Facultad de Estudios Estadísticos – Universidad Complutense de Madrid
Ciclo de conferencias 2014 - 2015

CIENCIAS ACTUARIALES
EL USO DE LA ESTADISTICA EN EL PRECIO DEL RIESGO

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Actuario
Abril 2015

OBJETIVO

MOTIVAR

IDENTIFICAR LA UTILIDAD Y EL VALOR DE LOS CONOCIMIENTOS ESTADISTICOS EN EL MUNDO DE LOS SEGUROS

UNIVERSIDAD
COMPLUTENSE
MADRID

Orientación Preuniversitaria

GRADO EN
ESTADÍSTICA
APLICADA

LOS ESTADÍSTICOS TRABAJAMOS EN: *SECTOR PRIVADO*

- Laboratorios Farmacéuticos
- Empresas de Comunicación
- Consultorías
- Bancos
- Empresas de ingeniería
- Grandes Almacenes

Campus de Excelencia Internacional

CONTENIDO

EL MERCADO DE SEGUROS.

- Productos
- La profesión Actuarial

RIESGO. Prima y Provisiones Técnicas

- Modelo individual del riesgo.
- Modelo colectivo del riesgo

RIESGO. Capital

- Concepto y antecedentes
- Solvencia II

MODELOS

AHORRO E INVERSION

INTRODUCCION

Today´s actuary can get a certain amount of social respect anywhere ... by pretending to be an economist
actuarialjokes.com

El Diccionario de la Real Academia de la Lengua define al “actuario” como *aquella persona versada en los cálculos matemáticos y en los conocimientos* ***estadísticos***, ***jurídicos*** *y* ***financieros*** *concernientes a los* *<u>seguros</u>* *y a su régimen, la cual asesora a las entidades aseguradoras y sirve como perito en las operaciones de estas.*

Los seguros se asocian a lo contingente, lo azaroso y lo incierto. Se asocia a la antigua necesidad humana de protegerse frente a los eventos dañinos a través de la solidaridad. **Protegerse del riesgo**. La estadística se junta con los datos, la probabilidad y la inferencia. Era inevitable el matrimonio entre ambas. La solidaridad se convirtió en “ley de los grandes números”, el daño en “esperanza matemática”, y lo actuarial en una profesión

El desarrollo de los mercados internacionales de capitales y el potencial de las nuevas tecnologías permiten la innovación de nuevos y atractivos productos. La ingeniería financiera desarrollada a finales del siglo pasado introdujo nuevos paradigmas de valoración en los productos financieros. La solidaridad paso a negocio. La estadística se cansó de su viejo y rancio actuario y se fue con el joven triunfador de las finanzas. Los actuarios tuvieron que ponerse al día. Un famoso actuario definió a estas nuevas promociones como “Actuarios de Tercera Generación”.

El mundo de los servicios financieros se ha vuelto complicado. **Reguladores y accionistas luchan contra el riesgo**. Unos quieren proteger a los ciudadanos, los otros la supervivencia y rentabilidad del ciclo productivo. El riesgo es ahora un mercado y un negocio, hasta da nombre a una honorable profesión: el Director de Riesgos. Pero hay cosas que nunca cambian, la estadística sigue siendo su arma, eso sí, actualizada a los nuevos tiempos en forma de procesos estocásticos, modelos lineales generalizados o las siempre atractivas “copulas”.

El MERCADO. Actuario, una profesión

european
actuarial
academy

What is an actuary?

In spite of the very important role of actuaries, in most countries the profession is not well known, sometimes not even known at all. We can sum up their activities as follows:

Actuaries cope with uncertainty and financial risks.

To that end actuaries design schemes to control the financial risks or future uncertainties, which, as you know: are plentiful ...

Actuaries apply mathematical, statistical, economic and financial analyses and theories to solve a wide range of real business problems: in insurance, pensions, investments and banking. Part of their job is to try and predict what money will do when it is invested in insurance, pensions, stock markets or other financial services. Actuaries are therefore among the most influential professionals in the financial world. A lot of the actuaries' work can be described as 'risk management', that is, assesment of the probability of events and of the associated costs .

Actuaries have always combined excellent mathematical skills with the ability to think clearly and logically. They also need to be sharp-focused and detail-oriented. Actuaries must be interested in such subjects as probability and risk identification and assessment, and have the ability to understand such complex financial topics as derivatives. Actuaries should also be on easy terms with their professional environment as well as solution-oriented.

Where do actuaries work?

The traditional areas in which actuaries operate are: life and general insurance, pensions, consultancy and investment. Actuaries are also moving into other fields where their analytical skills are useful.

In carrying out this work, actuaries draw on resources provided by other professionals: legal, accounting, medical, economic, etc. They refer to information provided in journals, mathematical tables, statistical and other records and use sophisticated computer programmes to process the relevant data.

https://actuarial-academy.com/about/profession

El MERCADO

Ley de Ordenación y Supervisión de los Seguro Privado

http://www.boe.es/buscar/act.php?id=BOE-A-2004-18908

El MERCADO. Los productos

Artículo 6. Ramos de seguro.

1. En el seguro directo <u>*distinto del seguro de vida*</u> la clasificación de los riesgos por ramos, así como la denominación de la autorización concedida simultáneamente para varios ramos y, finalmente, la conceptuación de riesgos accesorios, se ajustará a lo siguiente:

a) Clasificación de los riesgos por ramos.

1. Accidentes.
2. Enfermedad (comprendida la asistencia sanitaria y la dependencia)
3. Vehículos terrestres (no ferroviarios).
4. Vehículos ferroviarios.
5. Vehículos aéreos.
6. Vehículos marítimos, lacustres y fluviales
7. Mercancías transportadas
8. Incendio y elementos naturales
9. Otros daños a los bienes
10. Responsabilidad civil en vehículos terrestres automóviles
11. Responsabilidad civil en vehículos aéreos
12. Responsabilidad civil en vehículos marítimos, lacustres y fluviales
13. Responsabilidad civil en general
14. Crédito
15. Caución
16. Pérdidas pecuniarias diversas
17. Defensa jurídica.
18. Asistencia
19. Decesos.

El MERCADO. Los productos

Artículo 6. Ramos de seguro.

2. El seguro directo *sobre la vida* se incluirá en un solo ramo, el ramo de vida, con el ámbito de todos los ramos del seguro directo sobre la vida enumerados en las directivas comunitarias reguladoras de la actividad del seguro directo sobre la vida.

A. Ámbito del ramo de vida.

El ramo de vida comprenderá:

a) El seguro sobre la vida, tanto para caso de **muerte** como de **supervivencia**, o ambos conjuntamente, incluido en el de supervivencia el seguro de renta; el seguro sobre la vida con contraseguro; el seguro de «nupcialidad», y el seguro de «natalidad». Asimismo, comprende cualquiera de estos seguros cuando estén vinculados con fondos de inversión. Igualmente, podrá comprender el seguro de dependencia.

b) Las operaciones de capitalización del artículo 3.1.b) de esta ley.

c) Las operaciones de gestión de fondos colectivos de jubilación y de gestión de operaciones tontinas.

4.3.3 Tontines

An early example of interest and survivorship accumulation is illustrated by the concept of a *tontine*. This is a scheme named after Lorenzo Tonti, a Neapolitan banker, who introduced it in 1653. It refers to a plan whereby a group of individuals all contribute to a fund, with the provision that the last one living among the group takes the entire amount. We will consider a somewhat modified version.

Non-life insurance directives

These directives harmonise the essential rules on insurance. They provide for:

1

- a common prudential framework
- single license and exclusive prudential supervision by the competent authorities in EU countries
- that insurance undertakings may operate in any other EU country
- the protection of insured persons and policyholders, in particular by determining the law applicable to insurance contracts concluded in the EU
- clarity about what information must be provided to policy holders before entering into a contract / through the duration of the contract.
- EU directive 73/239/EEC – taking up and pursuit of the business amended by:
  - EU directive 84/641/EEC – tourist assistance
  - EU directive 87/343/EEC – credit insurance and suretyship insurance
  - EU directive 88/357/EEC – exercise of freedom to provide services
  - EU directive 2002/13/EC – solvency margin for non-life insurance undertakings
  - EU directive 90/618/EEC motor vehicle liability insurance
  - EU directive 92/49/EEC – coordination of laws relating to direct insurance
  - EU directive 95/26/EC – reinforcing prudential supervision
  - EU directive 2000/64/EC – exchange of information with non-EU countries
- EU directive 73/240/EEC – abolition of restrictions on freedom of establishment
- EU directive 78/473/EEC – Community co-insurance
- EU directive 87/344/EEC – legal expenses insurance

Life assurance directives

The long list of life assurance directives has been combined into a consolidated EU directive (2002/83) on life assurance – providing all the provisions in a single text.

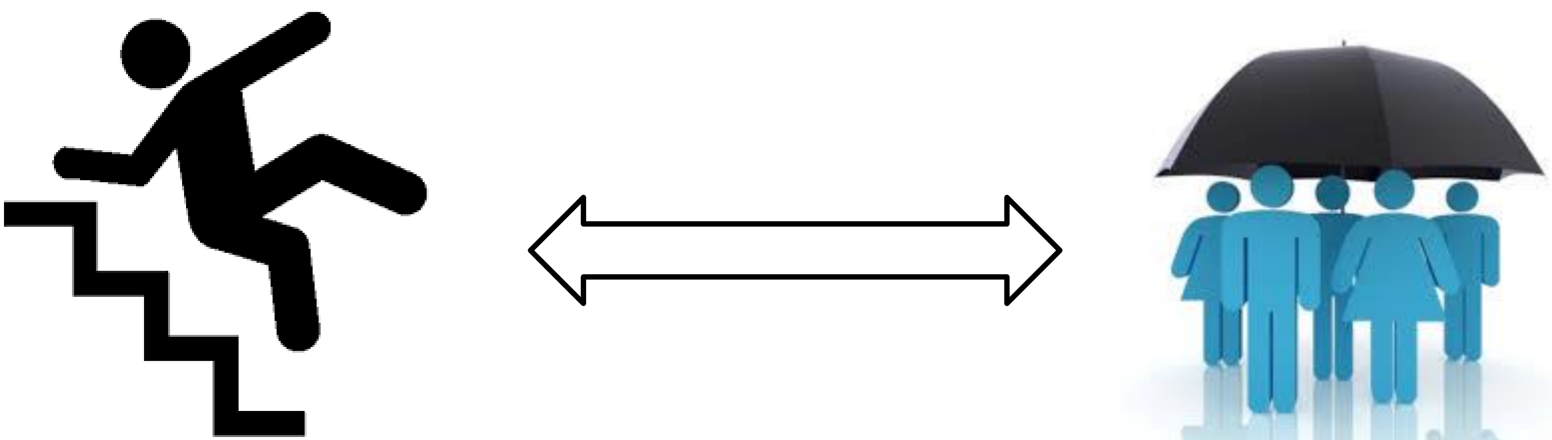
RIESGO: Prima y Provisión

El “*negocio financiero”*, entendido de forma general, se dedica a la **transferencia de fondos y riesgos** de unos agentes a otros, en operaciones con duración variable en el tiempo. Si las transacciones así generadas tuviesen un final feliz, es decir, que los fondos retornasen al agente que los colocó sin ningún contratiempo, entonces el precio por realizar esos intercambios sólo reflejaría la traslación de las unidades monetarias de unos periodos a otros y la recompensa por trasladar el uso y disfrute de esos fondos hacia el futuro

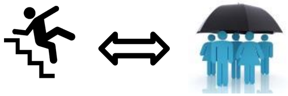
Sin embargo, esto no siempre es así y existe la posibilidad de que los fondos no retornen en la forma pactada (contingencia) o que se produzca un conjunto de circunstancias que lo impiden. Por esta razón, junto con los fondos, las operaciones financieras transfieren riesgos, y así, **la remuneración de los mismos deberá reflejar la posible ocurrencia de esas contingencias**

RIESGO: Prima y Provisión

La esencia del seguro es la transferencia o desplazamiento del riesgo de unidades económicas relativamente pequeñas (las personas) a unidades muy grandes o al total de la sociedad



RIESGO: Prima y Provisión



El seguro es un sistema <u>no para reducir el riesgo (eso es imposible), sino para redistribuir su incidencia</u>, para amortiguar sus efectos sobre los individuos aislados y extenderlos entre un número grande de personas que están dispuestas a compartir riesgos ajenos (pago prima), a cambio de una cierta compensación (recibo indemnización).

Primera idea de Riesgo: la posibilidad de que "*algo malo*" pase. Algo malo es algo que supone una "*pérdida económica*"

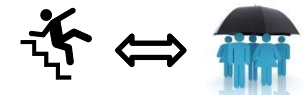
Una familiar fallece
Mi casa se quema
Mi coche se estrella
Me pongo enfermo
Pierdo mi trabajo
Me demandan
Mis clientes no me pagan
Pierdo mi cosecha
Mi cirujano se equivoca
Se cancela o retrasa mi vuelo

¿Para quién? : Las personas

El MERCADO. Los productos

www.mutua.es

RIESGO: Prima y Provisión



1. Asegurado
2. Presupuesto

DATOS DEL ASEGURADO

Datos del tomador

- ¿Es Vd. mutualista? * : Si / No (No)

Datos del asegurado

- Fecha nacimiento *: 01/04/1970
- Sexo * : Hombre / Mujer (Hombre)
- Altura * : 180 cms
- Peso * : 70 Kg
- Profesión *: Estadístico, Matemático, Actuario (Sin activid

Riesgos y garantías cubiertos

- Cobertura por fallecimiento*: 100000 €

RIESGO: Prima y Provisión

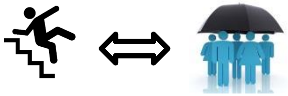
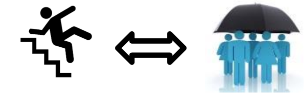


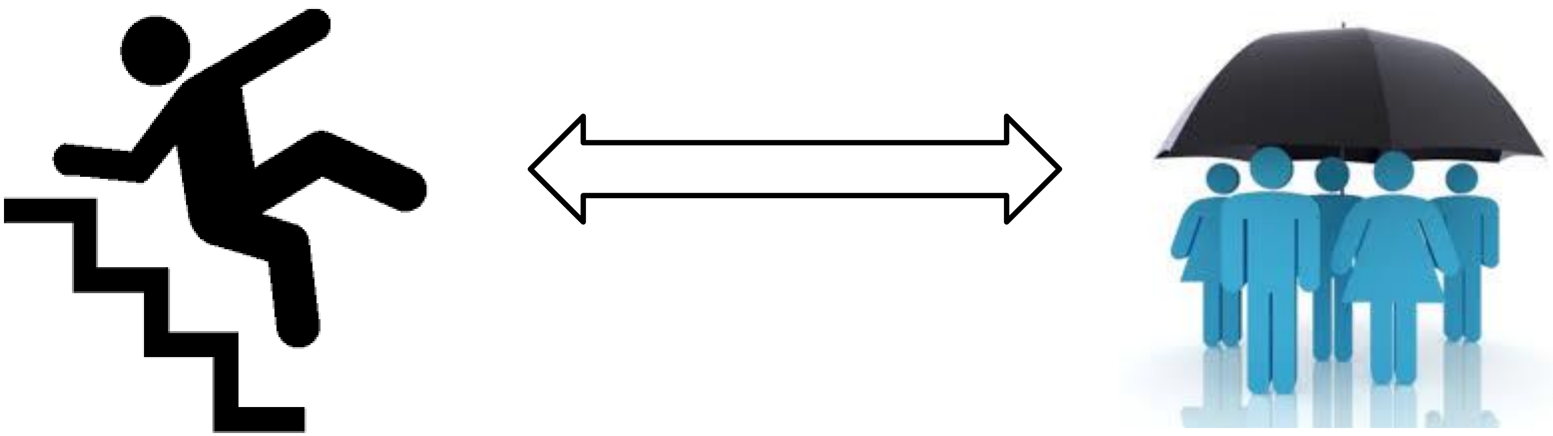
TABLA DE PRIMAS

Año	Prima	Prima con descuentos	Capital de fallecimiento
2015	352,86 €	300,43 €	100.000,00 €
2016	391,12 €	274,28 €	100.000,00 €
2017	434,17 €	304,42 €	100.000,00 €
2018	481,12 €	337,28 €	100.000,00 €
2019	533,81 €	374,17 €	100.000,00 €
2020	591,75 €	414,72 €	100.000,00 €
2021	655,45 €	459,31 €	100.000,00 €
2022	721,09 €	505,26 €	100.000,00 €
2023	788,20 €	552,24 €	100.000,00 €
2024	858,38 €	601,37 €	100.000,00 €

RIESGO: Prima y Provisión



La esencia del seguro es la transferencia o desplazamiento del riesgo de unidades económicas relativamente pequeñas (las personas) a unidades muy grandes o al total de la sociedad



1 Siniestro (€100.000)

283 personas (€352,86)

Frecuencia = 1/283 = 0,0035

RIESGO: Prima y Provisión

"An actuary is someone who expects everyone to be dead on time."

actuarialjokes.com

Actuarial Life Table

Office of the Chief Actuary

Life Tables

A period life table is based on the mortality experience of a population during a relat
short period of time. Here we present the 2010 period life table for the Social Sec
population. For this table, the period life expectancy at a given age is the average remai
number of years expected prior to death for a person at that exact age, born on January
using the mortality rates for 2010 over the course of his or her remaining life.

Period Life Table, 2010

	Male			Female		
Exact age	Death probability [a]	Number of lives [b]	Life expectancy	Death probability [a]	Number of lives [b]	Life expectancy
0	0.006680	100,000	76.10	0.005562	100,000	80.94
1	0.000436	99,332	75.62	0.000396	99,444	80.39
2	0.000304	99,289	74.65	0.000214	99,404	79.43
3	0.000232	99,259	73.67	0.000162	99,383	78.44
4	0.000172	99,235	72.69	0.000132	99,367	77.46
5	0.000155	99,218	71.70	0.000117	99,354	76.47
6	0.000143	99,203	70.71	0.000106	99,342	75.47
7	0.000131	99,189	69.72	0.000099	99,332	74.48
8	0.000115	99,176	68.73	0.000093	99,322	73.49
9	0.000096	99,164	67.74	0.000090	99,313	72.50
10	0.000082	99,155	66.74	0.000090	99,304	71.50
11	0.000086	99,147	65.75	0.000096	99,295	70.51
12	0.000125	99,138	64.76	0.000111	99,285	69.52

http://www.ssa.gov/oact/STATS/table4c6.html

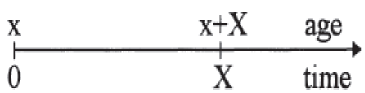
2.1. Life tables

A **life table** is a positive decreasing function l_ξ of $\xi \geq 0$ with $l_\infty = 0$, where of course $l_\infty := \lim_{\xi \to \infty} l_\xi = 0$. We imagine a closed group of persons with the same age, called lives, observed over time. Then l_ξ is the number of persons alive at age ξ. This is a convenient interpretation (the construction of **practical life tables** is not based on it).

In **theoretical life tables**, l_ξ is not necessarily an integer. The theoretical l_ξ is a continuous function of ξ and the derivative l_ξ' is supposed to exist whenever it is used.

2.2. Future lifetime X

We consider a **life** x aged x at the origin 0 of time. Hence x denotes the person and also her or his age. We denote the future lifetime of x by X. Hence, X is a random variable and x dies at the instant X, at age x+X.



$$ {}_t p_x := P(X>t) \qquad (1)$$

denotes the probability that x will attain age x+t.

$$ {}_t q_x := P(X\leq t) \qquad (2)$$

denotes the probability that x will die within t years. By (2), **the function ${}_t q_x$ of $t \geq 0$ is the distribution function of the random variable X.**

3.7 A sample table

To do the spreadsheet problems in the following chapters that require a life table, we introduce a sample table. It is given by

$$q_x = \begin{cases} 1 - e^{-0.000\,05(1.09)^x}, & x = 0, 1, \ldots, 118, \\ 1, & x = 119. \end{cases} \qquad (3.11)$$

2.6. Analytic life tables

With the advent of high speed computers, the advantages of analytic life tables has decreased in recent years. But some of them remain interesting, for instance Makeham's life table. Analytical life tables are defined for all ages $x \geq 0$. In practice, they used on particular age intervals only.

de Moivre (1729)

$$l_x = (86-x) \quad (0 \leq x \leq 86),$$

(and $l_x = 0$ if $x \geq 86$). Of course, this table is completely obsolete.

Gompertz (1925) $\qquad l_x = a\, g^{c^x},$

where $a>0$, $0<g<1$, $c>1$. In the last member, c^x must be evaluated, not g^c. Gompertz's life table results from the force of mortality function

$$\mu_\xi = \beta c^\xi,$$

by (9), after the introduction of new parameters (take s=1 in the following life table by Makeham).

Makeham (1860) $\qquad l_x = a\, s^x g^{c^x},$

where $a>0$, $0<s<1$, $0<g<1$, $c>1$. In the last member, c^x must be evaluated, not g^c. Makeham's life table results from the force of mortality function

$$\mu_\xi = \alpha + \beta c^\xi, \qquad (15)$$

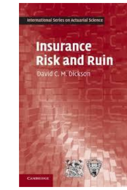
by (9), after the introduction of new parameters. Indeed,

$$\int_{(0,x)} \mu_\xi d\xi = \int_{(0,x)} (\alpha + \beta c^\xi) d\xi = \alpha x + \beta \int_{(0,x)} c^\xi d\xi$$
$$= \alpha x + \beta \int_{(0,x)} dc^\xi / \log c = \alpha x + (\beta/\log c) c^x - (\beta/\log c)$$

and then it is enough to take

$$a := l_0 e^{\beta/\log c}, \quad s := e^{-\alpha}, \quad g := e^{-\beta/\log c}.$$

RIESGO: Prima y Provisión



3

Principles of premium calculation

3.1 Introduction

Although we have previously used the term premium, we have not formally defined it. A premium is the payment that a policyholder makes for complete or partial insurance cover against a risk. In this chapter we describe and discuss some ways in which premiums may be calculated, but we consider premium calculation from a mathematical viewpoint only. In practice, insurers have to take account not only of the characteristics of risks they are insuring, but other factors su

We de … k X. When we … k are distribute … on of X, and a … nium calculation principle. … $= \phi(X)$

$$\Pi_X = E[X].$$

where ϕ is some function. In this chapter we start by describing some desirable properties of premium calculation principles. We then list some principles and consider which of the desirable properties they satisfy.

3.2 Properties of premium principles

There are many desirable properties for premium calculation principles. The following list is not exhaustive, but it does include most of the basic properties for premium principles.

(1) *Non-negative loading*. This property requires that $\Pi_X \geq E[X]$, that is that the premium should not be less than the expected claims. In Chapter 7 we will see the importance of this property in the context of ruin theory.

(2) *Additivity*. This property requires that if X_1 and X_2 are independent risks, then the premium for the combined risk, denoted $\Pi_{X_1+X_2}$, should equal $\Pi_{X_1} + \Pi_{X_2}$. If this property is satisfied, then there is no advantage, either to an individual or an insurer, in combining risks or splitting them, as the total premium does not alter under such courses of action.

(3) *Scale invariance*. This property requires that if $Z = aX$ where $a > 0$ then $\Pi_Z = a\Pi_X$. As an example of how this might apply, imagine that the currency of the United Kingdom changes from sterling to euros with one pound sterling being converted to a euros. Then, if a British insurer uses a scale invariant premium principle, a premium of £100 sterling would change to $100a$ euros.

(4) *Consistency*. This property requires that if $Y = X + c$ where $c > 0$, then we should have $\Pi_Y = \Pi_X + c$. Thus, if the distribution of Y is the distribution of X shifted by c units, then the premium for risk Y should be that for risk X increased by c.

(5) *No ripoff*. This property requires that if there is a (finite) maximum claim amount for the risk, say x_m, then we should have $\Pi_X \leq x_m$. If this condition is not satisfied, then there is no incentive for an individual to effect insurance.

3.3 Examples of premium principles

3.3.1 The pure premium principle

The pure premium principle sets

$$\Pi_X = E[X].$$

Thus, the pure premium is equal to the insurer's expected claims under the risk.

From an insurer's point of view, the pure premium principle is not a very attractive one. The premium covers the expected claims from the risk and contains no loading for profit or against an adverse claims experience. It is unlikely that an insurer who calculates premiums by this principle will remain in business very long. In the examples given below, the premium will exceed the pure premium, and the excess over the pure premium is referred to as the premium loading.

It is a straightforward exercise to show that the pure premium principle satisfies all five properties in Section 3.2.

http://www.cambridge.org/gb/academic/subjects/statistics-probability/statistics-econometrics-finance-and-insurance/insurance-risk-and-ruin

RIESGO: Prima y Provisión. Modelo Individual del riesgo

El pago esperado del total de todas las indemnizaciones como la suma de los valores esperados de cada contrato (variables aleatorias individuales)

$$X = x(1) + x(2) + x(3) + \ldots\ldots + x(283) + \ldots\ldots\ldots\ldots\ldots\ldots + x(N)$$

$$E[X] = E[x(1)] + E[x(2)] + E[x(3)] + \ldots\ldots + E[x(283)] + \ldots\ldots\ldots\ldots\ldots\ldots + E[x(N)]$$

Contratos Hipótesis Independencia	Suma Asegurada Individual	Suma Asegurada Cartera	Binomial Prob	Prima E(x)	V(x)	Desv(x)	Coef. Var
Nº	x		p(x)	(x)*Prob	n*p(x)*(1-p(x))		
1	100.000	100.000	0,350%	350	34.877.500	5.906	1687,35%
100	100.000	10.000.000	0,35%	35.000	3.487.750.000	59.057	168,73%
200	100.000	20.000.000	0,35%	70.000	6.975.500.000	83.519	119,31%
500	100.000	50.000.000	0,35%	175.000	17.438.750.000	132.056	75,46%
1.000	100.000	100.000.000	0,35%	350.000	34.877.500.000	186.755	53,36%
2.000	100.000	200.000.000	0,35%	700.000	69.755.000.000	264.112	37,73%
3.000	100.000	300.000.000	0,35%	1.050.000	104.632.500.000	323.469	30,81%
5.000	100.000	500.000.000	0,35%	1.750.000	174.387.500.000	417.597	23,86%
10.000	100.000	1.000.000.000	0,35%	3.500.000	348.775.000.000	590.572	16,87%
20.000	100.000	2.000.000.000	0,35%	7.000.000	697.550.000.000	835.195	11,93%
50.000	100.000	5.000.000.000	0,35%	17.500.000	1.743.875.000.000	1.320.559	7,55%
100.000	100.000	10.000.000.000	0,35%	**35.000.000**	**3.487.750.000.000**	**1.867.552**	**5,34%**
500.000	100.000	50.000.000.000	0,35%	175.000.000	17.438.750.000.000	4.175.973	2,39%
1.000.000	100.000	100.000.000.000	0,35%	350.000.000	34.877.500.000.000	5.905.718	1,69%

RIESGO: Prima y Provisión. Diversificación

Esperanza matemática = pagos esperados = Provisión Técnica = Prima
Transferencia de riesgo = Ley Grandes Números = Diversificación.

Ley débil [editar]

La **ley débil de los grandes números** establece que si $X_1, X_2, X_3, \ldots$ es una sucesión infinita de variables aleatorias independientes que tienen el mismo valor esperado μ y varianza σ^2, entonces el promedio

$$\overline{X}_n = (X_1 + \cdots + X_n)/n$$

converge en probabilidad a μ. En otras palabras, para cualquier número positivo ε se tiene

$$\lim_{n\to\infty} \mathrm{P}\left(\left|\overline{X}_n - \mu\right| < \varepsilon\right) = 1.$$

como es la importancia de la estadística

Ley fuerte [editar]

La **ley fuerte de los grandes números** establece que si $X_1, X_2, X_3, \ldots$ es una sucesión infinita de variables aleatorias independientes e idénticamente distribuidas que cumplen $E(|X_i|) < \infty$ y tienen el valor esperado μ, entonces

$$\mathrm{P}\left(\lim_{n\to\infty} \overline{X}_n = \mu\right) = 1,$$

es decir, el promedio de las variables aleatorias converge a μ casi seguramente (en un conjunto de probabilidad 1).

Esta ley justifica la interpretación intuitiva de que el valor esperado de una variable aleatoria como el "promedio a largo plazo al hacer un muestreo repetitivo".

http://es.wikipedia.org/wiki/Ley_de_los_grandes_n%C3%BAmeros

RIESGO: Prima y Provisión. Modelo Colectivo del riesgo

"If you put **5 red, 4 yellow, and 3 green balls in an urn**, what's the probability of…..you coming home with me tonight?"

actuarialjokes.com

Hay situaciones donde es difícil aplicar un modelo individual:

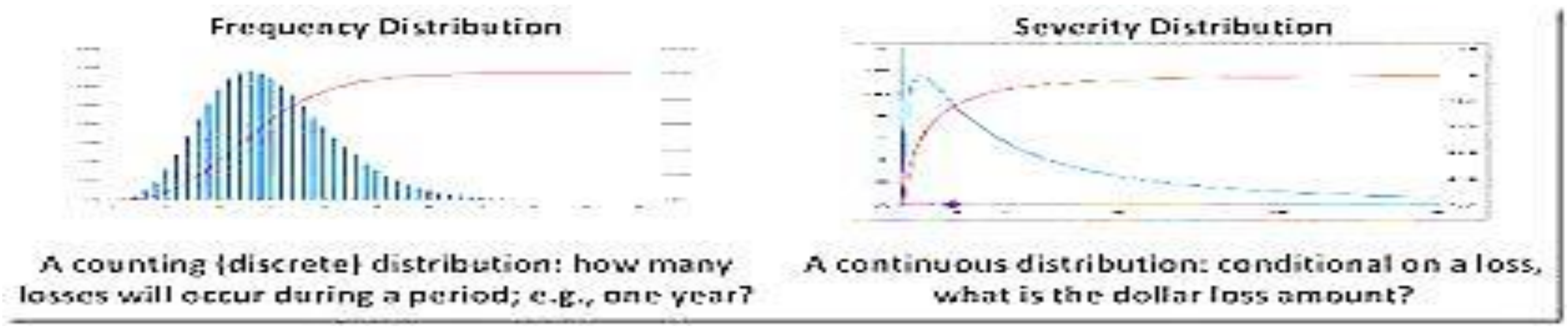
- Pueden existir varios siniestros en un mismo periodo de tiempo, accidentes de trafico, daños en el hogar, visitas al medico
- El importe de la perdida económica es aleatoria. Daños en un bien, importe de las facturas médicas o legales

Juego:

- Lanza dos moneda. Por cada cara lanza un dado. Resultado: (0,1,2,3,…..,12)
- Lanzo 10.000 monedas. Por cada cara lanza un dado. Resultado ¿?
- Lanzo 100.000 monedas. Por cada cara escojo una carta. Resultado ¿?
- Tengo 100.000 clientes. Cada uno puede tener una accidente con su coche. Cada colisión tiene un coste

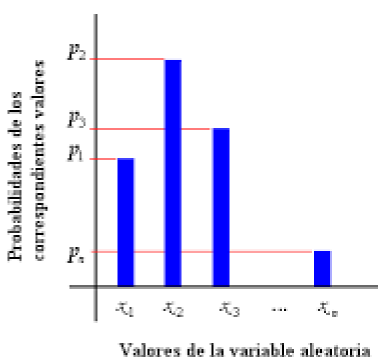
¿Cuál es la Función de Probabilidad?

El modelo colectivo. 2 componentes: Frecuencia y Severidad

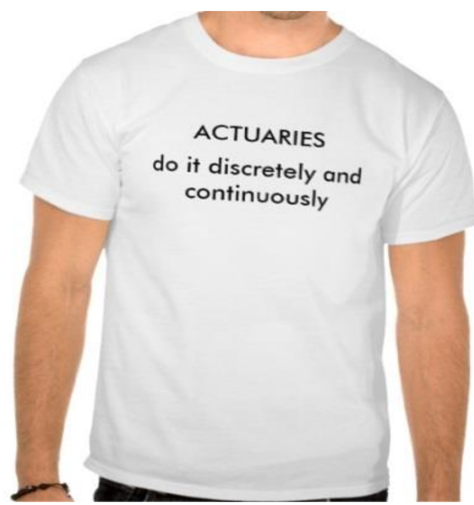


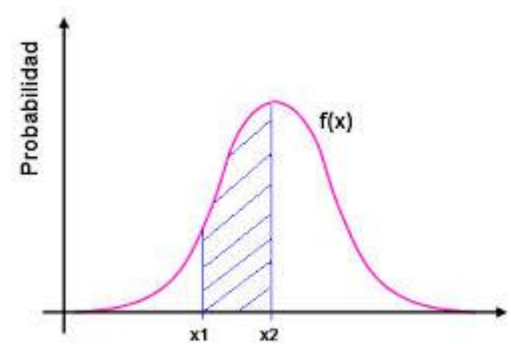
Numero de siniestros. Discreta

Perdida económico. Continua

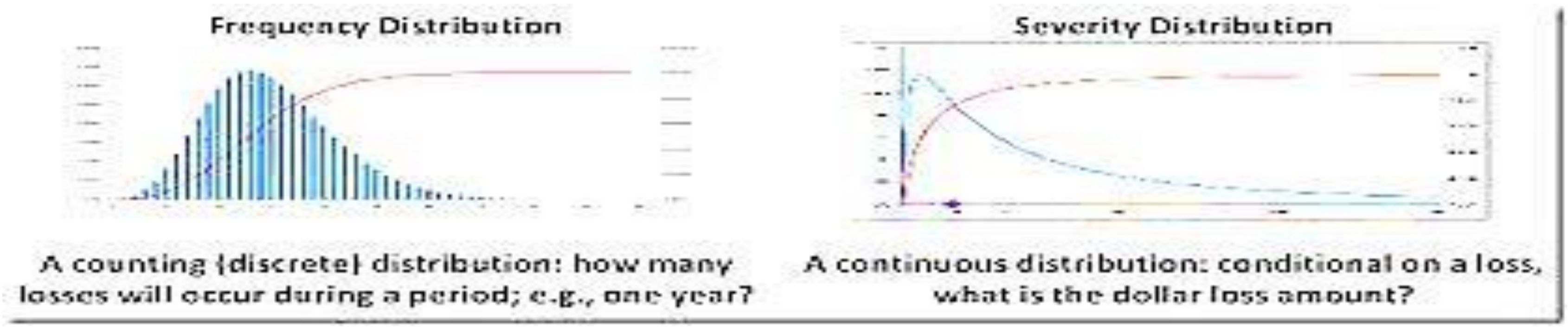


X	$p_i = p(X = x_i)$
x_1	p_1
x_2	p_2
$\vdots$	$\vdots$
x_n	p_n





El modelo colectivo identifica 2 factores: Frecuencia y Severidad



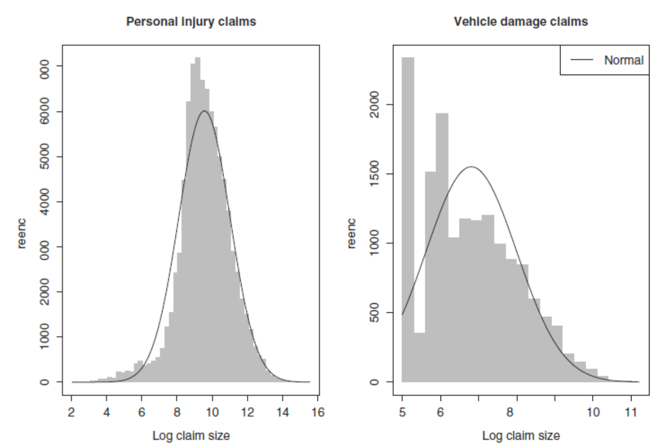
Binomial
Poisson
Binomial Negativa

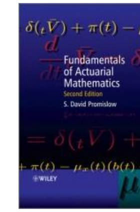
N: numero de Siniestros
$X(\mathrm{i})$= Cuantía de cada siniestro
$S = X1 + X2 + \cdot\cdot\cdot + XN$.
S =Distribución compuesta

Normal
Lognormal
Gamma
Pareto

Ventajas del modelo. Cambios de las condiciones:

- Obligación cinturón de velocidad: cambio en severidad, no en frecuencia
- Mejora iluminación carreteras: cambio en frecuencia , no en severidad





17.2 The mean and variance of S

As mentioned, although it may be difficult to calculate the distribution of S exactly, it is quite simple to find its mean and variance, given the corresponding quantities for N and X. Throughout this chapter we will let p denote the probability function of N. From the law of total probability (A.30),

$$E(S) = \sum_n E(S|N=n)p(n).$$

When $N = n$, S is just the sum of n independent copies of a random variable with the distribution of X. By using the fact that N and X are independent, we can write

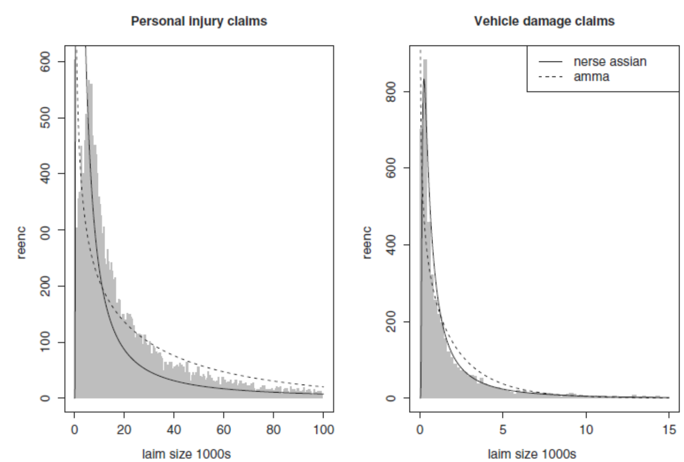
$$E(S|N=n) = nE(X|N=n) = nE(X),$$

leading to

$$E(S) = E(X)E(N),$$

an intuitively obvious result. Similarly,

$$E(S^2) = \sum_n E(S^2|N=n)p(n).$$



RIESGO: Capital

La esencia del seguro es la <u>transferencia</u> o desplazamiento del riesgo de unidades económicas relativamente pequeñas (las personas) a unidades muy grandes o al total de la sociedad

Segunda idea de Riesgo: la posibilidad de que "*algo malo*" pase. Algo malo es algo que supone una "*pérdida económica*"

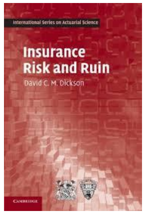
¿Para quién? : Las Compañías que aseguran

RIESGO: Capital

Segunda idea de Riesgo: la posibilidad de que “*algo malo*” pase. Algo malo es algo que supone una “*perdida económica*” <u>para la empresa aseguradora.</u>

Contratos Hipótesis Independencia	Suma Asegurada Individual	Suma Asegurada Cartera	Binomial Prob	Prima E(x)	V(x)	Desv(x)	Coef. Var
Nº	x		p(x)	(x)*Prob	n*p(x)*(1-p(x))		
1	100.000	100.000	0,350%	350	34.877.500	5.906	1687,35%
100	100.000	10.000.000	0,35%	35.000	3.487.750.000	59.057	168,73%
200	100.000	20.000.000	0,35%	70.000	6.975.500.000	83.519	119,31%
500	100.000	50.000.000	0,35%	175.000	17.438.750.000	132.056	75,46%
1.000	100.000	100.000.000	0,35%	350.000	34.877.500.000	186.755	53,36%
2.000	100.000	200.000.000	0,35%	700.000	69.755.000.000	264.112	37,73%
3.000	100.000	300.000.000	0,35%	1.050.000	104.632.500.000	323.469	30,81%
5.000	100.000	500.000.000	0,35%	1.750.000	174.387.500.000	417.597	23,86%
10.000	100.000	1.000.000.000	0,35%	3.500.000	348.775.000.000	590.572	16,87%
20.000	100.000	2.000.000.000	0,35%	7.000.000	697.550.000.000	835.195	11,93%
50.000	100.000	5.000.000.000	0,35%	17.500.000	1.743.875.000.000	1.320.559	7,55%
100.000	100.000	10.000.000.000	0,35%	**35.000.000**	**3.487.750.000.000**	**1.867.552**	**5,34%**
500.000	100.000	50.000.000.000	0,35%	175.000.000	17.438.750.000.000	4.175.973	2,39%
1.000.000	100.000	100.000.000.000	0,35%	350.000.000	34.877.500.000.000	5.905.718	1,69%

RIESGO: Recargos



3.3.2 The expected value principle

The expected value principle sets

$$\Pi_X = (1+\theta)E[X],$$

where $\theta > 0$ is referred to as the premium loading factor. The loading in the premium is thus $\theta E[X]$.

The expected value principle is a very simple one. However, its major deficiency is that it assigns the same premium to all risks with the same mean. Intuitively, risks with identical means but different variances should have different premiums.

The expected value principle satisfies the non-negative loading property since $(1+\theta)E[X] \geq E[X]$. (Strictly this requires that $E[X] \geq 0$, but this is invariably the case in practice.) Similarly, the principle is additive since

$$(1+\theta)E[X_1 + X_2] = (1+\theta)E[X_1] + (1+\theta)E[X_2],$$

and is scale invariant since for $Z = aX$,

$$\begin{aligned}\Pi_Z &= (1+\theta)E[Z] \\ &= a(1+\theta)E[X] \\ &= a\Pi_X.\end{aligned}$$

The expected value principle is not consistent, since for $Y = X + c$,

$$\Pi_Y = (1+\theta)(E[X]+c) > \Pi_X + c.$$

An alternative way of showing that a premium calculation principle does not satisfy a particular property is to construct a counter example. Thus, we can see that the no ripoff property is not satisfied by letting $\Pr(X = b) = 1$ where $b > 0$. Then as $\theta > 0$, $\Pi_X = (1+\theta)b > b$.

3.3.3 The variance principle

Motivated by the fact that the expected value principle takes account only of the expected claims, the variance principle sets

$$\Pi_X = E[X] + \alpha V[X],$$

where $\alpha > 0$. Thus, the loading in this premium is proportional to $V[X]$.

Since $\alpha > 0$, the variance principle clearly has a non-negative loading. The principle is additive since $V[X_1 + X_2] = V[X_1] + V[X_2]$ when X_1 and X_2 are independent, so that

$$\Pi_X = (1+\theta)E[X],$$

$$= \Pi_X + c.$$

However, the variance principle is not scale invariant since for $Z = aX$,

$$\begin{aligned}\Pi_Z &= E[Z] + \alpha V[Z] \\ &= aE[X] + \alpha a^2 V[X] \\ &\neq a\Pi_X,\end{aligned}$$

nor does it satisfy the no ripoff property. To see this, let

$$\Pr(X = 8) = \Pr(X = 12) = 0.5.$$

Then $E[X] = 10$ and $V[X] = 4$. Hence $\Pi_X = 10 + 4\alpha$ which exceeds 12 when $\alpha > 0.5$.

3.3.4 The standard deviation principle

The standard deviation principle sets

$$\Pi_X = E[X] + \alpha V[X]^{1/2},$$

where $\alpha > 0$. Thus, under this premium principle, the loading is proportional to the standard deviation of X. Although the motivation for the standard deviation principle is the same as for the variance principle, these two principles have different properties.

As in the case of the variance principle, as $\alpha > 0$ the standard deviation principle clearly has a non-negative loading. The principle is consistent since for $Y = X + c$,

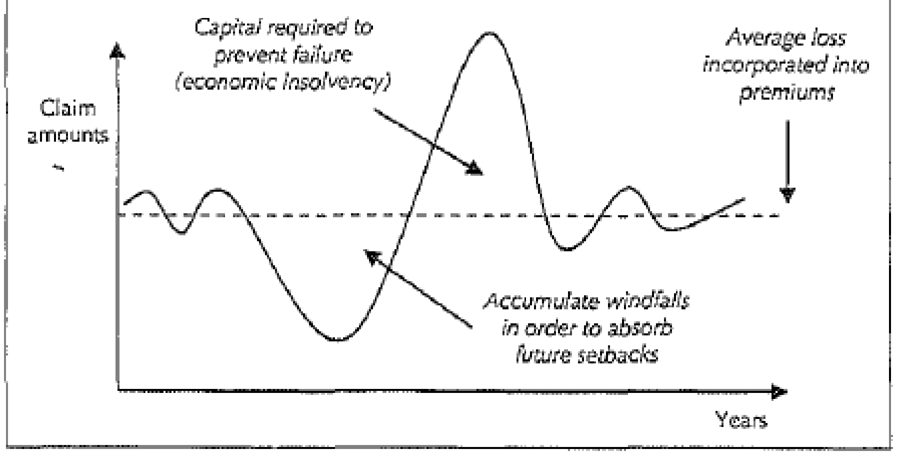
$$\begin{aligned}\Pi_Y &= E[Y] + \alpha V[Y]^{1/2} \\ &= E[X] + c + \alpha V[X]^{1/2} \\ &= \Pi_X + c,\end{aligned}$$

Pensemos que el negocio tiene un **<u>Ciclo de Explotación Inverso</u>**. Primero se cobra y luego se paga.

Las Primas (Provisiones) deben reflejar los <u>pagos esperados</u>. La siniestralidad es una variable aleatoria y los resultados están sujetos a variabilidad, sujetos a pagos no esperados.

Para hacer frente al impacto negativo que suponen la desviación en los pagos esperados, las entidades financieras, bancos y aseguradoras, deben disponer de un <u>excedente de fondos</u> que sea capaz de absorber las posibles pérdidas que se puedan producir.

Figure 2.7 Rationale to hold capital



¿Cuál debería ser el importe óptimo de esos fondos?.

A mayor cantidad de fondos, mayor seguridad y tranquilidad pero menor cantidad disponible para operar, con lo que se pierde en rentabilidad

RIESGO: Capital. Margen de Solvencia

Condiciones para el ejercicio de la actividad aseguradora

Sección 1.ª Garantías financieras

Artículo 16. *Provisiones técnicas.*

1. Las entidades aseguradoras tendrán la obligación de constituir y mantener en todo momento provisiones técnicas suficientes para el conjunto de sus actividades.

A estos efectos, deberán estar adecuadamente calculadas, contabilizadas e invertidas en activos aptos para su cobertura.

Son provisiones técnicas las de primas no consumidas, de riesgos en curso, de seguros de vida, de participación en los beneficios, de prestaciones, la reserva de estabilización y aquellas otras que, con arreglo al reglamento de desarrollo de esta Ley, sean necesarias al objeto de cumplir la finalidad a que se refiere el párrafo anterior.

2. La cuantía de dichas provisiones se determinará con arreglo a hipótesis prudentes y razonables.

Reglamentariamente se fijarán los métodos y procedimientos de cálculo de las provisiones técnicas, así como el importe de éstas que debe cubrir la entidad aseguradora.

3. Los activos representativos de las provisiones técnicas deberán tener en cuenta el tipo de operaciones efectuadas por la entidad aseguradora para garantizar la seguridad, el rendimiento y la liquidez de las inversiones de la entidad, con una adecuada distribución diversificada de dichas inversiones.

4. En el seguro de vida, la entidad aseguradora deberá tener a disposición de quienes estén interesados las bases y los métodos utilizados para el cálculo de las provisiones técnicas, incluida la provisión de participación de los asegurados en los beneficios.

5. Reglamentariamente se determinarán los activos aptos para la cobertura de las provisiones técnicas, los porcentajes máximos de éstas que puedan estar invertidos en cada tipo de estos activos, las demás condiciones que deban reunir dichas inversiones, así como los criterios de valoración de éstas y las normas y límites para el cumplimiento del principio de congruencia monetaria.

Artículo 17. *Margen de solvencia.*

1. Las entidades aseguradoras deberán disponer en todo momento de un margen de solvencia suficiente respecto al conjunto de sus actividades.

2. El margen de solvencia estará constituido por el patrimonio de la entidad aseguradora libre de todo compromiso previsible y con deducción de los elementos inmateriales.

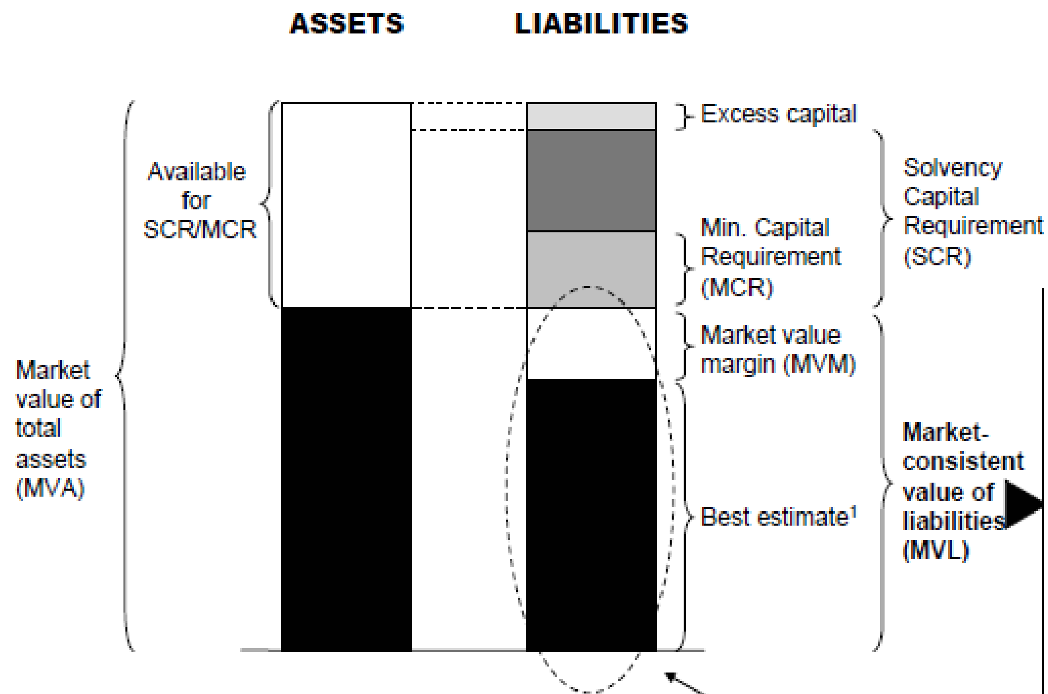
3. Los grupos consolidables de entidades aseguradoras deberán disponer en todo

RIESGO: Capital. Margen de Solvencia

El Capital refleja los **<u>pagos no esperados</u>** que aseguran el complimiento de las obligaciones para los clientes y la continuidad de la explotación a los accionistas.

¿Dónde tiene la empresa los fondos?. Balance

Overview of the Economic Balance Sheet



1. It should be noted that for prices directly inferred from market prices, the price already includes the risk margin (or market value margin)

RIESGO: Capital. Solvencia II

EIOPA
EUROPEAN INSURANCE
AND OCCUPATIONAL PENSIONS AUTHORITY

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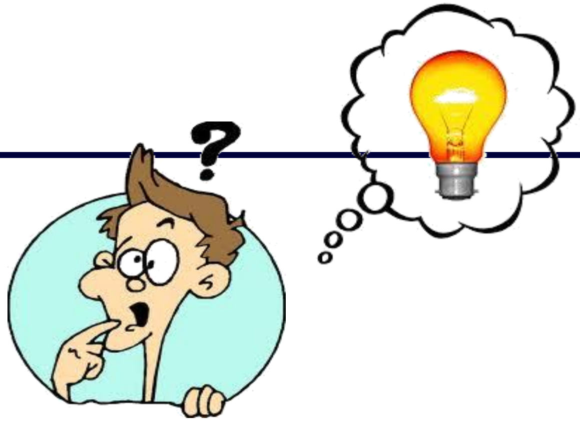
The objective of EIOPA is to protect the public interest. Among its tasks in the field of insurance is to contribute to the establishment of high-quality common regulatory and supervisory standards and practices in the European Union. EIOPA's powers include issuing guidelines and recommendations and developing draft regulatory and implementing technical standards. EIOPA is also entitled to provide opinions to the European Parliament, the Council of the European Union and the European Commission on insurance related issues.

One of the main aims of EIOPA for the coming years is the preparation of the new supervisory regime for insurance and reinsurance undertakings and particularly the conduct of all the necessary work for the implementation of the EU Directive on the taking-up and pursuit of the business of Insurance and Reinsurance (Solvency II).

EIOPA is also advising the European Commission on the revision of the Insurance Mediation Directive, and will support the development of industry training standards in this area. In this respect, EIOPA is also working on the development of common disclosure rules, and regulation of the sale of Packaged Retail Investment Products (PRIPs).

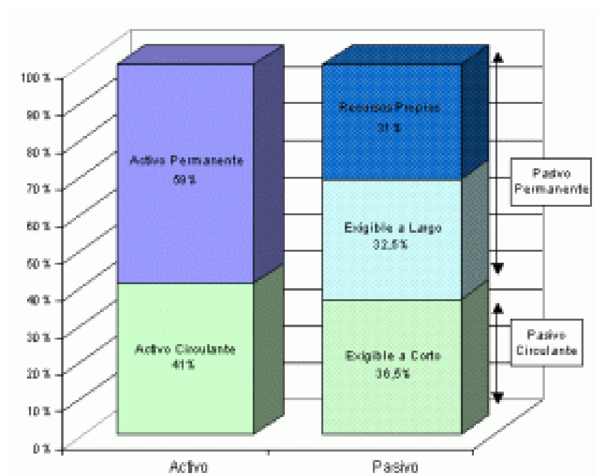
EIOPA also provides input into the European Commission's policy-making with regards to Insurance Guarantee Schemes (IGS) with a view to contributing to the assessment of the need for a European network of national Insurance Guarantee Schemes which is adequately funded and sufficiently harmonised.

RIESGO: Capital. Solvencia II

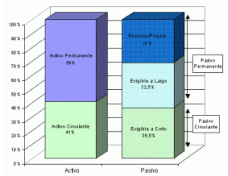


Fondos (Capital) como la desviación negativa del valor de la Compañía

Lo que tengo (Activo) – Lo que debo (Pasivo)

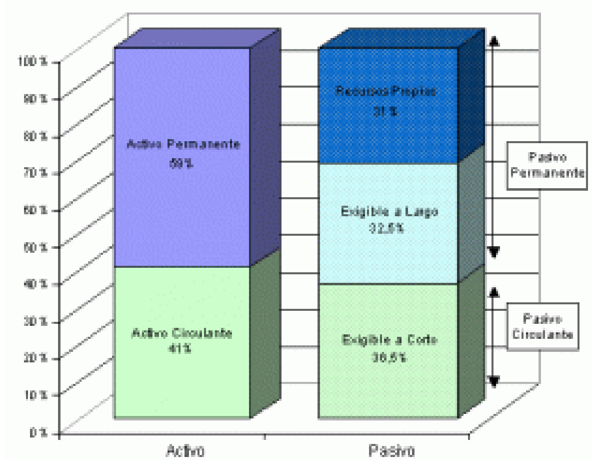


RIESGO: Capital. Solvencia II



$ in millions	2010	2011	2012	2013	2014
Cash and Investments	$1,567.1	$2,086.6	$2,696.4	$4,587.8	$5,664.9
Reinsurance Recoverable	775.4	1,098.6	1,318.4	1,929.8	2,440.6
Premiums Receivable, Net	727.6	933.0	1,251.3	1,594.0	1,826.2
Goodwill and Intangable Assets	204.1	392.5	532.8	665.4	667.7
Deferred Policy Acquisition Costs	224.7	281.0	349.1	468.4	628.4
Other Assets	706.8	970.8	1,288.4	2,033.8	2,619.6
Total Assets	$4,205.7	$5,762.4	$7,436.5	$11,279.1	$13,847.4
Loss and LAE Reserve	1,263.5	1,879.2	2,426.4	4,368.2	5,664.2
Unearned Premiums	1,025.0	1,366.2	1,773.6	2,681.0	3,447.2
Debt	144.8	279.6	302.0	560.2	757.9
Reinsurance Payables, Accrued Expenses and Other Liabilities	1,031.4	1,277.1	1,686.5	2,090.2	1,781.3
Total Liabilities	$3,464.7	$4,802.1	$6,188.5	$9,699.6	$11,650.6
Redeemable Non-Controlling Interest	0.6	0.6	0.6	0.6	0.6
AmTrust Financial Shareholders' Equity	716.5	890.6	1,144.1	1,441.0	2,037.0
Non-Controlling Interest	23.9	69.1	103.3	137.9	159.2
Total Shareholders' Equity	741.0	960.3	1,248.0	1,579.5	2,196.2

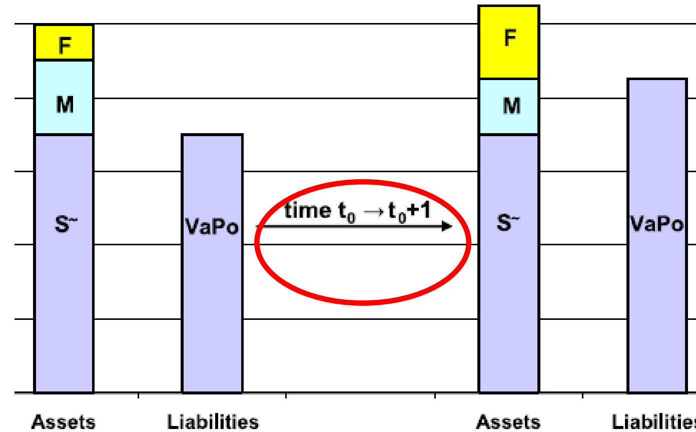
RIESGO: Capital. Solvencia II

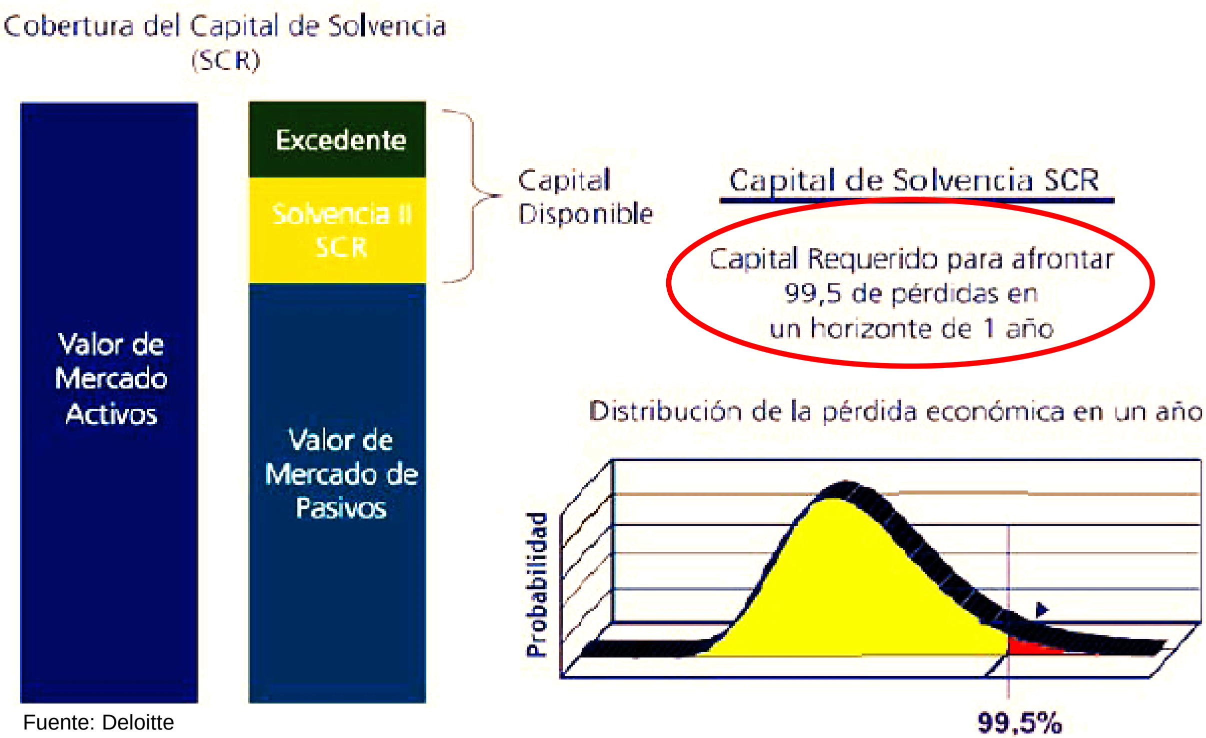


Fondos (Capital) como la desviación negativa del valor de la Compañía en un periodo de tiempo hasta un determinado nivel de confianza.

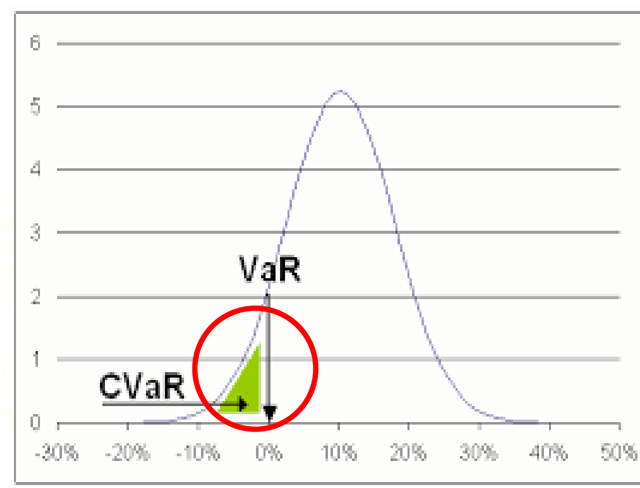
¿Cuál es la función de densidad de probabilidad del valor de la Compañía?

RIESGO: Capital. Solvencia II



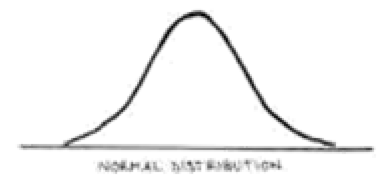


Value at Risk (VaR)

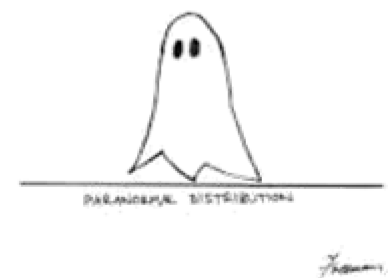


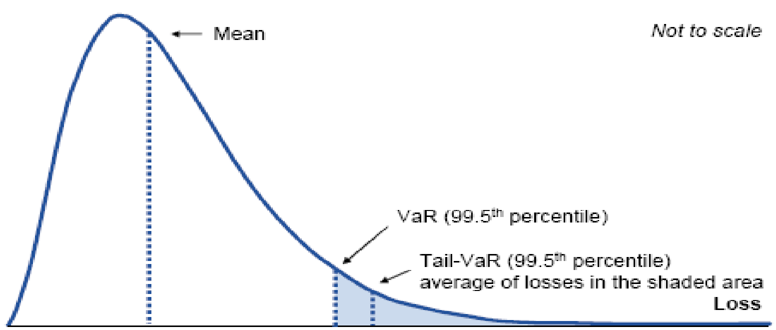
Fuente: Deloitte

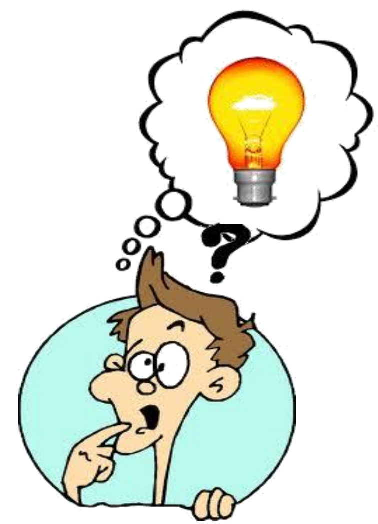
¿Cuál es la función de densidad de probabilidad del valor
1 de la Compañía?



Podemos descomponer las diferentes causas de incertidumbre en **factores de riesgo** de los cuales podemos estudiar sus funciones de distribución de probabilidad. Parece lógico pensar que si entendemos el cambio de todos los factores de riesgo entenderemos pues el cambio de movimiento del Balance Económico.

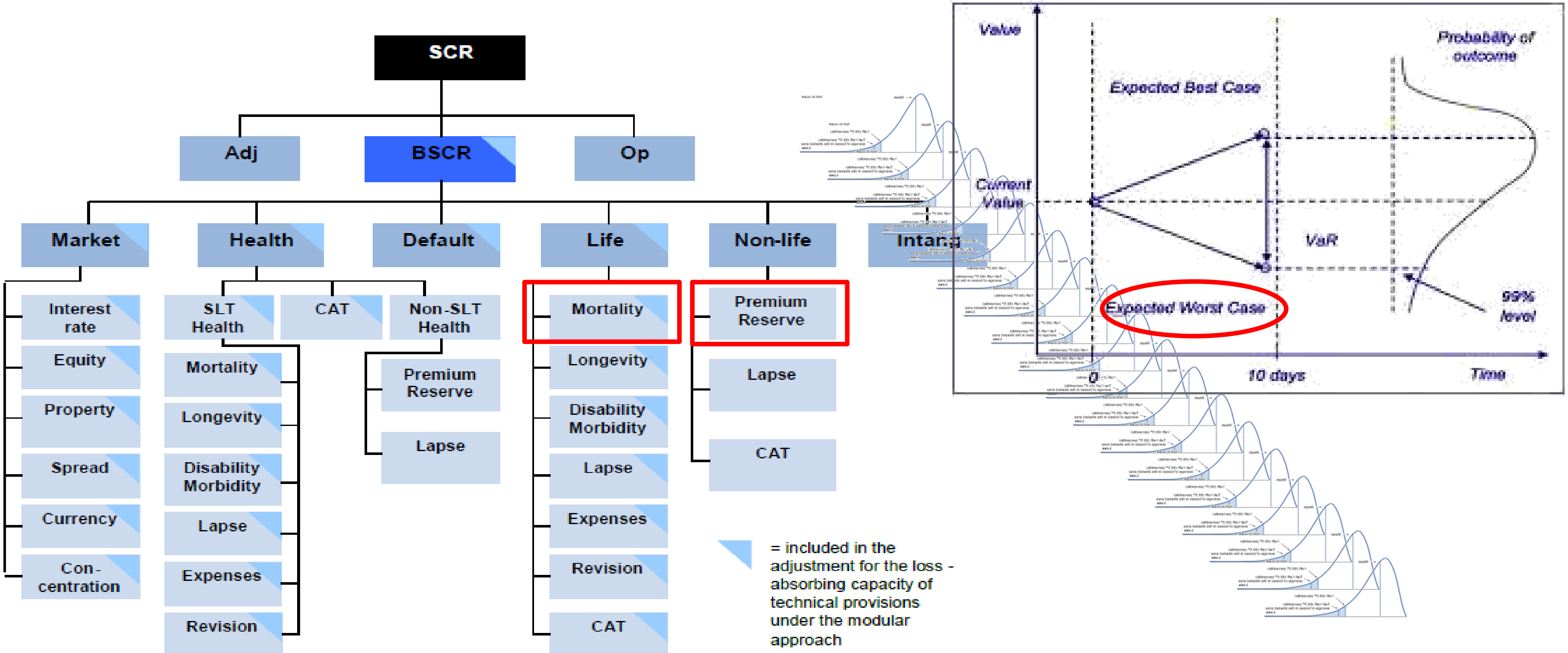






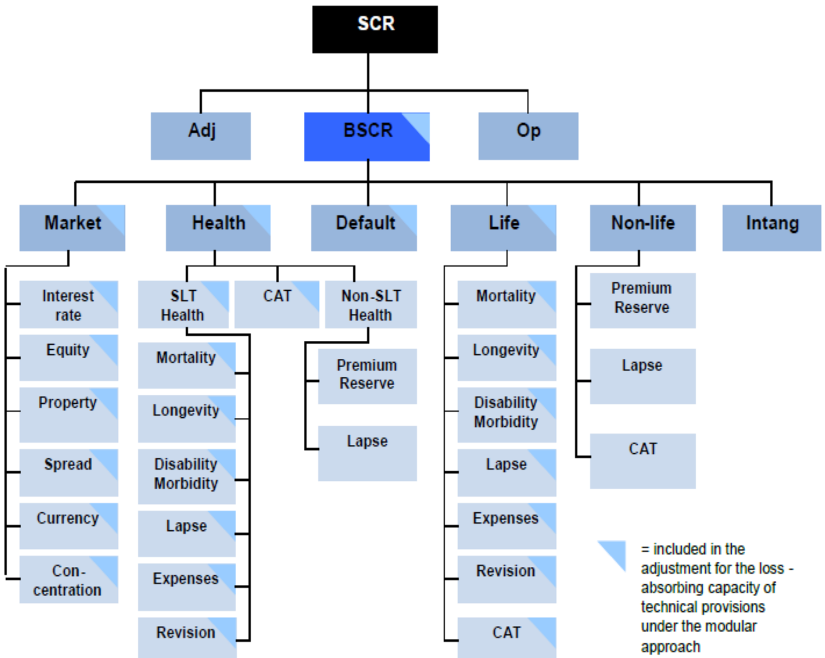
La clasificación de módulos y submódulos es la descomposición bajo Solvencia II de los diferentes factores de riesgo que afectan al negocio asegurador.

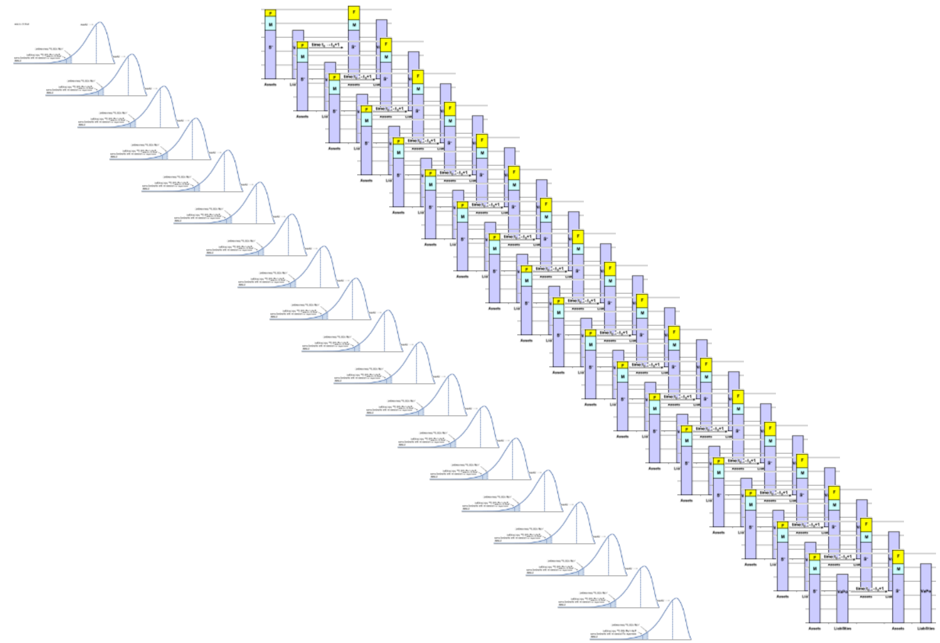
RIESGO: Capital. Solvencia II



Caracterizar la función de probabilidad de cada uno de los factores de riesgo de la cartera de la Entidad y "encontrar" el percentil.

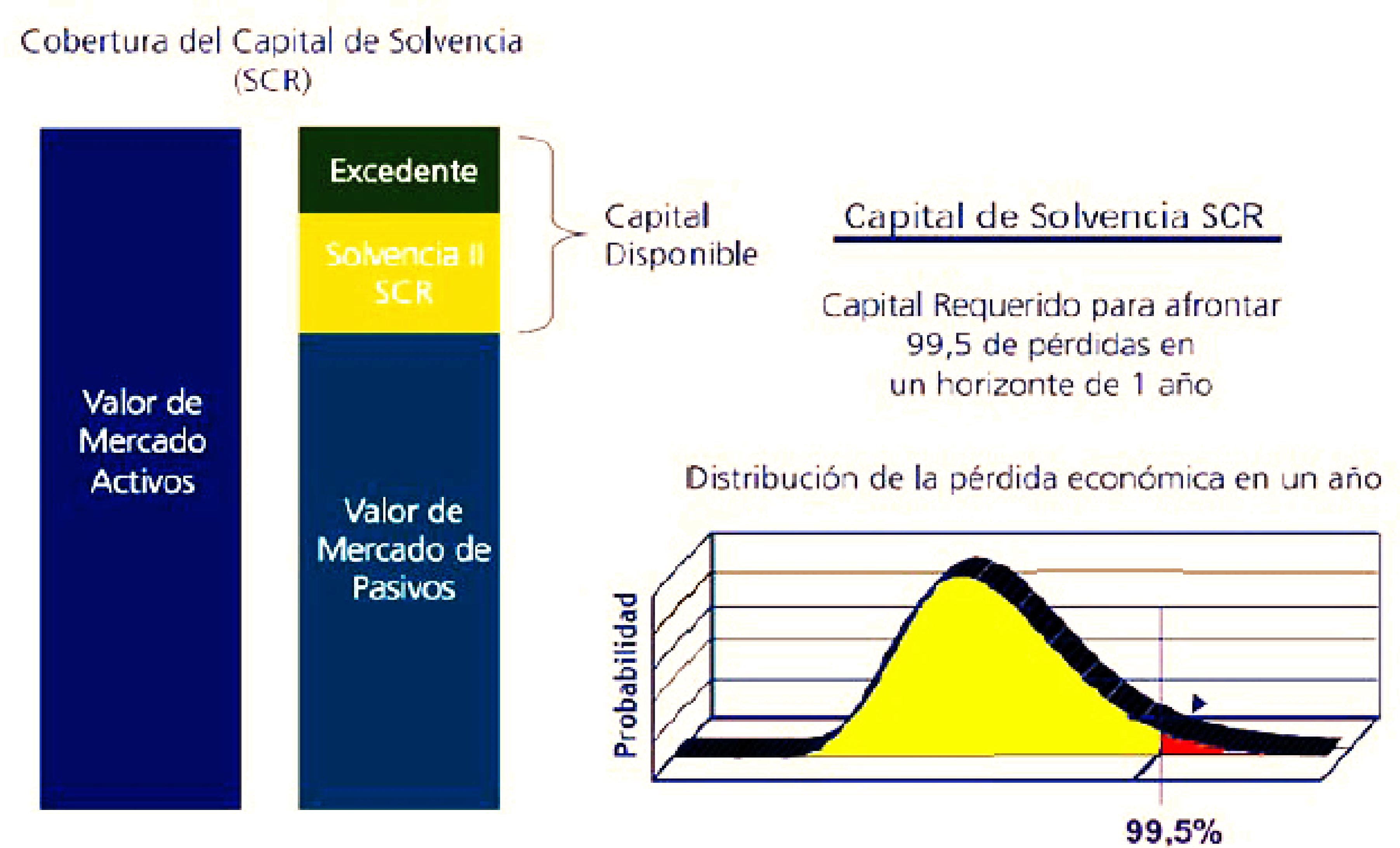
RIESGO: Capital. Solvencia II





Revaluar el Balance con el escenario “*Worst Case*” de cada uno de los factores de riesgo

RIESGO: Capital. Solvencia II



Fuente: Deloitte

RIESGO: Capital. Solvencia II

Correlación entre Riesgos

No todos lo malo ocurre a la vez (¡Gracias a Dios!)

La formula estándar tiene en cuenta una matriz de correlaciones para agregar los resultados

SCR.1.31. The BSCR is determined as follows:

$$BSCR = \sqrt{\sum_{ij} Corr_{ij} \times SCR_i \times SCR_j} + SCR_{intangible}$$

where

$Corr_{i,j}$ = the entries of the correlation matrix *Corr*

SCR_i, SCR_j = Capital requirements for the individual SCR risks according to the rows and columns of the correlation matrix *Corr*.

$SCR_{intangible}$ = the capital requirement for intangible asset risk calculated in accordance with SCR.4

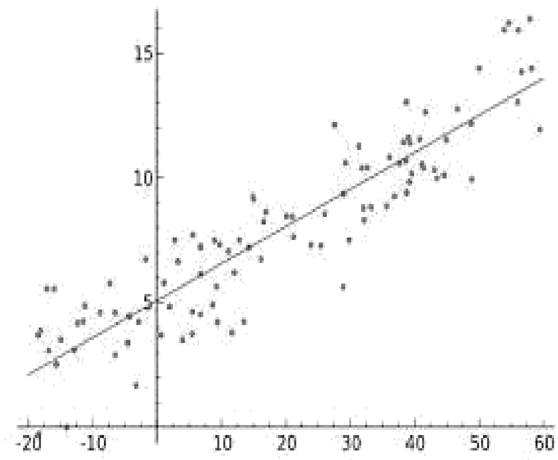
SCR.1.32. The factor $Corr_{i,j}$ denotes the item set out in row i and in column j of the following correlation matrix *Corr*:

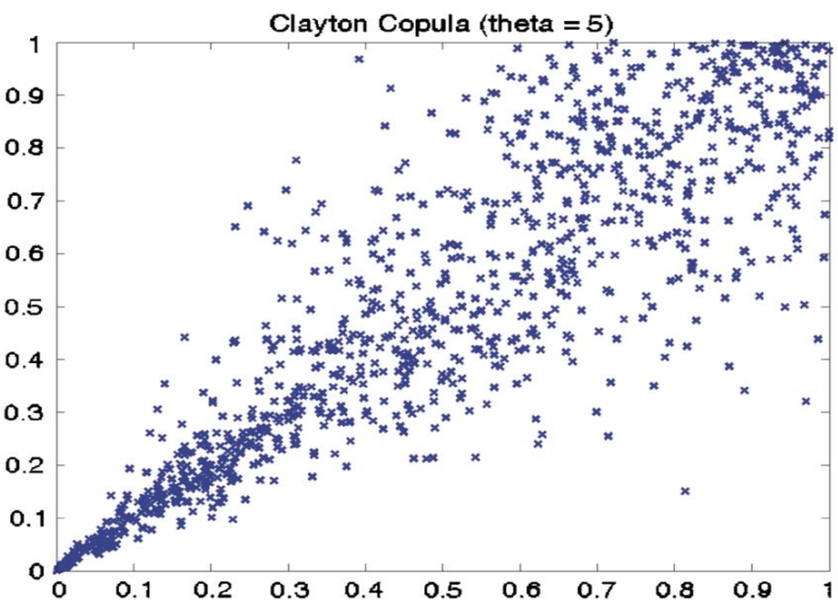
j / i	Market	Default	Life	Health	Non-life
Market	1				
Default	0.25	1			
Life	0.25	0.25	1		
Health	0.25	0.25	0.25	1	
Non-life	0.25	0.5	0	0	1

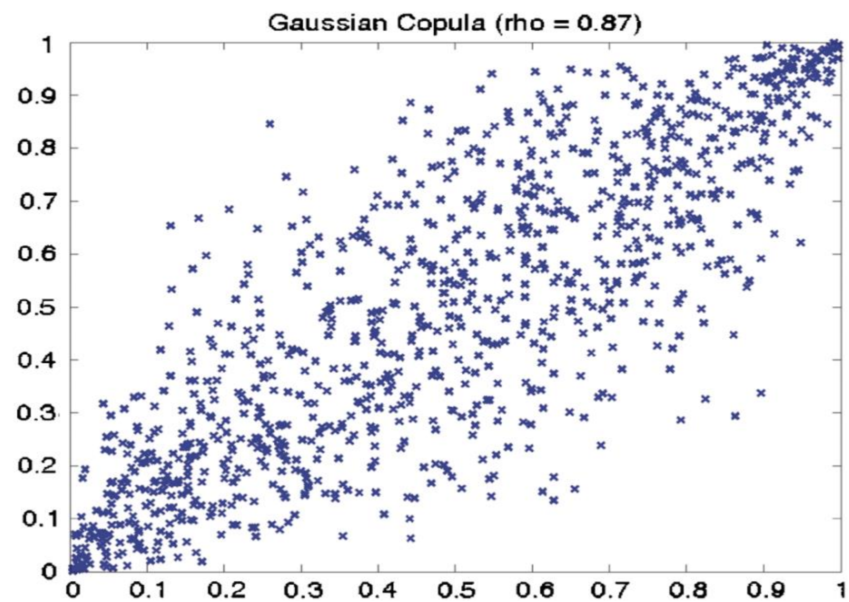
La agregación matricial implica una relación lineal entre los diferentes riesgos.

La vida real nos muestra que las relaciones no son lineales.

<u>Cuando algo va bien y cuando algo va mal no tiende a afectar al resto de variables de la misma manera.</u>

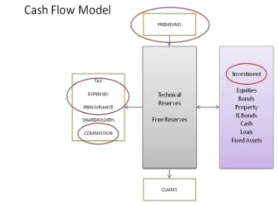






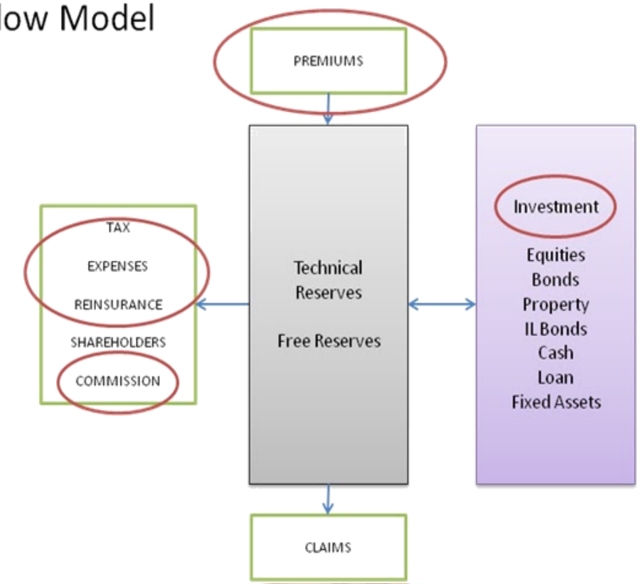
http://www.theactuary.com/archive/old-articles/part-2/copulas-for-insomniacs-3F/

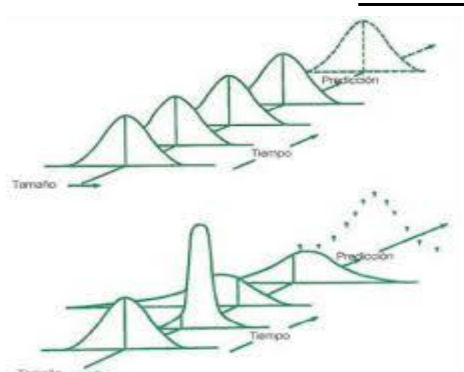
MODELOS

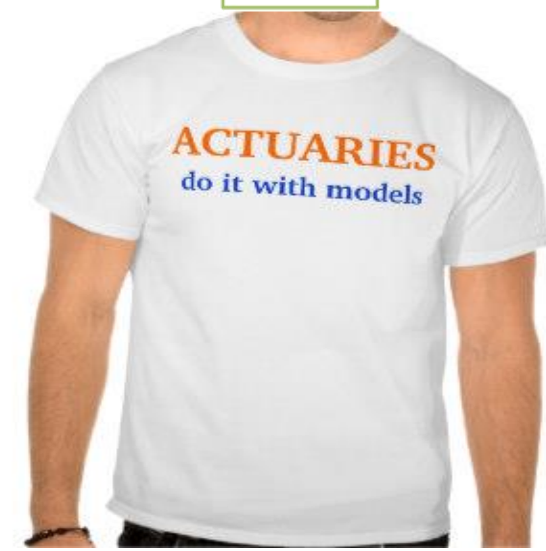


Cash Flow Model

$ in millions	2010	2011	2012	2013	2014
Cash and Investments	$1,567.1	$2,086.6	$2,696.4	$4,587.8	$5,664.9
Reinsurance Recoverable	775.4	1,098.6	1,318.4	1,929.8	2,440.6
Premiums Receivable, Net	727.6	933.0	1,251.3	1,594.0	1,826.2
Goodwill and Intangable Assets	204.1	392.5	532.8	665.4	667.7
Deferred Policy Acquisition Costs	224.7	281.0	349.1	468.4	628.4
Other Assets	706.8	970.8	1,288.4	2,033.8	2,619.6
Total Assets	$4,205.7	$5,762.4	$7,436.5	$11,279.1	$13,847.4
Loss and LAE Reserve	1,263.5	1,879.2	2,426.4	4,368.2	5,664.2
Unearned Premiums	1,025.0	1,366.2	1,773.6	2,681.0	3,447.2
Debt	144.8	279.6	302.0	560.2	757.9
Reinsurance Payables, Accrued Expenses and Other Liabilities	1,031.4	1,277.1	1,686.5	2,090.2	1,781.3
Total Liabilities	$3,464.7	$4,802.1	$6,188.5	$9,699.6	$11,650.6
Redeemable Non-Controlling Interest	0.6	0.6	0.6	0.6	0.6
AmTrust Financial Shareholders' Equity	716.5	890.6	1,144.1	1,441.0	2,037.0
Non-Controlling Interest	23.9	69.1	103.3	137.9	159.2
Total Shareholders' Equity	741.0	960.3	1,248.0	1,579.5	2,196.2

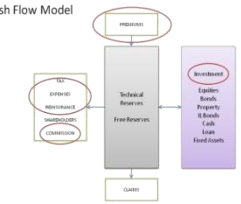






MODELOS

Cash Flow Model



Datos del asegurado

- Fecha nacimiento *: 01/04/1970
- Sexo * : (•) Hombre ( ) Mujer
- Altura * : 180 cms
- Peso * : 100 Kg
- Profesión *: Estadístico, Matemático, Actuario (Sin activid

Riesgos y garantías cubiertos

- Cobertura por fallecimiento*: 100000 €
- ¿Para que va a destinar su seguro de vida? * : (•) Cubrir a mi familia ( ) Cubrir préstamo

Año	Prima
2015	441,08 €
2016	488,90 €
2017	542,72 €
2018	601,40 €
2019	667,27 €
2020	739,69 €
2021	[illegible]

Datos del asegurado

- Fecha nacimiento *: 01/04/1970
- Sexo * : (•) Hombre ( ) Mujer
- Altura * : 180 cms
- Peso * : 70 Kg
- Profesión *: Trabajadores de la labra de piedras, mármole

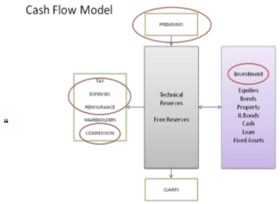
Riesgos y garantías cubiertos

- Cobertura por fallecimiento*: 100000 €

Año	Prima
2015	502,86 €
2016	541,12 €
2017	584,17 €
2018	631,12 €
2019	683,81 €
2020	741,75 €
2021	805,45 €

MODELOS

$$y \approx \beta_0 + \beta_1 x\,.$$



4.1 History and terminology of linear modeling

There is a smooth line of development from Gauss' original idea of simple least squares to present day generalized linear modeling. This line of thought and development is surveyed in the current chapter.

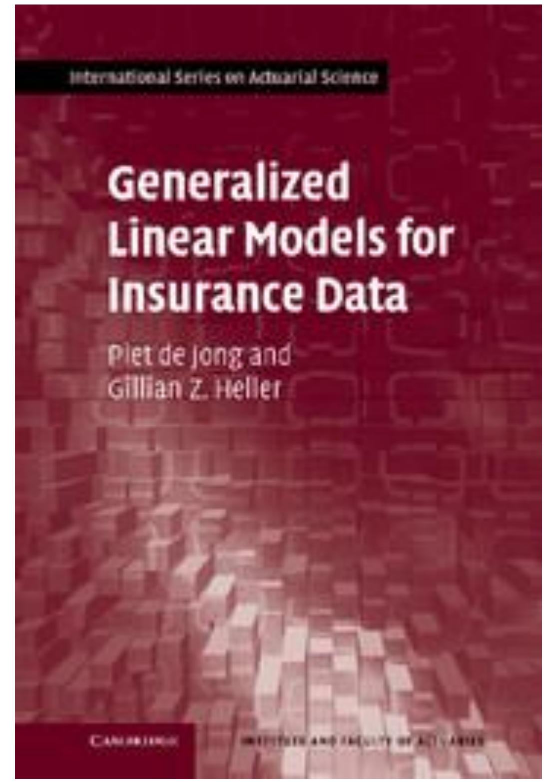
(i) **Simple linear modeling.** The aim is to explain an observed variable y by a single other observed variable x. The variable y is called the response variable and x the explanatory variable. Alternative terminology used in the literature for y are dependent, outcome, or (in econometrics) endogenous variable. Alternative names for x are covariate, independent, predictor, driver, risk factor, exogenous variable, regressor or simply the "x" variable. When x is categorical it is also called a factor.

(ii) **Multiple linear modeling.** Here simple least squares is extended by supposing that x contains more than one explanatory variable, the combination of which serve to explain the response y.

(iii) **Transforming the response.** A small extension is to replace y, the variable to be explained, by a transformation $g(y)$. In this case the aim is to explain observed values of the transformed response $g(y)$ by the observed explanatory variables in x. Typical transformations are the logarithm or logit. For obvious reasons g is constrained to be monotonic.

(iv) **Classical linear modeling.** A more subtle, conceptual, change is to replace the response y by its expected value $\mathrm{E}(y)$, or more specifically, $\mathrm{E}(y|x)$. In this case the statistical average of y is modeled in terms of x.

(v) **Generalized linear modeling.** Here $g\{\mathrm{E}(y|x)\}$ is explained in terms of x. Similar to above, g is a monotonic function and called the "link."



4.4 Multiple linear modeling

With multiple regression it is supposed that

$$y \approx \beta_0 + \beta_1 x_1 + \cdots + \beta_p x_p\,.$$

MODELOS

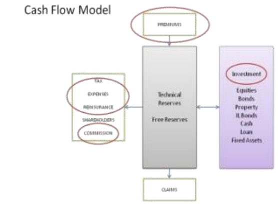


Table 1.1 *Major methods of statistical analysis for response and explanatory variables measured on various scales and chapter references for this book.*

Response (chapter)	Explanatory variables	Methods
Continuous (Chapter 6)	Binary	t-test
	Nominal, >2 categories	Analysis of variance
	Ordinal	Analysis of variance
	Continuous	Multiple regression
	Nominal & some continuous	Analysis of covariance
	Categorical & continuous	Multiple regression
Binary (Chapter 7)	Categorical	Contingency tables Logistic regression
	Continuous	Logistic, probit & other dose-response models
	Categorical & continuous	Logistic regression
Nominal with >2 categories (Chapter 8 & 9)	Nominal	Contingency tables
	Categorical & continuous	Nominal logistic regression
Ordinal (Chapter 8)	Categorical & continuous	Ordinal logistic regression
Counts (Chapter 9)	Categorical	Log-linear models
	Categorical & continuous	Poisson regression
Failure times (Chapter 10)	Categorical & continuous	Survival analysis (parametric)
Correlated responses (Chapter 11)	Categorical & continuous	Generalized estimating equations Multilevel models

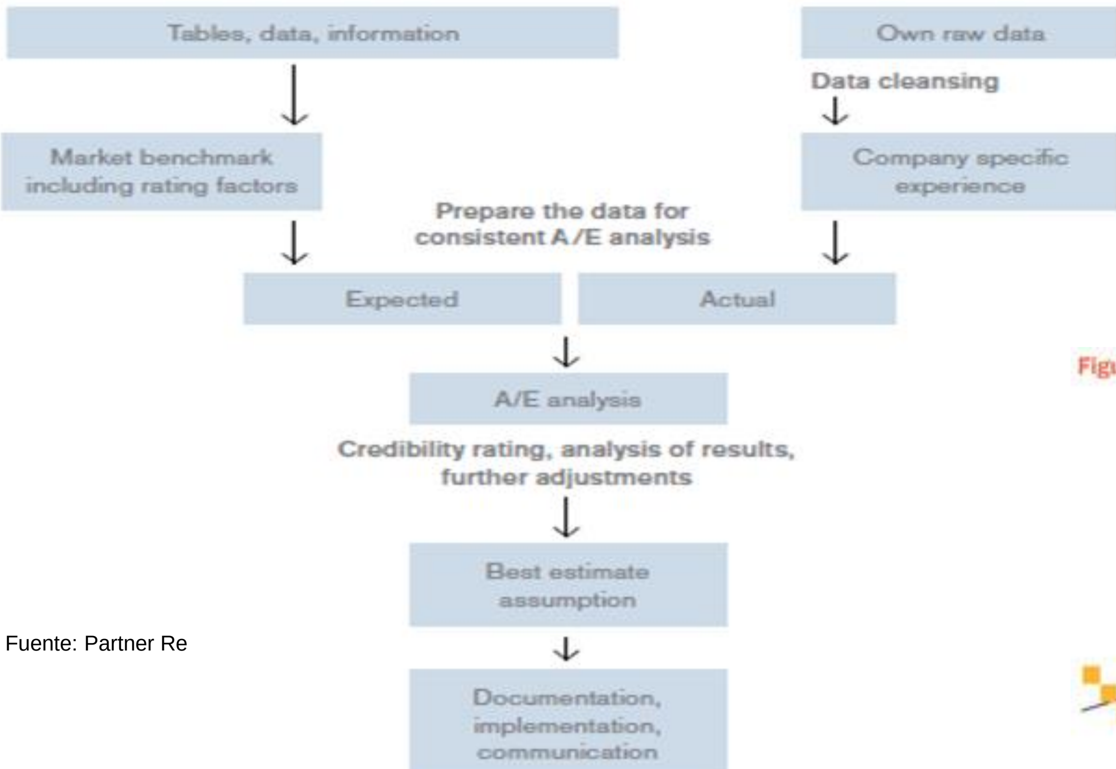
$\underline{Y}$	Claim frequencies	Claim numbers or counts	Average claim amounts	Probability (eg of renewing)
Link function $g(x)$	ln(x)	ln(x)	ln(x)	ln(x/(1-x))
Error	Poisson	Poisson	Gamma	Binomial
Scale parameter ϕ	1	1	Estimated	1
Variance function $V(x)$	x	x	x^2	x(1-x)*
Prior weights $\underline{\omega}$	Exposure	1	# of claims	1
Offset $\underline{\xi}$	0	ln(exposure)	0	0

MODELOS

How much is 2 plus 2?
-A **marketing** guy will say "22"
-An **accountant** will say "4.00"
-A **mathematician** will say "I can demonstrate it equals 4 with the following proof … "
-An **actuary** will ask "What do you want it to equal?"

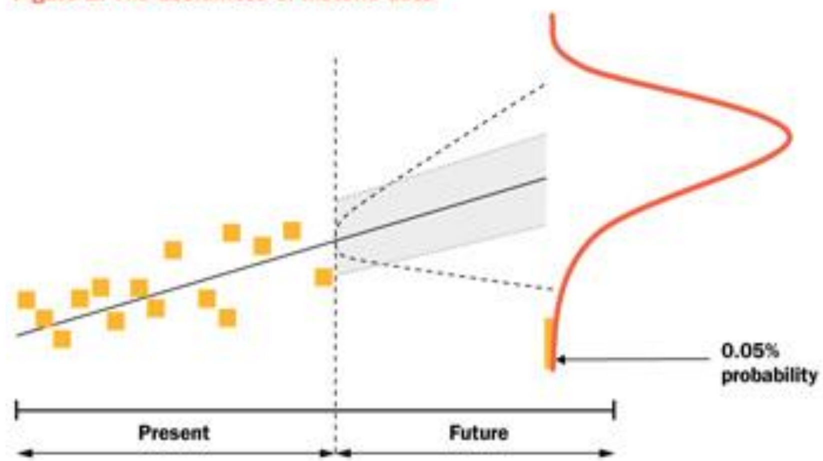
"I've never met an actuary whose rates didn't depend on the cost of his character
actuarialjokes.com

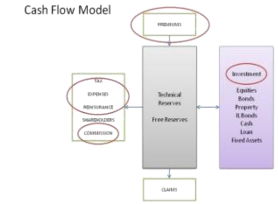
Figure 1 provides a top-level indication of the various steps involved in establishing a Best Estimate.
In chapter 1 we reviewed the various data sources, the chapters that follow look into each further stage of this diagram in more detail.



Fuente: Partner Re

Figure 1. The usefulness of historic data





1. Full Credibility Standard: The number of exposure units needed for full credibility, calculated according to limited fluctuation credibility theory. This output can only be calculated for count basis.

2. Credibility of Sample (Z): The credibility of the observed proportion, calculated as Z = √ (exposure units in study / full credibility standard) for count, and as Z= (standard normal value of sample/standard normal value of desired confidence level).

3. Credibility-Weighted Estimate: The credibility weighted best estimate of the rate, calculated as Z * Observed Rate + (1 – Z) * Initial Assumption

Full Credibility Standards

Determine the minimum number of claims which yields full credibility for a particular risk classification or cell. Denote this number as n_0. Assume normal approximation of claim count, that is, assume $\frac{N-\mu_N}{\sigma_N} \text{---}> N(0,1)$.

- ***n_0* on claim count basis (Frequency)**

  Let's assume significant level $p = 90\%$, then the $\frac{1+p}{2}$ percentile of Standard Normal[3] is: $y_p = \Phi^{-1}(\frac{1+p}{2}) = 1.645$. The goal here is to compute the full credibility number n_0 such that the chance of being within $\pm k$ of the mean is p. That is,

  $p = \text{Prob}\{-k\sqrt{n_0} \le \frac{N-\mu_N}{\sigma_N} \le k\sqrt{n_0}\}$. Let $\lambda_0 = \left(\frac{y_p}{k}\right)^2 = 1082.41$. By classical credibility model, we have full credibility (Z=1) if *claim* counts $\ge n_0$, *where*

  $$n_0 = \lambda_0 \frac{\sigma_N^2}{\mu_N} \text{ and } \lambda_0 = 1082.41$$

Credibility Model

This simple model calculates the credibility of any observed binomial event using limited fluctuation ("classical credibility") theory.
Examples of applicable events would be a mortality rate for life insurance, an incidence rate for accident and health insurance, or an annual lapse rate.

1. Enter initial assumption (pricing basis, rate manual basis — 0,0035
2. Enter the observed number of events: — 935
3. Enter the number of exposure units studied: — 300.000

Experience rate: — 0,0031

Therefore, partial credibility factor could be defined as:

$$Z = min\left\{\sqrt{\frac{n}{n_0}}, 1\right\}$$

Where n is experience claim counts and n_0 is full credibility standards defined above.

Confidence Level	1% Fluctuation			2% Fluctuation			5% Fluctuation			10% Fluctuation		
	Full Credibility Standard	Credibility of Sample (Z)	Credibility-Weighted Estimate	Full Credibility Standard	Credibility of Sample (Z)	Credibility-Weighted Estimate	Full Credibility Standard	Credibility of Sample (Z)	Credibility-Weighted Estimate	Full Credibility Standard	Credibility of Sample (Z)	Credibility-Weighted Estimate
80%	4.676.075	25%	0,003403	1.169.019	51%	0,003306	187.043	100%	0,003117	46.761	100%	0,003117
90%	7.703.069	20%	0,003424	1.925.767	39%	0,003349	308.123	99%	0,003122	77.031	100%	0,003117
95%	10.937.182	17%	0,003437	2.734.296	33%	0,003373	437.487	83%	0,003183	109.372	100%	0,003117
99%	18.890.498	13%	0,003452	4.722.625	25%	0,003403	755.620	63%	0,003258	188.905	100%	0,003117

MODELOS

Modeling the volatility

Mortality rates will vary around a deterministic best-estimate due to fluctuation risks, such as an unpredictable single hot summer or cold winter, trend risks, such as an unexpected new medical treatment, and pandemic risks, shock risks which may then disappear after a period of time. Fluctuation risks become insignificant when studying a large portfolio, pandemic risks can be easily built into models of deviation over time; it is trend risks that are the true drivers of volatility in the result.

The observed mortality in our model is described as the expected mortality (deterministic) multiplied by a stochastic process (C_t) that is time dependent. In simple terms:

$$\hat{q}_{x,t} = q_{x,t} \times C_t + \varepsilon_{x,t}$$

Where for age x at time t:

- $\hat{q}_{x,t}$ is the observed mortality
- $q_{x,t}$ is the expected mortality (best-estimate)
- C_t is a Log-Normal stochastic process centered around 1
- $\varepsilon_{x,t}$ is the random noise

Fuente: Partner Re

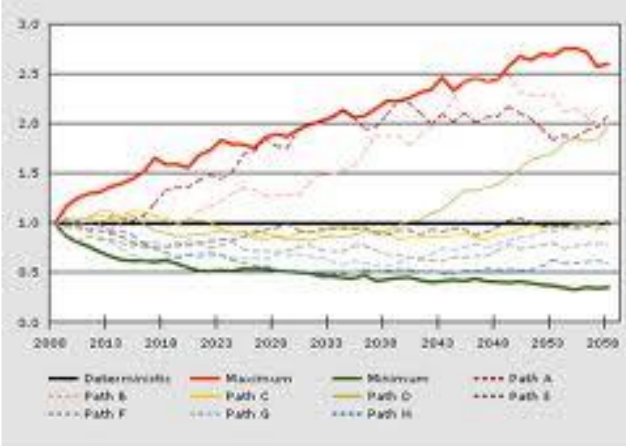
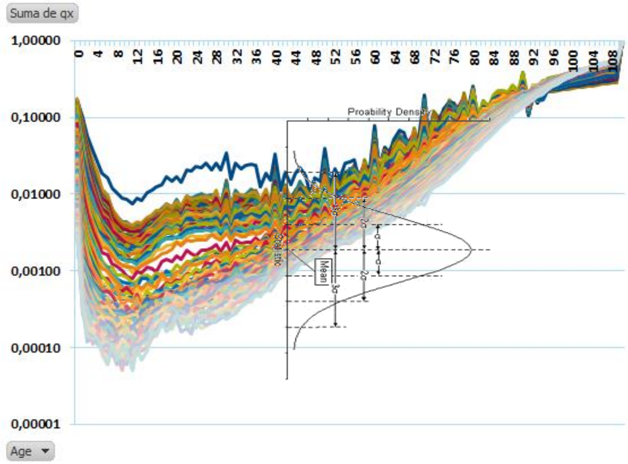


Figure 1 Plots over a 50-year time period of the cumulative deviation (Ct) between the deterministic mortality scenario (black line) and the stochastic projection. These curves provide a more robust and complete measure of volatility around the mortality best-estimate than single-point stress scenarios.

Modelo	Fórmula
Lee-Carter	$\ln m(t,x) = \beta_x^{(1)} + \beta_x^{(2)}\kappa_t^{(2)}$
Renshaw y Haberman	$\ln m(t,x) = \beta_x^{(1)} + \beta_x^{(2)}\kappa_t^{(2)} + \beta_x^{(3)}\gamma_{t-x}^{(3)}$
APC	$\ln m(t,x) = \beta_x^{(1)} + \kappa_t^{(2)} + \gamma_{t-x}^{(3)}$
B-splines y *P-splines*	$\ln m(t,x) = \sum_{i,j} \theta_{ij} B_{ij}^{ay}(x,t)$
CBD	$\log itq(t,x) = \beta_x^{(1)}\kappa_t^{(1)} + \beta_x^{(2)}\kappa_t^{(2)}$
CBD con efecto cohorte	$\log itq(t,x) = \beta_x^{(1)}\kappa_t^{(1)} + \beta_x^{(2)}\kappa_t^{(2)} + \beta_x^{(3)}\gamma_{t-x}^{(3)}$
CBD con efecto cohorte y componente cuadrático	$\log itq(t,x) = \kappa_t^{(1)} + (x-\bar{x})\kappa_t^{(2)} + \kappa_t^{(3)}\left[(x-\bar{x})^2 - \hat{\sigma}_x^2\right] + \gamma_{t-x}^{(4)}$
CBD con efecto cohorte decreciente con el tiempo	$\log itq(t,x) = \beta_x^{(1)}\kappa_t^{(1)} + \beta_x^{(2)}\kappa_t^{(2)} + \beta_x^{(3)}\gamma_{t-x}^{(3)}$

Tabla 1. Ecuaciones de los 8 modelos de mortalidad considerados. Las funciones β_x, κ_t, γ_{t-x} corresponden a efectos de edad, periodo y cohorte, respectivamente.



MODELOS

Table 2.2 Observed historical cumulative claims $C_{i,j}$ and estimated CL factors $\widehat{f}_j$

	0	1	2	3	4	5	6	7	8	9
0	5 946 975	9 668 212	10 563 929	10 771 690	10 978 394	11 040 518	11 106 331	11 121 181	11 132 310	11 148 124
1	6 346 756	9 593 162	10 316 383	10 468 180	10 536 004	10 572 608	10 625 360	10 636 546	10 648 192	
2	6 269 090	9 245 313	10 092 366	10 355 134	10 507 837	10 573 282	10 626 827	10 635 751		
3	5 863 015	8 546 239	9 268 771	9 459 424	9 592 399	9 680 740	9 724 068			
4	5 778 885	8 524 114	9 178 009	9 451 404	9 681 692	9 786 916				
5	6 184 793	9 013 132	9 585 897	9 830 796	9 935 753					
6	5 600 184	8 493 391	9 056 505	9 282 022						
7	5 288 066	7 728 169	8 256 211							
8	5 290 793	7 648 729								
9	5 675 568									
$\widehat{f}_j$	1.4925	1.0778	1.0229	1.0148	1.0070	1.0051	1.0011	1.0010		

"IBNR is like having a love child that you did not know about; sooner or later he is going to turn up on your doorstep"
actuarialjokes.com

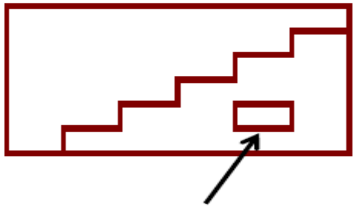
Table 2.3 Predicted cumulative CL claims $\widehat{C}_{i,j}^{CL}$ and CL reserves $\widehat{C}_{i,J}^{CL} - C_{i,I-i}$

	0	1	2	3	4	5	6	7	8	9	CL reserves
0											0
1										10 663 318	15 126
2									10 646 884	10 662 008	26 257
3								9 734 574	9 744 764	9 758 606	34 538
4							9 837 277	9 847 906	9 858 214	9 872 218	85 302
5						10 005 044	10 056 528	10 067 393	10 077 931	10 092 247	156 494
6					9 419 776	9 485 469	9 534 279	9 544 580	9 554 571	9 568 143	286 121
7				8 445 057	8 570 389	8 630 159	8 674 568	8 683 940	8 693 030	8 705 378	449 167
8			8 243 496	8 432 051	8 557 190	8 616 868	8 661 208	8 670 566	8 679 642	8 691 971	1 043 242
9		8 470 989	9 129 696	9 338 521	9 477 113	9 543 206	9 592 313	9 602 676	9 612 728	9 626 383	3 950 815
										Total	6 047 061

MODELOS

Premium Reserve

C_{ij} = Incremental claims in origin year i and development year j



C_{ij}

$$C_{ij} \sim ODP(\mu_{ij}, \phi_j)$$

$$E[C_{ij}] = \mu_{ij}$$

$$Var[C_{ij}] = \phi_j \mu_{ij}$$

Variance proportional to expected value

With constant scale parameters, $\phi_j = \phi \ \forall \ j$

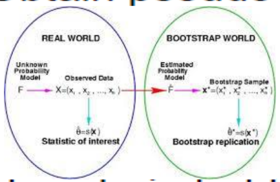
Stochastic Reserving in General Insurance

Peter England, PhD
EMB

GIRO 2002

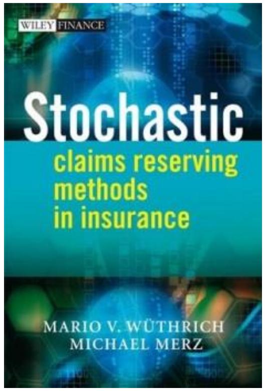
Bootstrapping the Chain Ladder
Over-dispersed Poisson model

1. Fit chain ladder model and obtain fitted incremental values
2. Obtain (scaled) Pearson residuals $\longrightarrow$ $r_{ij} = \dfrac{C_{ij} - \mu_{ij}}{\sqrt{\phi_j \mu_{ij}}}$
3. Resample residuals[1]
4. Obtain pseudo data, given $r_{ij}^*, \mu_{ij}, \phi_j$



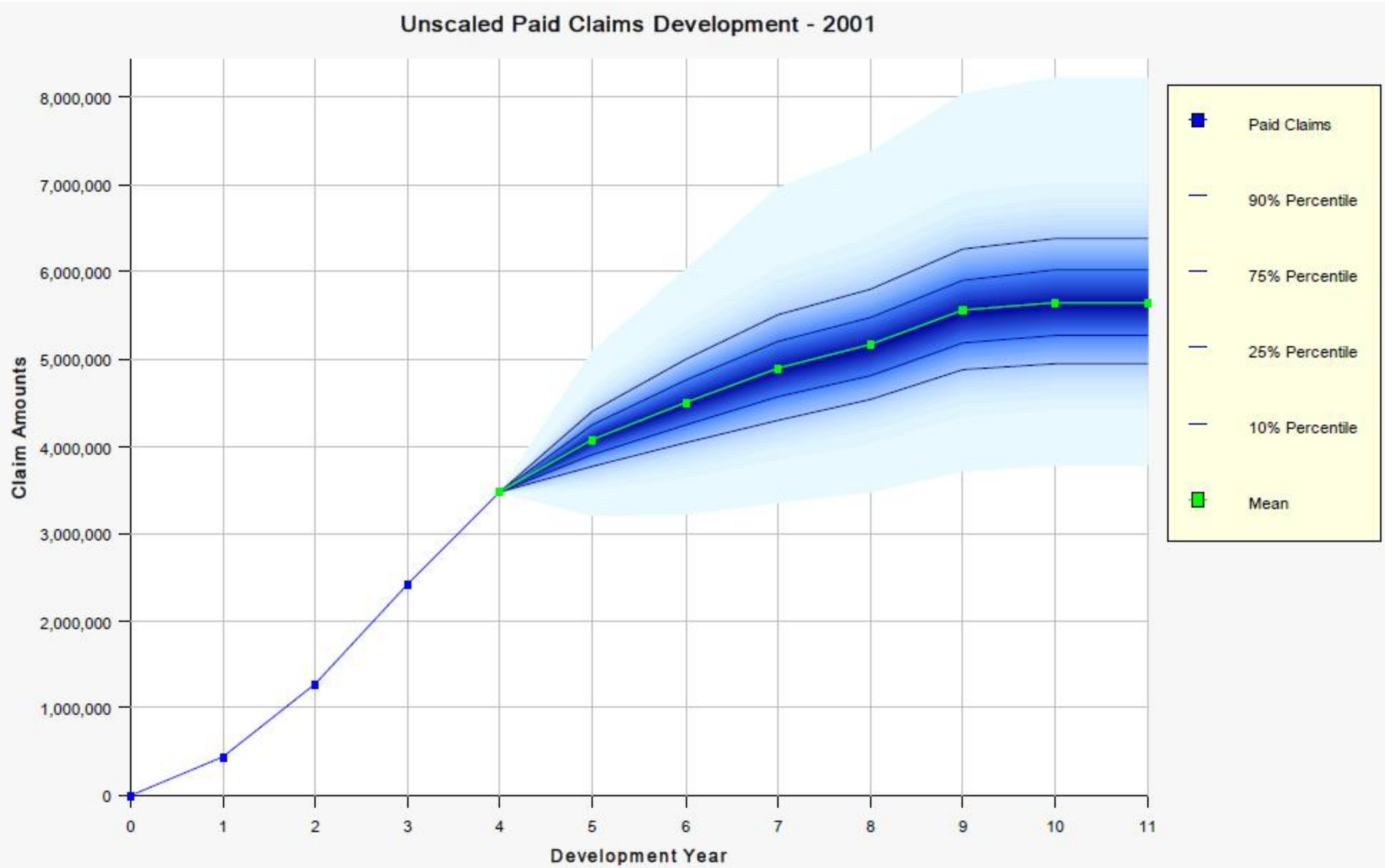
$$C_{ij}^* = r_{ij}^* \sqrt{\phi_j \mu_{ij}} + \mu_{ij}$$

5. Use chain ladder to re-fit model, and estimate future incremental payments



MODELOS


Premium
Reserve


Unscaled Paid Claims Development - 2001
Claim Amounts
8,000,000
7,000,000
6,000,000
5,000,000
4,000,000
3,000,000
2,000,000
1,000,000
0
0
1
2
3
4
5
6
7
8
9
10
11
Development Year
Paid Claims
90% Percentile
75% Percentile
25% Percentile
10% Percentile
Mean

MODELOS

The most common model for equity returns is geometric Brownian motion (GBM), which uses a stochastic process called a Wiener process, denoted by W_t. The Wiener process starts from zero,

has independent and normally distributed increments, and is continuous in time t as defined by increments of dW_t. The stochastic differential equation (SDE) for GBM is as follows:

$$\boxed{dS_t = \mu_t S_t dt + \sigma_t S_t dW_t} \qquad \text{(II.A-14)}$$

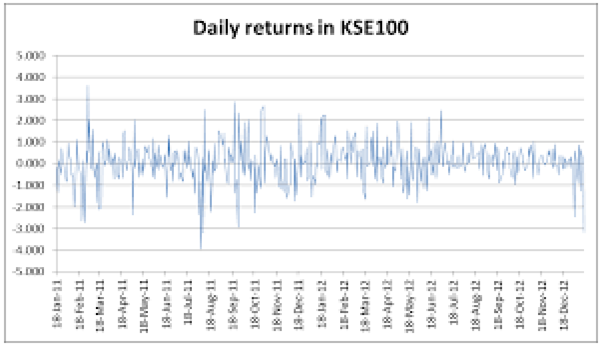
There are two components of this equation. The drift component is the coefficient of dt and the diffusion component is the coefficient of dW_t. Here the drift component μ_t is a deterministic function of t, and the diffusion component σ_t is a deterministic function of t. To remove excess noise from the simulation of S_t, the log transform is applied using Ito's lemma. After applying the log transform the equation is as follows:

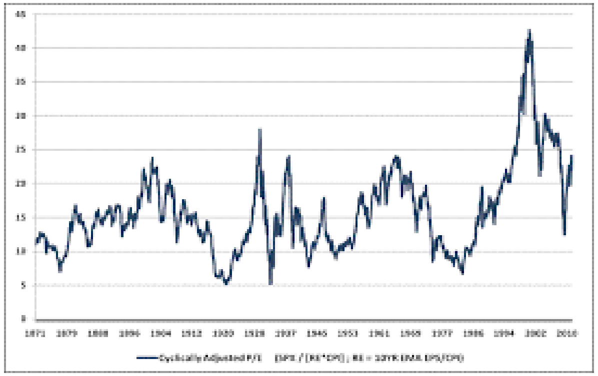
$$dS_t = S_t \exp\{(\mu_t - \sigma_t^2 / 2)dt + \sigma_t dW_t\} \qquad \text{(II.A-15)}$$

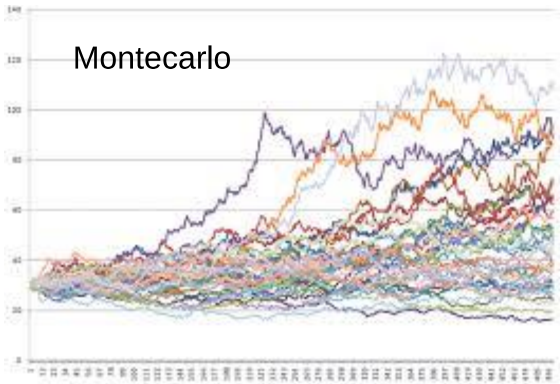
From Equation II.A-15 it is clear that the S_t term has been removed from the diffusion coefficient and reduces noise when simulating the process. To generate scenarios, the discrete-time approximation of Equation II.A-15 becomes

$$S_t = S_{t-1} \exp\{(\mu_t - \sigma_t^2 / 2)dt + \sigma_t \phi_t\} \qquad \text{(II.A-16)}$$

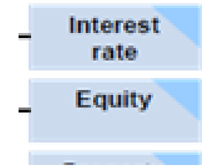
where ϕ_t is a normal random number drawn at time t, S_t is the equity level at time t, μ_t is the drift component at time t, and σ_t is the volatility at time t. Equation II.A-16 represents the most common simulation technique for generating equity scenarios. From this equation we can see that the equity level cannot drift below zero.







MODELOS



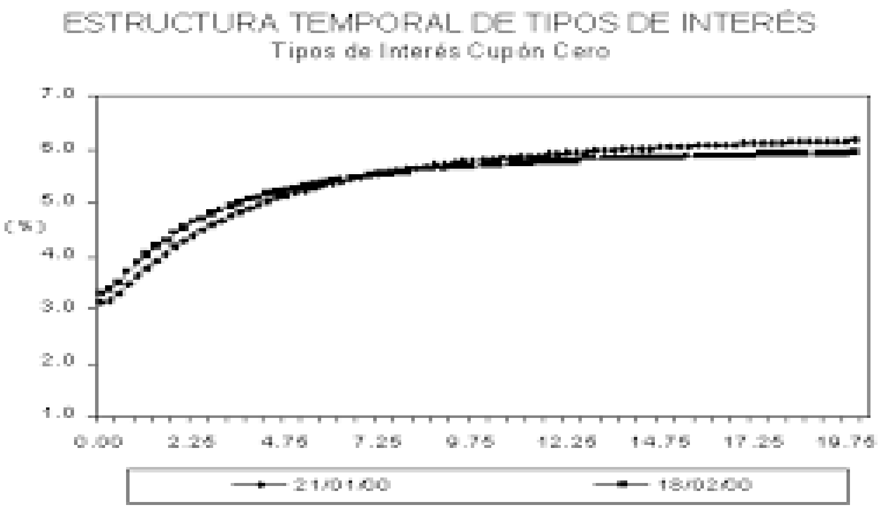
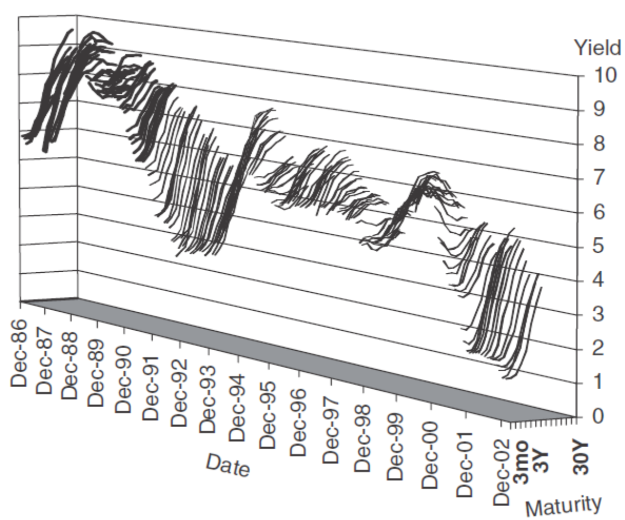


FIGURE 13-2 Movements in the U.S. Yield Curve



30.8.1 Vasicek

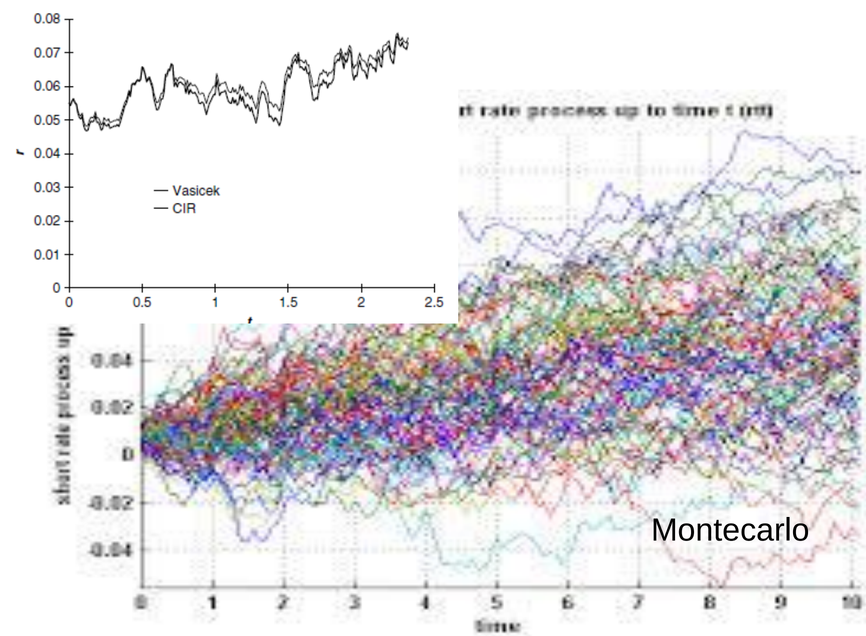
The Vasicek model takes the form of (30.6) and (30.7) but with $\alpha = 0$, $\beta > 0$ and with all other parameters independent of time:

$$dr = (\eta - \gamma r)\,dt + \beta^{1/2} dX.$$

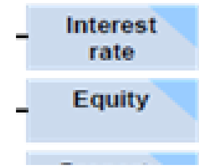
30.8.2 Cox, Ingersoll & Ross

The CIR model takes (30.6) and (30.7) as the interest rate model but with $\beta = 0$, and again no time dependence in the parameters:

$$dr = (\eta - \gamma r)\,dt + \sqrt{\alpha r}\,dX.$$

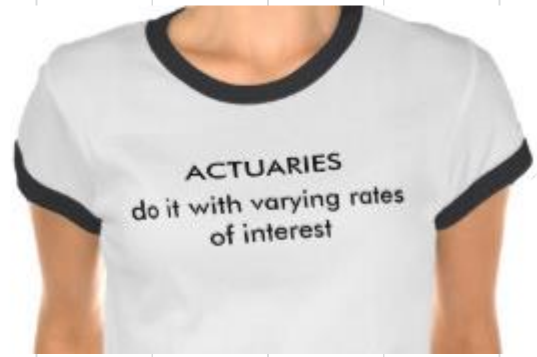


MODELOS



Interest Rate One-Factor Equilibrium Models

Source: Hull, John C., Options, Futures & Other Derivatives. Fourth edition (2000). Prentice-Hall. P. 567.
Models:
Vasicek, O. 1977 "An Equilibrium Characterization of the term structure." Journal of Financial Economics 5: 1
Cox, Ingersoll, and Ross. "A Theory of the Term Structure of Interest Rates". Econometrica, 53 (1985). 385-4



Vasicek Model (discrete version)

$$\Delta r = \alpha(b - r)\Delta t + \sigma\varepsilon\sqrt{\Delta t}$$

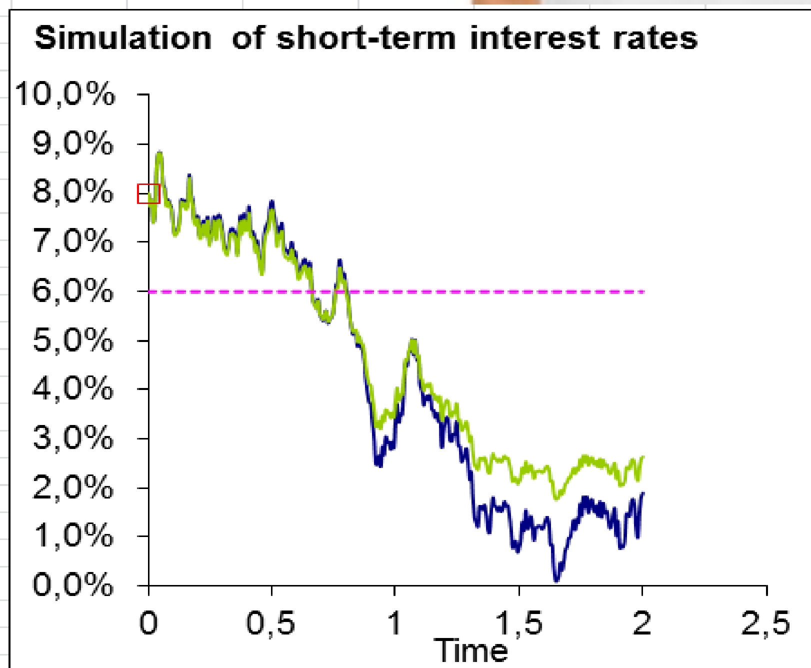
Stochastic process for short-term interest rate r:
α: "strength" at which r is pulled back to γ
b: long-term equilibrium of short-term rates
σ: volatility superimposed (annualized)

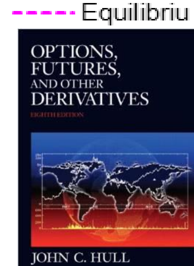
ε: is a random drawing from a standardized normal distribution,

Cox, Ingersoll, Ross Model (discrete version)

$$\Delta r = \alpha(b - r)\Delta t + \sqrt{r}\sigma\varepsilon\sqrt{\Delta t}$$

Parameters as for the Vasicek model.
Because the volatility is proportional to the square root of r, r cannot become negative. As the rates increase, their volatility increases. At the same time, the model has the same mean-reverting or "pull-back" properties as the Vasicek model.





Numerical examples (press F9 to generate new random numbers)

	Vasicek	CIR	
Rate r_0 at t=0	8,00%	8,00%	
Total simulation time (T)	2	2	year(s)
"Pullback" α	0,07	0,07	
Equilibrium b	6,00%	6,00%	
Volatility σ	3,00%	10,61%	
Δ t	0,0067		

AHORRO. ¿Seguros o Finanzas?

Artículo 6. Ramos de seguro.

2. El seguro directo *sobre la vida* se incluirá en un solo ramo, el ramo de vida, con el ámbito de todos los ramos del seguro directo sobre la vida enumerados en las directivas comunitarias reguladoras de la actividad del seguro directo sobre la vida.

A. Ámbito del ramo de vida.

El ramo de vida comprenderá:

a) El seguro sobre la vida, tanto para caso de **muerte** como de **supervivencia**, o ambos conjuntamente, incluido en el de supervivencia el seguro de renta; el seguro sobre la vida con contraseguro; el seguro de «nupcialidad», y el seguro de «natalidad». Asimismo, comprende cualquiera de estos seguros cuando estén vinculados con fondos de inversión. Igualmente, podrá comprender el seguro de dependencia.

b) Las operaciones de capitalización del artículo 3.1.b) de esta ley.

c) Las operaciones de gestión de fondos colectivos de jubilación y de gestión de operaciones tontinas.

6.4.2 The risk–savings decomposition

We now look at a useful decomposition of the policy into a risk portion and a savings portion. Multiply Equation (6.7) by $v(k, k+1)$ and rearrange to obtain

$$\pi_k = v(k, k+1) q_{x+k} \eta_k + [v(k, k+1)_{k+1}V - {}_kV].$$

ARTÍCULO 7. VALORES GARANTIZADOS
El Tomador del seguro tiene derecho, previa solicitud por escrito, a proceder al:

Rescate Total:

Consiste en percibir un capital con cargo a la provisión matemática constituida. El valor de rescate será igual a la provisión matemática acumulada en la fecha en la que se ejercite el derecho. Si se hubiera convenido algún tipo de penalización en las Condiciones Particulares, dicha penalización se aplicará sobre la provisión matemática resultante. El rescate total de la póliza implicará la extinción del contrato.

$$p_x = \text{probability of dying in year } x$$

$$(1 - p_x) = \text{probability of not dying in year } x$$

$$\prod_{x=1}^{n}(1 - p_x) = \text{probability of still being alive at end of year } n$$

$$\prod_{x=1}^{\infty}(1 - p_x) = \text{probability of living forever}$$

The solution to mortality:

$$\prod_{x=1}^{\infty}(1 - p_x) > 0$$

MINISTERIO DE ECONOMÍA Y COMPETITIVIDAD

2919 *Resolución de 9 de marzo de 2015, de la Dirección General de Seguros y Fondos de Pensiones, por la que se publica el tipo de interés máximo a utilizar en el cálculo de la provisión de seguros de vida, de aplicación al ejercicio 2015.*

El Reglamento de Ordenación y Supervisión de los Seguros Privados, aprobado por Real Decreto 2486/1998, de 20 de noviembre, regula, en su artículo 33.1, el tipo de interés aplicable para el cálculo de la provisión de seguros de vida. Este artículo ha sido modificado por el Real Decreto 128/2015, de 27 de febrero, por el que se modifica el Reglamento de Ordenación y Supervisión de los Seguros Privados en materia de tipo de interés aplicable para el cálculo de la provisión de seguros de vida. El mismo artículo también establece que la Dirección General de Seguros y Fondos de Pensiones publicará en su página web la resolución en la que se determine el tipo de interés resultante.

En su virtud, esta Dirección General hace público que el tipo de interés máximo aplicable para el cálculo de la provisión de seguros de vida durante el ejercicio 2015 es el 1,91 por 100.

Madrid, 9 de marzo de 2015.–La Directora General de Seguros y Fondos de Pensiones, M.ª Flavia Rodríguez-Ponga Salamanca.

AHORRO. ¿Seguros o Finanzas?

TANTOS

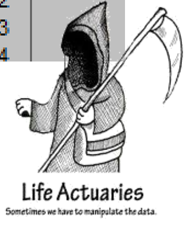
http://en.wikipedia.org/wiki/Actuarial_notation

Prepagable: A principio del periodo
Pospagable: A final del periodo

$v = (1+i)^{-1}$ $\quad {}_np_x = l_{x+n}/l_x$ $\quad {}_nq_x = {}_nd_x/l_x$ $\quad p_x = l_{x+1}/l_x$

TANTOS FINANCIEROS — **Interés Técnico** | TANTOS DE MORTALIDAD — **Tablas de Mortalidad**

Edad x	Tantos financieros para Pricing – Captialización – Pre 1/V^t	Tantos financieros para Pricing – Captialización – Pos 1/V^t	Tantos financieros para Pricing – Descuento – Pre V^t	Tantos financieros para Pricing – Descuento – Pos V^t	Tantos para analisis – Pre 1 / V^(n-t)	Tantos para analisis – Pos 1 / V^(n-t)	Tantos para analisis – 1 / V^1	Mortalidad para Pricing – tPx	Mortalidad para Pricing – /tQx	Mortalidad para Pricing – 1/Qx+t	Mortalidad para analisis – Inversa nPx+t	Mortalidad para analisis – 1Px	tl
								1	0,00000000				
45	1,000	1,019	**1**	**0,981258**	1,32816498	1,30327247	1,0191	**0,99730160**	**0,00269840**	**0,00269840**	1,08156425	0,99730160	0
46	1,019	1,039	**0,9813**	**0,962867**	1,30327247	1,2788465	1,0191	**0,99436814**	**0,00563186**	**0,00294140**	1,07864576	0,99705860	0
47	1,039	1,058	**0,9629**	**0,944821**	1,2788465	1,25487833	1,0191	**0,99116756**	**0,00883244**	**0,00321870**	1,07547303	0,99678130	0
48	1,058	1,079	**0,9448**	**0,927113**	1,25487833	1,23135936	1,0191	**0,98766498**	**0,01233502**	**0,00353380**	1,07201141	0,99646620	0
49	1,079	1,099	**0,9271**	**0,909737**	1,23135936	1,20828119	1,0191	**0,98384123**	**0,01615877**	**0,00387150**	1,06822313	0,99612850	0
50	1,099	1,120	**0,9097**	**0,892687**	1,20828119	1,18563555	1,0191	**0,97969178**	**0,02030822**	**0,00421760**	1,06408751	0,99578240	0
51	1,120	1,142	**0,8927**	**0,875956**	1,18563555	1,16341434	1,0191	**0,97520695**	**0,02479305**	**0,00457780**	1,05959961	0,99542220	0
52	1,142	1,163	**0,876**	**0,859539**	1,16341434	1,1416096	1,0191	**0,97037226**	**0,02962774**	**0,00495760**	1,05474898	0,99504240	0
53	1,163	1,186	**0,8595**	**0,843429**	1,1416096	1,12021352	1,0191	**0,96516855**	**0,03483145**	**0,00536260**	1,04951995	0,99463740	0
54	1,186	1,208	**0,8434**	**0,827622**	1,12021352	1,09921845	1,0191	**0,95957202**	**0,04042798**	**0,00579850**	1,04389180	0,99420150	0
55	1,208	1,231	**0,8276**	**0,812111**	1,09921845	1,07861686	1,0191	**0,95355473**	**0,04644527**	**0,00627080**	1,03783879	0,99372920	0
56	1,231	1,255	**0,8121**	**0,79689**	1,07861686	1,0584014	1,0191	**0,94708477**	**0,05291523**	**0,00678510**	1,03133071	0,99321490	0
57	1,255	1,279	**0,7969**	**0,781955**	1,0584014	1,03856481	1,0191	**0,94012654**	**0,05987346**	**0,00734700**	1,02433303	0,99265300	0
58	1,279	1,303	**0,782**	**0,767299**	1,03856481	1,0191	1,0191	**0,93264115**	**0,06735885**	**0,00796210**	1,01680725	0,99203790	0
59	1,303	1,328	**0,7673**	**0,752919**	1,0191	1	1,0191	**0,92458677**	**0,07541323**	**0,00863610**	1,00871133	0,99136390	0
60	1,328	1,354	**0,7529**	**0,738807**	0	0	1,0191	**0,91591933**	**0,08408067**	**0,00937440**	1,00000000	0,99062560	0
61	1,354	1,379	**0,7388**	**0,724961**	0	0	1,0191	**0,90659270**	**0,09340730**	**0,01018280**	0,99062560	0,98981720	0
62	1,379	1,406	**0,725**	**0,711373**	0	0	1,0191	**0,89655962**	**0,10344038**	**0,01106680**	0,98053826	0,98893320	0
63	1,406	1,433	**0,7114**	**0,698041**	0	0	1,0191	**0,88583865**	**0,11416135**	**0,01195790**	0,96968684	0,98804210	0
64	1,433	1,460	**0,698**	**0,684958**	0	0	1,0191	**0,87448388**	**0,12551612**	**0,01281810**	0,95809142	0,98718190	0



AHORRO. ¿Seguros o Finanzas?

DIFERIDO PURO (Ahorro) - Pago contingente a la Supervivenc http://en.wikipedia.org/wiki/Life_insurance

Life long

PRESTACIONES

Cupon Cero $\quad {}_nE_x = {}_np_x v^n.$

100.000 75.292 **69.614**

5.678

CONTRAPRESTACIONES

15,00 13,18 **12,83**

EQUILIBRIO FINANCIERO - ANALISIS

Prin.Equivalencia.Estáti

http://en.wikipedia.org/wiki/Force_of_mortality

PRIMA ~~5.711~~ → **5.424**

Puntos Básicos 69

TIPO MEDIO

85.667 **100.000** 81.361 **100.000**

1,91% **2,60%**

LONGEVIDAD

Puntos basicos: 35

97.445

2.555 **2,25%**

Pay Off	VA.Finan	VA Finan.Act		Prima	VA.Finan	VA Finan.Act	Imposición	Capitaliz.	Prima	Capitaliz.	Tipo Equivalente	Tipo Equivalente	Provisión Prima Unica	Provisión Pima Periodica	Mort.Real Capitaliz.	Tipo Equival
0	0	0	0	1	1,000	1,000	5.711	7.585	5.424	7.792	1,91%	2,44%	71.135	5.543	7.492	2,18%
0	0	0	0	1	0,981	0,979	5.711	7.443	5.424	7.625	1,91%	2,46%	72.708	11.209	7.341	2,19%
0	0	0	0	1	0,963	0,957	5.711	7.304	5.424	7.460	1,91%	2,48%	74.336	17.006	7.193	2,20%
0	0	0	0	1	0,945	0,936	5.711	7.167	5.424	7.297	1,91%	2,50%	76.024	22.939	7.047	2,21%
0	0	0	0	1	0,927	0,916	5.711	7.032	5.424	7.135	1,91%	2,52%	77.778	29.017	6.903	2,22%
0	0	0	0	1	0,910	0,895	5.711	6.901	5.424	6.974	1,91%	2,55%	79.599	35.248	6.760	2,23%
0	0	0	0	1	0,893	0,875	5.711	6.771	5.424	6.814	1,91%	2,57%	81.492	41.640	6.620	2,24%
0	0	0	0	1	0,876	0,854	5.711	6.644	5.424	6.656	1,91%	2,59%	83.463	48.202	6.481	2,25%
0	0	0	0	1	0,860	0,834	5.711	6.520	5.424	6.499	1,91%	2,62%	85.515	54.945	6.343	2,26%
0	0	0	0	1	0,843	0,814	5.711	6.398	5.424	6.343	1,91%	2,64%	87.657	61.880	6.208	2,27%
0	0	0	0	1	0,828	0,794	5.711	6.278	5.424	6.188	1,91%	2,67%	89.895	69.023	6.074	2,29%
0	0	0	0	1	0,812	0,774	5.711	6.160	5.424	6.034	1,91%	2,70%	92.238	76.387	5.941	2,30%
0	0	0	0	1	0,797	0,755	5.711	6.045	5.424	5.881	1,91%	2,73%	94.695	83.991	5.810	2,32%
0	0	0	0	1	0,782	0,735	5.711	5.931	5.424	5.728	1,91%	2,76%	97.278	91.854	5.680	2,33%
100.000	75.292	69.614	92.459	1	0,767	0,716	5.711	5.820	5.424	5.576	1,91%	2,80%	100.000	100.000	5.552	2,35%
0	0	0	0	0	0,000	0,000	0	0	0	0	0,00%	0,00%	0	0	0	0,00%
0	0	0	0	0	0,000	0,000	0	0	0	0	0,00%	0,00%	0	0	0	0,00%
	0	0	0	0	0,000	0,000	0	0	0	0	0,00%	0,00%	0	0	0	0,00%
	0	0	0	0	0,000	0,000	0	0	0	0	0,00%	0,00%	0	0	0	0,00%

n n-1 n-2 n-3

μ

AHORRO. ¿Seguros o Finanzas?

Ya lo ve, si quiere invertir en su futuro con un producto garantizado, **Generación F Único** es la solución:

100% de su inversión **garantizada**, al cumplirse el periodo de garantía

100% de la revalorización de su inversión, **sin medias ni porcentajes**

Está claro: invertir en bolsa es la mejor manera de rentabilizar su dinero. Sobre todo si lo hace con su capital 100% garantizado. Además, **Generación F Único** le ofrece unas expectativas de revalorización a las que ahora puede acceder sin fórmulas que limiten sus ganancias y sabiendo que, al final del periodo contratado, le garantizamos su capital.

Con **Generación F Único** no tendrá que preocuparse, porque al cumplirse el periodo de garantía, siempre recibirá el mayor de estos 2 valores:

- El valor de la inversión
- El 100% del capital invertido

Fuente: www.nnseguros.es/

Planning for retirement is challenging. Regardless of the encounters you may face, you may want to achieve some fundamental goals: **Protect** your Income, **Preserve** Your Legacy, **Participate** in the Market.

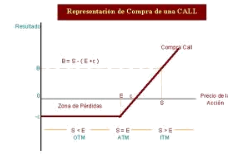
The Accumulator® variable annuity is a long-term retirement product that allows you the ability to invest for growth potential on a tax-deferred basis. In the most basic terms, annuities are contracts between you and an insurance company to accumulate funds and then to provide lifetime payments. There are contract limitations and fees and charges associated with Accumulator®, which include, but are not limited to, operations, distribution, withdrawal and administrative expense and charges for optional benefits.

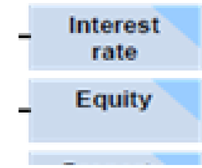
Accumulator® provides for guaranteed benefits through optional riders available for an additional fee; the Guaranteed Minimum Income Benefit (GMIB) can **protect** retirement income, and the Guaranteed Minimum Death Benefits (GMDB) which provide the ability to **preserve** the value of your death benefit for your legacy. A variety of equity portfolios allows you to **participate** in the market.

PROTECT

Protect your income, with predictable guaranteed payments, regardless of market turbulence.

Fuente: www. us.axa.com





EQUITY-LINKED INSURANCE AND OPTIONS

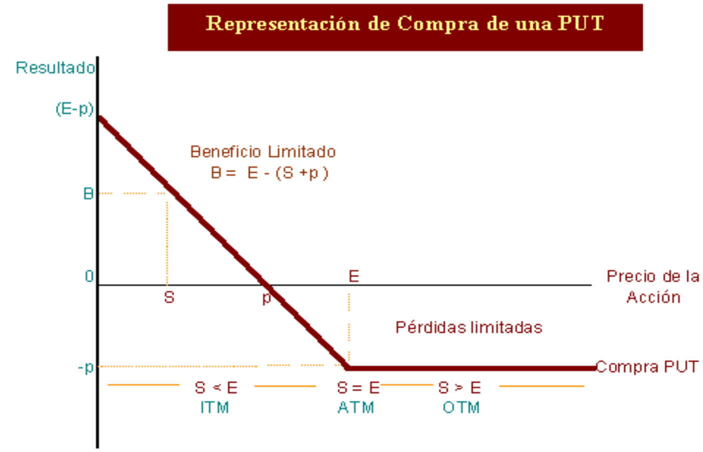
Call and Put Options

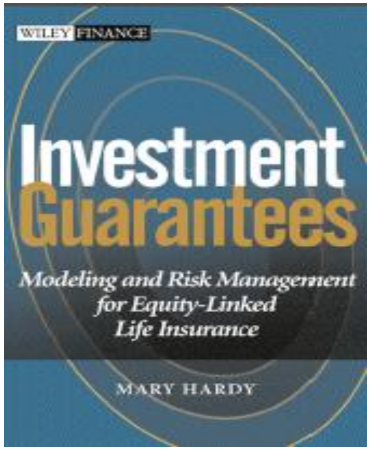
Although the risks associated with equity-linked insurance are new to insurers, at least, relative to life-contingent risks, they are very familiar to practitioners and academics in the field of derivative securities. The payoffs under equity-linked insurance contracts can be expressed in terms of *options*.

There are many books on the theory of option pricing and risk management. In this book we will review the relevant fundamental results, but the development of the theory is not covered. It is crucially important for practitioners in equity-linked insurance to understand the theory underpinning option pricing. The book by Boyle et al. (1998) is specifically written with actuaries and actuarial applications in mind. For a general, readable introduction to derivatives without any technical details, Boyle and Boyle (2001) is highly recommended.

The simplest forms of option contracts are:

- A *European call option* on a stock gives the purchaser the right (but not the obligation) to purchase a specified quantity of the underlying stock at a fixed price, called the *strike price*, at a predetermined date, known as the *expiry* or *maturity date* of the contract.
- A *European put option* on a stock gives the purchaser the right to sell a specified quantity of the underlying stock at a fixed strike price at the expiry date.





$$dS_t = \overbrace{S_t\,\mu\,dt}^{\text{drift}} + \underbrace{S_t\,\sigma\,\varepsilon\sqrt{\Delta t}}_{\text{uncertainty}}$$

$$S_t = S_0 \exp\left(\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W_t\right).$$

GRACIAS