Fluctuations and energetics in small systems

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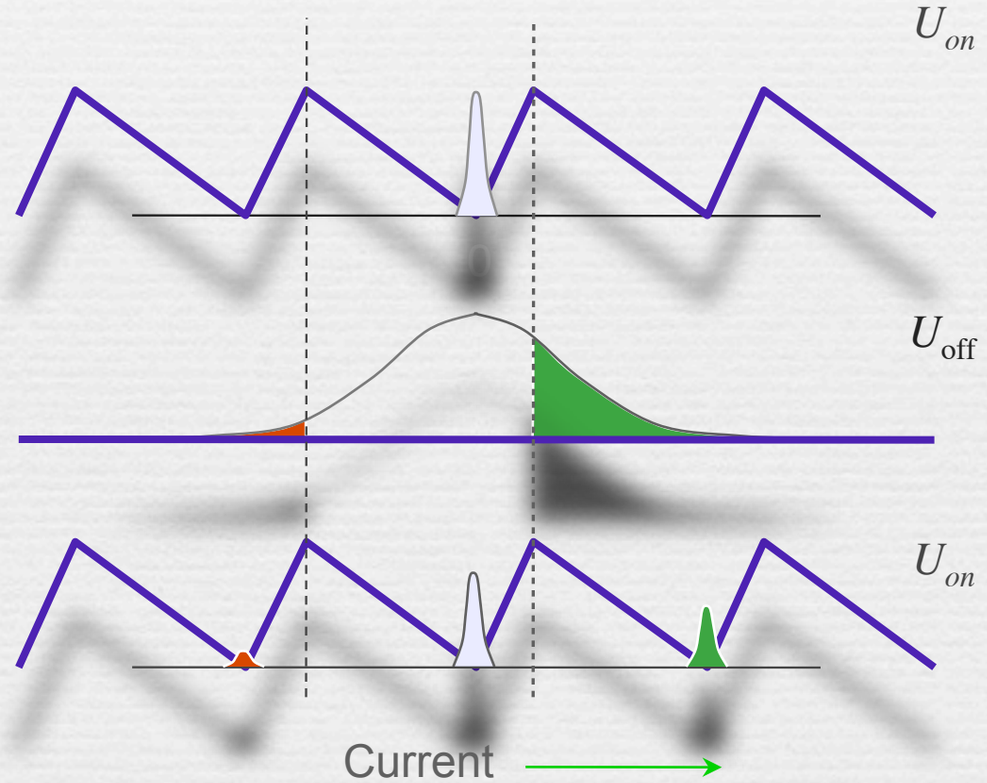
- Fluctuations and work: two examples.
- Irreversibility and dissipation.
- GISC activities.

Brownian motors

The flashing ratchet

A Brownian particle in a flashing asymmetric potential

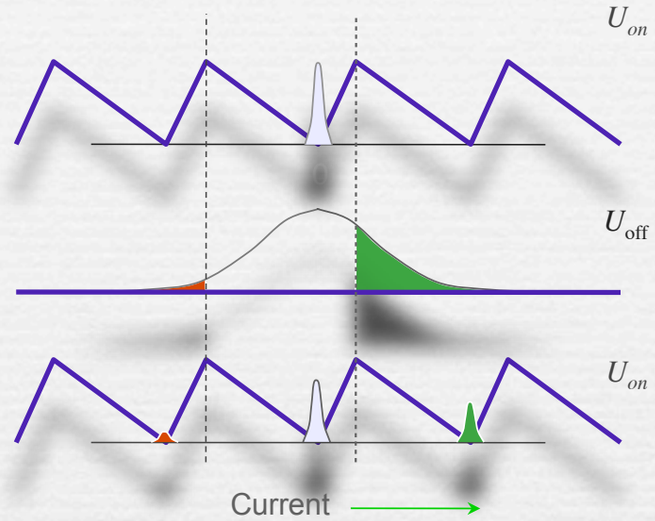
Ajdari and Prost, 1992

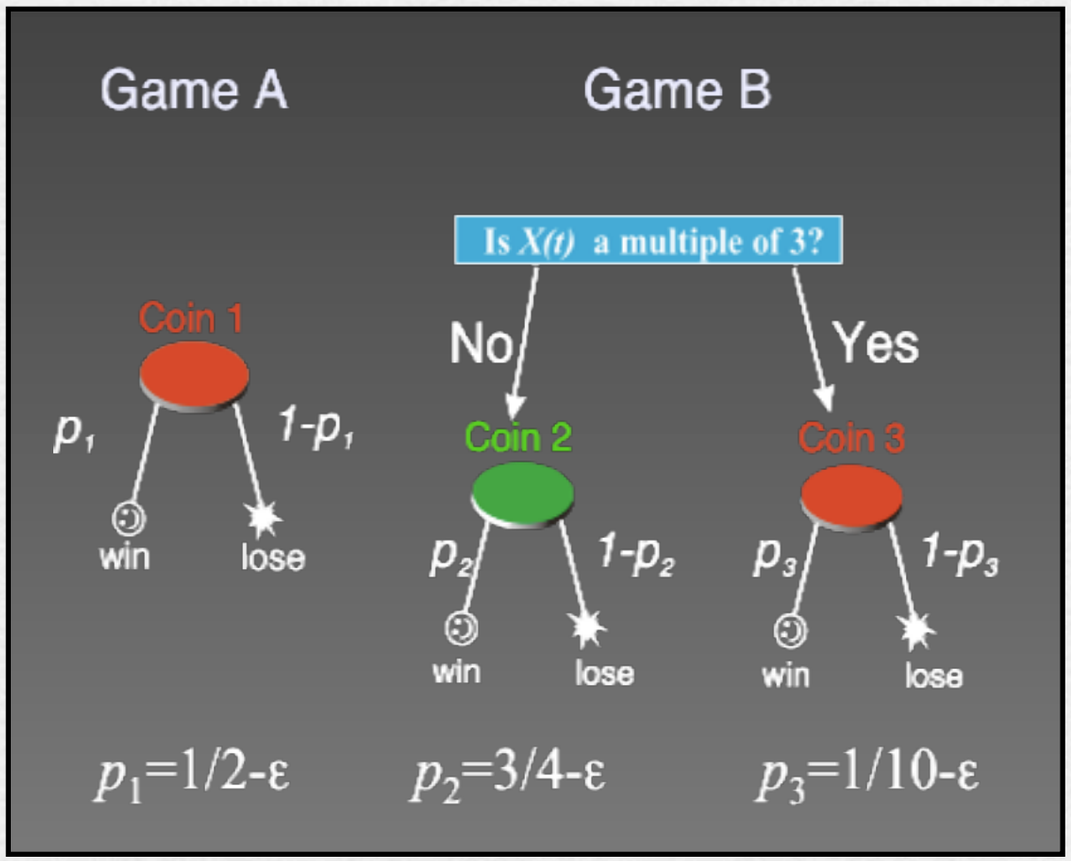


Simulation: http://www.elmer.unibas.ch/bm/index.html

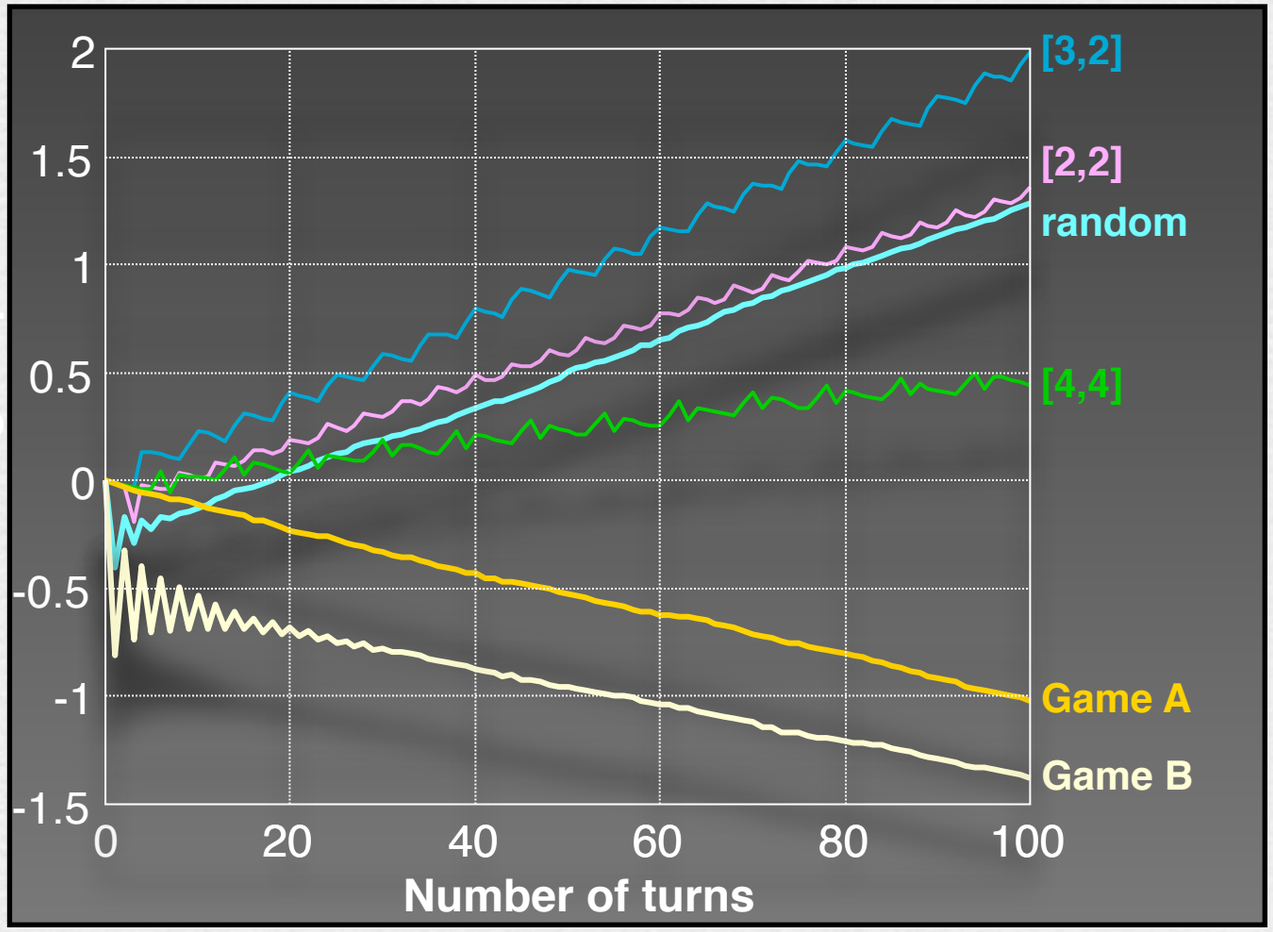
Paradoxical games

The flashing ratchet



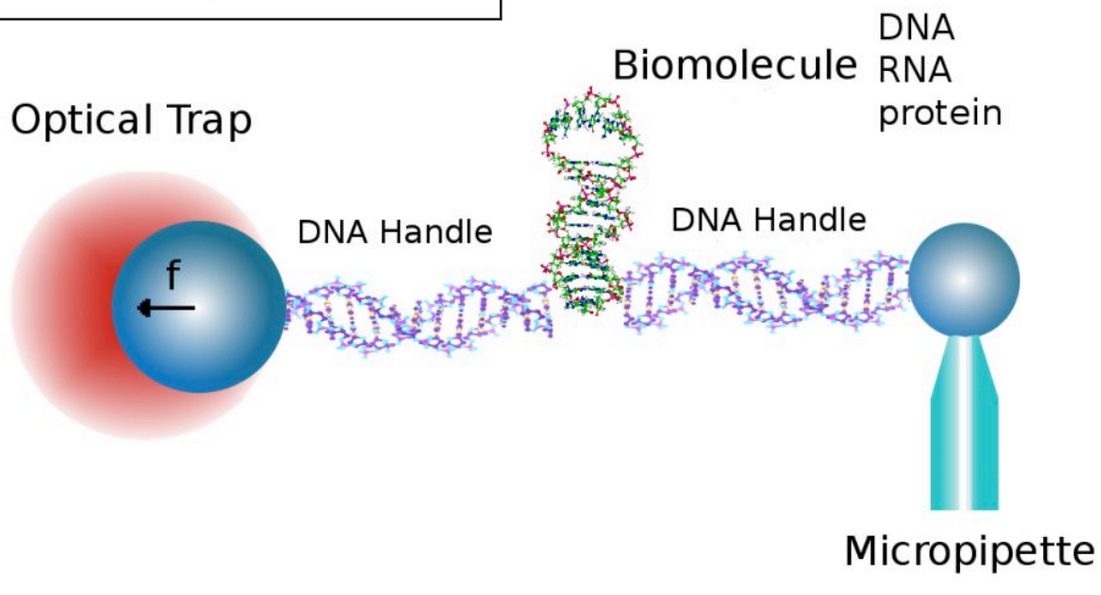


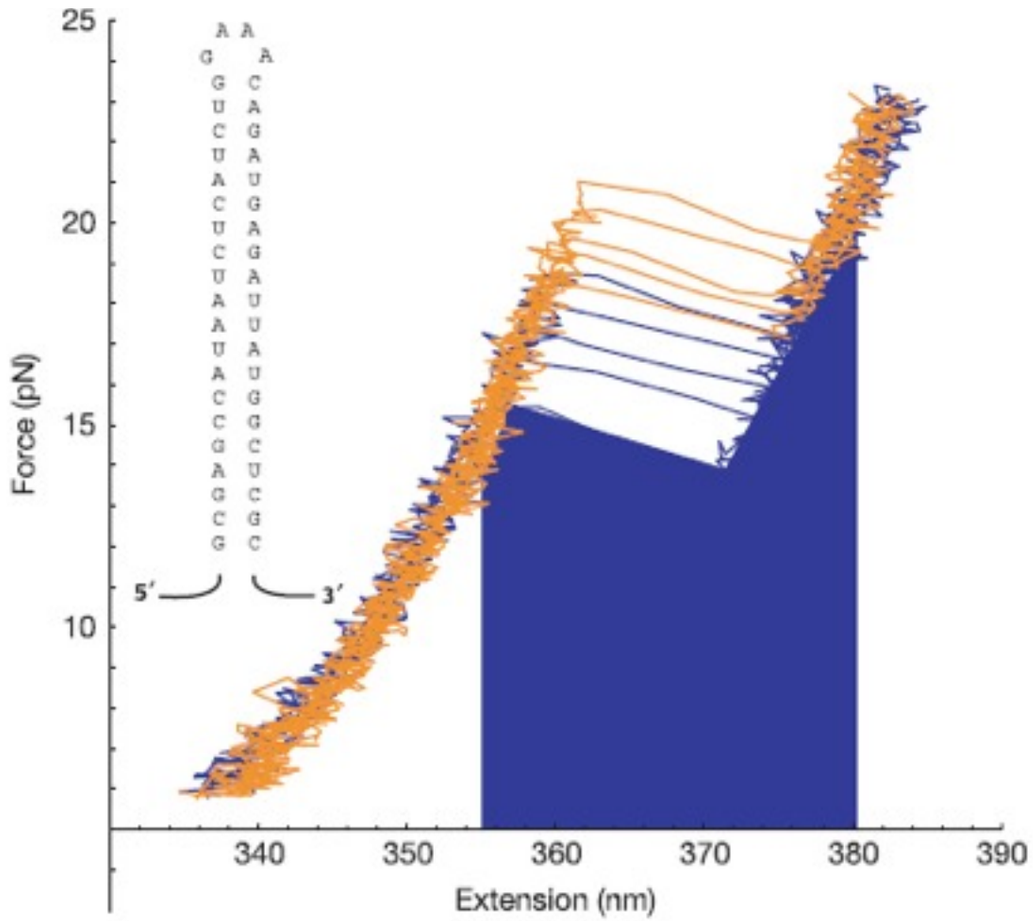
Parrondo Paradox

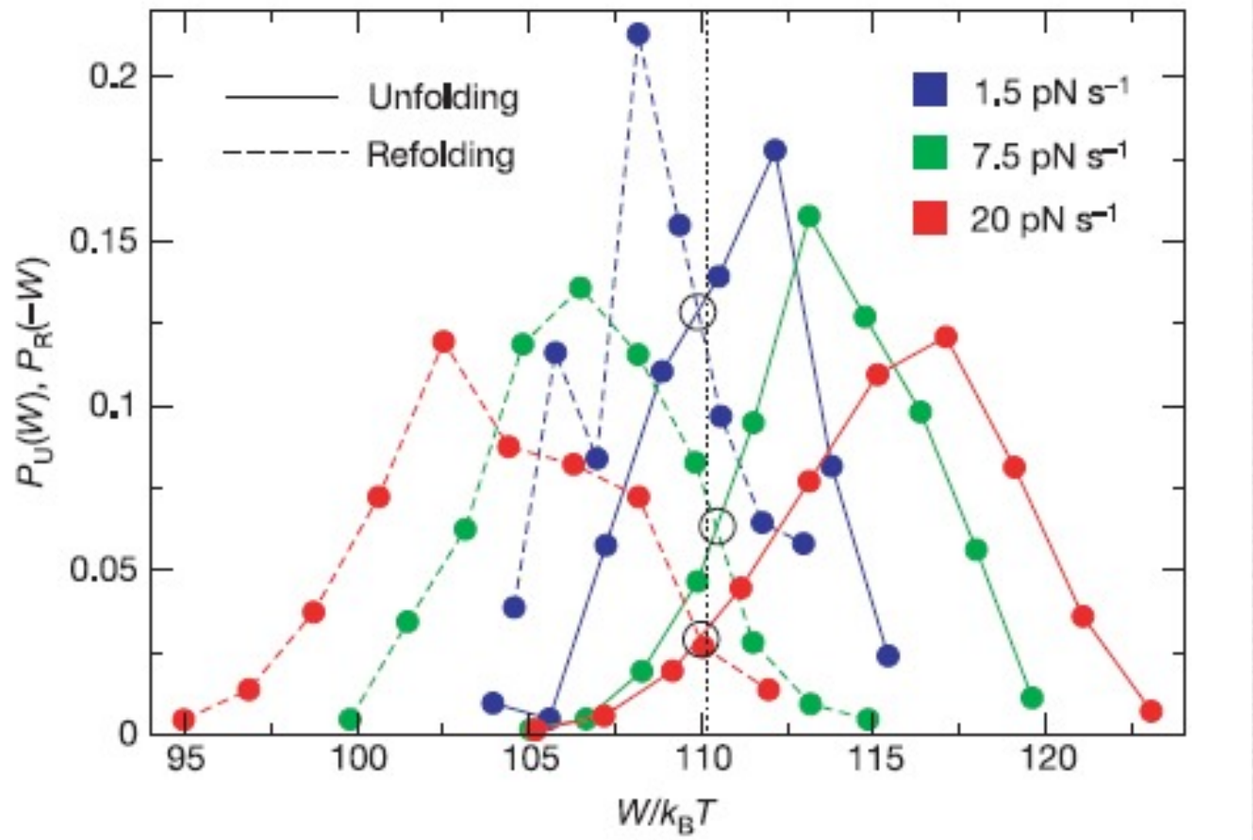


Fluctuations and work

Collin et al, Nature 2005







$$\frac{P_{\mathrm{U}}(W)}{P_{\mathrm{R}}(-W)} = \exp\left(\frac{W-\Delta G}{k_{\mathrm{B}}T}\right)$$

Crooks theorem

Information theory

$$\frac{P_{\mathrm{U}}(W)}{P_{\mathrm{R}}(-W)} = \exp\left(\frac{W - \Delta G}{k_{\mathrm{B}} T}\right)$$

$$\langle W \rangle - \Delta G = k_B T \int dW \, P_U(W) \ln \frac{P_U(W)}{P_R(-W)}$$

$$D(P_U(W) || P_R(-W))$$

Kullbak-Leibler distance
or relative entropy

$$\langle W \rangle - \Delta G = k_B T D(p_{\mathrm{forward}} || p_{\mathrm{backward}}) \leq k_B T D(\tilde{p}_{\mathrm{forward}} || \tilde{p}_{\mathrm{backward}})$$

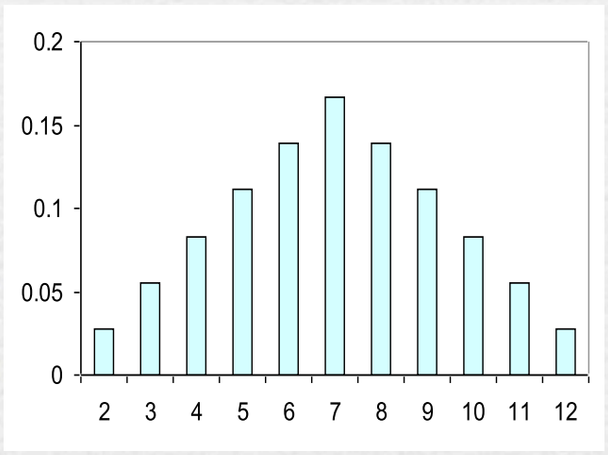
(Kawai, Parrondo, van den Broeck, PRL 2007)

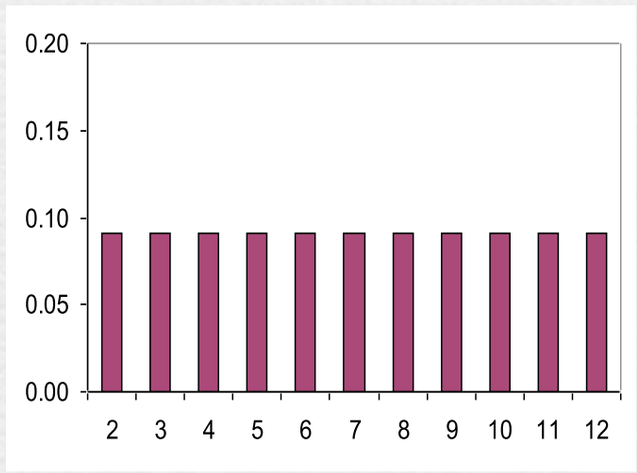
Relative entropy: a measure of indistinguishability

OR

?

5,7,12,9,6,2,5,2,9,2,12,8,10,4,...





$$D(p||q) \equiv \sum_i p_i \log_2 \left(\frac{p_i}{q_i} \right)$$

$D($  $||$  $)= 0.185$

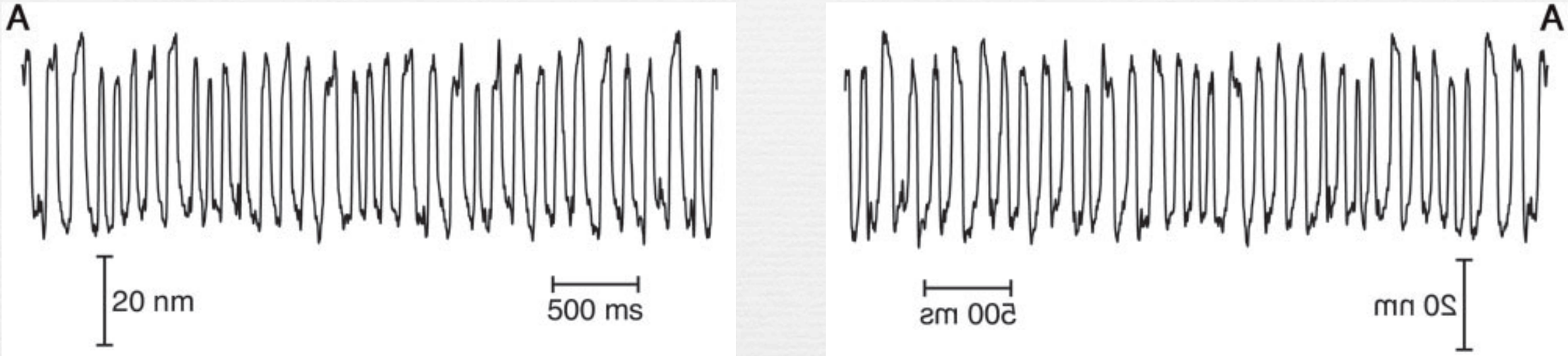
$D($  $||$  $)= 0.220$

Stein's Lemma:

the probability of incorrectly guessing p from n data actually distributed as q is (asympt.): $2^{-nD(p||q)}$

We need approximately 5 data to distinguish between dice and lottery with an error 1/2

Estimating dissipation



$$\frac{\langle W \rangle - \Delta G}{k_B T} = D(p_{\text{forward}} || p_{\text{backward}}) \leq D(\tilde{p}_{\text{forward}} || \tilde{p}_{\text{backward}})$$

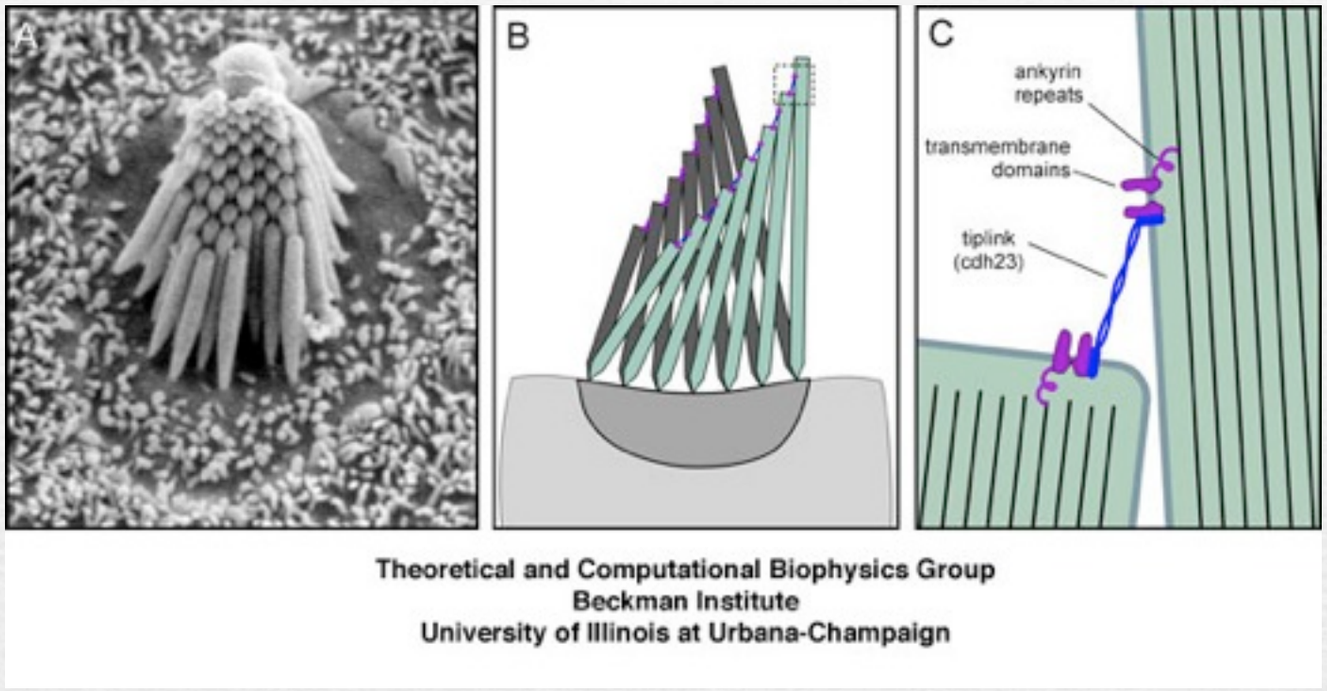
Dissipation
(entropy production)

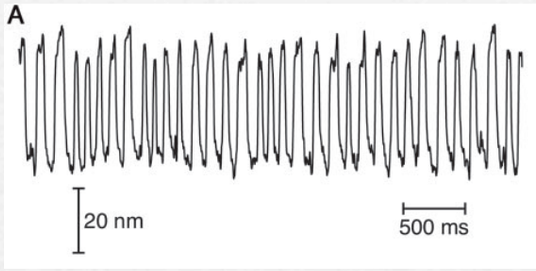
Irreversibility

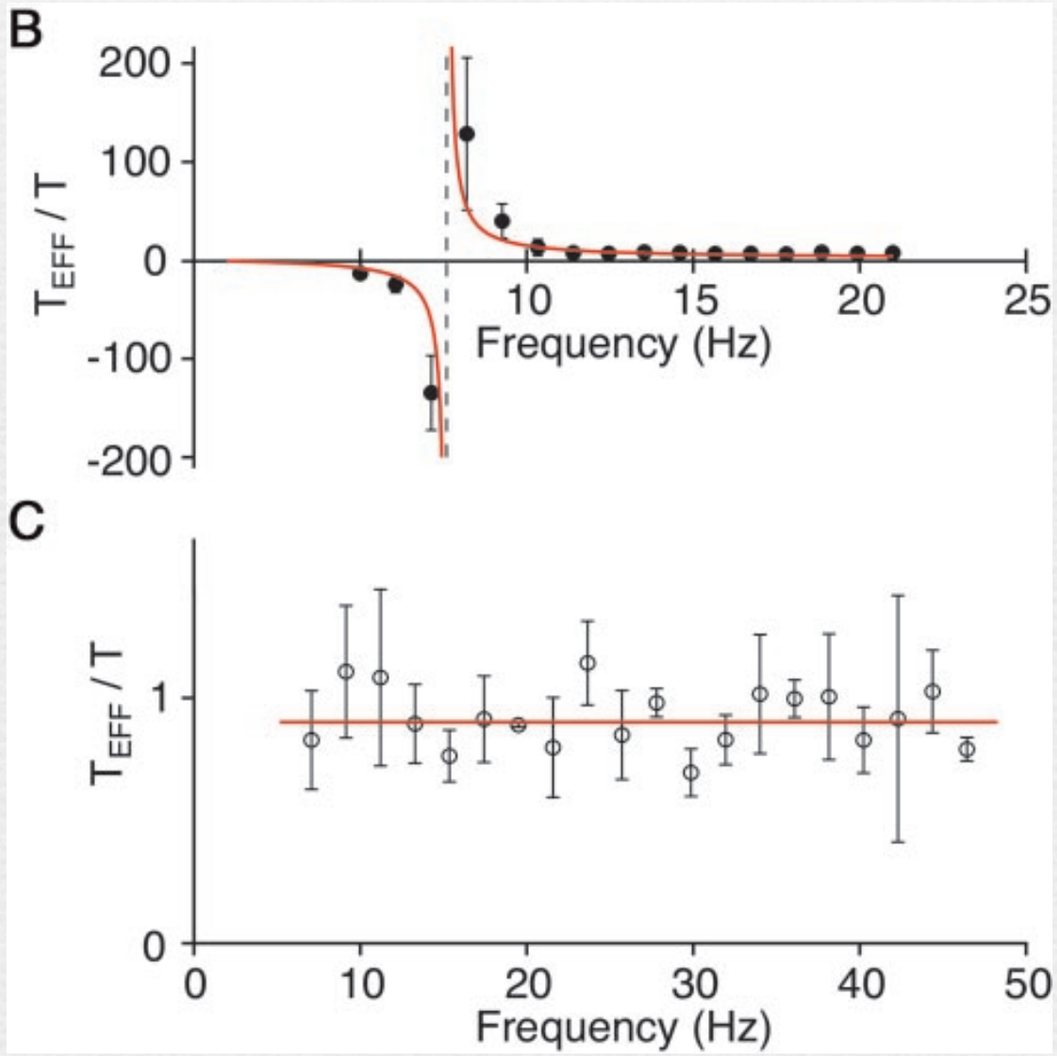
Physics

Information theory

Detecting active processes







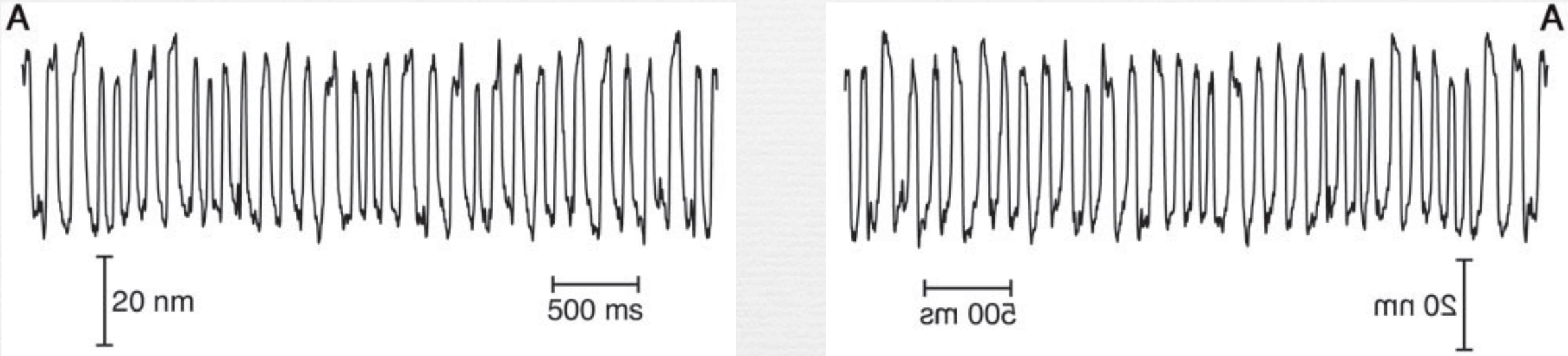
Linear response Spontaneous fluctuations

$$\chi_{x,F}(t-t') = -\frac{1}{kT}\frac{\partial}{\partial t}\left[\langle x(t)x(t')\rangle_{\rm eq} - \langle x^2\rangle_{\rm eq}\right]$$

Effective temperature

Martin, Hudspeth, Jülicher, PNAS 2001

Estimating dissipation



$$\frac{\langle W \rangle - \Delta G}{k_B T} = D(p_{\text{forward}} || p_{\text{backward}}) \leq D(\tilde{p}_{\text{forward}} || \tilde{p}_{\text{backward}})$$

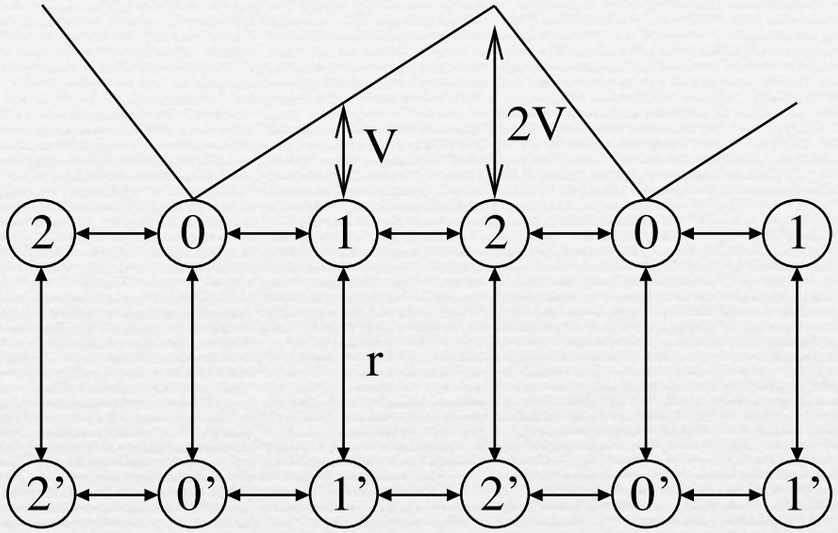
Dissipation
(entropy production)

Irreversibility

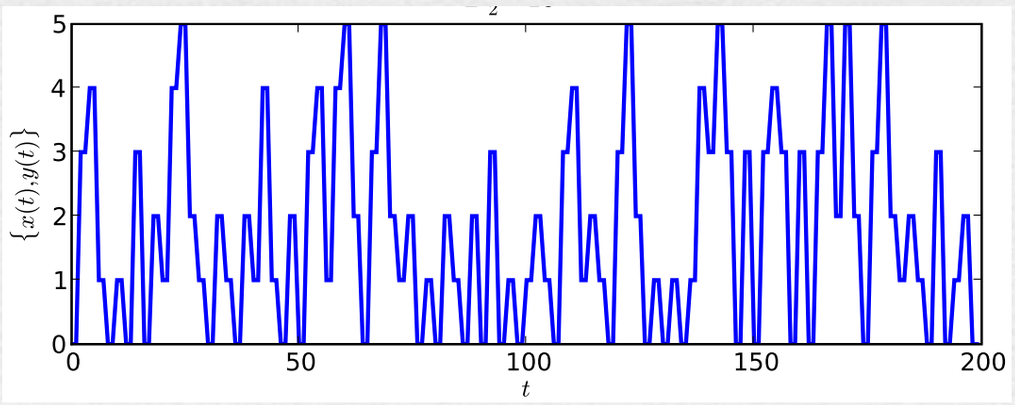
Physics

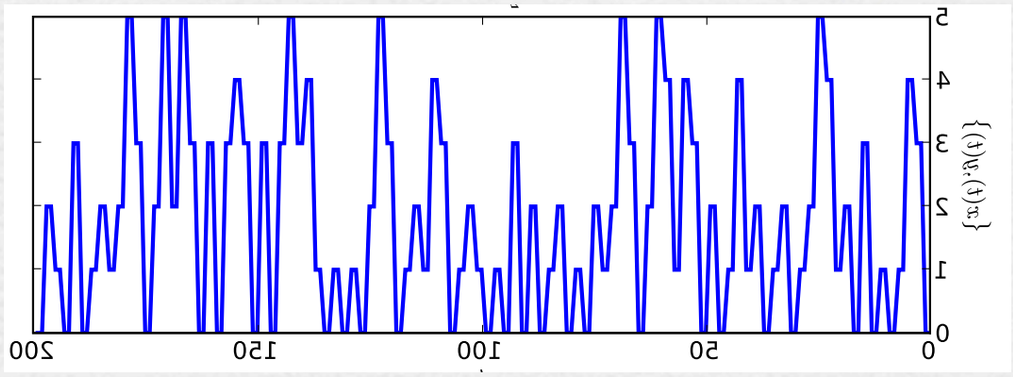
Information theory

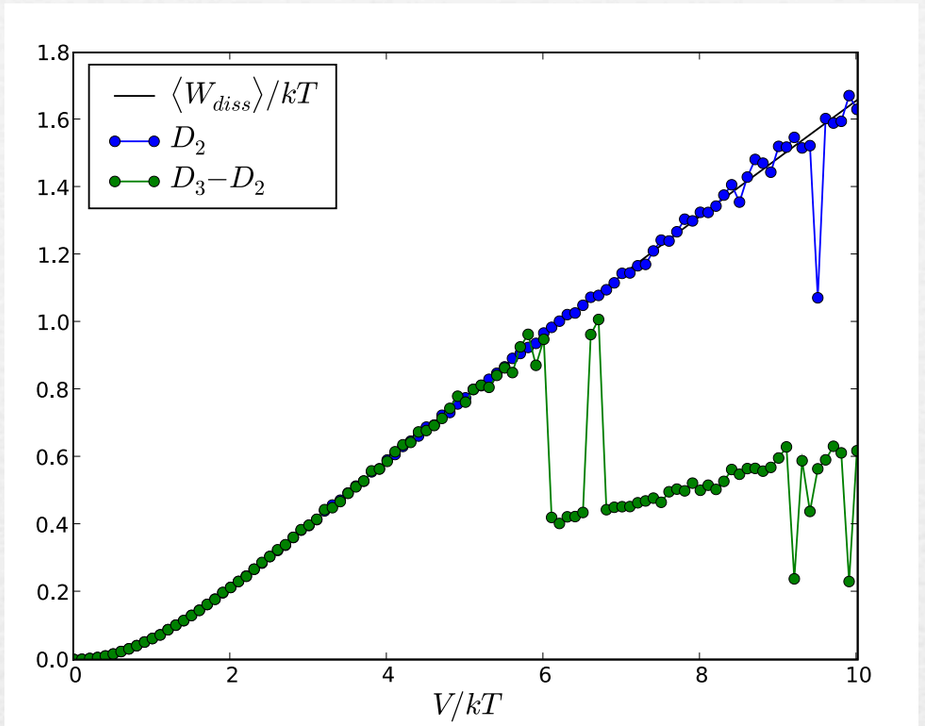
Estimating dissipation (E. Roldán)

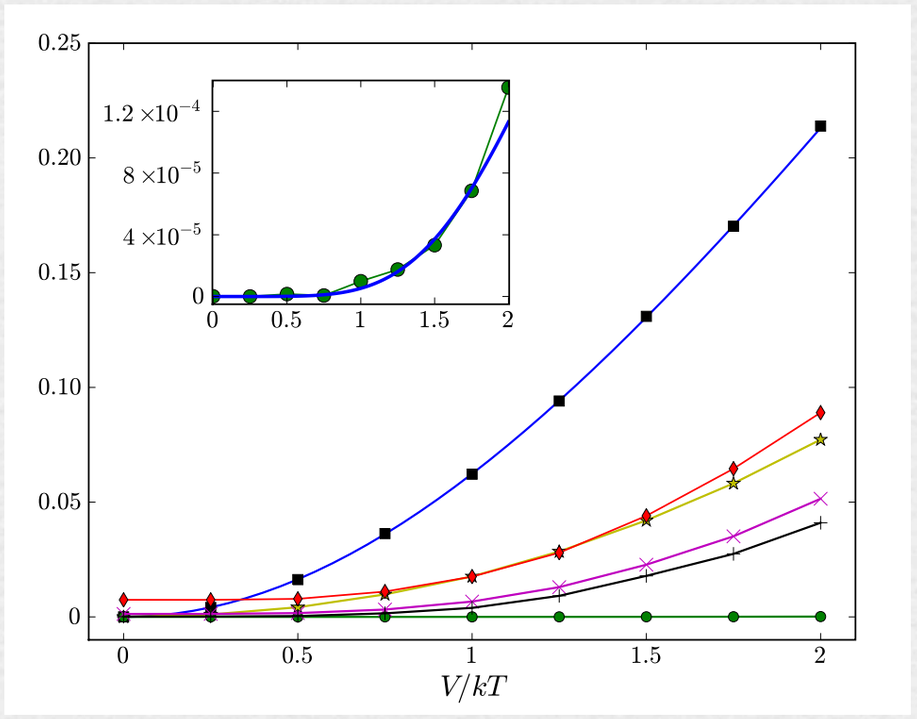


dissipation = 0.01 kT









Energetics and fluctuations in small systems

- *Applications*: biology, nanosciences.

- *Maths*: stochastic processes, stochastic control, information theory, time series analysis,...

GISC

3 Universities, 12 senior researchers, 5 posdocs, 11 PhD students

www.gisc.es

- Granular media (Brito, UCM).
- Transport in disordered systems and DNA (Domínguez-Adame, UCM)
- Game Theory (Sánchez, Cuesta, UC3M)
- Econophysics, networks (Moro, UC3M)
- Liquids, wetting (Rascón, Martínez-Ratón, UC3M)
- Growth phenomena (Cuerno, UC3M)