The Rise and Fall of S&P500 Variance Futures*

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Abstract

Volatility is an indispensible component of sensible portfolio risk management. The volatility of an asset of composite index can be traded by using volatility derivatives, such as volatility and variance swaps, options and futures. The most popular volatility index is VIX, which is a key measure of market expectations of volatility, and hence is a key barometer of investor sentiment and market volatility. Investors interpret the VIX cash index as a "fear" index, and of VIX options and VIX futures as derivatives of the "fear" index. VIX is based on S&P500 call and put options over a wide range of strike prices, and hence is not model based. Speculators can trade on volatility risk with VIX derivatives, with views on whether volatility will increase or decrease in the future, while hedgers can use volatility derivatives to avoid exposure to volatility risk. VIX and its options and futures derivatives has been widely analysed in recent years. An alternative volatility derivative to VIX is the S&P500 variance futures, which is an expectation of the variance of the S&P500 cash index. Variance futures are futures contracts written on realized variance, or standardized variance swaps. The S&P500 variance futures are not model based, so the assumptions underlying the index do not seem to have been clearly understood. As these two variance futures are thinly traded, their returns are not easy to model accurately using a variety of risk models. This paper analyses the S&P500 3-month variance futures before, during and after the GFC, as well as for the full data period, for each of three alternative conditional volatility models and three densities, in order to determine whether exposure to risk can be incorporated into a financial portfolio without taking positions on the S&P500 index itself.

Keywords: Risk management, financial derivatives, futures, options, swaps, 3-month variance futures, 12-month variance futures, risk exposure, volatility.

JEL Classifications: C22, G32, G01.

Introduction

Volatility is an indispensible component of sensible portfolio risk management. As such, significant research has been undertaken in the conditional, stochastic and realized volatility literature to model and forecast various types of volatility, where the choice of model is frequently based on the data frequency used. The volatility of an asset of composite index can be traded by using volatility derivatives, such as volatility and variance swaps, options and futures. As swaps are traded over-the-counter rather than exchange traded, they have much lower liquidity and associated limitations in data availability.

The first and most popular volatility index is VIX (see Whaley (1993)), which is a key measure of market expectations of volatility, and hence is a key barometer of investor sentiment and market volatility. VIX is presently based on S&P500 call and put options over a wide range of strike prices, and hence is not model based. The original CBOE volatility index, VXO, is based on the Black-Scholes implied volatilities from S&P100 index, and hence is model based. The Black-Scholes model assumes normality, which is typically unrealistic for financial market data. In 2003, together with Goldman Sachs, CBOE updated and reformulated VIX to reflect a model-free method of measuring expected volatility, one that continues to be widely used by financial theorists. The Chicago Board Options Exchange (CBOE) introduced VIX futures on 26 March 2004, and VIX options on 24 February 2006. Both VIX options and futures are very highly traded.

As discussed in Chang et al. (2011), the volatility index data are closing daily prices (settlement prices) for the 30-day maturity CBOE VIX futures (ticker name VX), which may be obtained from the Thomson Reuters-Data Stream Database. The settlement price is calculated by the CBOE as the average of the closing bid and ask quote so as to reduce the noise due to any microstructure effects. The contracts are cash settled on the Wednesday 30 days prior to the third Friday on the calendar month immediately following the month in which the contract expires. The underlying asset is the VIX index that was originally introduced by Whaley (1993) as an index of implied volatility on the S&P100. In 2003 the updated VIX was introduced based on the S&P500 index.

VIX is a measure of the implied volatility of 30-day S&P500 options. Its calculation is independent of an option pricing model and is calculated from the prices of the front month and next-to-front month S&P500 at-the-money and out-the-money call and put options. The level of VIX represents a measure of the implied volatilities of the entire smile for a constant 30-day to

maturity option chain. VIX is quoted in percentage points. In order to invest in VIX, an investor can take a position in VIX futures or VIX options.

Chicago Board Options Exchange (2003) define VIX as a measure of the expected volatility of the S&P500 over the next 30-days, with the prices of VIX futures being based on the current expectation of what the expected 30-day volatility will be at a particular time in the future (on the expiration date). Although the VIX futures should converge to the spot at expiration, it is possible to have significant disparities between the spot VIX and VIX futures prior to expiration.

Speculators can trade on volatility risk with VIX derivatives, with views on whether volatility will increase or decrease in the future, while hedgers can use volatility derivatives to avoid exposure to volatility risk. Thus, exposure to risk can be incorporated into a financial portfolio without taking positions on the S&P500 index itself. Volatility risk can occur for a long trading position, which is exposed to the risk of falling market prices, or for a short trading position, which is exposed to the risk of rising market prices. Value-at-Risk (VaR) forecasts typically focus on losses due to falling market prices, whereby investors are assumed to have long positions.

VIX is a cash index and hence is not traded, much like the various S&P indexes, but VIX futures and options lead to indirect trading in VIX. VIX futures can be hedged using VIX futures of different maturities, while VIX options can be hedged using VIX futures (see, for example, Sepp (2008)). Optimal hedge ratios can be calculated using consistently estimated dynamic conditional correlations (see, for example, Caporin and McAleer (2011)).

VIX and its options and futures derivatives has been widely analysed in recent years. For example, Brenner et al. (2006) derive an approximate analytical VIX futures pricing formula and analyse VIX futures. Sepp (2008) analyses the skewness in the implied volatilities of VIX options. Huskaj (2009) calculates the VaR of VIX futures, and shows that long memory, heavy tails and asymmetry are important in modelling VIX futures returns. McAleer and Wiphatthanananthakul (2010) examine the empirical behaviour of alternative simple expected volatility indexes, and compare them with VIX. Chang et al. (2011) analyse the VaR of VIX futures under the Basel Accord before, during and after the GFC, and also for the full sample period. Ishida et al. (2011) propose a new method for estimating continuous-time stochastic volatility (SV) models for the S&P500 stock index process using intraday high-frequency observations of both the S&P500 index and VIX.

An alternative volatility derivative to VIX is the S&P500 variance futures, which is an expectation of the variance of the S&P500 cash index. Variance futures are futures contracts written on realized variance, or standardized variance swaps, and may alternatively be interpreted as dynamic Bayesian priors. The CBOE Futures Exchange (CFE) introduced the S&P500 3-month variance futures on 18 May 2004, and the S&P500 12-month variance futures on 23 March 2006, the difference between the 3-month and 12-month variance futures being the data period for calculating the variance. The S&P500 12-month variance futures were delisted as of 17 March 2011. As contract values are available until 18 March 2011, S&P500 12-month variance futures did not reach its fifth anniversary.

Investors clearly understood the meaning and value of the VIX cash index as a "fear" index, and of VIX options and VIX futures as derivatives of a "fear" index. These are the most popular financial derivatives traded in financial markets worldwide. On the other hand, S&P500 3-month and 12-month variance futures do not seem to have been understood clearly as derivative measures of market volatility or risk, especially as they are, in effect, dynamic Bayesian priors that are neither easy to specify nor interpret. It is, therefore, not surprising that S&P500 3-month and 12-month variance options have not been created or listed.

The S&P500 variance futures are not model based, so the assumptions underlying the index do not seem to have been clearly understood. As these two variance futures are thinly traded, their returns are not easy to model accurately using a variety of risk models. As standard risk models cannot be used to model the risks and dynamic correlations of these two S&P500 variance futures, optimal hedge ratios would also be difficult to calculate. Therefore, S&P500 variance futures cannot be used for hedging purposes.

In comparison with substantial empirical analyses of the VIX cash index, VIX futures and VIX options, the empirical assessment of S&P variance futures has been virtually non-existent. Zhang and Huang (2010) analyse the CBOE S&P500 3-month variance futures. The authors use a mean-reverting stochastic volatility model for the S&P500 index and present a linear relation between the price of variance futures and the square of the VIX cash index. They analyse the relationship for 3-month, 6-month and 9-month fixed time-to-maturity variance futures. To date, there seems to have been no analysis of S&P500 12-month variance futures, or volatility modeling of variance futures of any maturity.

The remainder of the paper is organized as follows. Section 2 reviews some of the most widely-used univariate models of conditional volatility for analysing and forecasting risk. In Section 3 the data used for empirical analysis are presented, and the S&P500 3-month variance futures for the full sample period, as well as before, during and after the GFC, are analysed for three alternative densities. Section 4 presents some concluding remarks.

2. Univariate Models of Conditional Volatility

McAleer et al. (2010) and Chang et al. (2011), among others, discuss how Authorized Deposit-taking Institutions (ADIs) can use internal models to determine their Value-at-Risk (VaR) thresholds by using alternative univariate time series models for estimating conditional volatility. In what follows, we present several well-known conditional volatility models that can be used to evaluate strategic market risk disclosure, namely GARCH, GJR and EGARCH, with Gaussian, Student-t (with estimated degrees of freedom), and Generalized Normal distribution errors, where the parameters are estimated.

These conditional volatility models are chosen as they are widely used in the literature. For an extensive discussion of the theoretical properties of several of these models (see Ling and McAleer (2002a, 2002b, 2003a), Li et al. (2002), McAleer (2005), and Caporin and McAleer (2010)). We include a section on these models to present them in a unified framework and notation, and to make explicit the specific versions we are using.

2.1 GARCH

For a wide range of financial data series, time-varying conditional variances can be explained empirically through the autoregressive conditional heteroskedasticity (ARCH) model, which was proposed by Engle (1982). When the time-varying conditional variance has both autoregressive and moving average components, this leads to the generalized ARCH(p,q), or GARCH(p,q), model of Bollerslev (1986). It is very common in practice to impose the widely estimated GARCH(1,1) specification in advance.

Consider the stationary AR(1)-GARCH(1,1) model for daily returns, y_t:

$$y_t = \varphi_1 + \varphi_2 y_{t-1} + \varepsilon_t, \qquad |\varphi_2| < 1 \tag{1}$$

for $t = 1,...,n$, where the shocks to returns are given by:

$$\varepsilon_t = \eta_t \sqrt{h_t}, \quad \eta_t \sim iid(0,1)$$
$$h_t = \omega + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1}, \tag{2}$$

and $\omega > 0, \alpha \geq 0, \beta \geq 0$ are sufficient conditions to ensure that the conditional variance $h_t > 0$, while $\alpha + \beta < 1$ is sufficient for a finite unconditional variance which, in turn, is sufficient to establish asymptotic properties. The stationary AR(1)-GARCH(1,1) model can be modified to incorporate a non-stationary ARMA(p,q) conditional mean and a stationary GARCH(r,s) conditional variance, as in Ling and McAleer (2003b).

2.2 GJR

In the symmetric GARCH model, the effects of positive shocks (or upward movements in daily returns) on the conditional variance, h_t, are assumed to be the same as the effect of negative shocks (or downward movements in daily returns) of equal magnitude. In order to accommodate asymmetric behaviour, Glosten, Jagannathan and Runkle (1992) proposed a model (hereafter GJR), for which GJR(1,1) is defined as follows:

$$h_t = \omega + (\alpha + \gamma I(\eta_{t-1}))\varepsilon_{t-1}^2 + \beta h_{t-1}, \tag{3}$$

where $\omega > 0, \alpha \geq 0, \alpha + \gamma \geq 0, \beta \geq 0$ are sufficient conditions for $h_t > 0$, $\alpha + \beta + \gamma/2 < 1$ is sufficient for a finite unconditional variance which, in turn, is sufficient to establish asymptotic properties, and $I(\eta_t)$ is an indicator variable defined by:

$$I(\eta_t) = \begin{cases} 1, & \varepsilon_t < 0 \\ 0, & \varepsilon_t \geq 0 \end{cases} \tag{4}$$

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where η_t has the same sign as ε_t. The indicator variable differentiates between positive and negative shocks, so that asymmetric effects in the data are captured by the coefficient γ. For financial data, it is hypothesized that $\gamma \geq 0$ because negative shocks are expected to have a greater impact on risk than do positive shocks of similar magnitude. The asymmetric effect, γ, measures the contribution of shocks to both short run persistence, $\alpha + \gamma/2$, and to long run persistence, $\alpha + \beta + \gamma/2$.

Although GJR permits asymmetric effects of positive and negative shocks of equal magnitude on conditional volatility, the special case of leverage, whereby negative shocks increase volatility while positive shocks decrease volatility (see Black (1976) for an argument using the debt/equity ratio), cannot be accommodated, in practice (for further details on asymmetry versus leverage in the GJR model, see Caporin and McAleer (2010)). The reason why leverage does not exist in the GJR model is that restriction on the ARCH parameter arising from positive shocks, namely $\alpha < 0$, is not consistent with the interpretation of the model. Moreover, a negative and significant estímate of α is not found in practice.

2.3   EGARCH

An alternative model to capture asymmetric behaviour in the conditional variance is the Exponential GARCH, or EGARCH(1,1), model of Nelson (1991), namely:

$$logh_t = \omega + \alpha \left| \frac{\varepsilon_{t\text{-}1}}{h_{t\text{-}1}} \right| + \gamma \frac{\varepsilon_{t\text{-}1}}{h_{t\text{-}1}} + \beta logh_{t\text{-}1}, \quad |\beta| < 1 \tag{5}$$

where the parameters α, β and γ have different interpretations from those in the GARCH(1,1) and GJR(1,1) models discussed above.

EGARCH captures asymmetries differently from GJR. The parameters α and γ in EGARCH(1,1) represent the magnitude (or size) and sign effects of the standardized residuals, respectively, on the conditional variance, whereas α and $\alpha + \gamma$ represent the effects of positive and negative shocks, respectively, on the conditional variance in GJR(1,1). Unlike GJR, EGARCH

can accommodate leverage, namely $\gamma < 0$ and $\gamma < \alpha < -\gamma$, depending on the restrictions imposed on the size and sign parameters, though leverage is not guaranteed (for further details, see Caporin and McAleer (2010)).

As noted in McAleer et al. (2007), there are some important differences between EGARCH and the previous two models, as follows: (i) EGARCH is a model of the logarithm of the conditional variance, which implies that no restrictions on the parameters are required to ensure $h_t > 0$; (ii) moment conditions are required for the GARCH and GJR models as they are dependent on lagged unconditional shocks, whereas EGARCH does not require moment conditions to be established as it depends on lagged conditional shocks (or standardized residuals); (iii) Shephard (1996) observed that $|\beta| < 1$ is likely to be a sufficient condition for consistency of QMLE for EGARCH(1,1); (iv) as the standardized residuals appear in equation (7), $|\beta| < 1$ would seem to be a sufficient condition for the existence of moments; and (v) in addition to being a sufficient condition for consistency, $|\beta| < 1$ is also likely to be sufficient for asymptotic normality of the QMLE of EGARCH(1,1).

3.   Data and Empirical Results

According to Datastream, from which the data are obtained. variance futures are cash settled, exchange traded futures contracts based on the realized variance of the S&P500 index. Daily data on S&P500 3-month variance futures, with 3 month maturity, are obtained for the period 18 May 2004 to 1 April 2011, while daily data on S&P500 12-month variance futures, with 3 month maturity, are obtained for the period 24 March 2006 to 17 March 2011.

The three conditional volatility models discussed in the previous section are estimated under the following distributional assumptions on the conditional shocks: (1) Gaussian, (2) Student-*t*, with estimated degrees of freedom, and (3) Generalized Normal. As the models that incorporate the t distributed errors are estimated by QMLE, the resulting estimators are consistent and asymptotically normal, so they can be used for estimation, inference and forecasting.

Figures 1-4 plot the S&P500 3-month and 12-month variance futures, and S&P500 3-month and 12-month variance futures returns. In Figure 1, there is little evidence of volatility in 3-month variance futures until the impact of the Global Financial Crisis (GFC) in the third quarter of 2008, with a substantial reduction in 2009. In Figure 2, the high volatility for 12-month variance futures persists toward the end of 2009, well after the GFC had been presumed to have ended, after which there is much lower volatility. The 3-month variance futures returns in Figure 3 show that positive returns were far more numerous, and of greater magnitude, than negative returns. The 12-month variance futures returns in Figure 4 also show that positive returns were far more numerous than negative returns, but the most extreme return is a single negative return toward the end of 2009.

For the reasons given above, only the 3-month variance futures returns will be used to estimate volatility in the empirical analysis.

Tables 1-2 show the price and returns correlations for the 3-month and 12-month variance futures. Not surprisingly, the 3-month and 12-month variance futures prices are more highly correlated at 0.64 than are the corresponding 3-month and 12-month variance futures returns correlations at 0.52. Neither of these correlations is particularly high.

The GARCH volatility estimates for the 3-month variance futures are presented in Table 3 for three probability densities for the full sample ("All"), as well as the subsamples given as Before, During and After the GFC. The estimates for All and Before GFC are very similar, with the After GFC estimates being quite different from remaining estimates, especially for α and β, and hence $\alpha + \beta$. Negative estimates of α are obtained for the normal and Student-t distributions, which is uncommon for financial data. The estimates of α and β are similar across the three distributions only for the During GFC subperiod. The estimate of $\alpha + \beta$ exceeds unity for the All and Before GFC subperiod under the Student-t density.

The GJR volatility estimates for the 3-month variance futures are presented in Table 4 for three probability densities for the full sample and the three subsamples, namely Before, During and After the GFC. Depending on the probability density and the sample period, negative estimates of α, γ and β are obtained, which is uncommon for financial data. Asymmetry seems to be significant, but whether it is positive or negative, as well as its magnitude, depends on the probability density and the sample period considered. Apart from the results for the normal

density, the estimates seem closest for the All and Before GFC subperiod. The estimate of $\alpha + \beta + \gamma /2$ exceeds unity for all three densities for at least one subperiod.

Table 5 gives the EGARCH volatility estimates for the 3-month variance futures for the three probability densities for the full sample and the three subsamples. The estimates of α, γ and β are substantially different between the normal density, on the one hand, and the Student-t and generalized normal densities, on the other. As the estímate of γ is negative, and the estímate of α is bounded by γ, there is leverage for All, as well as Before and After GFC subperiods for the normal density, but there is no leverage for the Student-t and generalized normal densities. For the normal density and After GFC subperiod, the estímate of β exceeds unity.

Recursive estimates of the parameters for the full sample period are given in Figures 5, 6 and 7 for the GARCH, GJR and EGARCH models, respectively, for the normal, Student-t and generalized normal densities. Consistent with the results presented in Tables 3-5 above, the estimates are highly variable, and differ according to the probability density. For the GARCH and GJR models, the results for the Student-t density seem to be the least variable, with some semblance of persistence rather than randomness. The estimates for the EGARCH model display some similarity under the Student-t and generalized normal densities.

4.   Concluding Remarks

Volatility is an indispensible component of sensible portfolio risk management. The volatility of an asset of composite index can be traded by using volatility derivatives, such as volatility and variance swaps, options and futures. The most popular volatility index is VIX, with VIX and its options and futures derivatives having been widely analysed in recent years.

An alternative volatility derivative to VIX is the S&P500 variance futures, which is an expectation of the variance of the S&P500 cash index. Variance futures are futures contracts written on realized variance, or standardized variance swaps. The S&P500 variance futures are not model based, so the assumptions underlying the index do not seem to have been clearly understood. As these two variance futures are thinly traded, their returns are not easy to model accurately using a variety of risk models.

This paper analysed the S&P500 3-month variance futures before, during and after the GFC, as well as for the full data period from 18 May 2004 to 1 April 2011, for each of three alternative and widely-used conditional volatility models and three densities, in order to determine whether exposure to risk can be incorporated into a financial portfolio without taking positions on the S&P500 index itself.

The estimates typically differed according to the conditional volatility model, the normal, Student-t and generalized normal densities, and the data subset. Asymmetry and leverage were found to exist in some cases. Recursive estimates of the parameters for the full sample period for the GARCH, GJR and EGARCH models for the normal, Student-t and generalized normal densities showed the estimates to be highly variable, especially with respect to the probability density.

Overall, it was shown that S&P500 3-month variance futures could be factored into a financial portfolio as a risk component without taking a direct position on the S&P500 cash index. However, further research will show whether this relationship is generally stable under significant changes in market volatility of the S&P500 cash index.

References

Black, F. (1976), Studies of stock market volatility changes, in 1976 Proceedings of the American Statistical Association, Business & Economic Statistics Section, pp. 177-181.

Bollerslev, T. (1986), Generalised autoregressive conditional heteroscedasticity, Journal of Econometrics, 31, 307-327.

Brenner, M., E.Y. Ou and J.E. Zhang (2006), Hedging volatility risk, Journal of Banking and Finance, 30, 811-821.

Caporin, M. and M. McAleer (2010), Model selection and testing of conditional and stochastic volatility models, to appear in L. Bauwens, C. Hafner and S. Laurent (eds.), Handbook on Financial Engineering and Econometrics: Volatility Models and Their Applications, Wiley, New York. Available at SSRN: http://ssrn.com/abstract=1676826.

Caporin, M. and M. McAleer (2011), Do we really need both BEKK and DCC? A tale of two multivariate GARCH models, to appear in Journal of Economic Surveys.

Chang, C.-L., J.-A. Jimenez-Martin, M. McAleer and T. Perez Amaral (2011), Risk management of risk under the Basel Accord: Forecasting value-at-risk of VIX futures, Managerial Finance, 37, 1088-1206. Available at SSRN: http://ssrn.com/abstract=1765202.

Chicago Board Options Exchange (2003), VIX: CBOE volatility index, Working paper, Chicago.

Engle, R.F. (1982), Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation, Econometrica, 50, 987-1007.

Glosten, L., R. Jagannathan and D. Runkle (1992), On the relation between the expected value and volatility of nominal excess return on stocks, Journal of Finance, 46, 1779-1801.

Huskaj, B. (2009), A value-at-risk analysis of VIX futures: Long memory, heavy tails, and asymmetry. Available at SSRN: http://ssrn.com/abstract=1495229.

Ishida, I., M. McAleer and K. Oya (2011), Estimating the leverage parameter of continuous-time stochastic volatility models using high frequency S&P500 and VIX, Managerial Finance, 37, 1048-1067.

Li, W.K., S. Ling and M. McAleer (2002), Recent theoretical results for time series models with GARCH errors, Journal of Economic Surveys, 16, 245-269. Reprinted in M. McAleer and L. Oxley (eds.), Contributions to Financial Econometrics: Theoretical and Practical Issues, Blackwell, Oxford, 2002, pp. 9-33.

Ling, S. and M. McAleer (2002a), Stationarity and the existence of moments of a family of GARCH processes, Journal of Econometrics, 106, 109-117.

Ling, S. and M. McAleer (2002b), Necessary and sufficient moment conditions for the GARCH(r,s) and asymmetric power GARCH(r,s) models, Econometric Theory, 18, 722-729.

Ling, S. and M. McAleer, (2003a), Asymptotic theory for a vector ARMA-GARCH model, Econometric Theory, 19, 278-308.

Ling, S. and M. McAleer (2003b), On adaptive estimation in nonstationary ARMA models with GARCH errors, Annals of Statistics, 31, 642-674.

McAleer, M. (2005), Automated inference and learning in modeling financial volatility, Econometric Theory, 21, 232-261.

McAleer, M., F. Chan and D. Marinova (2007), An econometric analysis of asymmetric volatility: theory and application to patents, Journal of Econometrics, 139, 259-284.

McAleer, M., J.-Á. Jiménez-Martin and T. Pérez-Amaral (2010), Has the Basel II Accord encouraged risk management during the 2008-09 financial crisis?, Available at SSRN: http://ssrn.com/abstract=1397239.

McAleer, M. and C. Wiphatthanananthakul (2010), A simple expected volatility (SEV) index: Application to SET50 index options, Mathematics and Computers in Simulation, 80, 2079-2090.

Nelson, D.B. (1991), Conditional heteroscedasticity in asset returns: A new approach, Econometrica, 59, 347-370.

Sepp, A. (2008), VIX option pricing in a jump-diffusion model, Risk Magazine, April, 84-89.

Shephard, N. (1996), Statistical aspects of ARCH and stochastic volatility, in O.E. Barndorff-Nielsen, D.R. Cox and D.V. Hinkley (eds.), Statistical Models in Econometrics, Finance and Other Fields, Chapman & Hall, London, 1-67.

Whaley, R.E. (1993), Derivatives on market volatility: Hedging tools long overdue, Journal of Derivatives, 1, 71-84.

Zhang, J.E. and Y. Huang (2010), The CBOE S&P500 three-month variance futures, Journal of Futures Markets, 30, 48-70.

Figure 1

S&P500 3-Month Variance Futures
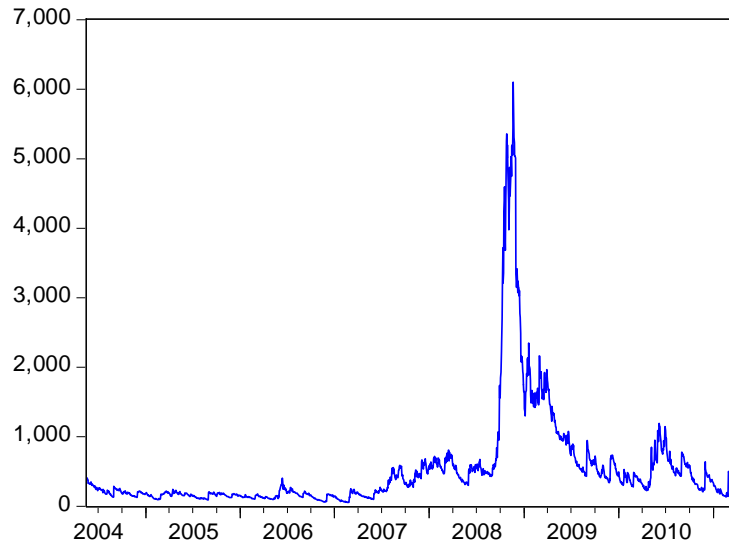## (18/05/2004 – 01/04/2011)

Figure 2

S&P500 12-Month Variance Futures
(24/03/2006 – 01/04/2011)
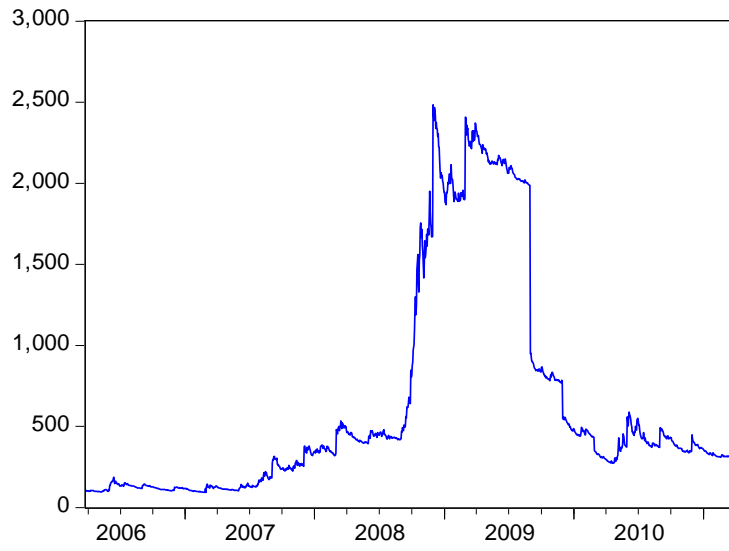
Figure 3

S&P500 3-Month Variance Futures Returns
(18/05/2004 – 01/04/2011)
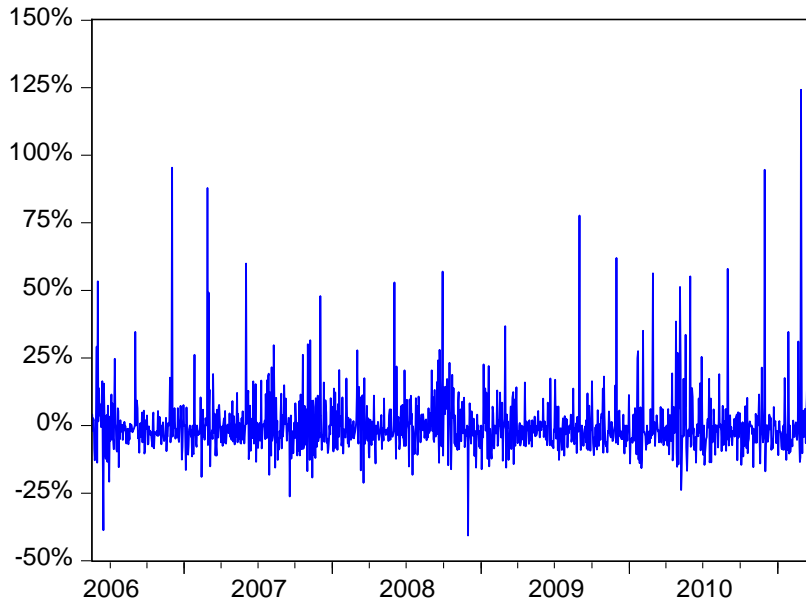
Figure 4

S&P500 12-Month Variance Futures Returns
(24/03/2006 – 01/04/2011)
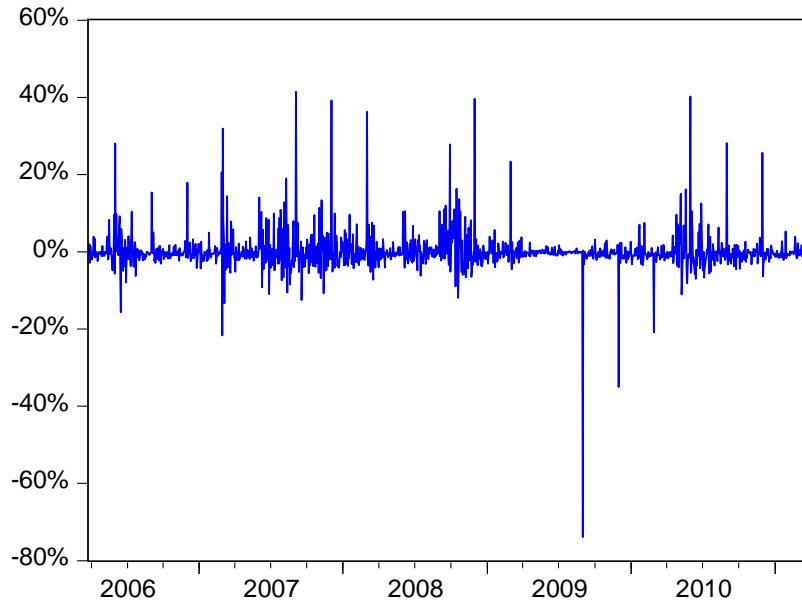
Table 1
Price Correlations

Variable	3-Month VF	12-Month VF
3-Month VF	1	0.64
12-Month VF	0.64	1

Note: VF denotes variance futures.

Table 2
Returns Correlations

Variable	3-Month VF returns	12-Month VF returns
3-Month VF returns	1	0.52
12-Month VF returns	0.52	1

Note: VF denotes variance futures.

Table 3

GARCH Estimates for 3-month VF Before, During and After GFC

Density	Parameter	All	Before	During	After
Normal	α	-0.0030	-0.0044	0.0099	-0.0110
	β	0.978	0.9805	0.9224	0.6822
	$\alpha + \beta$	0.9754	0.9761	0.9321	0.6712

Density	Parameter	All (2.16)	Before (2.15)	During (2.5)	After (11)
Student-t	α	0.1252	0.1697	0.0906	-0.0128
	β	0.8952	0.8838	0.9049	0.4147
	$\alpha + \beta$	1.0204	1.0535	0.9954	0.4018

Density	Parameter	All	Before	During	After
Generalized Normal	α	0.0358	0.0329	0.0397	0.0694
	β	0.8360	0.8512	0.9102	0.4115
	$\alpha + \beta$	0.8718	0.8841	0.9499	0.4809

Notes: All denotes the full sample period. The entries in parentheses for the Student-t distribution are the estimated degrees of freedom.

Table 4

GJR Estimates for 3-month VF Before, During and After GFC

Density	Parameter	All	Before	During	After
Normal	α	-0.0065	0.0726	0.0591**	-0.0111
	γ	0.3488	-0.1642	-0.1396	0.3843
	β	-0.1726	0.5808	0.5808**	0.9594
	$\alpha + \beta + \gamma\,/2$	-0.0047	0.5715	0.5701	1.1404

Density	Parameter	All (2.19)	Before (2.27)	During (17.54)	After (2.16)
Student-t	α	0.0971**	0.0772	0.0301**	0.5375**
	γ	0.5382**	0.4568	-0.1269**	0.9899**
	β	0.8750	0.8677	0.4826**	0.3151**
	$\alpha + \beta + \gamma\,/2$	1.2412	1.1733	0.4492**	1.3475**

Density	Parameter	All	Before	During	After
Generalized Normal	α	0.0146	0.0150	0.0514**	-0.0012
	γ	0.1689	0.1942	-0.1191**	0.2790
	β	0.8738	0.8468	0.7041**	0.9083
	$\alpha + \beta + \gamma\,/2$	0.9729	0.9589	0.6960**	1.04664

Notes: All denotes the full sample period. The entries in parentheses for
the Student-t distribution are the estimated degrees of freedom.
** These estimates are not statistically significant.

Table 5

EGARCH Estimates for 3-month VF Before, During and After GFC

Density	Parameter	All	Before	During	After
Normal	α	0.1192	-0.0321	-0.5012	0.0070
	γ	-0.1674	-0.1730	0.2580	-0.1318
	β	-0.8083	-0.8941	-0.4134**	1.0020

Density	Parameter	All (2.29)	Before (2.36)	During (2.9)	After (2.21)
Student-t	α	0.2315	0.2514	-0.1908	0.2685
	γ	-0.0314	-0.0571	0.2615	0.0307
	β	0.9477	0.9500	0.8921	0.6978

Density	Parameter	All	Before	During	After
Generalized Normal	α	0.1272	0.1284	-0.1859**	0.1647
	γ	-0.0161	-0.0207	0.2142	0.0013
	β	0.9282	0.9241	0.9019	0.7463

Notes: All denotes the full sample period. The entries in parenthesis for the Student-t distribution are the estimated degrees of freedom.
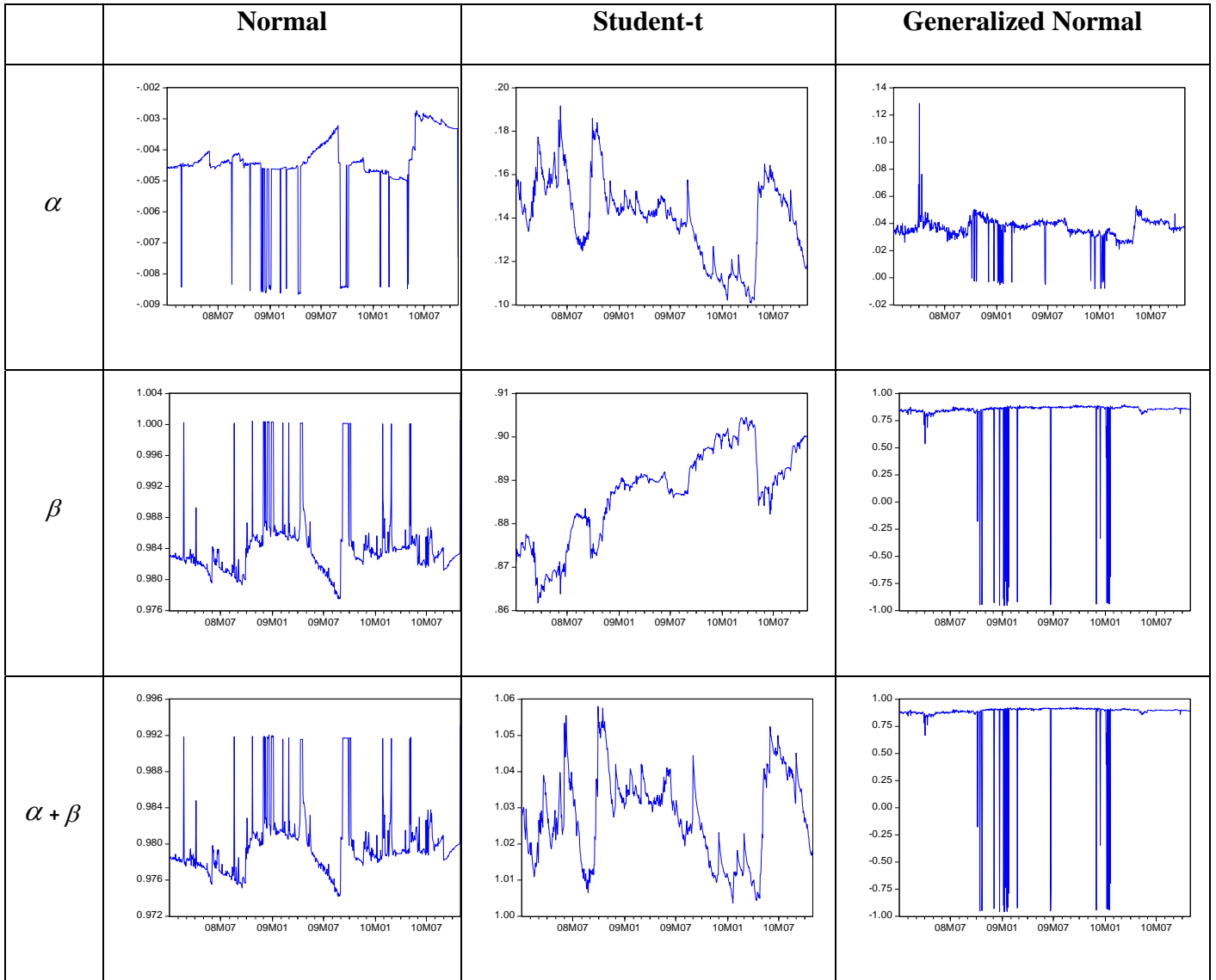** These estimates are not statistically significant.

Figure 5

α and β Estimates: GARCH

	Normal	**Student-t**	**Generalized Normal**
α			
β			
$\alpha + \beta$			

Figure 6

α, β and γ Estimates: GJR

	Normal	Student-t	Generalized Normal
α			
β			
γ			
$\alpha + \beta + \gamma/2$			

Figure 7

α , β and γ Estimates: EGARCH