

Extended theory about the allocation of time Description and application to the increase in the retirement age policies

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Abstract

This paper revises mainstream economic models which include time use in an explicit and endogenous manner, suggesting a extended theory which escape from the main problem existing in the literature. In order to do it, we start by presenting in section 2 the mainstream time use models in economics, showing their main features. Once this is done, we introduce the reader in the main problems this kind of well-established models imply, within section 3, being the most highlighted the problem of joint production. Subsequently, we propose an extended theory which solves the problem of joint production; this is extensively described in section 4. Last, but not least, we apply this model to offer a time use analysis of the effect of a policy which increases the retirement age in a life-cycle perspective for a representative individual.

Keywords: allocation of time, joint production, time use, retirement age, life cycle.

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1 Introduction and literature review

Economic theory did poorly cover the topic called allocation of time before the 1960s. Nevertheless, we find in Reid (1934) and Mincer (1962) the first mentions to a time use arguments linked to economics. However, no model was apparently able to include ideas revolving around the use of time and microeconomic theories.

In parallel, some other economists had been thinking of a new way to approach microeconomic behaviour, in contrast to the standard economic model¹. So a theoretical modelling can be found in the doctoral dissertation by Duncan Ironmonger, defended at the University of Cambridge in 1962, however published ten years later, in 1972. A similar theoretical setting is proposed by Lancaster (1966). Nonetheless, neither Ironmonger (1972) nor Lancaster (1966) refer explicitly to time as a central input in their models². Their models deal with the maximization of utility, defined over commodities or wants, which are obtained by using different inputs or characteristics. Basically, Ironmonger (1972) argues that in order to produce each want some inputs are required, while Lancaster (1966) states that each good is different if the characteristics are different.

The particularization of the setting in Ironmonger (1972) to the issue of allocation of time came with Becker (1965) in his well-known theory of allocation of time. Becker (1965) argues that consumers maximize utility, which is defined over what calls commodities that are produced with market goods and time, and consumer faces both budget and time constraints.

Becker (1965) has generated large research in the social sciences; in economics and sociology, particularly. However, theoretical contributions in economic theory have not been so abundant since his trigger paper. Some exceptions are the papers by DeSerpa (1971) or Evans (1972), that can be seen in essence as particular cases of Becker (1965).

The contribution by Pollak and Wachter (1975) can be considered the first critique to Becker's model. They posed some problems of time use models in general, and in particular of the benchmark model proposed by Becker (1965). Pollak and Wachter (1975) left some open questions that will be addressed in sections 3 and 4. Pollak and Wachter (1975) generated some interesting direct replies, like Barnett (1977).

Another perspective is given by Gronau (1977), which uses a very simple theoretical model to provide interesting insights and interpretations of real situations supported by empirical information.

Some other discussions are Flemming (1973)

¹Based on maximizing utility defined over market goods, given that consumer has limited resources and market goods are costly at market prices in a competitive market.

²Reid (1934) is considered the genesis of this line of thought suggesting a new way to approach microeconomic behaviour and is also considered an antecedent of Becker (1965). However the contribution by Margaret Reid has not been very highlighted by a quite considerable part of related economic literature.

and Juster and Stafford (1991). Particularly interesting is the survey paper by Juster and Stafford (1991), an up-to-date account of both theoretical and empirical research on the matter.

Little research on time use has employed dynamic models. Fischer (2001) analyses procrastination using time inputs as a key variable. Gonzalez-Chapela (2004) comprises essays on time allocation dealing with dynamic models. A particular model can be found in Gonzalez-Chapela (2007).

2 Mainstream time use models in microeconomics

In this section we illustrate how classic microeconomic models incorporate time use as a choice variable to provide a more realistic perspective of some decision problems. Firstly, we present a simple possible version of the leisure model. Secondly, we show a simple version of the Becker model.

Economic theory has produced a well known textbook model, frequently used in labour economics among other areas. Such model, often known as leisure model, includes the use of time as a choice variable and can be presented as follows, in line with the notation in Varian (1992) or Mas-Colell et al. (1995):

$$\begin{cases} \max_{x,T_1} & U = U(x,T_1), \\ s.t. & px \le w(T-T_1) + V, \end{cases}$$
(1)

where T_1 is the leisure time, w is the wage per unit of time and V is the non-labour income, x is consumption and p is the price of such consumption.

The solution of this model is obtained by the usual marginalist analysis, and in essence deals with leisure time as an extra good that in practice is as if it were purchased in the market at the wage rate.

A simplified version of the benchmark model by Becker (1965) is as follows. The main innovation is the introduction of what Becker calls commodities, which determine utility; such commodities are either tangible (home-made products) or intangible (home-made services, personal needs or similar) outputs produced with inputs such as time use and market products. Consider two commodities $(Z_1 \text{ and }$ Z_2), where Z_1 is produced with time $(\vec{T_1})$ and goods $(\vec{x_1})$, and Z_2 is produced with more time (T_2) and different goods $(\vec{x_2})$. An individual has to work at a wage w the remaining time in order to purchase good x, and may have non-labour income (V) at her disposal. He solves the following problem to determine $\vec{x_1}, \vec{T_1}, \vec{x_2}, \vec{T_2}$

$$\begin{cases} \max_{\vec{x_1}, \vec{x_2}, \vec{T_1}, \vec{T_2}} & U(Z_1(\vec{x_1}, \vec{T_1}), Z_2(\vec{x_2}, \vec{T_2})), \\ s.t. & \vec{p_1}^T \vec{x_1} + \vec{p_2}^T \vec{x_2} \le I, \\ & \vec{T_1} + \vec{T_2} \le \vec{T}, \\ & T_{ij} \ge 0, \text{ for all } i, j. \end{cases}$$
(2)

where $I = \vec{w}^T (\vec{T} - \vec{T_1} - \vec{T_2}) + V.$

As a simple example, if we set a meal as Z_1 and listening to music as Z_2 , this model would require as $\vec{x_1}$ the vector of ingredients of the meal, let us say, meat (x_{11}) and potatoes (x_{21}) . In addition, in order to produce and enjoy a meal, time is needed: the vector of time inputs \vec{T}_1 may be composed by the cooking time in the kitchen (T_{11}) and the eating time in the dining room (T_{21}) . Similarly, in order to listen to music (Z_2) , a vector of goods (\vec{x}_2) composed by a CD player (x_{12}) and an album in CD format (x_{22}) is required, but also a vector of time (\vec{T}_2) composed by listening to music in the kitchen (T_{12}) and listening to music in the dining room (T_{22}) .

$$\vec{x_{1(2\times1)}} = \begin{pmatrix} x_{11} \\ x_{21} \end{pmatrix}, \tag{3}$$

$$\vec{x}_{2(2\times1)} = \begin{pmatrix} x_{12} \\ x_{22} \end{pmatrix}, \qquad (4)$$

$$\vec{T}_{1(2\times1)} = \begin{pmatrix} T_{11} \\ T_{21} \end{pmatrix}, \tag{5}$$

$$\vec{T}_{2(2\times1)} = \begin{pmatrix} T_{12} \\ T_{22} \end{pmatrix}.$$
 (6)

Of course all the inputs of time must add up to the total time available and the goods employed must be feasible. This analysis aim at a more detailed description and explanation of the consumer behaviour.

Notice that model \dot{a} la Becker is more general than the leisure model: first, the concept of commodities expands the arguments in the utility function and, second, the model \dot{a} la Becker defines different types of time (or as he termed them, aspects). The model in (2) collapse to (1) if $Z_1 = x$ –whose price is p– and $Z_2 = T_1$, where there obviously is just one type or aspect of time.

3 Major theoretical problems of time use models

The improvements in realism expected from time use models bring several challenges which still remain as theoretical obstacles.

This section comments on the main problems, according to the literature in economics. In order to do it, we start by summarizing achievements of the benchmark model Becker (1965), to focus on its critique later on.

The Becker model is the following:

$$\max_{\vec{x_i}, \vec{T_i}} U(Z_1(\vec{x_1}, \vec{T_1}), ..., Z_m(\vec{x_m}, \vec{T_m})),$$
s.t.
$$\sum_{\substack{i=1\\m}}^{m} \vec{p_i}^T \vec{x_i} \le \vec{w}^T \vec{T_w} + V,$$

$$\sum_{\substack{i=1\\T_i = 1}}^{m} \vec{T_i} = \vec{T} - \vec{T_w},$$

$$T_{ij} \ge 0, \text{ for all } i, j.$$
(7)

Notice that this model includes the vector of commodities or wants (\vec{Z}) as an argument in the utility function (U). Each want $(Z_i, i = 1, ..., m)$ is obtained by using some *ingredients*, let us say, such as a vector of goods $(\vec{x_i})$ and a vector of time inputs $(\vec{T_i})$. Becker (1965) moves forward firstly merging the budget constraint and the time constraints into what he named as the full income constraint:

$$\sum_{i=1}^{m} \vec{p_i}^T \vec{x_i} + \sum_{i=1}^{m} \vec{w}^T \vec{T_i} \le \vec{w}^T \vec{T} + V.$$
(8)

At this point Becker (1965) makes some strong assumptions:

$$\vec{x}_{i(n\times 1)} = \vec{b}_{i(n\times 1)} Z_i,\tag{9}$$

$$\vec{T}_{i(p\times 1)} = \vec{t}_{i(p\times 1)} Z_i, \tag{10}$$

where $\vec{b_i}$ and $\vec{t_i}$ are the vectors giving the input of goods and time, respectively, per unit of Z_i . Notice that these assumptions impose linear relations defined by fixed coefficients.

Therefore, the model can be rewritten in this alternative way:

$$\begin{cases} \max_{\vec{Z}_{i}} & U(Z_{1}, ..., Z_{m}), \\ s.t. & \sum_{i=1}^{m} \pi_{i} Z_{i} \leq \vec{w}^{T} \vec{T} + V = S, \end{cases}$$
(11)

where π_i would represent –according to Becker (1965) terminology– the *full price* of each unit of commodity Z_i . Such *full price* would include the value of both goods and time used for such commodity, as follows:

$$\pi_i = \vec{p_i}^T \vec{b_i} + \vec{w}^T \vec{t_i}.$$
 (12)

This nice alternative version of the model in (7) permits to solve the problem for Z'_is , and in turn to do comparative statics as in the classical microeconomic textbook model.

Pollak and Wachter (1975) observed two shortcomings in the model by Becker (1965): absence of joint production and the need of constant returns to scale in the production of each Z_i .

It is apparent that the model cannot apply to many simple situations. Let us think in terms of the example of cooking and listening to music from the previous section: the time devoted to listen to music in the kitchen cannot be the same (simultaneous) as the time spent cooking, which is something very realistic for many cookers who cook while they listen to music. This feature is known in the literature as *joint production*, and was first noticed by Pollak and Wachter (1975).

The need of constant returns to scale in the production of each Z_i is related to the way in which we can write the model from (7) to (11). As we already mentioned, linear relations are assumed in the production of each commodity in relation to both goods and time. Doing that the model is transformed into one in which each commodity or want had a price (π_i) ; therefore, the consumer must choose her desired level of each want or commodity taking into account that prices for such wants and commodities are well defined by the π_i 's. The budget constraint is substituted into what is called the *full income constraint*. The problem of constant returns to scale in the production of wants is a technical discussion established by Pollak and Wachter (1975). This discussion lead them to conclude the following: in order to get a simplified model in which each commodity has a price (π_i) which is independent from the choice variable of the problem (Z_i) , the production of each commodity must satisfy constant returns to scale, and joint production is not possible. Otherwise, the model cannot be rewritten as in (11).

The problem of joint production is even more tricky, apart from its influence in the issue described above. Pollak and Wachter (1975) discuss extensively the issue of joint production, -since it creates a more structural problem than analytical-, and provide bright insights; however no theoretical model solving joint production is given. The issue remains unsolved in the literature; it also creates numerous problems when researchers work with time use data, because of -as they call itsimultaneous activities, i.e. joint production. The following quotation illustrates the importance of this issue within our field of interest:

"The major problem in studying the allocation of time in the household production function model is centred on joint production rather than nonconstant returns to scale" (Pollak and Wachter, 1975).

There are other problems as the one suggested by DeSerpa (1971), which argues that consumption of goods (x) are constrained by some minimum amount of time that is needed for such consumption. Therefore, extra constraints must be added to the Becker model. Although DeSerpa (1971) is in essence a particular case of Becker (1965), it poses a plausible problem and also proposes its solution.

In sum, we can observe that the main limitation which arises in models \dot{a} la Becker comes up related to joint production. This is addressed in the following section.

4 Extended theory of allocation of time

In order to facilitate the presentation of the extended model, we will refer to the cooking example above when commenting on joint production issues.

Let us define the production of wants, -which are represented in the *m*-dimensional vector $\vec{Z} = (Z_1, ..., Z_m) \in \Re^m$ - as follows, for all i = 1, ..., m:

$$Z_i = f_i \left(X_{n \times q}, \Im_{p \times r} \right), \tag{13}$$

where

$$X_{n \times q} = \begin{pmatrix} x_{11} & \cdots & x_{1q} \\ \vdots & & \vdots \\ x_{n1} & \cdots & x_{nq} \end{pmatrix}, \quad (14)$$

$$\Im_{p \times r} = \begin{pmatrix} T_{11} & \cdots & T_{1r} \\ \vdots & & \vdots \\ T_{p1} & \cdots & T_{pr} \end{pmatrix}.$$
 (15)

The cooking example would imply a setting in which Z_1 and Z_2 would be functions of the following form,

$$Z_i = f_i \left(X_{2 \times 2}, \Im_{2 \times 1} \right), i = 1, 2.$$
(16)

This simply states that each want may be produced as a function of all the ingredients and musical components (the goods), however both commodities can be produced using the same time inputs, which are the cooking time T_{11} and the eating time T_{21} . It can be specified that for this particular example, the meal can be produced defining the matrix X only over the vector of ingredients, whereas the listening to music can be produced using just the vector of music components, following the Becker specification. However, Becker specification cannot model this situation in which both when cooking and when enjoying the meal the consumer is listening to music, which is what the setting in (13), and of course in (16), allows for.

Therefore, we can propose the extended theory of allocation of time, by implementing the setting in (13) into a Becker-based model, as follows:

$$\max_{X,\Im} \quad U\left(\vec{Z}(X_{n\times q},\Im_{p\times r})\right),$$

s.t. $G\left(X_{n\times q},\Im_{p\times r}\right) \leq \vec{w}^T \vec{T_w} + V,$
 $\sum_{\substack{r=1\\p}}^r \vec{T_r} = \vec{T},$
 $\sum_{\substack{p=1\\X_{n\times q}} \geq 0_{n\times q}, \Im_{p\times r} \geq 0_{p\times r},$ (17)

where

- G is a function which express the expenditure of resources made by this individual, in terms of money.
- $\vec{x_q}$ is a vector corresponding to q-th column in $X_{n \times q}$. where q is a generic use of the goods.
- $\vec{T_r}$ is a vector corresponding to *r*-th column in $\mathfrak{S}_{p \times r}$, where *r* is a generic use of time.
- \vec{T} is a *p*-dimensional vector whose elements, T_p , represent the amounts of time available for each type of time *p*.
- T is the total immutable time available (24 hours per day, 7 days a week, etc)
- \vec{w} is the *p*-dimensional vector of wage rates for any *p*-th type of time.
- *T*_w is the *p*-dimensional vector of working time for any *p*-th type of time. Note that this specific use of time is included in the matrix ℑ_{p×r} within all the *r* uses of time.
- V is any other income which does not comes from $\vec{T_w}$.

To take into account the minimum time assumption posed in DeSerpa $(1971)^3$, notice that any extra linear constraint can be modelled with this set of constraints:

$$B_{s\times(nq+pr)}Q_{(nq+pr)\times 1} \le 0_{s\times 1},\tag{18}$$

where

• $B_{s\times(nq+pr)}$ is a matrix of positive or negative coefficients (all elements equal to zero implies the presence of no extra constraint), and

•
$$Q_{(nq+pr)\times 1} = (x_{11}, \dots, x_{nq}, T_{11}, \dots, T_{pr})$$

It must be noticed that the matrix of time inputs can be considered as a grid in which each type of time is, say, a five-minutes slot, showed vertically, as an schedule. Each row in the matrix would be a 5-minute slot within the 24 hours of the day. All possible ways in which time can be used could be arranged by columns. This setting would account for joint production, e.g. I can be driving and listening to music between 8:11h and 8:15h, as well as cooking and listening to music between 20:11h and 20:15h. A parallel interpretation can be done for the goods.

A common way to express the left hand side of the budget constraint in 17 is as the expenditure in market goods. The model thus reads as follows:

$$\begin{cases}
\max_{X,\Im} & U\left(\vec{Z}(X_{n\times q},\Im_{p\times r})\right), \\
s.t. & \sum_{q=1}^{q} \vec{p}^{T} \vec{x_{q}} \leq \vec{w}^{T} \vec{T_{w}} + V, \\
& \sum_{r=1}^{r} \vec{T_{r}} = \vec{T} - \vec{T_{w}}, \\
& \sum_{p=1}^{p=1} \vec{T_{p}} = \vec{T}, \\
& X_{n\times q} \geq 0_{n\times q}, \Im_{p\times r} \geq 0_{p\times r}.
\end{cases}$$
(19)

Of course this way to express market expenditure could be replaced by any other expenditure function G.

It is obvious that Becker model is a particular case of the last problem (19), when the production of the *m*-th commodity or want is only depending on the *m*-th column of both $X_{n\times q}$ and $\mathfrak{F}_{p\times r}$, m = q = r and no extra constraints are regarded.

5 Increase in the retirement age?

This section is illustrates one possible application of a simple version of the extended theory. This application is related to the research in line with Heckman (1976), although with a static model. The core idea of the model can be said to be partially described with the empirical job in Easterlin (2006) and Bonke et al. (2009).

Our simple application seeks to model the following idea. A policy maker tries to

³DeSerpa (1971) introduces a set of linear constraints as fixed proportions of minimum time needed for the consumption of each market goods. These constraints take a generic form as $x_{nq} \ge \alpha_h T_{pr}$.

influence individual's decision over the use of time. Nevertheless, he just can control in some degree some parameters for the individual (as the total amount of time available, wages, and others). Of course, the individual decides on her use of time and has her own tastes and preferences; all these choices comprise what is known as the life-cycle. This sort of debate about policies in favour of increasing the retirement age is very controversial (and its currently taking place in e.g. France and Spain).

The implications for policy makers can be found in issues like retirement decisions under lifetime choice, retirement decision "in the margin" (anticipated retirement), work-life balance over the working (lifetime) period, or daily work-life balance in terms of time budgets.

The model is applied here to a work-life balance over the working lifetime period, e.g. 16-65 year old period, in which an individual must decide how much time to work in the labour market; the policy maker is considering to increase the retirement age up to 67 years, in order to get higher working time in the labour market by the individuals for, say, fiscal reasons.

5.1 THE MODEL

An individual's lifetime decisions can be fundamentally related to how much time she is willing to allocate into some activity throughout a considerable period of her life. With this in mind, she must solve a problem like

$$\max_{\substack{T_{11}\\s.t.\\G \leq (w_{11} - w_{12})T_{11} + w_{12}T + V, \\0 \leq T_{11} \leq T.} U(Z_1(T_{11}), Z_2(T_{11})),$$
(21)

Assume that $w_{11}T + V \ge T$, so that working the full lifetime period guarantees the minimum expenditure level G. In the typical case that $w_{11} > w_{12}$, this implies that

$$\begin{cases}
\max_{\substack{T_{11},T_{12}\\ s.t.}} & U(Z_1(T_{11},T_{12}),Z_2(T_{11},T_{12})), \\
s.t. & G \le w_{11}T_{11} + w_{12}T_{12} + V, \\
& T_{11} + T_{12} = T,
\end{cases}$$
(20)

where there is only one type of time, the lifetime period of working time $(T_1 = T)$, -determined by the government-, which can be spent in working (T_{11}) and not working (T_{12}) . The partition (work-life balance) of lifetime in both uses produces *jointly* job satisfaction (Z_1) and personal satisfaction (Z_2) . Individual's expenditure for the lifetime period of working time is denoted by G; so far, we will consider G as a fixed amount of money for the whole lifetime period of working life, although later on we will relax this assumption. Average wage rate per unit of working time is denoted with w_{11} , while average subsidized income obtained from the welfare state during the non-working time periods is denoted by w_{12} . All other non-labour income is represented by V.

The model in (20) can be reduced to a model in which the decision variable is the working time within the total time available, as follows:

$$T_{11}^{min} \equiv \frac{G - V - w_{12}T}{w_{11} - w_{12}} \le T_{11}^* \le T.$$
 (22)

This defines the feasible set for the retirement problem. There are two possible solutions, depending upon whether the budget constraint binds or not. When it does not bind, an interior solution must satisfy

$$\sum_{i=1}^{2} \frac{\partial U}{\partial Z_m} \frac{\partial Z_m}{\partial T_{11}} = 0, \qquad (23)$$

with $\frac{G - V - w_{12}T}{w_{11} - w_{12}} \le T_{11}^* \le T.$

On the other hand, when the budget constraint binds – also assuming $w_{11} > w_{12}$ – there are two possible solutions: $T_{11}^* = T$ or $T_{11}^* = T_{11}^{min}$.

This case has interesting implications in economic terms: the individual must spend more time working (T_{11}^{min}) than she likes (T_{11}^{**}) in figure 2) in order to meet her economic standards –represented by G– given the lifetime period of the working life $(T_1 = T)$ by law.

Assume that

$$\frac{\partial U}{\partial Z_i} > 0, i = 1, 2. \tag{24}$$

Notice that

$$\frac{\partial U}{\partial T_{11}} = \frac{\partial U}{\partial Z_1} \left(\frac{\partial Z_1}{\partial T_{11}} - \frac{\partial Z_1}{\partial T_{12}} \right) \\
+ \frac{\partial U}{\partial Z_2} \left(\frac{\partial Z_2}{\partial T_{11}} - \frac{\partial Z_2}{\partial T_{12}} \right) = (25) \\
= JMU_{T_{11}} - JMU_{T_{12}}.$$

where $JMU_{T_{1r}} = \frac{\partial U}{\partial Z_1} \frac{\partial Z_1}{\partial T_{1r}} + \frac{\partial U}{\partial Z_2} \frac{\partial Z_2}{\partial T_{1r}}$, r = 1, 2 denotes joint marginal utility of the use of time T_{1r} . Under the plausible assumption

$$JMU_{T_{11}} < JMU_{T_{12}}, \text{ for } T_{11} = T,$$
 (26)

the boundary solution in the case that the budget constraint is binding must be

$$T_{11}^* = T_{11}^{min} = \frac{G - V - w_{12}T}{w_{11} - w_{12}}.$$
 (27)

If it is further assumed

$$JMU_{T_{11}} > JMU_{T_{12}}, \text{ for } T_{11} = 0,$$
 (28)

which is also sensible, and the function $JMU_{T_{11}} - JMU_{T_{12}}|_{(T_{11},T_{12}=T-T_{11})}$ is decreasing with respect to T_{11} , figure 2 represents the situation in this case. Notice that T_{11}^{**} is the unconstrained choice of the individual in this case, but it cannot be implemented due to the budget constraint.

Under the assumptions above, the utility as a function of the working years (T_{11}) presents an inverted-U shape.

As we can observe in figure 2, this solution implies that the individual has to work during more time than she would actually like, compared to the case where meeting the desired expenditure level is not constraining her.



Figure 1: Solution when the budget constraint does not bind



Figure 2: Retirement age paradox, typical solution when budget constraint binds

Now assume that the government is considering to increase the retirement age with the purpose of making more sustainable the pension system. Such policy actually increases the potential working years T from T^0 (situation in red in figure 2) to T^1 (situation in black in figure 2). However, it can be observed in figure 2 that the overall effect of such policy consists of a reduction in actual working years, and the individual reduces working time from T_{11}^0 to T_{11}^1 . This reduction corresponds to the following expression, obtained from (27):

$$\frac{\partial T_{11}^*}{\partial T} = \frac{-w_{12}}{w_{11} - w_{12}}.$$
(29)

Since we have assumed that $w_{11}, w_{12} > 0$ satisfy $w_{11} > w_{12}$, it follows from (29) that $\frac{\partial T_{11}}{\partial T} < 0.$

Therefore, a policy maker may observe a reaction from individuals consisting of reducing their working years, which would be potentially harmful in terms of public policy (alleviating public finance), as expected *a priori*.

In order to illustrate the model analysis with some numerics, we next do some numerical analysis, using concrete functions Z and U, such that sufficient conditions from (26) and (28) hold. We have considered the following ones:

$$U(Z_1, Z_2) = a_0 + a_1 ln(Z_1) + a_2 ln(Z_2), \quad (30)$$

$$Z_1 = b_{11}ln(1+T_{11}) + b_{12}ln(1+T-T_{11}), (31)$$

$$Z_2 = b_{21}ln(1+T_{11}) + b_{22}ln(1+T-T_{11}).$$
(32)

The particular values of the parameters are shown in tables 1 and 2. We consider an increase in the retirement age of two years, from T = 49 to T = 51; this corresponds to an increase in the retirement age from 65 to 67 years. Running the model for this parametric values, we obtain the results displayed in figures 3 and 5.

As expected the numerical analysis in figure 3 matches the theory described earlier.

This model illustrates in a very simple manner a paradox. However, it may be argued that keeping G constant when T increases is not realistic. To account for that, consider a variation of the model in which G = gT, where g is the average expenditure per year in the working lifetime period; obviously satisfying $w_{11} > g$. This refinement generates the following model:

$$\max_{\substack{T_{11}\\s.t.}} U(Z_1(T_{11}), Z_2(T_{11})), \\ gT \le (w_{11} - w_{12})T_{11} + w_{12}T + V, \quad (33) \\ 0 \le T_{11} \le T.$$

Under the assumptions for U and Z_i considered above, the boundary solution in this case is given by

$$T_{11}^* = \frac{(g - w_{12})T - V}{w_{11} - w_{12}}.$$
 (34)

	Inputs				
Т	w_{11}	w_{12}	G	g	V
$49 \rightarrow 51$	30000	8000	1100000	22500	0

Table 1: Parametrical values for the inputs

Table 2: Parametrical values for utility function and satisfactions functions

Utility				Satisfa	action	\mathbf{S}
a_0	a_1	a_2	b_{11}	b_{12}	b_{21}	b_{22}
0	2	3	0.6	0.3	0.2	0.7



Figure 3: Numerical analysis of model in (21) representing the retirement age increase paradox.



Figure 4: Retirement age insufficiency, solution extended case

Since

$$\frac{\partial T_{11}^*}{\partial T} = \frac{g - w_{12}}{w_{11} - w_{12}},\tag{35}$$

and $w_{11} > w_{12}$ the paradox also occurs if $g < w_{12}$, which is not expected to hold in general. However, if $g \ge w_{12}$ weaker version of the paradox still may hold. We have that individuals will decide to increase their working time by less time than the time increase in the retirement age.

This is because, since

$$\frac{\partial T_{11}^*}{\partial T} = \frac{g - w_{12}}{w_{11} - w_{12}} \ge 0, \tag{36}$$

and $w_{11} > g \ge w_{12} > 0$, we have that $g - w_{12} < w_{11} - w_{12}$, so $\frac{\partial T_{11}}{\partial T} < 1$, and then

 $\Delta T_{11}^* < \Delta T.$

Thus, the increase in the retirement age will not be fully covered by working time.

It follows from our analysis that the public policy may not be as effective as expected. This conclusion is particularly interesting in a situation in which the retirement age would be increased by law to account for the increase in life expectancy.

We can illustrate this model numerically as with the previous model. We use the same functions in (30), (31) and (32). Also the values for the parameters are those given, except from G. Now we have G = gT, with g = 22500. We consider an increase in retirement age of two years, as above. Our results are illustrated in figure 5. We observe, again, that our theoretical analysis illustrated in figure 4 is replicated by the numerical analysis showed in figure 5. That is, an increase in the retirement age is not transferred completely in working time by the individual rational decision. So, for instance, if the retirement age is extended in 24 months, the individual would increase her working time only by about 16 months, an incomplete proportion of the increase in the retirement age. Under the assumptions above, such an expenditure level actually obliges her to work more than she would wish.

5.2 CONCLUSIONS FROM THIS SIMPLE ILLUSTRATION

Whenever money imposes probа lem/constraint in a lifetime perspective, policy decisions in favour to increasing retirement age may lead to an increase in individual welfare. Nevertheless, such decision may not fulfil the public goals expected by policy makers. Both theoretical predictions and numerical analysis confirm this assertion. This particular values are arbitrarily chosen for simplicity; we leave for future research a more detailed analysis of this model using actual life-cycle data. However, under the assumptions made about the parameters, our results seem robust.

6 Summarize and conclusions

Within this paper we have started by introducing main time use models built by economic theory. Later on, we have shown what the main problems of these models are, from a theoretical point of view, according to the literature. As a result, an extended theory of the allocation of time is provided in this paper, solving the theoretical problem known as joint production.

Based on the theory provided, a simple theoretical model is obtained, and applied to the case of the policies claiming for an increase of the retirement age. Moreover, we suggest a refinement for this model, which we also analyse. According to our theoretical predictions and our numerical analysis, we conclude that an increase in the retirement age with the purpose of alleviate public finance by increasing working time may not get effective results, which can even be contrary to the initial goals set by the policy makers.



Figure 5: Numerical analysis of model in (33) representing the Increasing the retirement age insufficiency

Case	with G	Case with $G = gT$		
T_{11}^{*}	ΔT_{11}^* vs. ΔT	$ T_{11}^*$	ΔT_{11}^* vs. ΔT	
$32.18 \rightarrow 31.45$	-0.73 < +2	$32.29 \rightarrow 33.61$	+1.32 < +2	

Table 3: Solutions, working time (T_{11}^*) for a policy consisting of an increase in retirement age of two years $(T = 49 \rightarrow T = 51)$

Bibliography

- Barnett, W. A. (1977), "Pollak and Wachter on the Household Production Function Approach," Journal of Political Economy, 85, pp. 1073–1082.
- Becker, G. S. (1965), "A Theory of the Allocation of Time," The economic journal, 75, 493–517.
- Bonke, J., Deding, M., and Lausten, M. (2009), "Time and Money," Journal of Happiness Studies, 10, 113.
- DeSerpa, A. C. (1971), "A Theory of the Economics of Time," *The Economic Journal*, 81, pp. 828–846.
- Easterlin, R. A. (2006), "Life cycle happiness and its sources: Intersections of psychology, economics, and demography," *Journal of Economic Psychology*, 27, 463–482.
- Evans, A. W. (1972), "On the theory of the valuation and allocation of time," Scottish Journal of Political Economy, 19, 1–17.
- Fischer, C. (2001), "Read this paper later: procrastination with time-consistent preferences," Journal of Economic Behavior and Organization, 46, 249–269.
- Flemming, J. S. (1973), "On the theory of the valuation and allocation of time: some comments," Scottish Journal of Political Economy, 20, 65–71.
- Gonzalez-Chapela, J. (2004), "Three Essays on Time Allocation," Universidad Pompeu Fabra, Departamento de Economia y Empresa.
- (2007), "On the Price of Recreation Goods as a Determinant of Male Labor Supply," Journal of Labor Economics, 25, 795–824.
- Gronau, R. (1977), "Leisure, Home Production, and Work-The Theory of the Allocation of Time Revisited," The Journal of Political Economy, 85.
- Heckman, J. J. (1976), "A Life-Cycle Model of Earnings, Learning, and Consumption," Journal of Political Economy, 84, pp. S11–S44.
- Ironmonger, D. S. (1972), New commodities and consumer behaviour, Cambridge University Press.
- Juster, F. T. and Stafford, F. P. (1991), "The Allocation of Time: Empirical Findings, Behavioral Models, and Problems of Measurement," *Journal of Economic Literature*, 29, 471.
- Lancaster, K. J. (1966), "A New Approach to Consumer Theory," *Journal of Political Economy*, 74, pp. 132–157.

- Mas-Colell, A., Whinston, M. D., and Green, J. R. (1995), *Microeconomic Theory*, Oxford University Press.
- Mincer, J. (1962), Labor Force Participation of Married Women, National Bureau of Economic Research, Inc, Aspects of Labor Economics, pp. 63–106.
- Pollak, R. A. and Wachter, M. L. (1975), "The Relevance of the Household Production Function and Its Implications for the Allocation of Time," *Journal of Political Economy*, 83, pp. 255–278.
- Reid, M. (1934), Economics of household production, J. Wiley & Sons, Inc.

Varian, H. R. (1992), Microeconomic Analysis, Norton.

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