}\ell_{k_{t+1}} + \dfrac{U_{c_{t+1}}\mathcal{C}_{\tau_{t+1}} + U_{g_{t+1}}\mathcal{G}_{\tau_{t+1}} + U_{l_{t+1}}\ell_{\tau_{t+1}}}{y_{t+1}\left(1-(1-\tau_{t+1})\frac{1-\alpha}{\alpha}\left(\frac{\ell_{\tau_{t+1}}}{\ell_{t+1}}+\frac{1}{\tau_{t+1}}\right)\right)+\mathcal{C}_{\tau_{t+1}}} \times \\ \qquad \left[1-\delta+(1-\tau_{t+1})\left(\frac{y_{t+1}}{k_{t+1}}+\frac{1-\alpha}{\alpha}\frac{y_{t+1}}{\ell_{t+1}}\ell_{k_{t+1}}\right)-\mathcal{C}_{k_{t+1}}\right] \end{array}\right].$$

Appendix 2. The Ramsey problem

Under a CRRA utility, the Ramsey problem for our model economy is,

$$\underset{\{\tau_t,\eta_t\}}{Max}\sum_{t=0}^{\infty}\rho^t\frac{c_t^{1-\sigma}(1-l_t)^{\varepsilon(1-\sigma)}g_t^{\theta(1-\sigma)}-1}{1-\sigma}$$

subject to:

$$k_{t+1}=(1-\delta)k_t+B^{1/\alpha}(1-\tau_t)\left[(1-\eta_t)\tau_t\right]^{\frac{1-\alpha}{\alpha}}l_t^{\frac{1-\alpha}{\alpha}}k_t-c_t$$

$$c_t^{-\sigma}(1-l_t)^{\varepsilon(1-\sigma)}g_t^{\theta(1-\sigma)}=\rho c_{t+1}^{-\sigma}(1-l_{t+1})^{\varepsilon(1-\sigma)}g_{t+1}^{\theta(1-\sigma)}\left[1-\delta+\alpha B^{1/\alpha}(1-\tau_{t+1})\left[(1-\eta_{t+1})\tau_{t+1}\right]^{\frac{1-\alpha}{\alpha}}l_{t+1}^{\frac{1-\alpha}{\alpha}}\right]$$

$$\frac{\varepsilon c_t}{1-l_t}=\left(1-\tau_t\right)(1-\alpha)B^{1/\alpha}\left[\left(1-\eta_t\right)\tau_t\right]^{(1-\alpha)/\alpha}l_t^{(1-2\alpha)/\alpha}k_t$$

$$g_t=\eta_t\tau_t^{1/\alpha}(1-\eta_t)^{\frac{1-\alpha}{\alpha}}B^{1/\alpha}l_t^{\frac{1-\alpha}{\alpha}}k_t\ ,$$

The Ramsey optimal policy is the solution to the utility maximization problem, subject to the equilibrium conditions as constraints. Under the CRRA utility function, the Lagrangian for the Ramsey problem becomes:

$$\begin{aligned}L=\sum_{t=0}^{\infty}\rho^t\frac{c_t^{1-\sigma}(1-l_t)^{\varepsilon(1-\sigma)}g_t^{\theta(1-\sigma)}-1}{1-\sigma}&+\rho^t\mu_{1t}\left[\left(1-\delta+\Omega(\tau_t,\eta_t)\right)k_t-c_t-k_{t+1}\right]+\\ &\rho^t\mu_{2t}\left[\eta_t\left(1-\eta_t\right)^{\frac{1-\alpha}{\alpha}}\tau_t^{1/\alpha}B^{1/\alpha}l_t^{(1-\alpha)/\alpha}k_t-g_t\right]+\\ &\rho^t\mu_{3t}\left[\rho c_{t+1}^{-\sigma}(1-l_{t+1})^{\varepsilon(1-\sigma)}g_{t+1}^{\theta(1-\sigma)}\left(1-\delta+\alpha\,\Omega(\tau_{t+1},\eta_{t+1})\right)-c_t^{-\sigma}(1-l_t)^{\varepsilon(1-\sigma)}g_t^{\theta(1-\sigma)}\right]+\\ &\rho^t\mu_{4t}\left[\left(1-\tau_t\right)(1-\alpha)B^{1/\alpha}\left[\left(1-\eta_t\right)\tau_t\right]^{(1-\alpha)/\alpha}l_t^{(1-\alpha)/\alpha}k_t-\varepsilon c_t\frac{l_t}{1-l_t}\right].\end{aligned}$$

Taking derivatives with respect to $c_t, g_t, l_t, k_{t+1}, \tau_t, \eta_t$ to be equal to zero, we obtain the optimality conditions for the Ramsey problem:

$$\begin{aligned}&c_t^{-\sigma}(1-l_t)^{\varepsilon(1-\sigma)}g_t^{\theta(1-\sigma)}-\mu_{1t}+\mu_{3t}\sigma c_t^{-\sigma-1}(1-l_t)^{\varepsilon(1-\sigma)}g_t^{\theta(1-\sigma)}-\\ &\quad\sigma\mu_{3,t-1}c_t^{-\sigma-1}g_t^{\theta(1-\sigma)}(1-l_t)^{\varepsilon(1-\sigma)}\left(1-\delta+\alpha\,\Omega(\tau_t,\eta_t)\right)-\mu_{4t}\frac{\varepsilon l_t}{1-l_t}=0\,,\end{aligned}$$

$$\begin{aligned}&\theta c_t^{-\sigma}(1-l_t)^{\varepsilon(1-\sigma)}g_t^{\theta(1-\sigma)}-\mu_{2t}-\\ &\quad(1-\sigma)\theta c_t^{-\sigma}(1-l_t)^{\varepsilon(1-\sigma)}g_t^{\theta(1-\sigma)-1}\left[\mu_{3t}-\mu_{3,t-1}\left(1-\delta+\alpha\,\Omega(\tau_t,\eta_t)\right)\right]=0\,,\end{aligned}$$

$$
\left[-c_t + (1-\sigma)\mu_{3t}\right]\varepsilon c_t^{-\sigma}(1-l_t)^{\varepsilon(1-\sigma)} g_t^{\theta(1-\sigma)} + \left[\mu_{1t}(1-\tau_t)\tau_t^{(1-\alpha)/\alpha} + \mu_{2t}\eta_t\tau^{1/\alpha}\right]B^{1/\alpha}(1-\eta_t)^{\frac{1-\alpha}{\alpha}}\frac{1-\alpha}{\alpha}l_t^{(1-2\alpha)/\alpha}k_t -
$$

$$
\mu_{3t-1}c_t^{-\sigma}(1-l_t)^{\varepsilon(1-\sigma)-1}g_t^{\theta(1-\sigma)}\begin{bmatrix}\varepsilon(1-\sigma)\left(1-\delta+(1-\tau_t)\alpha B^{1/\alpha}\left[(1-\eta_t)\tau_t\right]^{\frac{1-\alpha}{\alpha}}l_t^{(1-\alpha)/\alpha}\right) \\ -(1-\tau_t)(1-\alpha)B^{1/\alpha}\left[(1-\eta_t)\tau_t\right]^{\frac{1-\alpha}{\alpha}}l_t^{(1-\alpha)/\alpha}\end{bmatrix}+
$$

$$
\mu_{4t}\left[(1-\tau_t)(1-\alpha)B^{1/\alpha}\left[(1-\eta_t)\tau_t\right]^{\frac{1-\alpha}{\alpha}}\frac{1-\alpha}{\alpha}l_t^{(1-\alpha)/\alpha-1}k_t - \varepsilon c_t\frac{1}{(1-l_t)^2}\right] = 0
$$

$$
\begin{gathered}
\frac{1}{\rho}\mu_{1t} - \mu_{1,t+1}\left(1-\delta+\Omega(\tau_{t+1},\eta_{t+1})l_{t+1}^{(1-\alpha)/\alpha}\right) + \mu_{2,t+1}B^{1/\alpha}\tau_{t+1}{}^{1/\alpha}\eta_{t+1}(1-\eta_{t+1})^{\frac{1-\alpha}{\alpha}}l_{t+1}^{(1-\alpha)/\alpha} + \\
\mu_{4,t+1}(1-\tau_{t+1})(1-\alpha)B^{1/\alpha}\left[(1-\eta_{t+1})\tau_{t+1}\right]^{\frac{1-\alpha}{\alpha}}l_{t+1}^{(1-\alpha)/\alpha} = 0
\end{gathered},
$$

$$
\begin{gathered}
\mu_{1t}k_t\left(\frac{1-\alpha}{\alpha}\frac{1-\tau_t}{\tau_t}-1\right) + \mu_{2t}\eta_t k_t\frac{1}{\alpha} + \mu_{3t-1}c_t^{-\sigma}(1-l_t)^{\varepsilon(1-\sigma)}g_t^{\theta(1-\sigma)}\alpha\left(\frac{1-\alpha}{\alpha}\frac{1-\tau_t}{\tau_t}-1\right) + \\
\mu_{4t}(1-\alpha)\frac{1-\tau_t-\alpha}{\alpha\tau_t}k_t = 0
\end{gathered},
$$

$$
-\mu_{1t}\frac{1-\alpha}{\alpha}k_t + \mu_{2t}\frac{\tau_t}{1-\tau_t}k_t\left(1-\frac{\eta_t}{\alpha}\right) - \mu_{3t-1}c_t^{-\sigma}(1-l_t)^{\varepsilon(1-\sigma)}g_t^{\theta(1-\sigma)}\left(1-\alpha\right) - \mu_{4t}\frac{(1-\alpha)^2}{\alpha}k_t = 0\,.
$$

Transforming the multipliers by: $\tilde{\mu}_{1t} \equiv \mu_{1t}k_t^{\sigma-\theta(1-\sigma)}$, $\tilde{\mu}_{2t} \equiv \mu_{2t}k_t^{\sigma-\theta(1-\sigma)}$, $\tilde{\mu}_{3t} \equiv \frac{\mu_{3t}}{k_t}$, $\tilde{\mu}_{4t} \equiv \mu_{4t}k_t^{\sigma-\theta(1-\sigma)}$ and defining the rate of growth $\gamma_{t+1} = \frac{k_{t+1}}{k_t}$, the consumption to capital ratio $\chi_t = \frac{c_t}{k_t}$, and the ratio between public and private capital: $\phi_t = \frac{g_t}{k_t}$, we can get a system of equations in stationary ratios. First, from the global constraint of resources, we get an expression for the growth rate:

$$
\gamma_{t+1} = 1-\delta+\Omega(\tau_t,\eta_t)l_t^{(1-\alpha)/\alpha} - \chi_t\,, \qquad \text{[A4.1]}
$$

whereas from the government budget constraint, we can write the ratio of public to private capital:

$$
\phi_t = B^{1/\alpha}\tau_t^{1/\alpha}l_t^{(1-\alpha)/\alpha}\eta_t(1-\eta_t)^{(1-\alpha)/\alpha}\,. \qquad \text{[A4.2]}
$$

From the condition on the marginal rate of substitution between consumption and leisure:

$$
(1-\tau_t)(1-\alpha)B^{1/\alpha}\left[(1-\eta_t)\tau_t\right]^{(1-\alpha)/\alpha}l_t^{(1-\alpha)/\alpha} = \varepsilon\chi_t\frac{l_t}{1-l_t} \qquad \text{[A4.3]}
$$

while from the Euler equation for the competitive equilibrium, we get:

$$\chi_t^{-\sigma}\phi_t^{\theta(1-\sigma)}(1-l_t)^{\varepsilon(1-\sigma)}\gamma_{t+1}^{\sigma-\theta(1-\sigma)} = \rho\chi_{t+1}^{-\sigma}\phi_{t+1}^{\theta(1-\sigma)}(1-l_{t+1})^{\varepsilon(1-\sigma)}\left(1-\delta+\alpha\,\Omega(\tau_{t+1},\eta_{t+1})l_{t+1}^{(1-\alpha)/\alpha}\right), \qquad [A4.4]$$

and from the set of optimality conditions above, we finally get the system of equations characterizing the optimal Ramsey policy represented in stationary ratios:

$$\tilde{\mu}_{2t}-\sigma\chi_t^{-\sigma-1}(1-l_t)^{\varepsilon(1-\sigma)}\phi_t^{\theta(1-\sigma)}\left[\frac{1}{\sigma}\chi_t+\tilde{\mu}_{3t}-\tilde{\mu}_{3t-1}\frac{1}{\gamma_t}\left(1-\delta+\alpha\,\Omega(\tau_t,\eta_t)l_t^{(1-\alpha)/\alpha}\right)\right]+\tilde{\mu}_{4t}\varepsilon\frac{l_t}{1-l_t}=0, \qquad (A4.5)$$

$$\theta x_t-\frac{\tilde{\mu}_{2t}}{\chi_t^{-\sigma}\phi_t^{\theta(1-\sigma)-1}(1-l_t)^{\varepsilon(1-\sigma)}}-(1-\sigma)\theta\tilde{\mu}_{3t}-(1-\sigma)\theta\tilde{\mu}_{3t-1}\frac{1}{\gamma_t}\left[\left(1-\delta+\alpha\,\Omega(\tau_t,\eta_t)l_t^{(1-\alpha)/\alpha}\right)\right], \qquad (A4.6)$$

$$\begin{aligned}&-\varepsilon\chi_t^{-\sigma}(1-l_t)^{\varepsilon(1-\sigma)-1}\phi_t^{\theta(1-\sigma)}\left(\chi_t+\tilde{\mu}_{3t}(1-\sigma)-\tilde{\mu}_{3t-1}\frac{1}{\gamma_t}(1-\sigma)\left(1-\delta+\Omega(\tau_t,\eta_t)l_t^{(1-\alpha)/\alpha}\right)\right)+\\&B^{1/\alpha}\left[(1-\eta_t)\tau_t\right]^{\frac{1-\alpha}{\alpha}}\frac{1-\alpha}{\alpha}l_t^{(1-\alpha)/\alpha-1}\left[\tilde{\mu}_{1t}(1-\tau_t)+\tilde{\mu}_{2t}\eta_t\tau_t+\tilde{\mu}_{4t}(1-\tau_t)(1-\alpha)\right]-\tilde{\mu}_{4t}\chi_t\frac{1}{(1-l_t)^2}=0\end{aligned} \qquad (A4.7)$$

$$\begin{aligned}&-\tilde{\mu}_{1t}\frac{1}{\rho}\gamma_{t+1}^{\sigma-\theta(1-\sigma)}+\tilde{\mu}_{1t+1}\left(1-\delta+\Omega(\tau_{t+1},\eta_{t+1})l_{t+1}^{(1-\alpha)/\alpha}\right)+\\&\qquad\left[\tilde{\mu}_{2t+1}+\tilde{\mu}_{4t+1}\frac{1-\tau_{t+1}}{\tau_{t+1}}(1-\alpha)\right]B^{1/\alpha}\tau_{t+1}^{1/\alpha}(1-\eta_{t+1})^{\frac{1-\alpha}{\alpha}}l_{t+1}^{(1-\alpha)/\alpha}=0,\end{aligned} \qquad (A4.8)$$

$$\tilde{\mu}_{1t}\left(\frac{1-\alpha}{\alpha}\frac{1-\tau_t}{\tau_t}-1\right)+\tilde{\mu}_{2t}\eta_t\frac{1}{\alpha}+\tilde{\mu}_{3t-1}\frac{1}{\gamma_t}x_t^{-\sigma}\phi_t^{\theta(1-\sigma)}(1-l_t)^{\varepsilon(1-\sigma)}\frac{1-\tau_t-\alpha}{\tau_t}+\tilde{\mu}_{4t}(1-\alpha)\frac{1-\tau_t-\alpha}{\alpha\tau_t}=0, \qquad (A4.9)$$

$$-\tilde{\mu}_{1t}\frac{1-\alpha}{\alpha}+\tilde{\mu}_{2t}\frac{\tau_t}{1-\tau_t}\left(1-\frac{\eta_t}{\alpha}\right)-\tilde{\mu}_{3t-1}\chi_t^{-\sigma}\phi_t^{\theta(1-\sigma)}\left(1-\alpha\right)(1-l_t)^{\varepsilon(1-\sigma)}\frac{1}{\gamma_t}-\tilde{\mu}_{4t}\frac{(1-\alpha)^2}{\alpha}=0. \qquad (A4.10)$$

Along the balanced growth path, the system of equations for the Ramsey equilibrium becomes:

$$\gamma^{\sigma-\theta(1-\sigma)}=\rho\left(1-\delta+\alpha\,\Omega(\tau,\eta)l^{(1-\alpha)/\alpha}\right), \qquad (A4.11)$$

$$\chi=1-\delta+\Omega(\tau,\eta)l^{(1-\alpha)/\alpha}-\gamma, \qquad (A4.12)$$

$$\phi=B^{1/\alpha}\tau^{1/\alpha}\eta(1-\eta)^{(1-\alpha)/\alpha}l^{(1-\alpha)/\alpha}, \qquad (A4.13)$$

$$(1-\alpha)\Omega(\tau,\eta)l^{(1-\alpha)/\alpha}=\varepsilon\chi\frac{l}{1-l} \qquad (A4.14)$$

$$1-\chi^{\sigma}(1-l)^{-\varepsilon(1-\sigma)}\phi^{-\theta(1-\sigma)}\left(\tilde{\mu}_1+\tilde{\mu}_4\varepsilon\frac{l}{1-l}\right)+\tilde{\mu}_3\sigma\frac{1}{\chi}\left[1-\frac{1}{\gamma}\left(1-\delta+\alpha\,\Omega(\tau,\eta)l^{(1-\alpha)/\alpha}\right)\right]=0, \qquad (A4.15)$$

$$\theta\chi-\tilde{\mu}_2\frac{1}{\chi_t^{-\sigma}(1-l)^{\varepsilon(1-\sigma)}\phi^{\theta(1-\sigma)-1}}-\tilde{\mu}_3\left(1-\sigma\right)\left[1-\frac{1}{\gamma}\left(1-\delta+\alpha\,\Omega(\tau,\eta)l^{(1-\alpha)/\alpha}\right)\right], \qquad (A4.16)$$

$$\varepsilon\chi^{-\sigma}(1-l)^{\varepsilon(1-\sigma)-1}\phi^{\theta(1-\sigma)}\left\{-\chi+\tilde{\mu}_3(1-\sigma)\left[1-\frac{1}{\gamma}\left(1-\delta+\Omega(\eta,\tau)l^{(1-\alpha)/\alpha}\right)\right]\right\}+$$

$$\Omega(\eta,\tau)l^{(1-2\alpha)/\alpha}\frac{1-\alpha}{\alpha}\left[\tilde{\mu}_1+\eta\frac{\tau}{1-\tau}\tilde{\mu}_2+\tilde{\mu}_4(1-\alpha)\right]-\tilde{\mu}_4\varepsilon\frac{1}{(1-l)^2}=0, \tag{A4.17}$$

$$\tilde{\mu}_1\left[\gamma^{\sigma-\theta(1-\sigma)}-\rho\left(1-\delta+\Omega(\tau,\eta)l^{(1-\alpha)/\alpha}\right)\right]-$$

$$\rho B^{1/\alpha}\left[(1-\eta)\tau\right]^{(1-\alpha)/\alpha}l^{(1-\alpha)/\alpha}\left(\eta\tilde{\mu}_2\tau+\tilde{\mu}_4(1-\tau)(1-\alpha)\right)=0, \tag{A4.18}$$

$$\frac{1-\alpha-\tau}{\alpha\tau}\tilde{\mu}_1+\tilde{\mu}_2\eta\frac{1}{\alpha}+\tilde{\mu}_3\frac{1}{\gamma}\chi^{-\sigma}\phi^{\theta(1-\sigma)}(1-l)^{\varepsilon(1-\sigma)}\frac{1-\alpha-\tau}{\tau}+\tilde{\mu}_4(1-\alpha)\frac{1-\alpha-\tau}{\alpha\tau}=0, \tag{A4.19}$$

$$-\tilde{\mu}_1\frac{1-\alpha}{\alpha}+\tilde{\mu}_2\frac{\tau}{1-\tau}\left(1-\frac{\eta}{\alpha}\right)-\tilde{\mu}_3\chi^{-\sigma}\phi^{\theta(1-\sigma)}(1-l)^{\varepsilon(1-\sigma)}\left(1-\alpha\right)\frac{1}{\gamma}-\tilde{\mu}_4\frac{(1-\alpha)^2}{\alpha}=0\,. \tag{A4.20}$$

a system of 10 equations in $\{\gamma,\chi,l,\phi,\eta,\tau,\tilde{\mu}_1,\tilde{\mu}_2,\tilde{\mu}_3,\tilde{\mu}_4\}$ that allows us to compute the balanced growth path for the Ramsey policy (τ^R,η^R) as well as the implied allocation of resources, characterized by $(l^R,\gamma^R,\chi^R,\phi^R)$.

Appendix 3. The planner's problem

A benevolent planner solves the problem,

$$\underset{\{c_t,l_t,k_{t+1},k_{pt},g_t\}}{Max}\sum_{t=0}^{\infty}\rho^t\frac{c_t^{1-\sigma}(1-l_t)^{\varepsilon(1-\sigma)}g_t^{\theta(1-\sigma)}-1}{1-\sigma}$$

subject to:

$$k_{t+1}-(1-\delta)k_t+c_t+g_t+k_{p,t}=Bk_t^{\alpha}k_{p,t}^{1-\alpha}l_t^{1-\alpha}, \tag{A1.1}$$

leading to characterizing the marginal rate of substitution between consumption and leisure at any time period:

$$\frac{\varepsilon c_t}{1-l_t}=(1-\alpha)Bk_t^{\alpha}k_{p,t}^{1-\alpha}l_t^{-\alpha},$$

the marginal rate of substitution between private and public consumption:

$$g_t=\theta c_t, \tag{A1.2}$$

the intertemporal rate of substitution between current and future consumption:

$$\frac{c_t^{-\sigma}(1-l_t)^{\varepsilon(1-\sigma)}g_t^{\theta(1-\sigma)}}{c_{t+1}^{-\sigma}(1-l_{t+1})^{\varepsilon(1-\sigma)}g_{t+1}^{\theta(1-\sigma)}}=\rho\left[(1-\delta)+\alpha Bk_{t+1}^{\alpha-1}k_{p,t+1}^{1-\alpha}l_{t+1}^{1-\alpha}\right], \tag{A1.3}$$

As a consequence, output is related to private and public capital each period by,

$$y_t = \frac{1}{1-\alpha} k_{p,t}$$
$$k_{p,t} = \left[(1-\alpha) B l_t^{1-\alpha} \right]^{1/\alpha} k_t$$

Using these relationships we obtain the rate of growth γ_P from [A1.3],

$$\gamma_t = \left\{ \rho \left[(1-\delta) + \alpha (1-\alpha)^{\frac{1-\alpha}{\alpha}} B^{1/\alpha} l_t^{\frac{1-\alpha}{\alpha}} \right] \right\}^{\frac{1}{\sigma - \theta(1-\sigma)}},$$

From the global constraint of resources [A1.1] we have the relationship:

$$\gamma_t - (1-\delta) + (1+\theta)\chi_t = \alpha B^{1/\alpha} (1-\alpha)^{\frac{1-\alpha}{\alpha}} l_t^{\frac{1-\alpha}{\alpha}}. \qquad [A1.4]$$

Along the balanced growth path, the rate of growth will be constant, and as a consequence, l_t and χ_t will also be constant, with:

$$\gamma_P = \left\{ \rho \left[(1-\delta) + \alpha (1-\alpha)^{\frac{1-\alpha}{\alpha}} B^{1/\alpha} l_P^{\frac{1-\alpha}{\alpha}} \right] \right\}^{\frac{1}{\sigma - \theta(1-\sigma)}},$$

$$\chi_P = \frac{1}{1+\theta} \left[\alpha B^{1/\alpha} (1-\alpha)^{\frac{1-\alpha}{\alpha}} l_P^{\frac{1-\alpha}{\alpha}} + (1-\delta) - \gamma_P \right],$$

$$\phi_P = \frac{g_t}{k_t} = \theta \chi_P.$$

where the expressions for the ratios of public investment and consumption to private capital, χ_P, ϕ_P have been obtained from [A1.2] and [A1.4].

For the purpose of comparison with the Markov and Ramsey equilibria, we use as a measure of the size of the public sector, as $\tau_t^P = \frac{g_t + k_{p,t}}{y_t}$ while for the composition of public expenditures we use, $\eta_t^P = \frac{g_t}{g_t + k_{p,t}}$.

Appendix 4.- *Optimal Markov policy under logarithmic utility and full depreciation of private capital*

Proof of Proposition 3.-

The problem solved by the government is:

$$V(k_t)=\underset{\{\tau_t,\eta_t\}}{\text{Max}}\left[\ln \mathcal{C}(k_t,\tau_t,\eta_t)+\varepsilon \ln \ell(k_t,\tau_t,\eta_t)+\theta \ln \mathcal{G}(k_t,\tau_t,\eta_t)+\rho\, V(k_{t+1})\right]$$

where $k_{t+1}=\Omega(\tau_t,\eta_t)\ell(k_t,\tau_t,\eta_t)^{\frac{1-\alpha}{\alpha}}k_t-\mathcal{C}(k_t,\tau_t,\eta_t),$

$$\mathcal{C}(k_t,\tau_t,\eta_t)=(1-\rho\alpha)\,\Omega(\tau_t,\eta_t)\ell(k_t,\tau_t,\eta_t)^{\frac{1-\alpha}{\alpha}}k_t,$$

$$\mathcal{G}(k_t,\tau_t,\eta_t)=\frac{\eta_t\tau_t}{1-\tau_t}\Omega(\tau_t,\eta_t)\ell(k_t,\tau_t,\eta_t)^{\frac{1-\alpha}{\alpha}}k_t,$$

$$\ell(k_t,\tau_t,\eta_t)=\ell=\frac{1-\alpha}{1-\alpha+\varepsilon(1-\rho\alpha)},$$

$$\Omega(\tau_t,\eta_t)\equiv(1-\tau_t)\left[(1-\eta_t)\tau_t\right]^{\frac{1-\alpha}{\alpha}}B^{\frac{1}{\alpha}}.$$

where we have used the optimal labor decision rule obtained in Proposition 2.

First order conditions for this problem are:

$$\tau_t:\ -(1+\theta)\frac{\tau_t-(1-\alpha)}{\alpha\tau_t(1-\tau_t)}+\frac{\theta}{\tau_t(1-\tau_t)}-\rho\,V_{k_{t+1}}\left[\rho\alpha\,\Omega(\tau_t,\eta_t)\ell^{\frac{1-\alpha}{\alpha}}\frac{\tau_t-(1-\alpha)}{\alpha\tau_t(1-\tau_t)}k_t\right]=0\,, \tag{A4.1}$$

$$\eta_t:\ -(1+\theta)\frac{1-\alpha}{\alpha}\frac{1}{1-\eta_t}+\frac{\theta}{\eta_t}-\rho\,V_{k_{t+1}}\left[\rho\alpha\,\Omega(\tau_t,\eta_t)\ell^{\frac{1-\alpha}{\alpha}}\frac{1-\alpha}{\alpha}\frac{1}{1-\eta_t}k_t\right]=0\,, \tag{A4.2}$$

A comparison between (A4.1) and (A4.2) leads to a relationship between the optimal values of the tax rate and the government spending split in the Markov-perfect equilibrium:

$$\tau_t^M=\frac{1-\alpha}{1-\eta_t^M}\ \forall t\,. \tag{A4.3}$$

To examine the dynamic properties of the Markov solution, we consider the envelope condition :

$$V_{k_t}=(1+\theta)\frac{1}{k_t}+(1+\theta)\frac{1}{\Omega(\tau_t,\eta_t)}\frac{\partial\Omega(\tau_t,\eta_t)}{\partial\tau_t}\left(\frac{\partial\tau_t}{\partial k_t}+\frac{\partial\eta_t}{\partial k_t}\right)+$$
$$\theta\left(\frac{1}{\tau_t(1-\tau_t)}\frac{\partial\tau_t}{\partial k_t}+\frac{1}{\eta_t}\frac{\partial\eta_t}{\partial k_t}\right)+\rho V_{k_{t+1}}\left[\rho\alpha\left(\Omega(\tau_t,\eta_t)+\frac{\partial\Omega(\tau_t,\eta_t)}{\partial\tau_t}\left(\frac{\partial\tau_t}{\partial k_t}+\frac{\partial\eta_t}{\partial k_t}\right)\right)\right],$$

which, using conditions (A4.1) and (A4.2), it can be written as,

$$V_{k_t}=(1+\theta)\frac{1}{k_t}+\rho V_{k_{t+1}}\left[\rho\alpha\,\Omega(\tau_t,\eta_t)\,\ell^{\frac{1-\alpha}{\alpha}}\right]. \tag{A4.4}$$

Using (A4.2) and (A4.3) in (A4.4), we obtain the dynamic equation:

$$\tilde{\eta}_{t+1}-\frac{1}{\rho}\tilde{\eta}_t+\frac{1+\theta}{\rho}=0, \tag{A4.5}$$

where $\tilde{\eta}_t \equiv \frac{\theta\alpha}{(1-\alpha)}\frac{1-\eta_t}{\eta_t}$. This difference equation (A2.5) is unstable, since $1/\rho > 1$.

Hence the only stable solution is that $\tilde{\eta}_t$ stays constant over time, and the same applies to η_t, that is, $\eta_t = \eta, \forall t$. Then, from (A4.3) we have that $\tau_t = \tau \; \forall t$. Finally, $\mathcal{C}(k_t, \tau_t, \eta_t)/k_t$ and $\mathcal{G}(k_t, \tau_t, \eta_t)/k_t$ will hold constant for all *t*.

Finally, from (A4.5) we obtain the value of η:

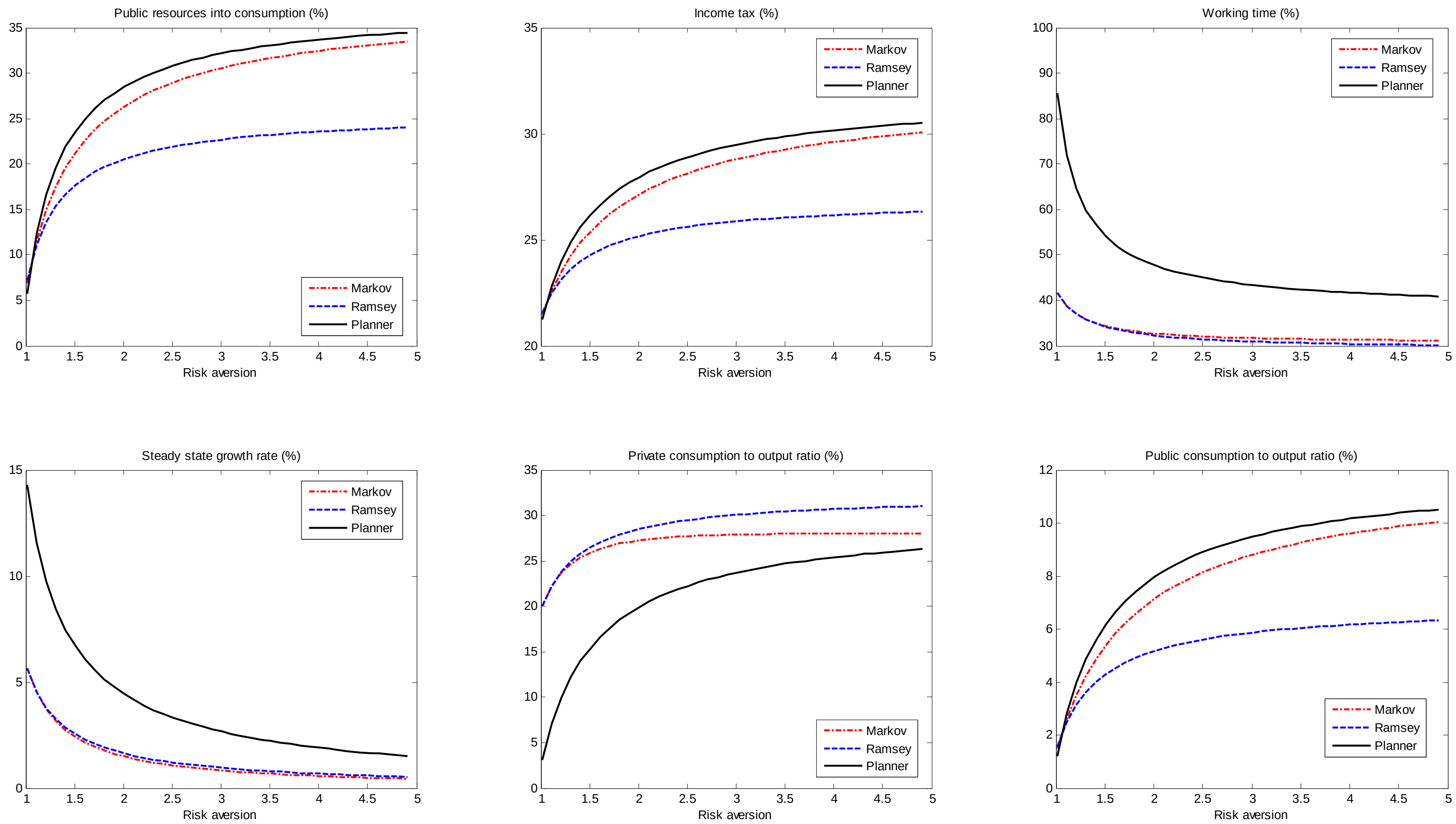
$$\eta^M = \frac{\alpha\theta(1-\rho)}{1-\alpha+\theta(1-\alpha\rho)},$$

and using (A4.3), we obtain the Markov perfect optimal tax rate:

$$\tau^M = 1 - \frac{\alpha(1+\rho\theta)}{1+\theta}. \square$$

Figure 1

Values of the main variables along the balanced growth path under the three equilibrium concepts, for different degrees of risk aversion

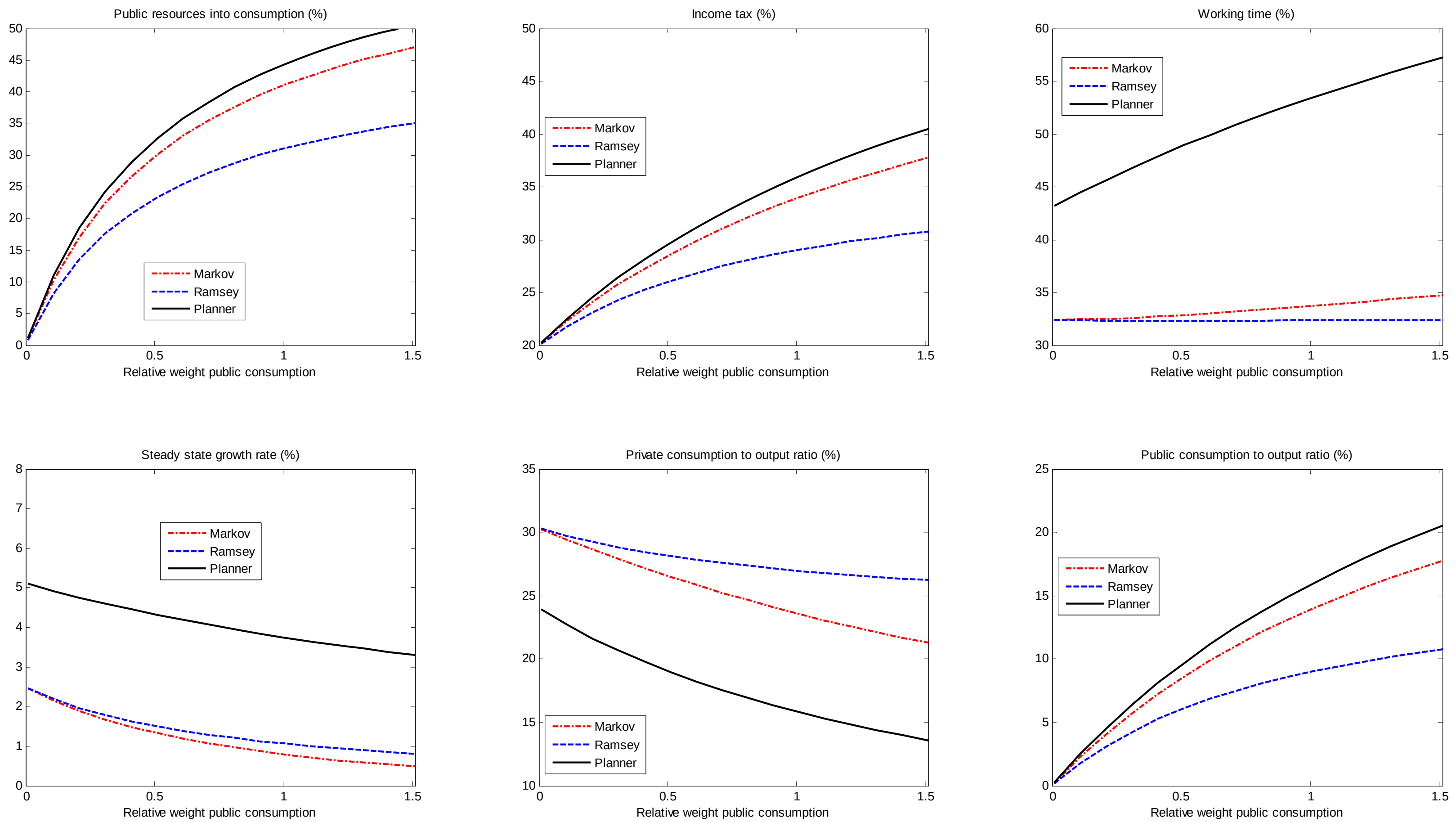


From left to right and from above to below, the graphs display: the share of public resources devoted to public consumption, the optimal income tax rate, the number of hours devoted to work, the growth rate along the balanced path, and the ratios of private and public consumption to output.

θ = 0.40	Relative weight of public consumption in utility
ε = 1.10	Relative weight of leisure time in utility
α = 0.80	Elasticity of private capital in production function
ρ = 0.99	Discount rate
δ = 0.10	Depreciation rate
B = 0.573	Productivity level

Figure 2

Values of the main variables along the balanced growth path under the three equilibrium concepts, for different values of the relative weight of public consumption in the utility function

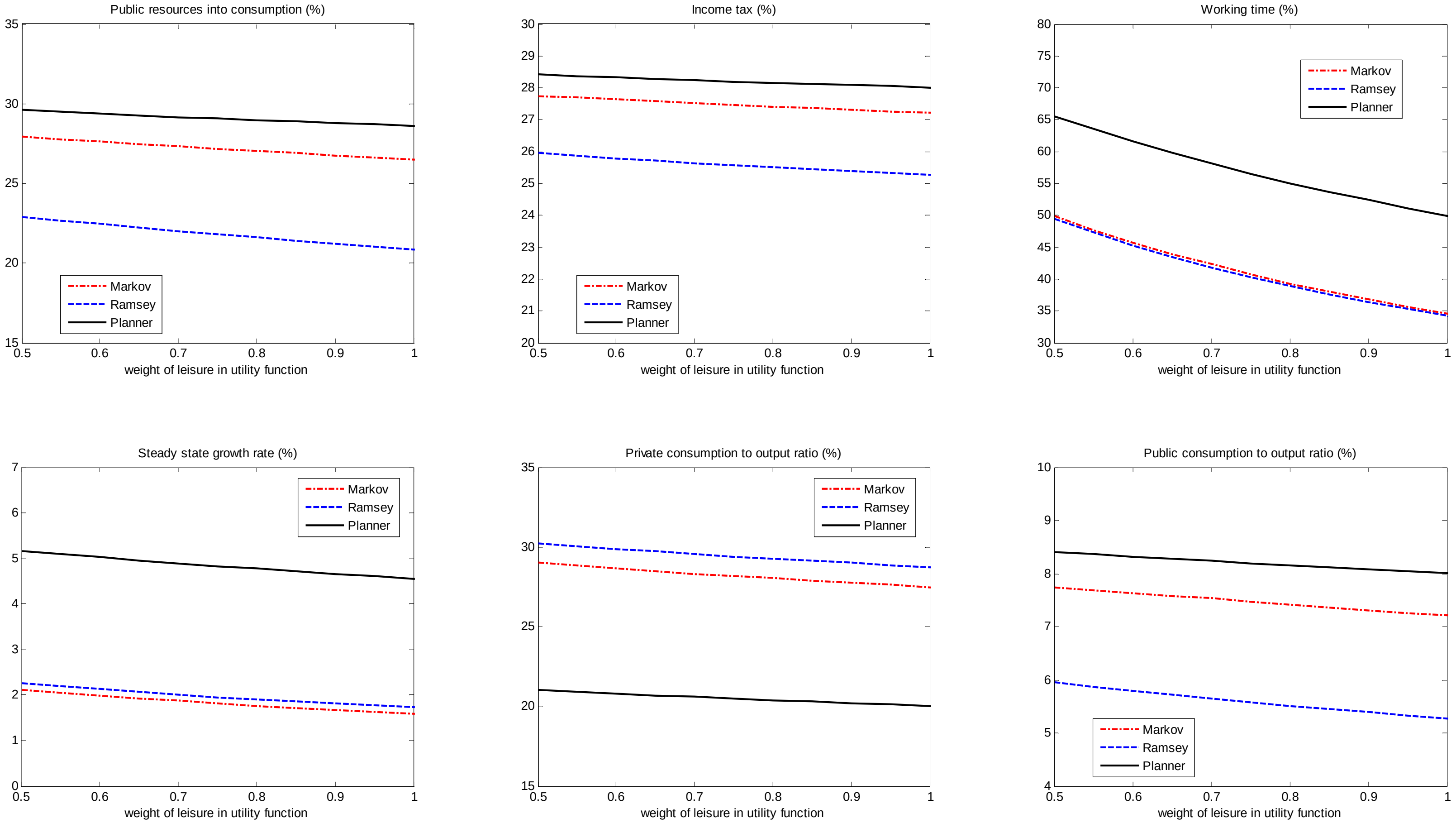


From left to right and from above to below, the graphs display: the share of public resources devoted to public consumption, the optimal income tax rate, the number of hours devoted to work, the growth rate along the balanced path, and the ratios of private and public consumption to output.

$\sigma = 2.00$	Risk aversion
$\varepsilon = 1.10$	Relative weight of leisure time in utility
$\alpha = 0.80$	Elasticity of private capital in production function
$\rho = 0.99$	Discount rate
$\delta = 0.10$	Depreciation rate
$B = 0.573$	Productivity level

Figure 3

Values of the main variables along the balanced growth path under the three equilibrium concepts, for different values of the relative weight of leisure in the utility function

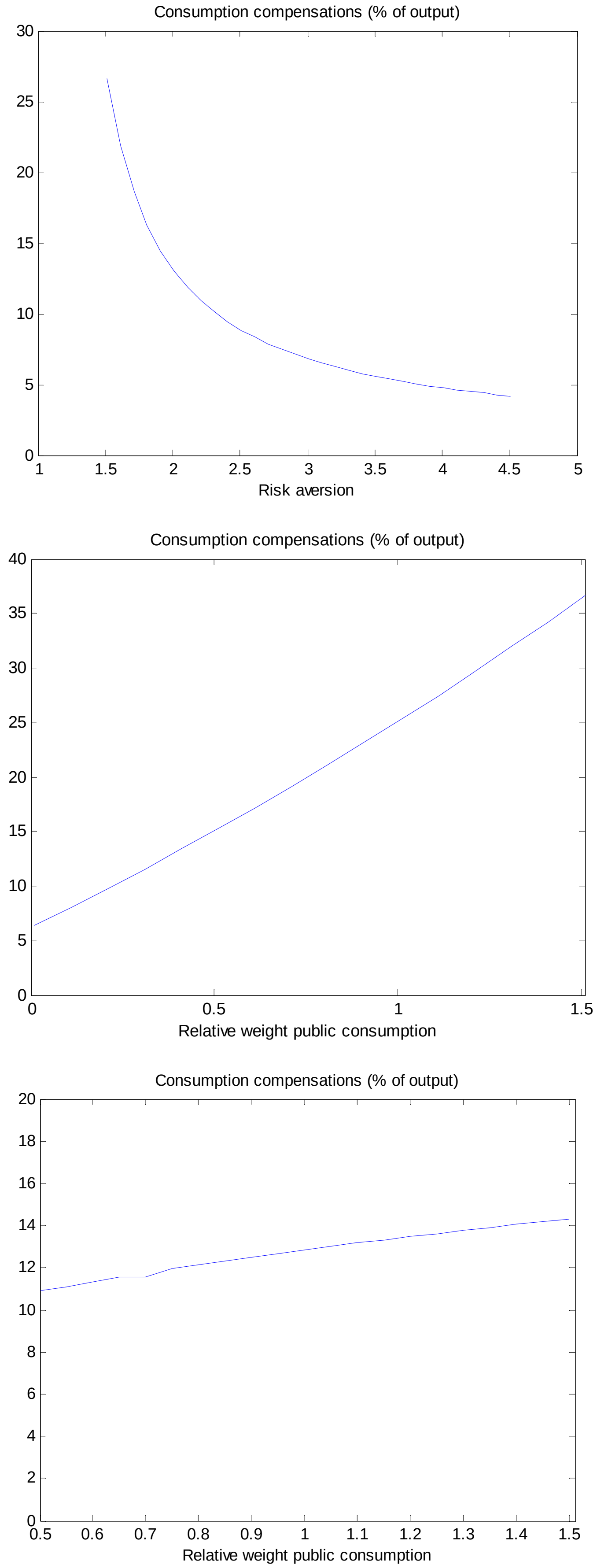


From left to right and from above to below, the graphs display: the share of public resources devoted to public consumption, the optimal income tax rate, the number of hours devoted to work, the growth rate along the balanced path, and the ratios of private and public consumption to output.

$\sigma = 2.00$	Risk aversion
$\theta = 0.40$	Relative weight of public consumption in utility
$\alpha = 0.80$	Elasticity of private capital in production function
$\rho = 0.99$	Discount rate
$\delta = 0.10$	Depreciation rate
$B = 0.573$	Productivity level

Figure 4

Consumption compensations



$$\varepsilon\mathcal{C}(k_t, k_{p,t}, \tau_t, \eta_t)\frac{\ell(k_t, k_{p,t}, \tau_t, \eta_t)}{1-\ell(k_t, k_{p,t}, \tau_t, \eta_t)} = (1-\alpha)(1-\tau_t)\Big[(1-\eta_t)\tau_t\Big]^{\frac{1-\alpha}{\alpha}} B^{1/\alpha}\Big[\ell(k_t, k_{p,t}, \tau_t, \eta_t)\Big]^{\frac{1-\alpha}{\alpha}} k_t,$$

$$\begin{aligned}
&\Big[\mathcal{C}(k_t, k_{p,t}, \tau_t, \eta_t)\Big]^{-\sigma}\Big[1-\ell(k_t, k_{p,t}, \tau_t, \eta_t)\Big]^{\varepsilon(1-\sigma)}\Big[\mathcal{G}(k_t, k_{p,t}, \tau_t, \eta_t)\Big]^{\theta(1-\sigma)} = \\
&\quad \rho\Big[\mathcal{C}(k_{t+1}, k_{p,t+1}, \tau_{t+1}, \eta_{t+1})\Big]^{-\sigma}\Big[1-\ell(k_{t+1}, k_{p,t+1}, \tau_{t+1}, \eta_{t+1})\Big]^{\varepsilon(1-\sigma)}\Big[\mathcal{G}(k_{t+1}, k_{p,t+1}, \tau_{t+1}, \eta_{t+1})\Big]^{\theta(1-\sigma)} \times \\
&\quad \left[1-\delta+\alpha(1-\tau_{t+1})[(1-\eta_{t+1})\tau_{t+1}]^{\frac{1-\alpha}{\alpha}} B^{1/\alpha}\Big[\ell(k_{t+1}, k_{p,t+1}, \tau_{t+1}, \eta_{t+1})\Big]^{\frac{1-\alpha}{\alpha}}\right]
\end{aligned}$$