ShApes, Geometry and Algebra Initial Training Network - FP7-PEOPLE-2007-1-1-ITN (2008-2012)

Ragni Piene

NILS Meeting 2010
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SAGA ShApes Geometry Algebra

MARIE CURIE ACTIONS

SEVENTH FRAMEWORK PROGRAMME

UNIVERSITETET I OSLO

CMA Centre of Mathematics for Applications

Background and history

Cathrine Tegnander (Master degree in algebraic geometry, PhD in PDEs) at SINTEF, working with Tor Dokken, wanted to integrate algebraic geometry and geometric modeling.

We contacted algebraic geometers in Nice, Dokken had contacts in INRIA and the company think3.

We applied for and obtained a FET assesment project *GAIA*, and then a full 3-year FET-Open project *GAIA II – Intersection algorithms for geometry based IT-applications using approximate algebraic methods*, now including also Johannes Kepler University in Austria and Universidad de Cantabria in Spain. This project ended in 2005. We applied for and obtained a new project:

ShApes, Geometry, and Algebra

starting in 2008 and lasting for four years.

SAGA goals

SAGA aims at advancing the mathematical foundations of CAD technology. This technology can be greatly enhanced by exploiting results and techniques from many different fields in mathematics: Algebraic Geometry, Symbolic Computation, Numerical Analysis and Approximation Theory.

The main goal of the project is to exploit the diverse knowledge of the partners (from universities, research institutes, and industry) for training a new generation of mathematicians.

To master the challenge of combining CAD and Algebraic Geometry, Europe needs new experts with experience from different schools of thought and applications, who have built their own Europe-wide contact net and have either gathered important industrial experience before pursuing an academic career or have academic research experience before entering industry.

The SAGA partners

The network has a total of 10 partners: two industrial companies, three research insitutes, and five universities.

- Missler Software, France (Dominique Laffret)
- Kongsberg SIM, Norway (Eigil Samset)
- INRIA, France (Bernard Mourrain)
- GraphiTech, Italy (Raffaele De Amicis)
- SINTEF, Norway (Tor Dokken, coordinator)
- Johannes Kepler Universitaet, Austria (Bert Jüttler)
- National and Kapodistrian Unversity of Athens, Greece (Ioannis Emiris)
- Universidad de Cantabria, Spain (Laureano Gonzalez-Vega)
- Vilniaus Universitetas, Lithuania (Rimvydas Krasauskas)
- University of Oslo, Norway (Ragni Piene)

Scientific Advisory Board

- Alicia Dickenstein, Buenos Aires
- Bianca Falcidieno, IMATI
- Thomas Grandine, Boeing
- Jens Gravesen, DTU
- Wen Ping Wang, Hongkong

The program

- WP1: Change of Representation
- WP2: Geometric Computing Algebraic Tools
- WP3: Algebraic Geometry for CAD Applications
- WP4: Practical Industrial Problems

WP1: Change of Representation

- Sparse approximate implicitization
- Approximate and exact parameterization
- Interpolation of curves by surfaces

Research problems related to methods for the conversion between the two main representations of curves and surfaces: the implicit and the parametric form. Parametric representations are interesting to generate points, implicit representations encompass a larger class of shapes and are more powerful for geometric queries. Implicitization and parameterization have been studied in classical algebraic geometry and in symbolic computation. Approximate techniques have emerged recently, and their practical value was demonstrated in the GAIA II project.

Figure: Approximation of a point set – tetrahedral

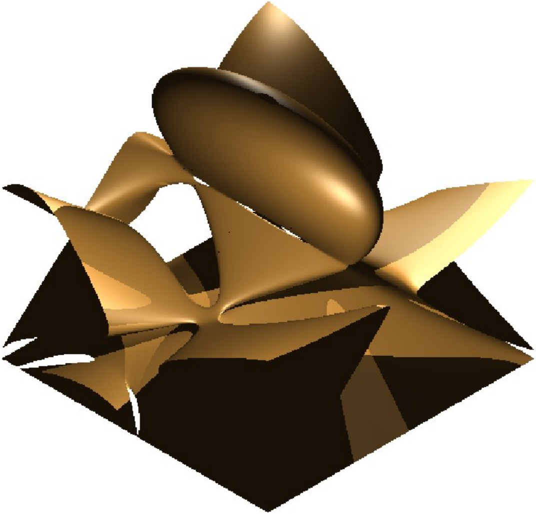
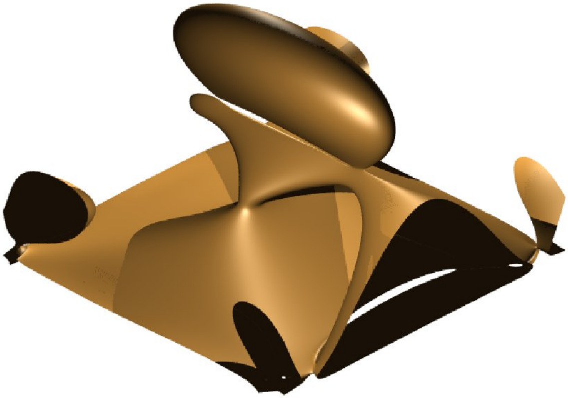


Figure: Approximation of a point set — tensor



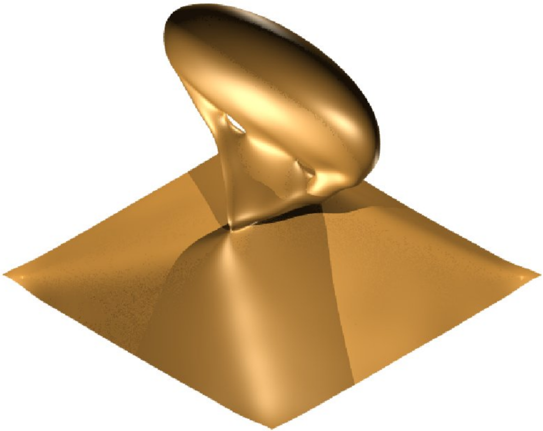


Figure: Approximation of a point set — splines

WP2: Geometric Computing Algebraic Tools

- Algebraic spline geometry
- Procedural surfaces and point represented surfaces
- Algebraic tools for geometric constructions

Geometric modelling usually involves piecewise algebraic representations of shapes. Their effective treatment leads to the resolution of polynomial systems of equations, which requires the use of stable and efficient tools. Need to combine accuracy, robustness and efficiency. Elimination techniques in computational algebraic geometry open a new window of applications of Algebraic Geometry to CAD.

The libraries SYNAPS (http://synaps.inria.fr) and the algebraic-geometric modeller AXEL (http://axel.inria.fr) provide a framework for this approach.

WP3: Algebraic Geometry for CAD Applications

- Improved blends between primitive surfaces
- Representing and dealing with offsets
- Computational geometry with higher order primitives
- Geometric interval arithmetic

The surfaces used in CAD are traditionally pieces of the simplest rational surfaces like quadrics — cones, cylinders, ruled surfaces. The majority of mechanical parts in CAD have a boundary representation that combines planes and primitive surfaces with smooth blends between them.

Offsets of curves and surfaces are natural tools, but the offsets of rational (parameterizable) objects need not be rational. To find the exact algebraic equations is computationally difficult. Develop a toolbox to deal exactly with offsets of conics, quadrics and cubics in a very efficient and robust way.

Figure: Self-intersecting pipe

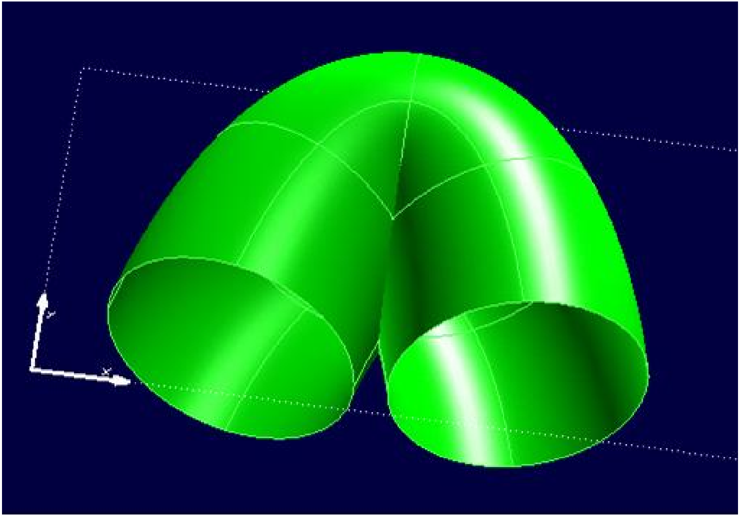


Figure: Face

Figure: Offset with selfintersections

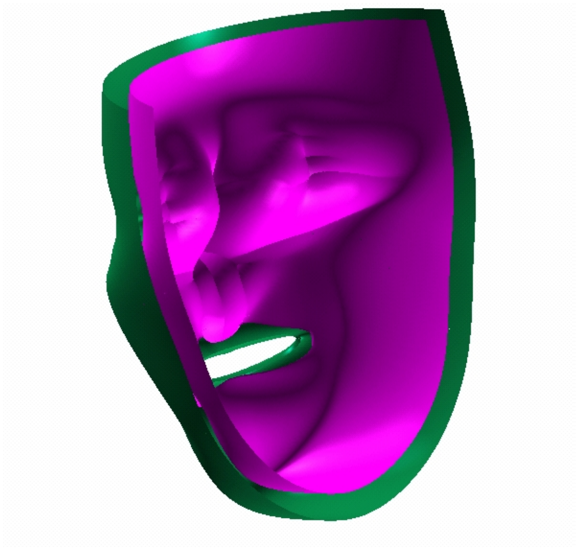


Figure: Offset with penetration

WP4: Practical Industrial Problems

- Constrained modeling
- Curve and surface generation in a kinematics sense
- Design by sketch
- Field visualization

The sketch problem for 2D-profiles, for constraints related to topology, angle, distances, Kinematics and dynamics for 3D rigid bodies, for constraints related to contact, joints, Surface generation, for constraints on points, curves, curvature, minimization of some tension, avoid oscillations,
Visualize huge CAD models real time with good frame rate and good quality on standard PCs. Visualize CAD models together with other 3D graphics data, e.g., terrain, seabed, reservoir data, weather conditions and more. Take advantage of new algorithms and 3D graphics hardware in real time CAD visualization.

Fellows

- Mobility criterion
- Eligibility criteria
- Secondment at another institution
- Career development plan

- 9 PhD students (36 months)
  All positions filled. 3 women.
  Come from Colombia, Greece, India, Lithuania, Niger, Norway, Portugal, UK, Vietnam.
- 7 Early Researchers/post docs (6–12 months)
  Not all positions filled (University of Oslo is available!)
- 16 Visiting Scientists (1 month)

Schools and workshops

- SAGA Autumn School and kick-off Workshop
  November 17-21, 2008
  International Center of Mathematical Meetings
  Castro Urdiales, Spain
- Winter School and Workshop
  SHAPES, ALGEBRA, GEOMETRY AND ALGORITHMS
  15-19 March 2010
  Auron, France
- Autumn School and Workshop
  3–10 October 2010
  Chania, Greece
- Further schools and workshops: Vilnius and Trento.

Workshop in Castro Urdiales

The meeting aimed at supporting the exchange of ideas between the fields of Algebraic Geometry, Numerical Analysis, and Geometric Modelling. Topics of interest included:

- Geometry of curves and surfaces
- Rational/implicit representation
- Resultant constructions and implicitization
- Singularities: detection and analysis
- Curves and surfaces classification
- Intersection problems
- Approximate and/or certified methods
- Computer Implementations of Algorithmic Algebraic Geometry
- Isogeometric Analysis

SAGA workshop in Castro Urdiales

Algebraic spline geometry — a PhD project at UO

The project of PhD student Nelly Villamizar from Columbia:

By replacing an (affine real) algebraic variety in $\mathbb{R}^n$ by a convex polyhedral domain Δ, and regular functions by algebraic spline functions, we would like to develop a new theory: "algebraic spline geometry." The purpose of this PhD project is precisely to undertake this task, guided by concrete modeling problems.

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Algebraic splines

An *algebraic spline function* is a piecewise polynomial function on polyhedral subdivisions of regions in $\mathbb{R}^n$.

Example

Consider the univariate spline function, defined on the interval $\Delta = [a, b] \subset \mathbb{R}$ by

$$f(x) = f_1(x) \text{ if } x \in [a, c], f(x) = f_2(x) \text{ if } x \in [c, b].$$

Then f is C^r iff $(x - c)^{r+1} | (f_1 - f_2)$.

If $c = 1$, $f_1 = x + 1$, $f_2 = x^2 - x + 2$, then f is C^1.

The spline function spaces

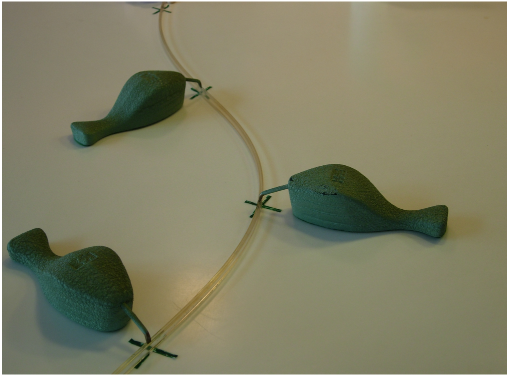
The dimension of the vector space $V_{n,k}$ of polynomials in n variables of degree $\leq k$ is known. The dimension of the space of C^r spline functions of degree $\leq k$ is equal to??

We have $\dim V_{1,k} = k + 1$. In the example, for $r + 1 \leq k$, the dimension of C^r spline functions is $2k + 2 - (r + 1)$.

More generally, consider a polyhedral subdivision of a region $\Delta \subset \mathbb{R}^n$.

- Study the ring $C^r(\Delta)$ of algebraic spline functions on Δ, and the vector spaces $C^r_k(\Delta)$ of functions of degree $\leq k$.
- Determine the dimension of this space.
- Study the Hilbert series (or generating function of this problem).
- Relate to the Ehrhardt function for convex lattice polytopes and toric geometry.

A draftsman's spline



http://www.saga-network.eu/

THANK YOU FOR YOUR ATTENTION!

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